

CS 401: Introduction to Advanced Studies (Data Structures)

Vijay K. Gurbani, Ph.D., Illinois Institute of Technology

Lecture 11: Sorting

# Sorting

- 20-25% of computing time devoted to sorting in commercial applications.
- Sorting is dominated by:
  - Programmer's time
  - Machine time (runtime complexity)
  - Space
- It is not possible to minimize all three!

# Sorting

### • Formally:

Sorting: a method that takes an input sequence,  $S = \langle a_1, a_2, ..., a_{n-1}, a_n \rangle$  and produces a permuted sequence,  $S' = \langle a'_1, a'_2, ..., a'_{n-1}, a'_n \rangle$  such that  $a'_1 \langle = a'_2 \text{ and } a'_2 \langle = a'_3 ... a'_n \langle = a'_{n+1}.$ 

### **Evaluation metrics for sorts**

- Runtime (of course)
- Space (also apparent)
- Stability (what is this?)

### **Evaluation metrics for sorts**

- Runtime (of course)
- Space (also apparent)
- Stability (what is this?)
  - Informally, stability means that if an element a comes before b in the unsorted input, then their relative ordering is preserved in the sorted output (i.e., a comes before b in the sorted output as well).
  - So, why is stability important? (Let's revisit this after an example).

### Evaluation metric for sorts

Sort following using airport as the key:

8:00a JFK 8:10a ORD 9:23a EWR 4:34p BRU 8:12a ORD 9:55a EWR



4:34p	BRU
9:23a	EWR
9:55a	EWR
8:00a	JFK
8:10a	ORD
8:12a	ORD

Which of the 4 output sorts is

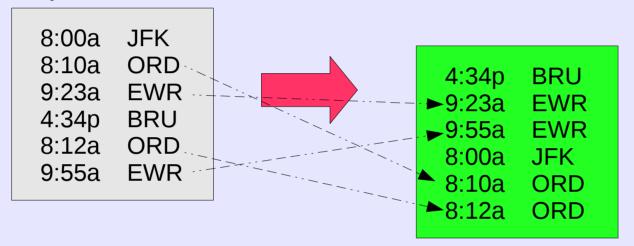
- Correct?
- Stable?

BRU
EWR
<b>EWR</b>
JFK
ORD
ORD

4:34p	BRU	
9:23a	EWR	
9:55a	EWR	
8:00a	JFK	
8:12a	ORD	
8:10a	ORD	

### **Evaluation metric for sorts**

Sort following using airport as the key:



4:34p BRU 9:55a EWR 9:23a EWR 8:00a JFK 8:10a ORD 8:12a ORD

So, why is stability important?

4:34p	BRU	
9:55a	<b>EWR</b>	
9:23a	<b>EWR</b>	
8:00a	JFK	
8:12a	ORD	
8:10a	ORD	

4:34p BRU 9:23a EWR 9:55a EWR 8:00a JFK 8:12a ORD 8:10a ORD

# Sorting

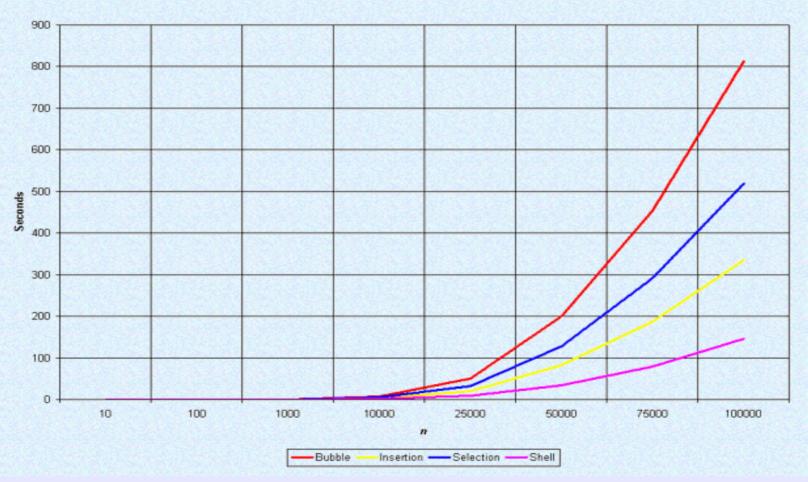
- Two types of sort algorithms:
  - Quadratic Sorts
    - Easy to understand.
    - Simple to implement.
    - Not very efficient on large data.
  - Divide and Conquer
    - More complex.
    - Efficient on large data.

# Sorting: Quadratic Sorts

- Bubble sort
- Insertion sort
- Selection sort

# Quadratic Sort Performance



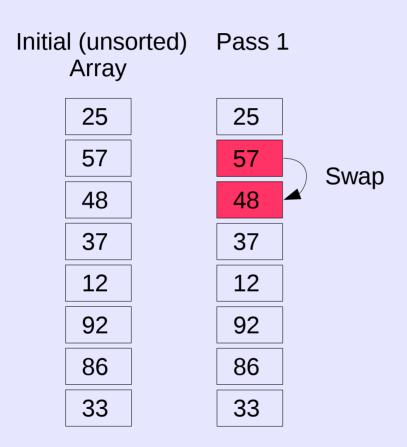


Slide source: http://www.personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/Sorting/sortingIntro.htm

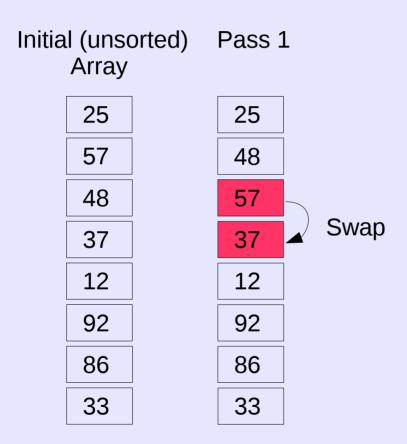
- Simple.
- Not efficient.
- Make multiple passes (n 1) through the unsorted array, swapping adjacent elements until the array is sorted.
- Can use some simple optimizations to speed up bubble sort.

#### Initial (unsorted) Array

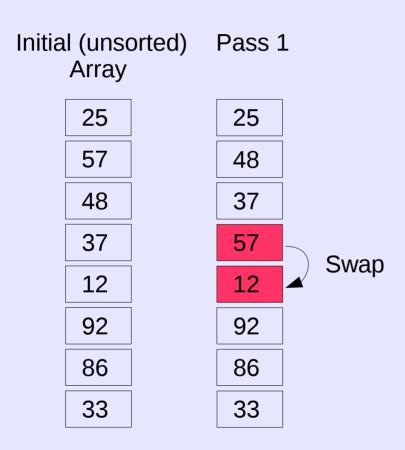
#### Pass 1 Initial (unsorted) Array



#### Initial (unsorted) Pass 1 Array

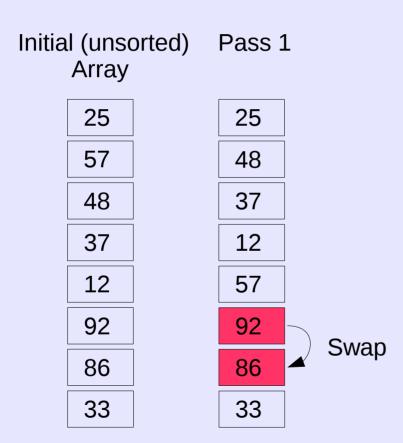


#### Initial (unsorted) Pass 1 Array

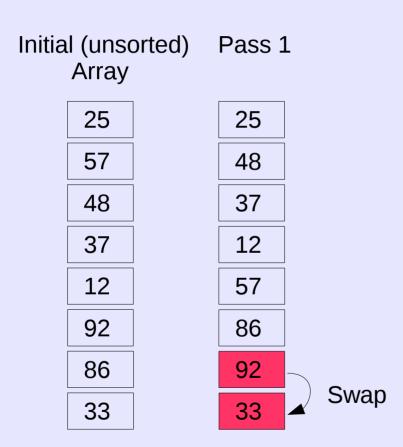


#### Initial (unsorted) Pass 1 Array

#### Initial (unsorted) Pass 1 Array



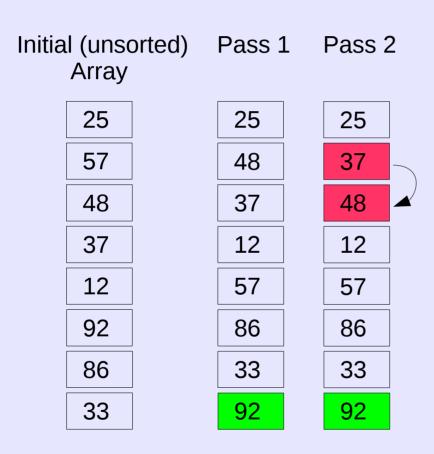
#### Initial (unsorted) Pass 1 Array

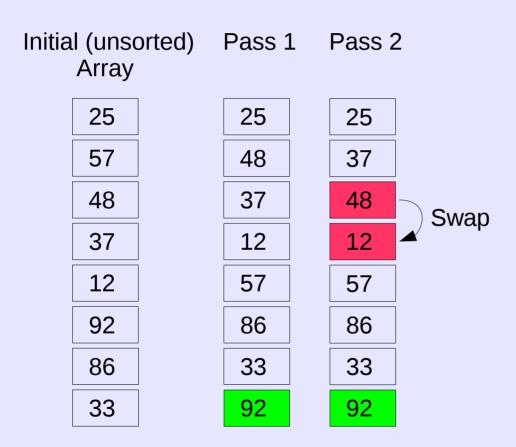


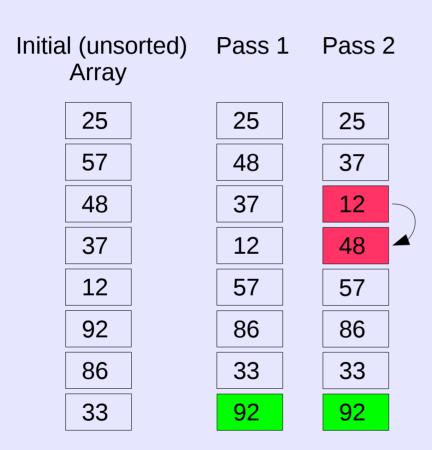
Initial (unsorted) Pass 1 Array

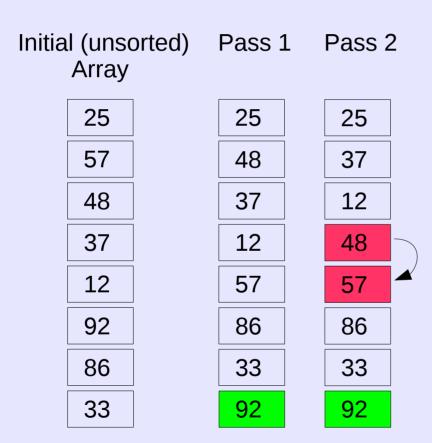




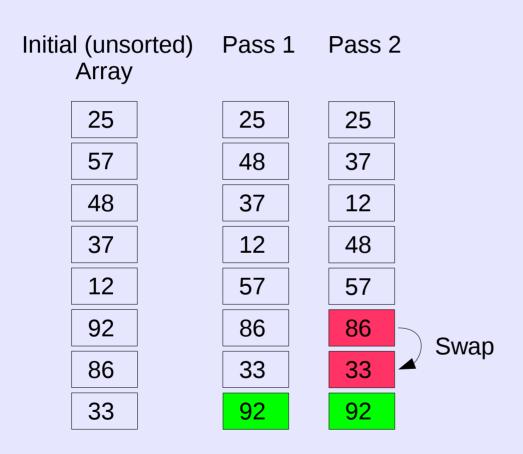


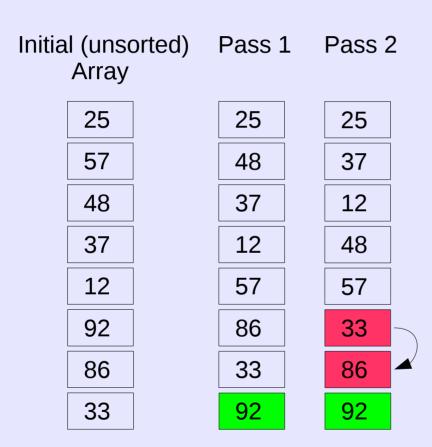


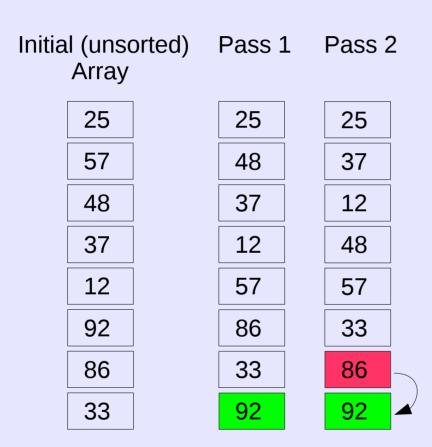




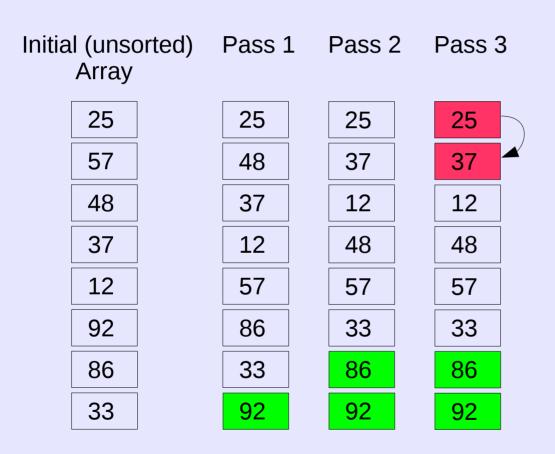


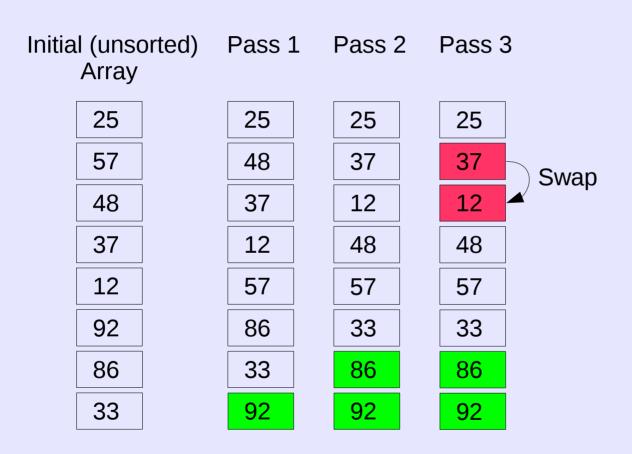


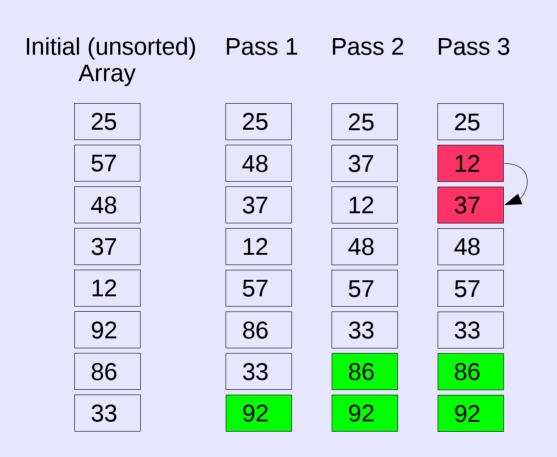


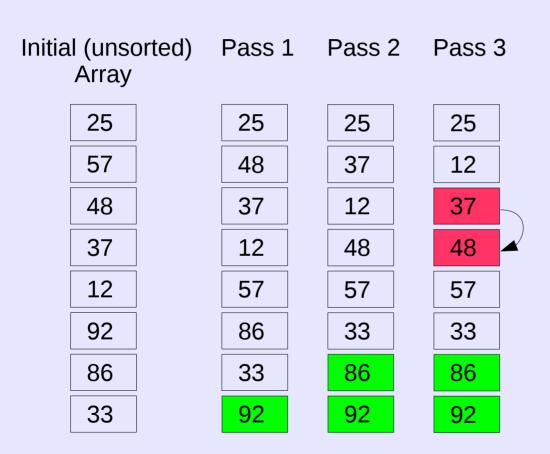


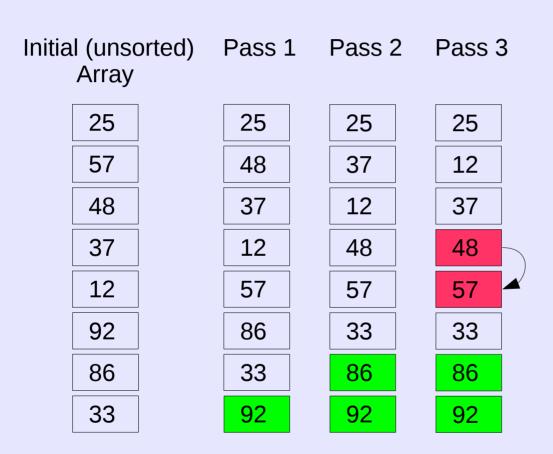
Initial (unsorted) Array	Pass 1	Pass 2
25	25	25
57	48	37
48	37	12
37	12	48
12	57	57
92	86	33
86	33	86
33	92	92

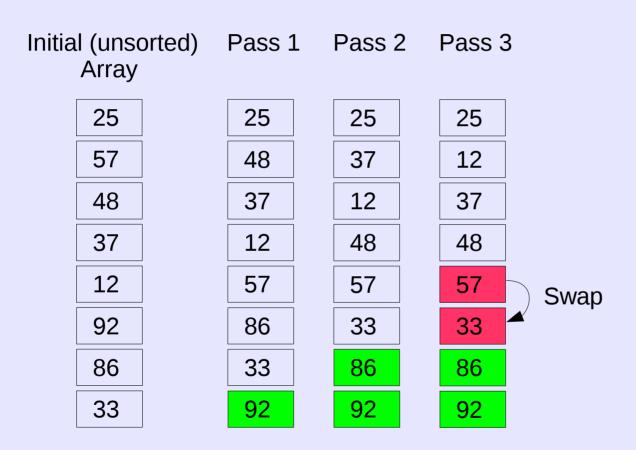


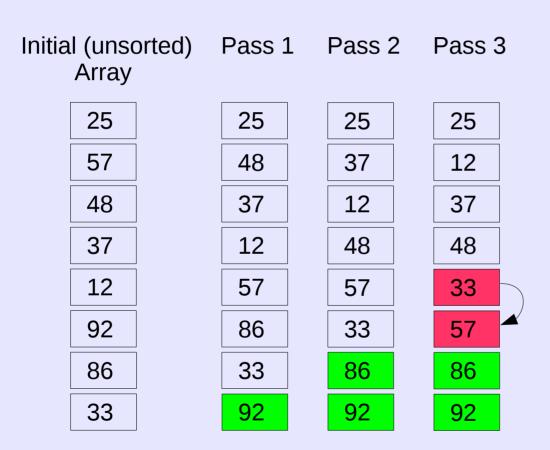


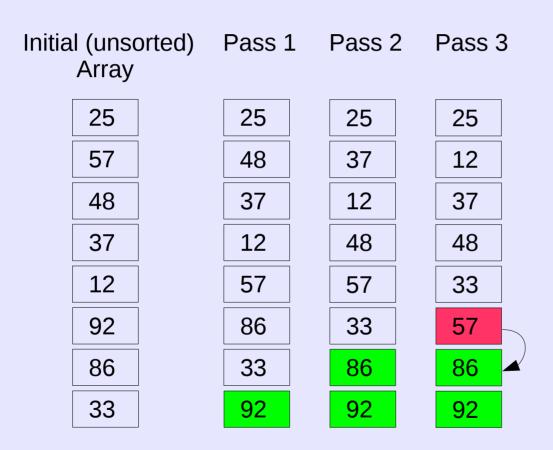








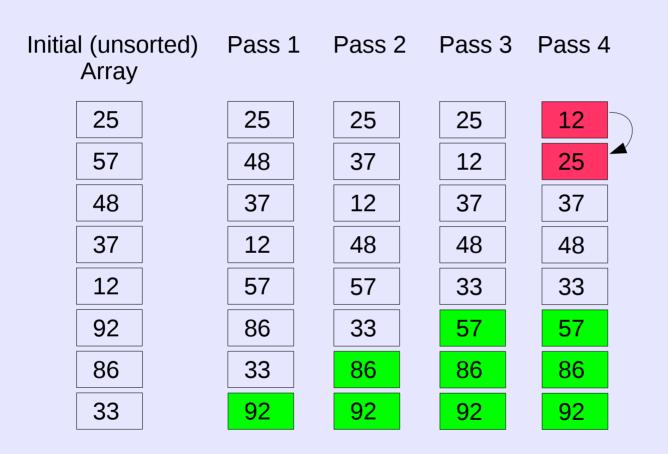


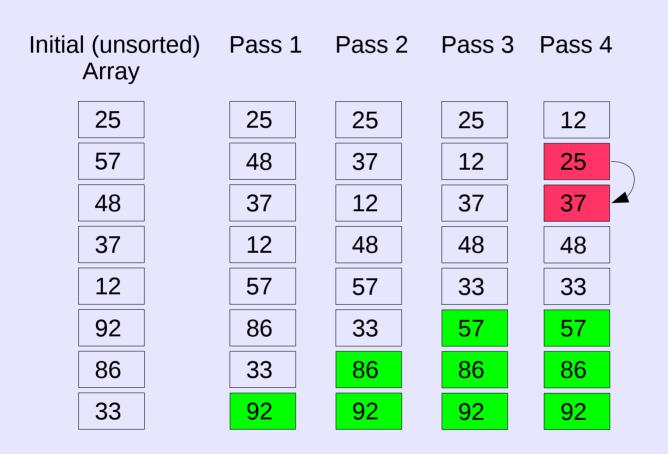


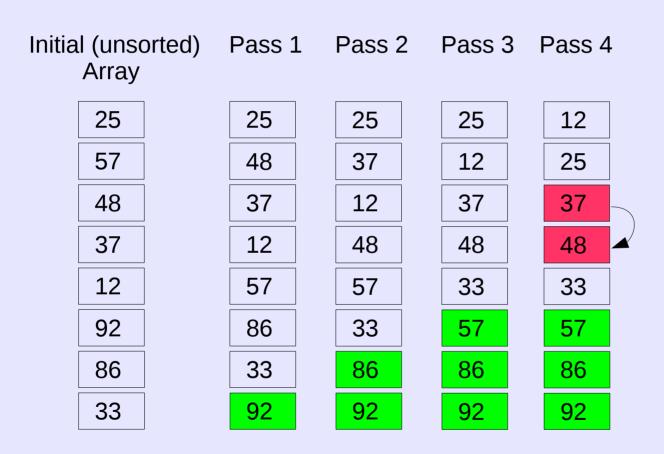
Initia	l (unso Array	orted)	Pass 1	L	Pass 2	2	Pass (	3
	25		25		25		25	
	57		48		37		12	
	48		37		12		37	
	37		12		48		48	
	12		57		57		33	
	92		86		33		57	
	86		33		86		86	
	33		92		92		92	

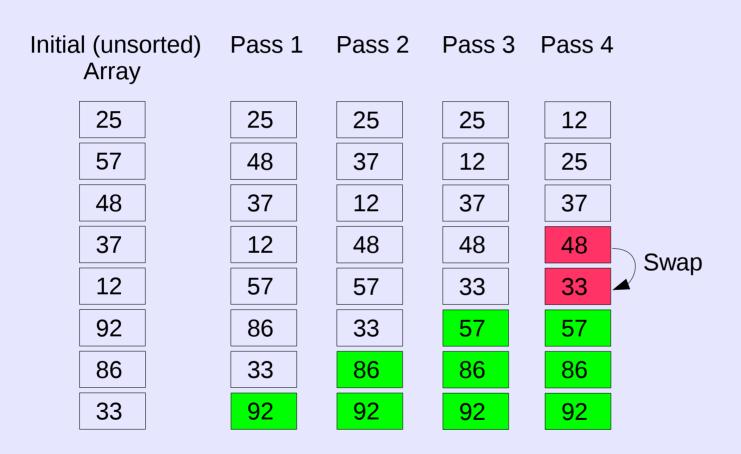
Initia	l (unso Array	orted)	Pass 1	Pass 2	Pass 3
	25		25	25	25
	57		48	37	12
	48		37	12	37
	37		12	48	48
	12		57	57	33
	92		86	33	57
	86		33	86	86
	33		92	92	92



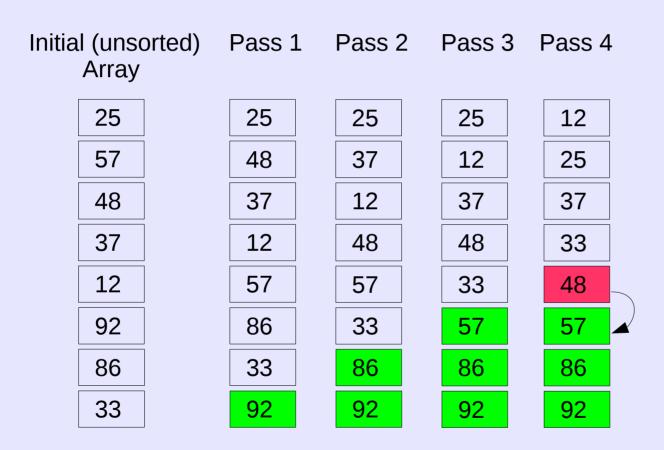










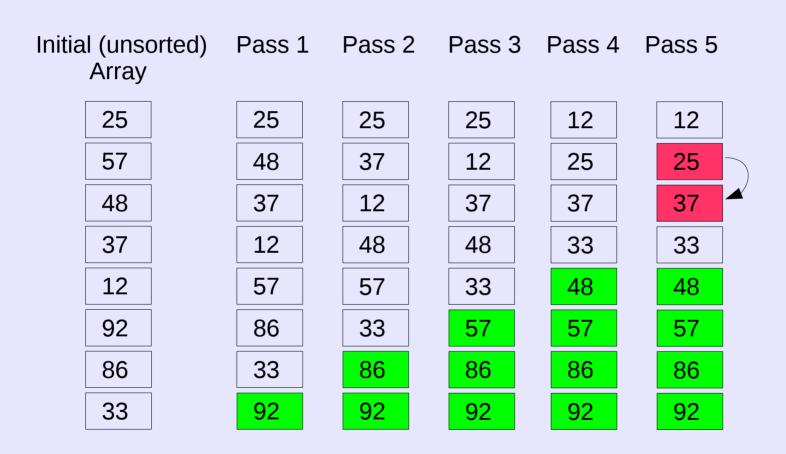


Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4
25	25	25	25	12
57	48	37	12	25
48	37	12	37	37
37	12	48	48	33
12	57	57	33	48
92	86	33	57	57
86	33	86	86	86
33	92	92	92	92

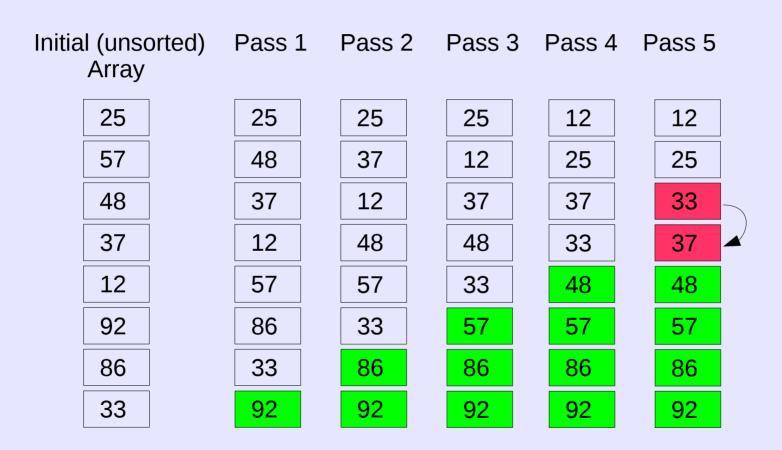
Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4
25	25	25	25	12
57	48	37	12	25
48	37	12	37	37
37	12	48	48	33
12	57	57	33	48
92	86	33	57	57
86	33	86	86	86
33	92	92	92	92

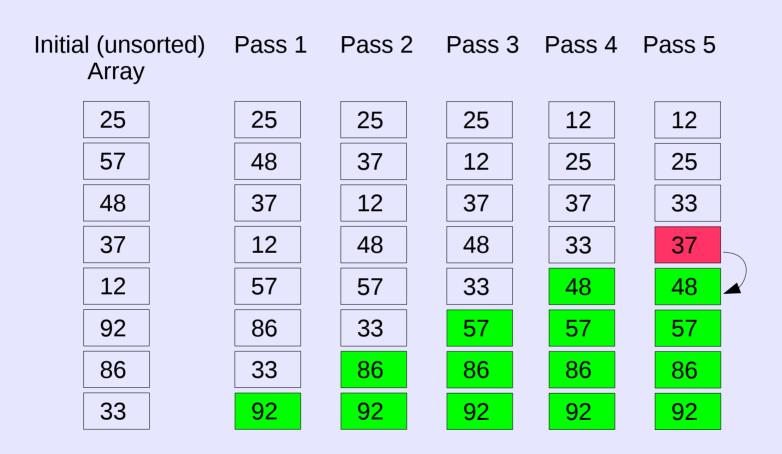
Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4
25	25	25	25	12
57	48	37	12	25
48	37	12	37	37
37	12	48	48	33
12	57	57	33	48
92	86	33	57	57
86	33	86	86	86
33	92	92	92	92











Initial (unso Array	rted) Pass 1	L Pass 2	Pass 3	Pass 4	Pass 5
25	25	25	25	12	12
57	48	37	12	25	25
48	37	12	37	37	33
37	12	48	48	33	37
12	57	57	33	48	48
92	86	33	57	57	<b>57</b>
86	33	86	86	86	86
33	92	92	92	92	92

Initial (unso Array	rted) Pass 1	. Pass 2	Pass 3	Pass 4	Pass 5
25	25	25	25	12	12
57	48	37	12	25	25
48	37	12	37	37	33
37	12	48	48	33	37
12	57	57	33	48	48
92	86	33	57	57	57
86	33	86	86	86	86
33	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5
25	25	25	25	12	12
57	48	37	12	25	25
48	37	12	37	37	33
37	12	48	48	33	37
12	57	57	33	48	48
92	86	33	57	57	57
86	33	86	86	86	86
33	92	92	92	92	92

Initial (un Arra	•	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5
25		25	25	25	12	12
57		48	37	12	25	25
48		37	12	37	37	33
37		12	48	48	33	37
12		57	57	33	48	48
92		86	33	57	57	57
86		33	86	86	86	86
33		92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6
25	25	25	25	12	12	12
57	48	37	12	25	25	25
48	37	12	37	37	33	33
37	12	48	48	33	37	37
12	57	57	33	48	48	48
92	86	33	57	57	57	57
86	33	86	86	86	86	86
33	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6
25	25	25	25	12	12	12
57	48	37	12	25	25	25
48	37	12	37	37	33	33
37	12	48	48	33	37	37
12	57	57	33	48	48	48
92	86	33	57	57	57	57
86	33	86	86	86	86	86
33	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6
25	25	25	25	12	12	12
57	48	37	12	25	25	25
48	37	12	37	37	33	33
37	12	48	48	33	37	37
12	57	57	33	48	48	48
92	86	33	57	57	57	57
86	33	86	86	86	86	86
33	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6
25	25	25	25	12	12	12
57	48	37	12	25	25	25
48	37	12	37	37	33	33
37	12	48	48	33	37	37
12	57	57	33	48	48	48
92	86	33	57	57	57	57
86	33	86	86	86	86	86
33	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6
25	25	25	25	12	12	12
57	48	37	12	25	25	25
48	37	12	37	37	33	33
37	12	48	48	33	37	37
12	57	57	33	48	48	48
92	86	33	57	57	57	57
86	33	86	86	86	86	86
33	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6
25	25	25	25	12	12	12
57	48	37	12	25	25	25
48	37	12	37	37	33	33
37	12	48	48	33	37	37
12	57	57	33	48	48	48
92	86	33	57	57	57	57
86	33	86	86	86	86	86
33	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6
25	25	25	25	12	12	12
57	48	37	12	25	25	25
48	37	12	37	37	33	33
37	12	48	48	33	37	37
12	57	57	33	48	48	48
92	86	33	57	57	57	57
86	33	86	86	86	86	86
33	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6
25	25	25	25	12	12	12
57	48	37	12	25	25	25
48	37	12	37	37	33	33
37	12	48	48	33	37	37
12	57	57	33	48	48	48
92	86	33	57	57	57	57
86	33	86	86	86	86	86
33	92	92	92	92	92	92

Initial (unsorted Array	d) Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6	Pass 7
25	25	25	25	12	12	12	12
57	48	37	12	25	25	25	25
48	37	12	37	37	33	33	33
37	12	48	48	33	37	37	37
12	57	57	33	48	48	48	48
92	86	33	57	57	57	57	57
86	33	86	86	86	86	86	86
33	92	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6	Pass 7
25	25	25	25	12	12	12	12
57	48	37	12	25	25	25	25
48	37	12	37	37	33	33	33
37	12	48	48	33	37	37	37
12	57	57	33	48	48	48	48
92	86	33	57	57	57	57	57
86	33	86	86	86	86	86	86
33	92	92	92	92	92	92	92

Initial (unsorted Array	) Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6	Pass 7
25	25	25	25	12	12	12	12
57	48	37	12	25	25	25	25
48	37	12	37	37	33	33	33
37	12	48	48	33	37	37	37
12	57	57	33	48	48	48	48
92	86	33	57	57	57	57	57
86	33	86	86	86	86	86	86
33	92	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6	Pass 7
25	25	25	25	12	12	12	12
57	48	37	12	25	25	25	25
48	37	12	37	37	33	33	33
37	12	48	48	33	37	37	37
12	57	57	33	48	48	48	48
92	86	33	57	57	57	57	57
86	33	86	86	86	86	86	86
33	92	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6	Pass 7
25	25	25	25	12	12	12	12
57	48	37	12	25	25	25	25
48	37	12	37	37	33	33	33
37	12	48	48	33	37	37	37
12	57	57	33	48	48	48	48
92	86	33	57	57	57	57	57
86	33	86	86	86	86	86	86
33	92	92	92	92	92	92	92

Initial (unsorted Array	l) Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6	Pass 7
25	25	25	25	12	12	12	12
57	48	37	12	25	25	25	25
48	37	12	37	37	33	33	33
37	12	48	48	33	37	37	37
12	57	57	33	48	48	48	48
92	86	33	57	57	57	57	57
86	33	86	86	86	86	86	86
33	92	92	92	92	92	92	92

Initial (unsorted) Array	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6	Pass 7
25	25	25	25	12	12	12	12
57	48	37	12	25	25	25	25
48	37	12	37	37	33	33	33
37	12	48	48	33	37	37	37
12	57	57	33	48	48	48	48
92	86	33	57	57	57	57	57
86	33	86	86	86	86	86	86
33	92	92	92	92	92	92	92

Initial (unso Array	orted)	Pass 1	Pass 2	Pass 3	Pass 4	Pass 5	Pass 6	Pass 7
25		25	25	25	12	12	12	12
57		48	37	12	25	25	25	25
48		37	12	37	37	33	33	33
37		12	48	48	33	37	37	37
12		57	57	33	48	48	48	48
92		86	33	57	57	57	57	57
86		33	86	86	86	86	86	86
33		92	92	92	92	92	92	92

# **Bubble Sort Optimizations**

- 2 optimizations
  - Note that at the end of each pass k, k elements are in place (sorted), (n – k) remain unsorted.
  - If in a pass you do no swaps, array is sorted.
- Runtime complexity analysis...

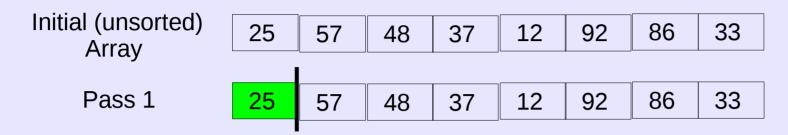
# **Bubble Sort Complexity**

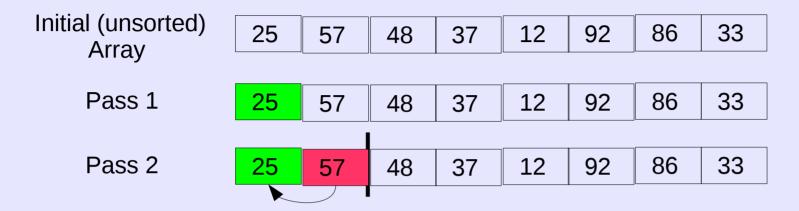
- Complexity:
  - Runtime complexity of Bubble Sort: O(n²)
  - Space complexity of Bubble Sort: O(1)
  - Stable: yes.

- Simple.
- Not efficient.
- Make multiple passes (n) through the unsorted array, considering a new element in each pass and inserting the new element in its proper place.
- Faster than Bubble Sort.

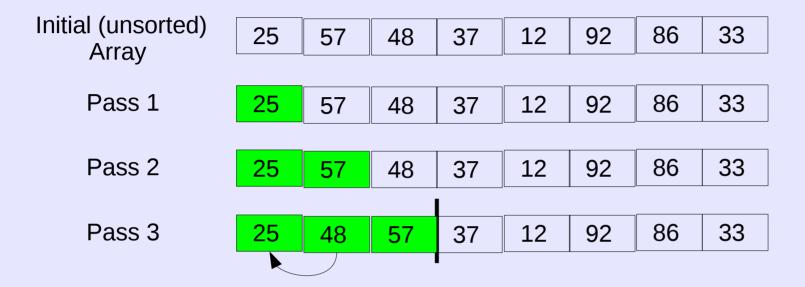
Initial (unsorted) Array

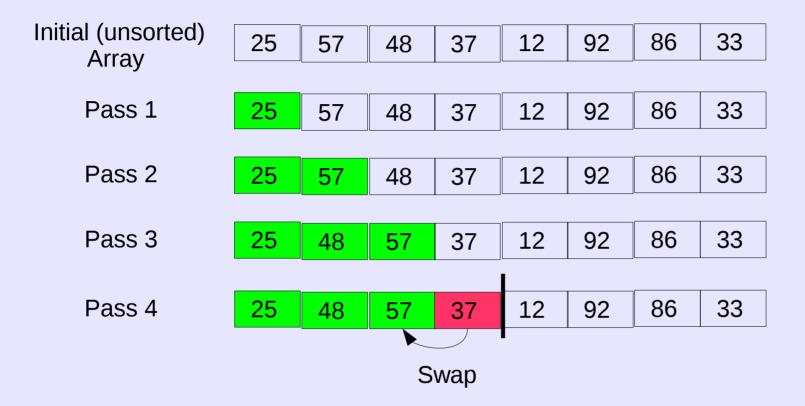
25 5	7 48	37	12	92	86	33
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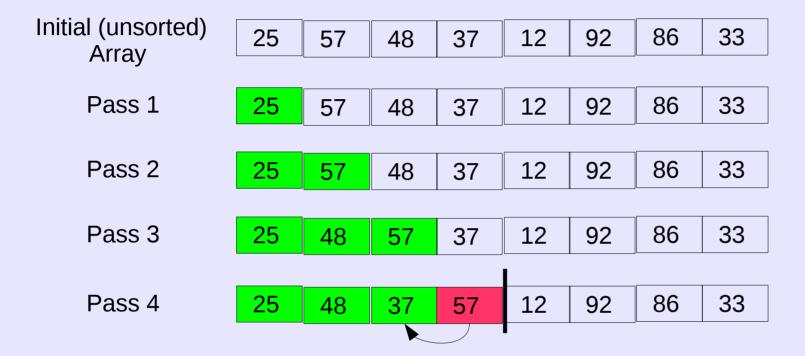


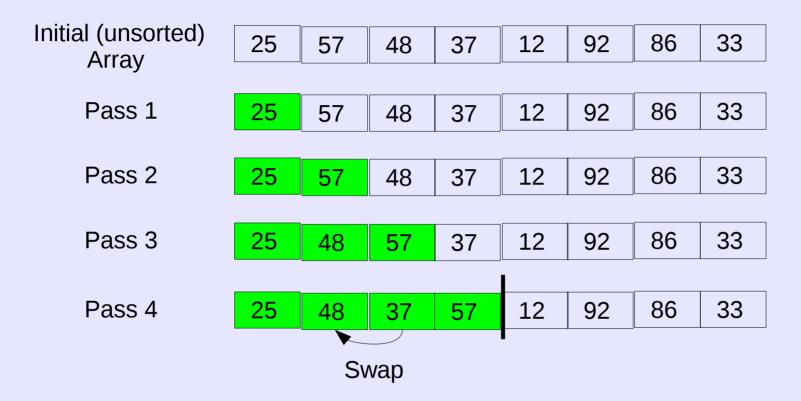








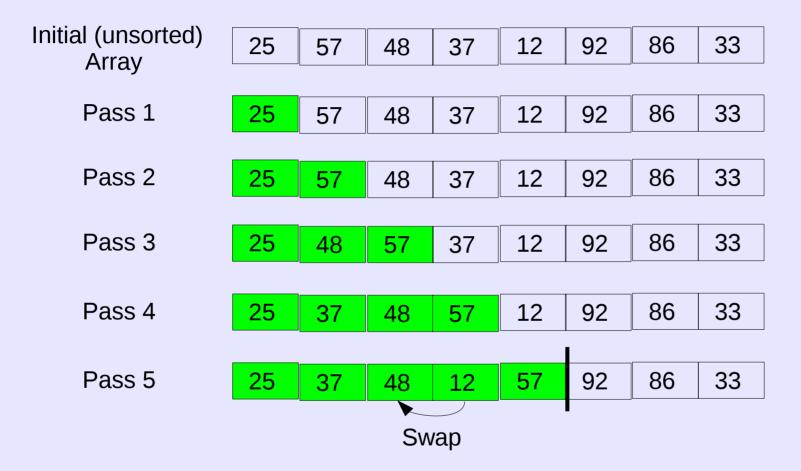




Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	<b>37</b>	48	57	12	92	86	33

Initial (unsorted) Array	25	57	48	37	12	92	86	33	
Pass 1	25	57	48	37	12	92	86	33	
Pass 2	25	57	48	37	12	92	86	33	
Pass 3	25	48	57	37	12	92	86	33	
Dogg 4	25	07	40		10	00	00		
Pass 4	25	37	48	57	12	92 •	86	33	
Pass 5	25	37	48	57	12	92	86	33	
	Swap								



Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	07	40	<b>-</b> 7	12	02	86	33
Pass 4	25	37	48	57	12	92 	00	33
Pass 5	25	37	12	48	57	92	86	33

Initial (unsorted) Array	25	57	48	37	12	92	86	33	
Pass 1	25	57	48	37	12	92	86	33	
Pass 2	25	57	48	37	12	92	86	33	
Pass 3	25	48	57	37	12	92	86	33	
Pass 4	25	37	48	57	12	92	86	33	
Pass 5	25	37	12	48	57	92	86	33	
Swap									

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	40	<b>E</b> 7	12	92	86	33
r ass 4	25	31	48	57	12	92 	80	33
Pass 5	25	12	37	48	57	92	86	33

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
r ass 4	23	37	40	57	12	92 	00	33
Pass 5	25	<b>12</b>	37	48	57	92	86	33
	Swa	ap						

25	57	48	37	12	92	86	33
25	57	48	37	12	92	86	33
25	57	48	37	12	92	86	33
25	48	57	37	12	92	86	33
25	37	48	57	12	92	86	33
12	25	37	48	57	92	86	33
	25 25 25	25 57 25 57 25 48 25 37	25 57 48   25 57 48   25 48 57   25 37 48	25 57 48 37   25 57 48 37   25 48 57 37   25 37 48 57	25 57 48 37 12   25 57 48 37 12   25 48 57 37 12   25 37 48 57 12	25 57 48 37 12 92   25 57 48 37 12 92   25 48 57 37 12 92   25 37 48 57 12 92	25 57 48 37 12 92 86   25 57 48 37 12 92 86   25 48 57 37 12 92 86   25 37 48 57 12 92 86

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	<b>57</b>	92	86	33

Initial (unsorted) Array	25	57	48	37	12	92	86	33			
Pass 1	25	57	48	37	12	92	86	33			
Pass 2	25	57	48	37	12	92	86	33			
Pass 3	25	48	57	37	12	92	86	33			
Pass 4	25	37	48	57	12	92	86	33			
Dace F											
Pass 5	12	25	37	48	57	92	86	33			
Pass 6	12	25	37	48	57	92	86	33			
Pass 7	12	25	37	48	57	92	86	33			
						Swap					

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	2E	27	40	<b>-</b>	10	02	86	33
Pass 4	25	37	48	57	12	92	00	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33

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Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33
Pass 8	12	25	37	48	57	86	92	33
		CS 401						∠ I wap

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33
Pass 8	12	25	37	48	57	86	33	92

Initial (unsorted) Array	25	57	48	37	12	92	86	33	
, o.y									
Pass 1	25	57	48	37	12	92	86	33	
					1				
Pass 2	25	57	48	37	12	92	86	33	
		_							
Pass 3	25	48	57	37	12	92	86	33	
Pass 4	25	37	48	57	12	92	86	33	
Pass 5	12	25	37	48	57	92	86	33	
Pass 6	12	25	37	48	57	92	86	33	
Pass 7	12	25	37	48	57	86	92	33	
<b>D</b>									
Pass 8	12	25	37	48	57	86	33	92	
				CS 401 Swap vgurbani@iit.edu					

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Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33
Pass 8	12	25	37	48	57	33	86	92
				C	S 401			

Initial (unsorted) Array	25	57	48	37	12	92	86	33	
ruray									
Pass 1	25	57	48	37	12	92	86	33	
Pass 2	25	57	48	37	12	92	86	33	
					<u> </u>				
Pass 3	25	48	57	37	12	92	86	33	
Pass 4	25	37	48	57	12	92	86	33	
Pass 5	12	25	37	48	57	92	86	33	
Pass 6	12	25	37	48	57	92	86	33	
D 7								00	
Pass 7	12	25	37	48	57	86	92	33	
D 0	4.0						0.0	00	
Pass 8	12	25	37	48	57	33	86	92	
	CS 401 Swap vgurbani@iit.edu								

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Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33
Pass 8	12	25	37	48	33	57	86	92
				С	S 401			

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33
Pass 8	12	25	37	48	33	57	86	92
			Swap	C	S 401 nni@iit.eo			

Initial (unsorted) Array	25	57	48	37	12	92	86	33
<b></b>								
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	27	40	<b>E</b> 7	10	02	86	33
Pass 4	25	37	48	57	12	92	80	33
							ı	
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33
Pass 8	12	25	37	33	48	57	86	92
					S 401			
				vgurba	ıni@iit.eo	du		

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
				l I	]			
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Doce 4	O.F.	07	40	F-7	10	00	06	22
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	10						02	22
Pass 1	12	25	37	48	57	86	92	33
Pass 8	12	25	37	33	48	57	86	92
		Sv	vap		S 401 ani@iit.e	du		

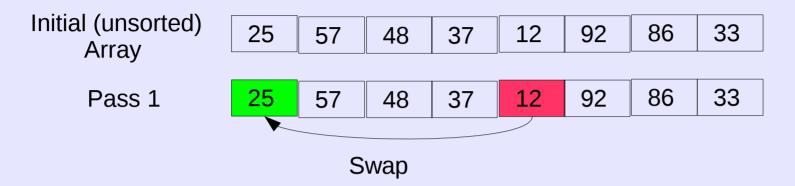
Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33
Pass 8	12	25	33	37	48	57	86	92

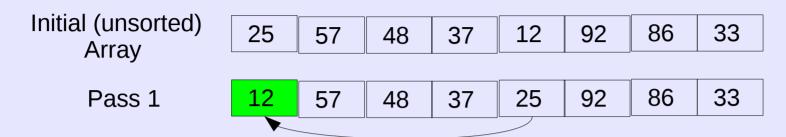
Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	25	57	48	37	12	92	86	33
Pass 2	25	57	48	37	12	92	86	33
Pass 3	25	48	57	37	12	92	86	33
Pass 4	25	37	48	57	12	92	86	33
Pass 5	12	25	37	48	57	92	86	33
Pass 6	12	25	37	48	57	92	86	33
Pass 7	12	25	37	48	57	86	92	33
Pass 8	12	25	33	37	48	57	86	92

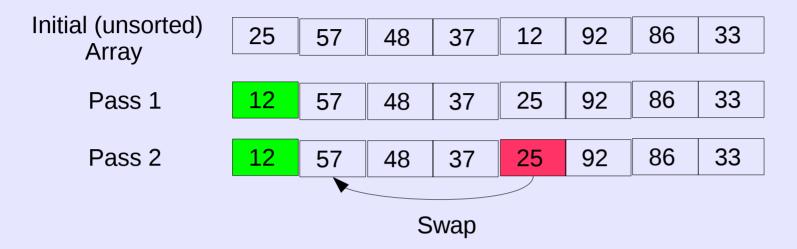
# **Insertion Sort Complexity**

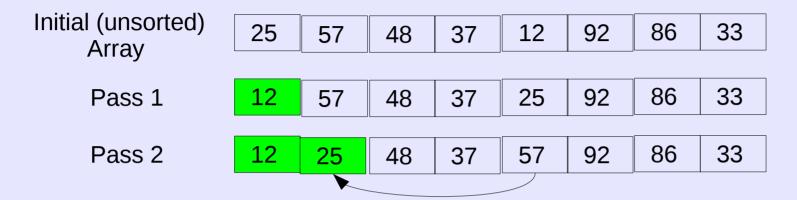
- Complexity:
  - Runtime complexity of Insertion Sort: O(n²)
  - Space complexity of Insertion Sort: O(1)
  - Stable: yes.

- Simple.
- Not efficient.
- Make multiple passes (n 1) through the unsorted array; in each pass, select smallest unsorted item and swap it with the item in the next position to be filled.
- Not as good as Insertion Sort.

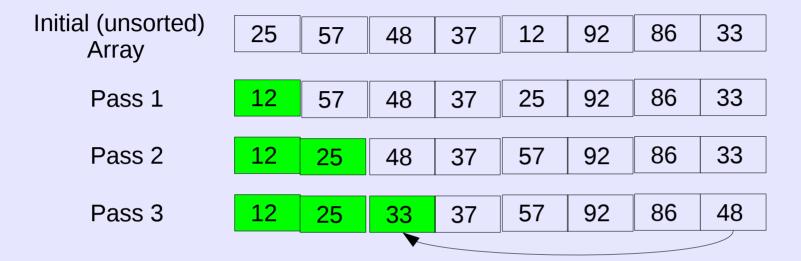






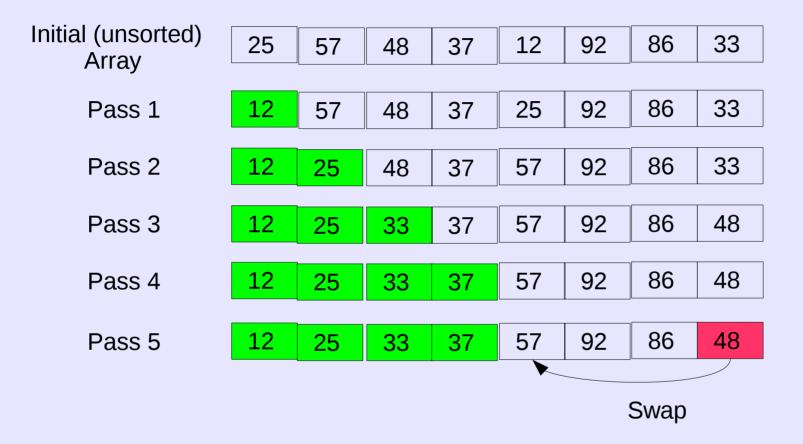






Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	12	57	48	37	25	92	86	33
Pass 2	12	25	48	37	57	92	86	33
Pass 3	12	25	33	37	57	92	86	48
Pass 4	12	25	33	37	57	92	86	48

In place



Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	12	57	48	37	25	92	86	33
Pass 2	12	25	48	37	57	92	86	33
Pass 3	12	25	33	37	57	92	86	48
Pass 4	12	25	33	37	57	92	86	48
Pass 5	12	25	33	37	48	92	86	57

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	12	57	48	37	25	92	86	33
Pass 2	12	25	48	37	57	92	86	33
Pass 3	12	25	33	37	57	92	86	48
Pass 4	12	25	33	37	57	92	86	48
Pass 5	12	25	33	37	48	92	86	57
Pass 6	12	25	33	37	48	92	86	57
							Swap	

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	12	57	48	37	25	92	86	33
Pass 2	12	25	48	37	57	92	86	33
Pass 3	12	25	33	37	57	92	86	48
Pass 4	12	25	33	37	57	92	86	48
Pass 5	12	25	33	37	48	92	86	57
Pass 6	12	25	33	37	48	57	86	92

Initial (unsorted) Array	25	57	48	37	12	92	86	33
Pass 1	12	57	48	37	25	92	86	33
Pass 2	12	25	48	37	57	92	86	33
Pass 3	12	25	33	37	57	92	86	48
Pass 4	12	25	33	37	57	92	86	48
Pass 5	12	25	33	37	48	92	86	57
Pass 6	12	25	33	37	48	57	86	92
Pass 7	12	25	33	37	48	57	86	92

In place

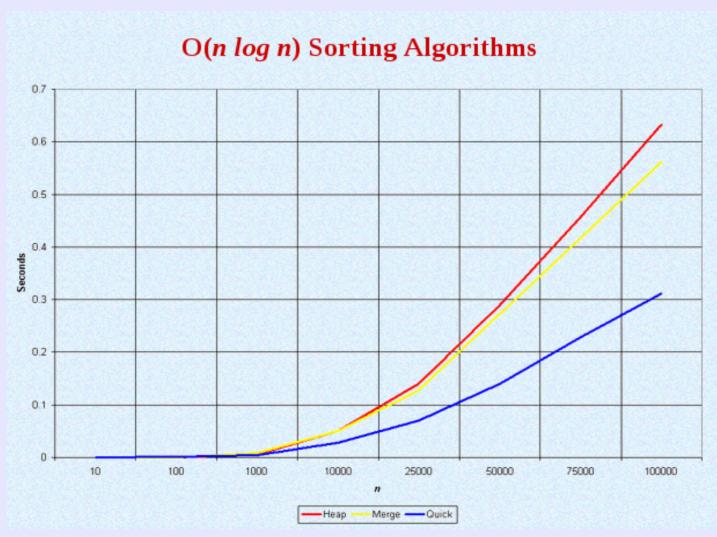
# Selection Sort Complexity

- Complexity:
  - Runtime complexity of Selection Sort: O(n²)
  - Space complexity of Selection Sort: O(1)
  - Stable: no.

## Sorting: Divide and Conquer Sorts

- Heap sort
- Merge sort
- Quick sort

# Divide and Conquer Sort Performance



Slide source: http://www.personal.kent.edu/~rmuhamma/Algorithms/MyAlgorithms/Sorting/sortingIntro.htm

# Heap sort