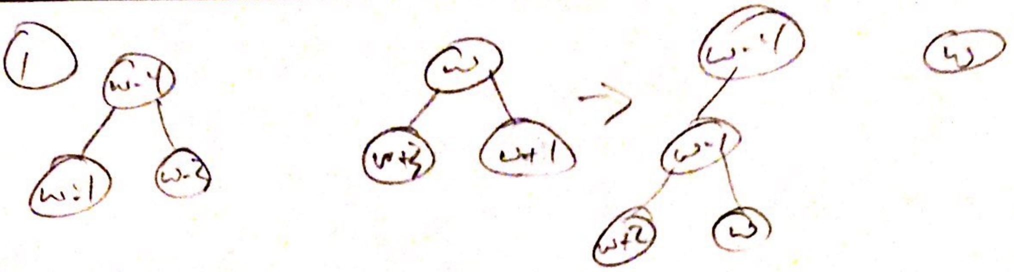
①



The weights of leaves in an optimal tree are either $w-1, w, w+2$ or $w, w+2, w+3$

b) A node of cost w is derived from nodes of cost $w-1$ and $w-3$ so $\boxed{M_w = M_{w-1} + M_{w-3}}$

② $T(n) = 2T(n-1) + T(n-2) \to r^2 = 2r+1$

$r^2 - 2r - 1 \to \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$ $\boxed{T(n) = a(1+\sqrt{2})^n + b(1-\sqrt{2})^n}$

ii) Homogeneous root is $r=4$ $T(n) = c4^n + r$

$r = c_2 2^n$ $T(n) = c_1 4^n + c_2 2^n$ $T(0) = c_1 4^0 + c_2 2^0 = 1$

$T(1) = 4T(0) + 2 = 6$ $T(1) = c_1 4 + c_2 2 = 6$

$c_1 + c_2 = 1$ $c_1 = 2$ $\boxed{T(n) = 2 \cdot 4^n - 2^n}$

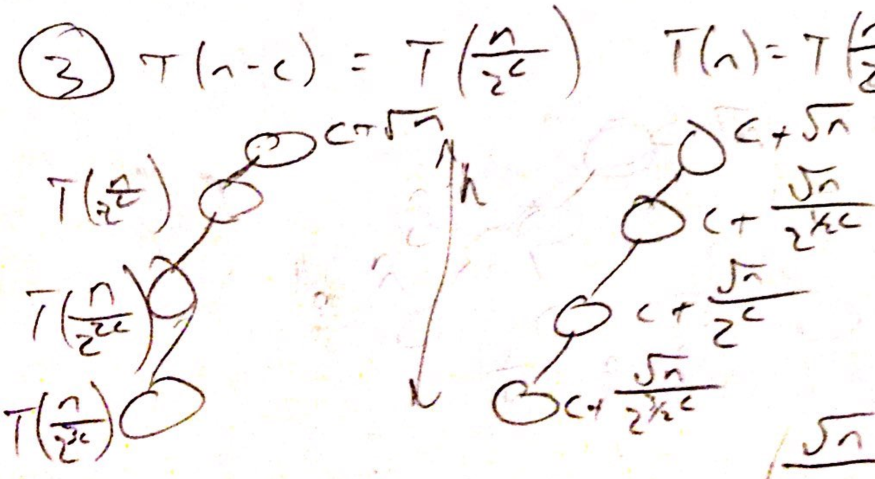
$4c_1 + 2c_2 = 6$ $c_2 = -1$

iii) $r^2 - 8r + 16 = 0$ $(r-4)^2 = 0$ $r_1 = r_2 = 2$

$T(n) = c_1 n 2^n + c_2 2^n$ $T(0) = c_2 = 2$

$T(1) = c_1 2 + 4 = 10$ $c_1 = 3$ $\boxed{T(n) = 3n2^n + 2 \cdot 2^n}$

③ $T(n-c) = T\left(\frac{n}{2^c}\right)$ $T(n) = T\left(\frac{n}{2^c}\right) + c + \sqrt{n}$



$c + \frac{\sqrt{n}}{2^{\frac{1}{2}ic}}$ is subproblem size

$c + \frac{\sqrt{n}}{2^{\frac{1}{2}ic}} = 1$

$\sqrt{n} = 2^{\frac{1}{2}ic}(1-c) \to \frac{\sqrt{n}}{1-c} = 2^{\frac{ic}{2}}$

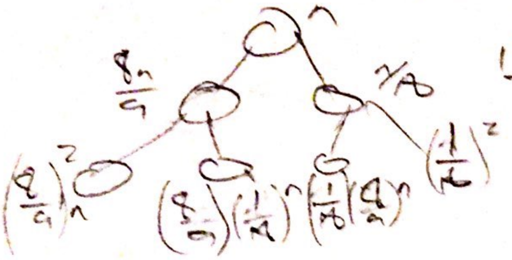
$\left(\frac{\sqrt{n}}{1-c}\right)^{2/c} = 2^i \to \frac{2}{c}\lg\left(\frac{\sqrt{n}}{1-c}\right) = i$

Tree has $\frac{2}{c}\lg\left(\frac{\sqrt{n}}{1-c}\right) + 1$ levels

$T(n) = (c + \sqrt{n}) + \left(c + \frac{\sqrt{n}}{2^{x_c}}\right) + \dots$ geometric series decreasing

Dominated by largest term $\boxed{T(n) = \Theta(\sqrt{n})}$

4) $T(n) = n + \frac{17n}{18} + \frac{289}{324}$ geometric decreasing $\boxed{T(n) = \Theta(n)}$ for $f(n) = n$

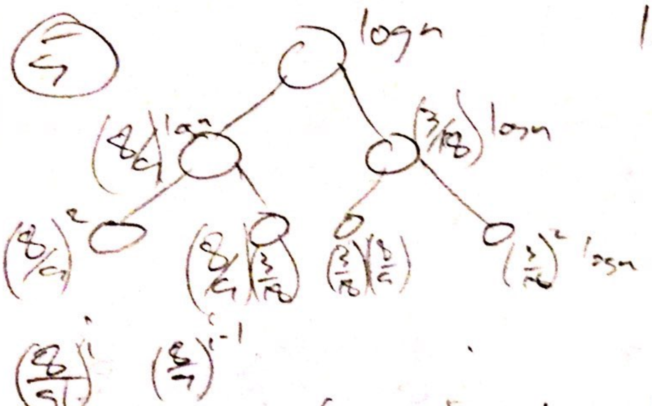


$\frac{17n}{18}$

$\frac{64}{81} + \frac{8}{162} + \frac{8}{162} + \frac{1}{324} = \frac{289}{324}$

$T(n) = n^2 + \left(\frac{17}{18}\right)^2 n^2 + \left(\frac{289}{324}\right)^2 n^2$ $\boxed{T(n) = \Theta(n^2)}$ $f(n) = n^2$

5)



$\log n + \frac{19}{18}\log n + \frac{361}{324}\log n$

Geometric series increasing

$\frac{64}{81} + \frac{16}{54} + \frac{1}{36} =$

$\frac{256}{324} + \frac{96}{324} + \frac{9}{324} = \frac{361}{324}$

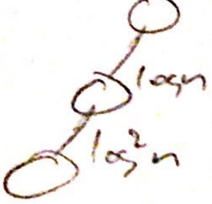
$\left(\frac{8}{9}\right)^i$ $\left(\frac{8}{9}\right)^{i-1}$

# of levels $= \log_{9/8} \log n$ $\left(\frac{8}{9}\right)^i + \left(\frac{8}{9}\right)^{i-1}\left(\frac{3}{18}\right) +$

$\boxed{T(n) \text{ is dominated by sum at } \log_{9/8} \log n \text{ level}}$ $f(n) = \log n$

$\boxed{T(n) \text{ is dominated by sum at } 2\log_{9/8} n \text{ level}}$ $f(n) = n^2$

6) a) $\boxed{1 \le i \le 10 \quad T(i) = 10}$ $\boxed{11 \le i \le 1024 \quad T(i) = 4}$



Decreasing geometric $\boxed{T(n) = \Theta(n) \text{ if } n > 1024}$

b) $\boxed{1 \le i \le 10 \quad T(i) = 10}$ $\boxed{11 \le i \le 1024 \quad T(i) = \Theta(n}$

$\boxed{T(n) = \Theta(n) \text{ if } n > 1024}$ Again decreasing geometric series