Project Part 2 - Progress Report

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1 Solved Issues

| Solved Issue | Member Responsible |
|---------------|--------------------|
| Initial State | Alec Landow |

We created a skeleton for the program with all of the functions we feel we need to implement. For each function we've either written comments on how to implement, written pseudocode, or written actual code. We are focusing on the implementing the algorithm defined in Mateus and Omar [?].

The first step in this algorithm is to create the initial state of the circuit. This consists of creating a quantum register consisting of s=lg(M) qubits to represent the position of each character, where M is the number of characters in the pattern we are searching for within a string of length N. In order to implement this we put the qubits representing the first character in a superposition by applying a Hadamard gate to each of them. Then for each subsequent register of s qubits we entangle that register with the previous register using CX gates. Entangling the registers encodes the order of the characters we are looking for. After entangling there are more CX gates applied - **Explain remainder of the CX gates**.

2 Open Problems

The remainder of the algorithm proceeds as follows. We still need to implement each of the below steps. The overall string is w of length N. The pattern is p of length M.

- 1. Select a number at random from the range consisting of the length of the string excluding the pattern, or N-M+1. Call this number r.
- 2. Iterate over the range of r, and in each iteration choose an index j at random from [1, M].

Only need to select r = N - M + 1 indices j at random from the pattern.

3. Apply respective query operator Q_j to the current state. The query operator is also known as the oracle function. The function it needs to implement is

$$f(x) = \begin{cases} 1, & \text{if the i}^{th} \text{ letter of string is } M_j \\ 0, & \text{otherwise} \end{cases}$$

If the j^{th} character from p matches a character in w, its state's amplitude will be amplified by the diffusor step below. Since we are selecting indices j at random each character's state amplitude may be amplified more than others.

The circuit for the oracle function also incorporates Gray codes. A gray code between two binary strings is a sequence of binary strings in which 2 consecutive strings only differ by one bit, eventually the source string evolves to the target string. A transposition gate (labelled CNOT in the paper) is developed to transition strings from string to string (state to state). It is still an open problem to determine the utility of this aspect of the oracle.

There will be a different query operator Q_j (circuit) for each character in the alphabet.

- 4. Apply Grover diffusion to current state. This is what amplifies the amplitude of the state.
- 5. Measure the state. The basis of the state is over the length of w which is N. So the measurement will correspond to an index in the string w which we interpret as the beginning of the closest match of the pattern p.
- 6. Repeat steps 1-5 \sqrt{N} times