Measuring the Mass of Jupiter Using Kepler's Third Law

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Abstract

The aim of this experiment is to determine the mass of Jupiter and to obtain an astrophotography of the Orion nebula. By measuring the orbital periods and semi-major axes of the four Galilean moons of Jupiter (i.e. Io, Europa, Ganymede and Callisto), Kepler's Third Law can be applied to determine Jupiter's mass. The equipment used includes a Celestron CGE1100 catadioptric telescope and a Canon EOS 6D camera. Data was gathered from astrographic images. To enhance the accuracy of the results, the observations took place at Diavolezza, GR, at a height of 2'973 meters above sea level, and over four consecutive nights. The results give an estimate of Jupiter's mass of $\sim 1.5 \cdot 10^{27}$kg, but there is a 20% discrepancy with the literature value which suggests a systematic error. Additionally, the Orion Nebula was photographed with various filters.

Contents

1 Introduction

In this experiment, the mass of the planet Jupiter is determined. For this purpose, the orbital periods and semi-major axes of the four biggest Jupiter moons is measured, also called the Galilean moons: Io, Europa, Ganymede and Callisto. For each moon, Kepler's third law is applied, which states

$$\frac{a^3}{T^2} = \frac{G(M+m)}{4\pi^2}, \tag{1}$$

where a is the semi-major axis, T the orbital period, M the Jupiter mass and m the moon's mass. It is assumed that the mass of the moons is negligible compared to the mass of Jupiter. Additionally, the orbits of the Galilean moons are approximately circular. This means that the eccentricity is small, such that the semi-major a can be replaced by the radius R. Therefore, the expression can be simplified to

$$\frac{R^3}{T^2} = \frac{GM}{4\pi^2}, \tag{2}$$

which gives us the Jupiter mass as a function of the orbital radius and period of the moons:

$$M = \frac{4\pi^2 R^3}{GT^2}, \tag{3}$$

The two unknown variables are extracted from astrographic pictures. Therefore, pictures of Jupiter and its moons are taken over several nights, at different time stamps to reconstruct the orbits of the moons. In order to get good results, the pictures are taken with as little light pollution as possible. Since the inclination of the four moons is very small, the elliptic orbit of the moons is projected on a plane as an oscillation. The oscillation x of the distance between the moons and the planet is measured as a function of time. The mathematical model for the oscillation x is

$$x(t) = R\cos(\omega t + \phi) \tag{4}$$

where ω is the angular frequency and ϕ the phase shift. We fit this model to the data points to extract R, ω and ϕ. Having determined the parameters R and $\omega = 2\pi/T$, we can use Eq. 2 to calculate the planet's mass. For reference, the literature value of the Jupiter mass is $M = 1.899 \cdot 10^{27}$kg [1] and the literature values for the variables [2] are

Table 1: Literature values of the Galilean moons

Io	Europa	Ganymede	Callisto
$T = 1.769$d	$T = 3.551$d	$T = 7.155$d	$T = 16.689$d
$a = 421'800$km	$a = 671'100$km	$a = 1'070'400$km	$a = 1'882'700$km
$\epsilon = 0.004$	$\epsilon = 0.009$	$\epsilon = 0.001$	$\epsilon = 0.007$

Additionally, the Orion Nebula is captured in photographs. This diffuse nebula is a star-formation region. The ionized light it emits falls within the visible spectrum, but is only faintly perceptible to the naked eye. Situated in the "Sword" of the Orion constellation, the Orion Nebula lies approximately 1,350 light-years away from Earth, making it one of the most active star-forming regions in our vicinity. We capture images of the nebula both in the visual band as well as with various filters which only let a specific wavelength pass. The following filters were used: H-alpha (656 nm), O3 (501 nm), and Si3 (309 nm). The H-alpha filter corresponds to a spectral line of neutral hydrogen, while O3 and Si3 refer to spectral lines stemming from doubly ionised oxygen and silicon respectively. By focusing on these specific wavelengths, we can infer that where the picture is bright, there must be atoms present which emit in the selected spectral line. This can provide us with more information about the structure of the nebula. For example, the distribution of ionised hydrogen gas in the nebula can be inferred from the H-alpha picture.

2 Measurement

2.1 Instruments

For this experiment, we used a Celestron CGE1100 catadioptric telescope. Its optical system is of the Schmidt-Cassegrain [3] type. As shown in Figure 1, it consists of two mirrors. The main mirror is spherical concave and collects the light rays to the secondary mirror. After the reflection on the secondary mirror, the light rays are directed through a hole in the main mirror to the eye piece. The CGE1100 has an 11-inch (279.4 mm) aperture and a focal length of 2800 mm. It is equipped with a computerized German Equatorial Mount, which allows it to track objects by driving the polar axis at constant speed. Its integrated GoTo capabilities permit us to easily locate and track astronomical objects with precision, a task which is further facilitated by the software-assisted polar alignment of the mount. [4]

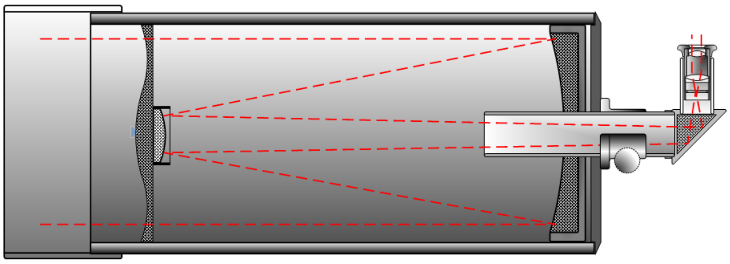

Figure 1: Schmidt-Cassegrain telescope

The camera used to capture images from the telescope is the Canon EOS 6D, which is a 20.2-megapixel (5,472 × 3,648) digital single-lens reflex camera. It is equipped with a CMOS sensor of 35.8mm × 23.9mm. [5]

2.2 Data Gathering

The measurements are taken at Diavolezza, GR at a height of 2'973 meters above sea level. The observation period starts on Monday night, 08.01.2024 and ends on Thursday night, 11.01.2024. Throughout the week, Jupiter was visible approximately between 18:00 (when the sun sets) and 02:00. The Orion Nebula, on the other hand, is visible the whole night. The observation time is only limited by the sunset and sunrise, as well as prevailing weather conditions.

On Monday evening we mounted the telescope and set up the equipment. It is necessary to position the tripod as horizontally as possible to ensure proper functioning of the setup. Additionally, the tripod should be stable even in rough weather. For these reasons a depression of sufficient size was dug out in the snow, wherein the tripod was placed on wooden boards. The feet of the tripod were then covered with snow and frozen in place. The next step was to mount the telescope tube, which was then balanced with the use of counterweights. Once this was in place, the finder scope was added and aligned using an easily visible object (on Monday evening, this was a mountain peak; on subsequent nights we also made use of a bright light placed some distance away). Since the weather on Monday was not good enough to perform the full alignment procedure, it was postponed until the weather cleared. To get an early data point, we aimed the telescope towards Jupiter without making use of the GoTo System. For this purpose, we used two screws on the tube as an iron-side to get the rough direction. Afterwards, we centered Jupiter in the finder scope. In the end, a fine adjustment was necessary to get Jupiter in the centre of the camera display. Unfortunately, the weather conditions remained unfavourable throughout Monday night, but we were nonetheless able to take one data point. On Tuesday the weather was similar, and we were able to collect a few data points whenever the clouds parted enough to give us a glimpse of Jupiter for a few minutes. However, the conditions were still not good enough to perform the alignment for the GoTo system. On the other hand, the weather for the next two days was excellent. To ensure proper tracking, as well as to enable the GoTo feature of the mount, the polar alignment was performed. The tripod was set up in such a way that one of the legs points approximately north. This approximate direction was then corrected using fine-adjustment screws on the mount, making use of Polaris to get the correct direction. Finally, a software-aided alignment procedure was initiated (three-star alignment), where various stars were centered in the telescope, such that the telescope software can create an accurate model of the night sky.

Thanks to the proper alignment and good weather, we were able to collect data points during the entirety of Jupiter's visibility period. Additionally, the GoTo system greatly facilitated the finding of the Orion Nebula, which is why we took the first pictures on Wednesday after the GoTo system was set up. During the breaks in the Jupiter measurements, we were able to quickly realign the telescope on the Orion Nebula, inserting the filters into the setup when necessary.

In Fig. 2 an unedited picture of Jupiter and the four Galliean moons is displayed. The pictures were taken by the camera through the telescope, with an exposure time of 1/30s and an ISO of 1600. For the Orion Nebula pictures we chose an exposure time of 10s and an ISO of 8000. The resulting images can be seen in Section 3.2.

Figure 2: Unedited picture of Jupiter with the four Galilean moons

3 Data Analysis and Results

3.1 Jupiter Mass

For the Jupiter measurements, our data analysis consists of firstly importing the raw Jupiter pictures into a Jupyter Notebook using `rawpy` [6] which can be seen on Fig.2. Then `photutils` [7] is used to reduce the background noise by subtracting the median and using a Gaussian filter. Furthermore, the picture is cropped and all pixels below a chosen threshhold are set manually to zero. For the source detection, the function DAOStarFinder from `photutils` is used to find the pixel coordinates of the moons. However, DAOStarFinder is not suitable to find the coordinates of Jupiter because it requires an excessive amount of computation time. Therefore, we make use of an algorithm which iteratively computes the centre of brightness of the data to find Jupiter's coordinates. In more detail, it computes the centre of brightness by calculating the weighted average of the x and y coordinates of all pixels, where the brightness of each pixel served as the weight. This step is repeated multiple times, but for each iteration, a radial distance from the centre is manually chosen such that Jupiter is still well inside this radius. Then a mask is created outside of this radius and the centre again computed to get higher precision. The outcome of these steps in our data analysis can be seen in Fig. 3.

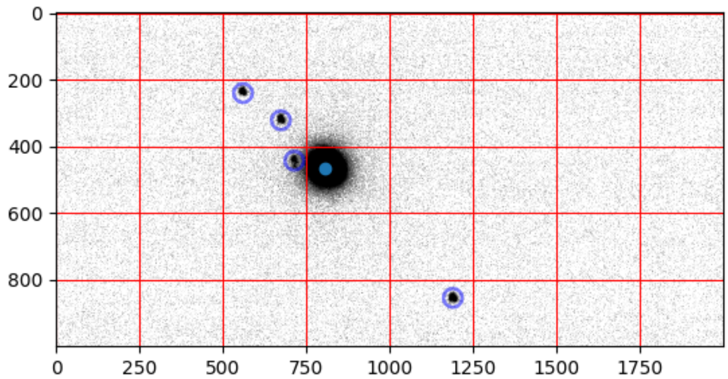

Figure 3: This photo was taken on Thursday, 11 January 2024 at 17:32 and we have from left to right: Europa, Io, Callisto, Jupiter and Ganymede.

As a next step, Stellarium is used to identify each moon at every data point and the relative pixel distances between Jupiter and the moons is calculated using the `numpy.linalg.norm` function and its coordinates. To convert the pixel distances into absolute distances in units of kilometers the following formula and variables are used:

$$D_{abs} = D_J \cdot \sin(\theta \cdot D_{pix}) \tag{5}$$

$s_{pix} = 6.54\ \mu$m
$f = 2800$ mm
$\theta = \frac{s_{pix}}{f}$

D_{pix} = pixel distance
D_J = Earth-Jupiter distance at time of measurement

Here, D_{abs} is the absolute distance, and θ describes how many radians of the sky are covered per pixel length. The exact computations can be seen in the Jupyter notebook on github [8].

Fig. 4 shows the measured data points and the best-fit for the distance between Jupiter and the moons as a function of time. Furthermore, a comparison is given between the measurements and the predicted curve when considering literature values for the period and semi-major axis.

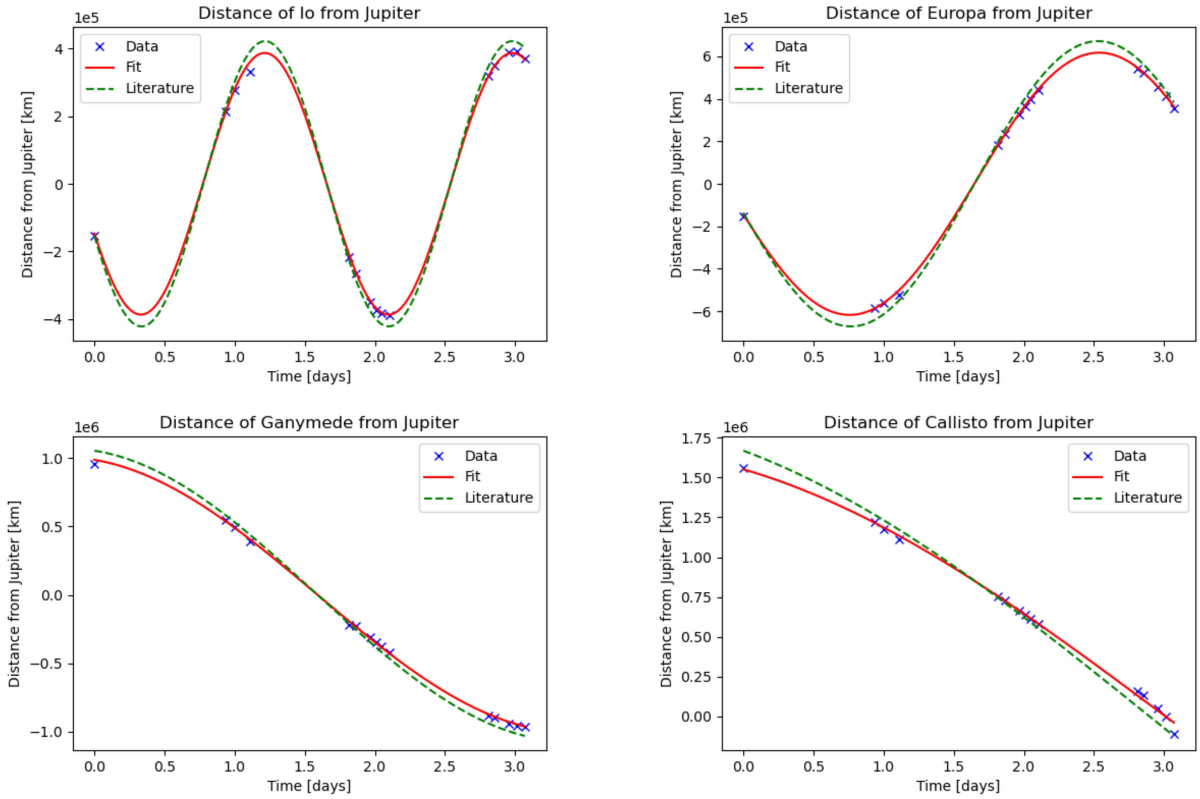

Figure 4: The blue crosses give the data points over a period of four days. The data clearly displays the expected sinusoidal oscillation. The red curve is the fit according to the model $x(t) = R\cos(\omega t + \phi)$. Furthermore, as reference the green curve gives the oscillation when literature values for the period and semi-major axis are considered.

The resulting fit parameters can be seen in the following Table 2:

Io	Europa	Ganymede	Callisto
$R = 386'960 \pm 3690$km	$R = 616'930 \pm 3064$km	$R = 1'009'000 \pm 16390$km	$R = 1'686'700 \pm 66390$km
$\omega = 3.56 \pm 0.01\text{d}^{-1}$	$\omega = 1.77 \pm 0.01\text{d}^{-1}$	$\omega = 0.86 \pm 0.02\text{d}^{-1}$	$\omega = 0.39 \pm 0.02\text{d}^{-1}$
$\phi = 0.39 \pm 0.02$	$\phi = 0.23 \pm 0.01$	$\phi = 4.51 \pm 0.04$	$\phi = 1.17 \pm 0.06$

Table 2: Fit parameters obtained from Fig. 4.

Using these parameters, we have calculated the resulting periods of the moons and the corresponding masses of Jupiter using the following formulas:

$$T = \frac{2\pi}{\omega}, \quad M = \frac{4\pi^2 R^3}{GT^2} \tag{6}$$

The results can be seen in the following Table 3:

Moon:	Period:	Jupiter Mass:
Io:	$T = 1.77 \pm 0.01$d	$M = (1.47 \pm 0.02) \cdot 10^{27}$kg
Europa:	$T = 3.56 \pm 0.01$d	$M = (1.47 \pm 0.06) \cdot 10^{27}$kg
Ganymede:	$T = 7.3 \pm 0.2$d	$M = (1.5 \pm 0.3) \cdot 10^{27}$kg
Callisto:	$T = 16.2 \pm 0.9$d	$M = (1 \pm 2) \cdot 10^{27}$kg

Table 3: The results obtained by using Equation 6 and the fit parameters in Table 2.

To estimate the error propagation of the Jupiter mass and the periods, we used the Gaussian error propagation formula. We assumed that the errors are sufficiently small, normally distributed and uncorrelated.

$$\sigma_T = \sqrt{\left(\frac{\partial T}{\partial \omega}\right)^2 \sigma_\omega^2} = \sqrt{\left(\frac{\pi^2}{T^2}\right)^2 \sigma_\omega^2}, \tag{7}$$

$$\sigma_M = \sqrt{\left(\frac{\partial M}{\partial R}\right)^2 \sigma_R^2 + \left(\frac{\partial M}{\partial \omega}\right)^2 \sigma_\omega^2} = \sqrt{\left(\frac{12\pi^2 R^2}{GT^2}\right)^2 \sigma_R^2 + \left(\frac{8\pi^2 R^3}{GT^3}\right)^2 \sigma_T^2}, \tag{8}$$

3.2 Orion Nebula

Fig. 5 shows the visual image of the Orion Nebula. In Fig. 6 we used an H-alpha filter, while in Fig. 7 the Orion Nebula with a O3 filter is shown. Finally, we took pictures with a Si3 filter. The result is shown in Fig. 8. From the figures it can be seen that the Si3 is concentrated closer to the stars and the O3 and H-alpha is more spread out. This is because the Orion Nebula is a star forming region and the interstellar gas gets ionised by the radiation of the stars. As hydrogen is easier to ionise than oxygen, which in turn is easier to ionise than silicon, the ionised hydrogen can be seen far away from the stars, while the oxygen is more concentrated and the silicon can only be seen in the very center. Additionally, it is possible that the distribution of elements is not homogenous throughout the nebula.

Figure 5: Orion Nebula

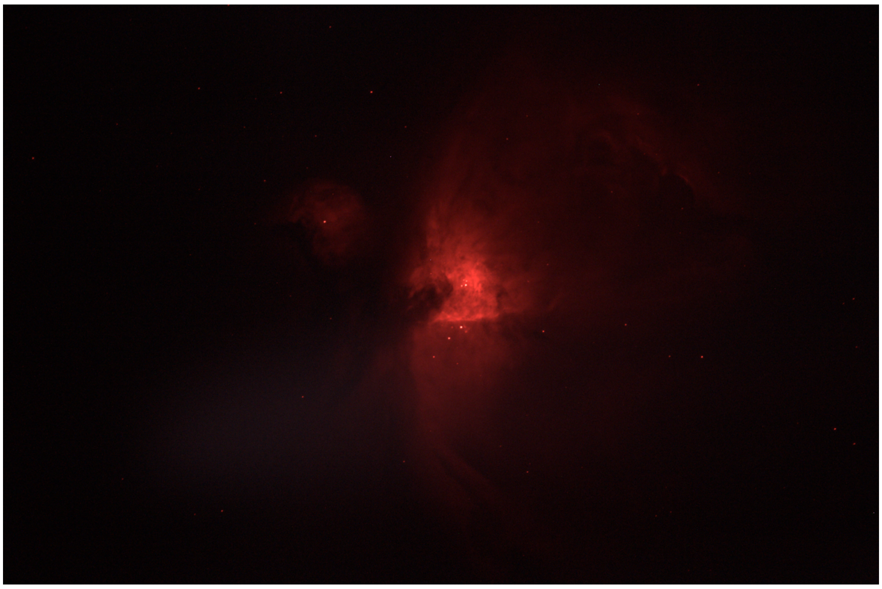

Figure 6: Orion H-alpha Filter

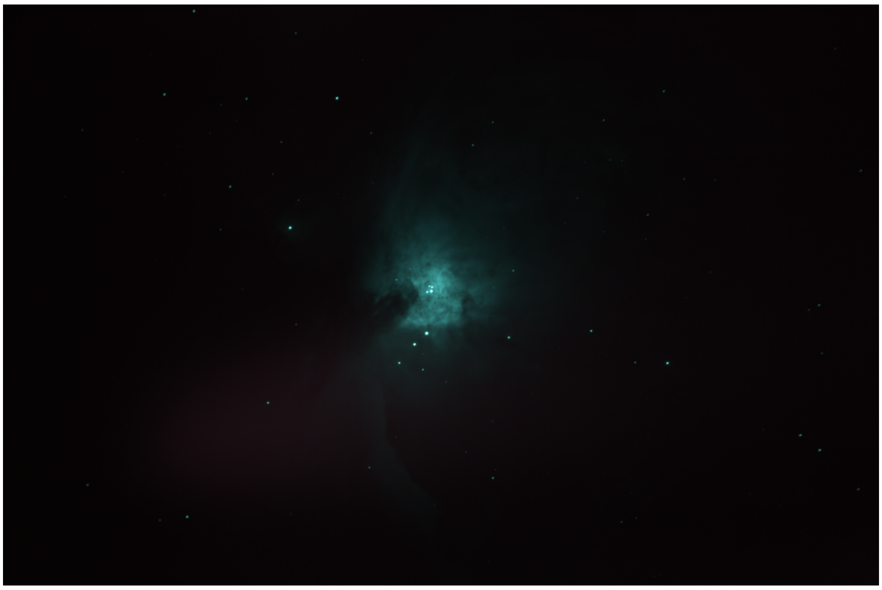

Figure 7: Orion O3 Filter

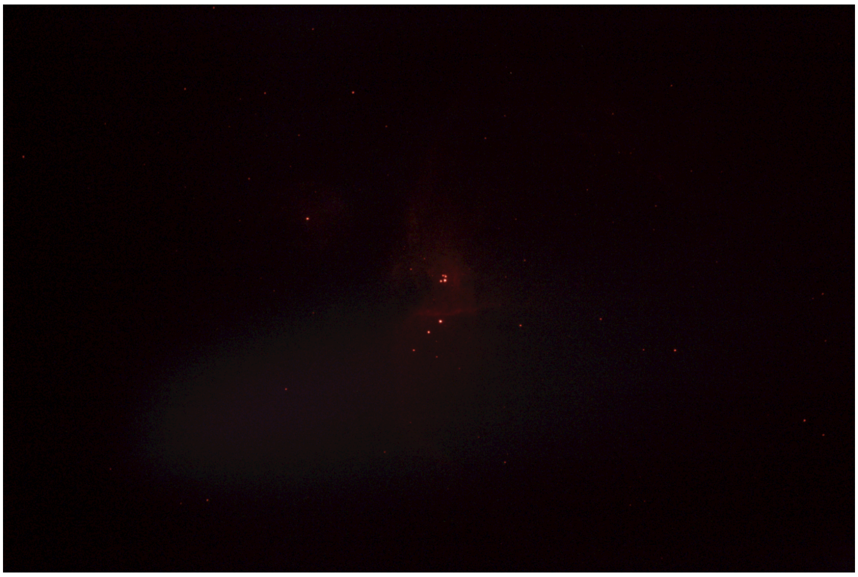

Figure 8: Orion Si3 Filter

For the Orion Nebula, we followed standard procedures for data reduction in astrophotography (see e.g. [9]). To reduce noise in our data we subtract dark frames from our raw data. The dark frames are taken with the same exposure time and ISO as the raw data. To produce the final image of the Orion Nebula we stacked 25 pictures together. For this task we used the `DeepSkyStacker` [10] software.

4 Discussion

As expected, the moons with the smaller period tend to give us more accurate results. This means that the deviation gets bigger if the cutout of the period gets smaller. However, we can see that the resulting T is accurate when compared to literature values, and most of the error seems to be stemming from the fitting of the radius. The uncertainty of the parameter R is about 6-10% off the literature value, resulting in the calculated mass being about 20-25% off the literature value. This deviation of Jupiter mass could not be explained by the uncertainties of the measurement. As a conclusion, it must be a systematic error.
To further investigate the possibility of a systematic bias in the focal length or pixel size, the dependence of the mass from these parameters was determined. Then the two parameters were varied, and the effect of this variation gives us an idea of whether the bias can be explained by an inaccuracy in the focal length or pixel size value. Fig. 9 and Fig. 10 show us that a very large deviation from the used values (2800 mm focal size and 6.54 μm) would be necessary to obtain the correct Jupiter mass of $1.898 \cdot 10^{27}$. More precisely, if the pixel size is held constant, a focal length of 2570 mm would result in agreement with literature values, while conversely a pixel size of 7.12 μm would achieve the same effect with the focal length fixed. This corresponds to a deviation of 8.2% for the focal length and 8.9% for the pixel size compared to the used values. Of course, any simultaneous variation of the two parameters which results in the appropriate ratio could also produce this effect. Nonetheless, it seems unlikely that this could explain the systematic error, since we expect the values given by the manufacturers of the telescope and camera to be at least somewhat accurate.

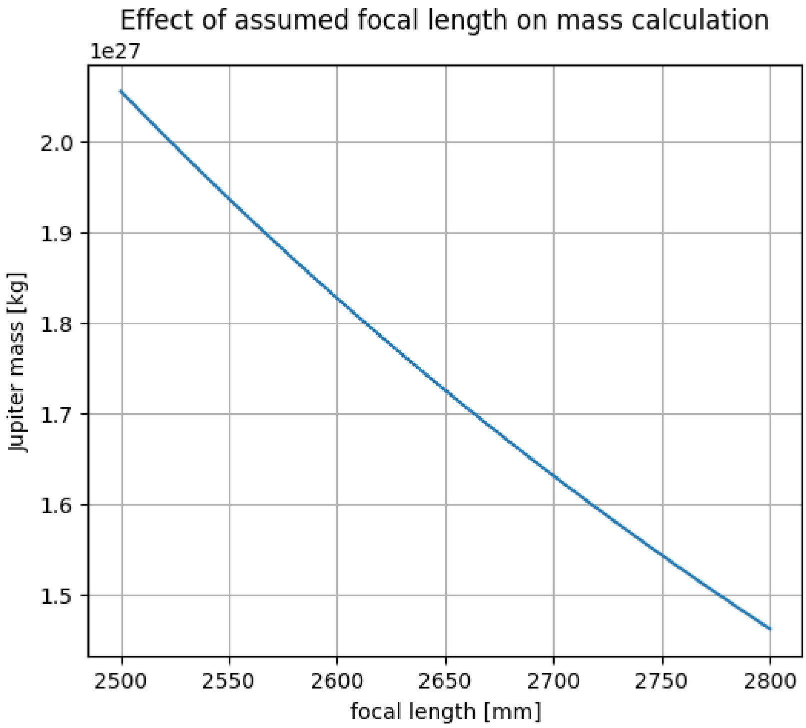

Figure 9: The mass of Jupiter vs. assumed focal length.

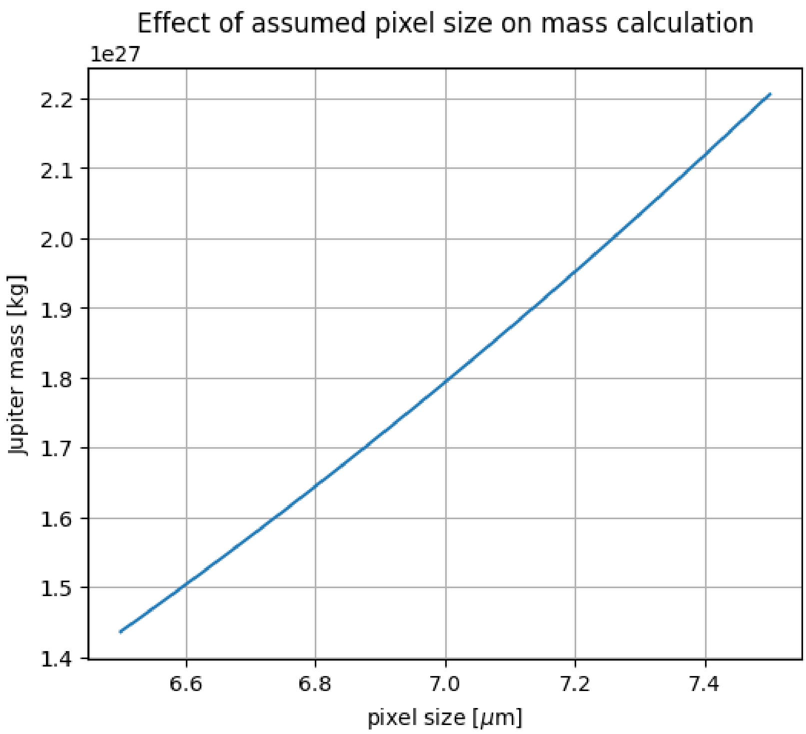

Figure 10: The mass of Jupiter vs. assumed pixel size.

5 Conclusion

The aim of this experiment was to determine the mass of Jupiter through the analysis of the orbital parameters of the Galilean moons. Utilizing a Celestron CGE1100 telescope and Canon EOS 6D camera, we conducted observations over multiple nights at Diavolezza, GR, at a height of 2'973 meters above sea level, which improves viewing conditions. In the data analysis, we processed raw images in python, identified the moon positions, and converted pixel distances to absolute values. While our data displays the expected sinusoidal oscillations and our results do give a rough estimate of Jupiter's mass, discrepancies from literature values suggested the presence of systematic errors. Despite efforts to address potential sources of error, further investigation is needed to eliminate the bias.
Possible improvements for the Jupiter measurements include: observing for longer periods of time, stacking multiple images and using standard data reduction procedures to increase the quality of the pictures (flat frames and dark frames), using a different mathematical model which does not neglect the eccentricity of the orbits, and using a more powerful telescope. However, as long as the source of the bias is not unambiguously determined, the quality of the results will remain low.
For the Orion Nebula pictures, a possible improvement for the future would be to take flat frames as well to improve the image quality. Additionally, more pictures could be stacked, and the pictures could be taken with longer exposure times (which would, however, require a more accurate alignment to reduce star streaking due to tracking errors). Finally, this part too could benefit from a more powerful telescope, which would allow us to see specific features of the nebula in more detail.

References

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