Quantum Simulation of the Agassi Model

Quantum Computing Exam Project

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Presentation Structure

Introduction to the model and challenges

Work done by the group

Obtained result and comparison with reference papers

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Introduction to the Model

$$\boxed{H = \epsilon J^0 - g \sum_{\sigma,\sigma'} A^\dagger_\sigma A_{\sigma'} - \frac{V}{2}\left[(J^+)^2 + (J^-)^2\right]}$$

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$$J^0 = \frac{1}{2} \sum_m c^\dagger_{1,m} c_{1,m} - \sum_m c^\dagger_{-1,m} c_{-1,m}$$

$$A^\dagger_1 = \sum_{m=1}^{j} c^\dagger_{1,m} c_{1,-m}$$

$$A^\dagger_{-1} = \sum_{m=1}^{j} c^\dagger_{-1,m} c_{-1,-m}$$

$$J^+ = \sum_m c^\dagger_{1,m} c_{-1,m} = (J^-)^\dagger$$

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- $4j$ number of particles
- $\sigma = \pm 1$ parity levels with degeneracy $2j$
- $m = \pm 1, \ldots, \pm j$ magnetic quantum number

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Goals

Main challenges

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Simulate the evolution for specific values of j

Study the phase transitions for specific values of j

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Simulate the evolution for specific values of j

Study the phase transitions for specific values of j

Main challenges

Decomposing the hamiltonian for any j

Efficiently determine phase transitions

Jordan-Wigner Transformation

From second quantization to qubits

$$c_i^\dagger \longrightarrow \mathbb{I}_1 \otimes \ldots \otimes \mathbb{I}_{i-1} \otimes \sigma_i^+ \otimes \sigma_{i+1}^z \otimes \ldots \otimes \sigma_N^z$$

$$c_i \longrightarrow \mathbb{I}_1 \otimes \ldots \otimes \mathbb{I}_{i-1} \otimes \sigma_i^- \otimes \sigma_{i+1}^z \otimes \ldots \otimes \sigma_N^z$$

Trotter-Suzuki Approximation

$$\lim_{N\to\infty}\left(\prod_j e^{-iH_j\frac{t}{N}}\right)^N = e^{-i\sum_j H_j t}$$

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Challenge: finding the best Hamiltonian decomposition

Higher Trotterization Order and Error Bound

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Starting from the Zassenhaus formula

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$$\hat{O}_2(t) = e^{-iH_1\frac{t}{2}} \dots e^{-iH_n\frac{t}{2}} e^{-iH_n\frac{t}{2}} \dots e^{-iH_1\frac{t}{2}}$$

$$\hat{O}_{2k}(t) = \hat{O}^2_{2k-2}\left(\left(4 - 4^{\frac{1}{2k-1}}\right)^{-1} t\right) \hat{O}_{2k-2}\left(\left(1 - 4\left(4 - 4^{\frac{1}{2k-1}}\right)^{-1}\right) t\right) \hat{O}^2_{2k-2}\left(\left(4 - 4^{\frac{1}{2k-1}}\right)^{-1} t\right)$$

Higher Trotterization Order and Error Bound

Starting from the Zassenhaus formula

$$\hat{O}_2(t) = e^{-iH_1\frac{t}{2}} \dots e^{-iH_n\frac{t}{2}} e^{-iH_n\frac{t}{2}} \dots e^{-iH_1\frac{t}{2}}$$

$$\hat{O}_{2k}(t) = \hat{O}^2_{2k-2}\left(\left(4 - 4^{\frac{1}{2k-1}}\right)^{-1} t\right) \hat{O}_{2k-2}\left(\left(1 - 4\left(4 - 4^{\frac{1}{2k-1}}\right)^{-1}\right) t\right) \hat{O}^2_{2k-2}\left(\left(4 - 4^{\frac{1}{2k-1}}\right)^{-1} t\right)$$

$$||\hat{O}_p(t) - e^{-iHt}|| \sim \sum_{\gamma_1,\dots,\gamma_{p+1}} ||[H_{\gamma_{p+1}}, [\dots[H_{\gamma_2}, H_{\gamma_1}]]..]||$$

Clique Partitioning

$$H = h_1 + h_2 + h_3 + \ \dots \ + h_N$$

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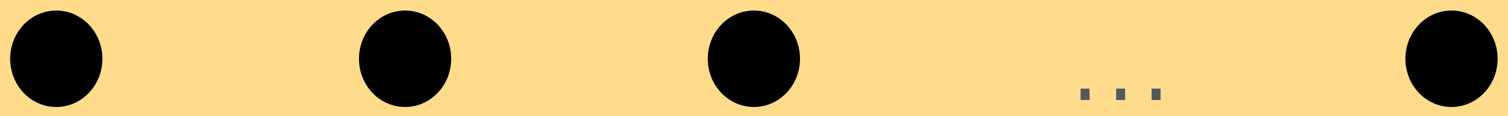

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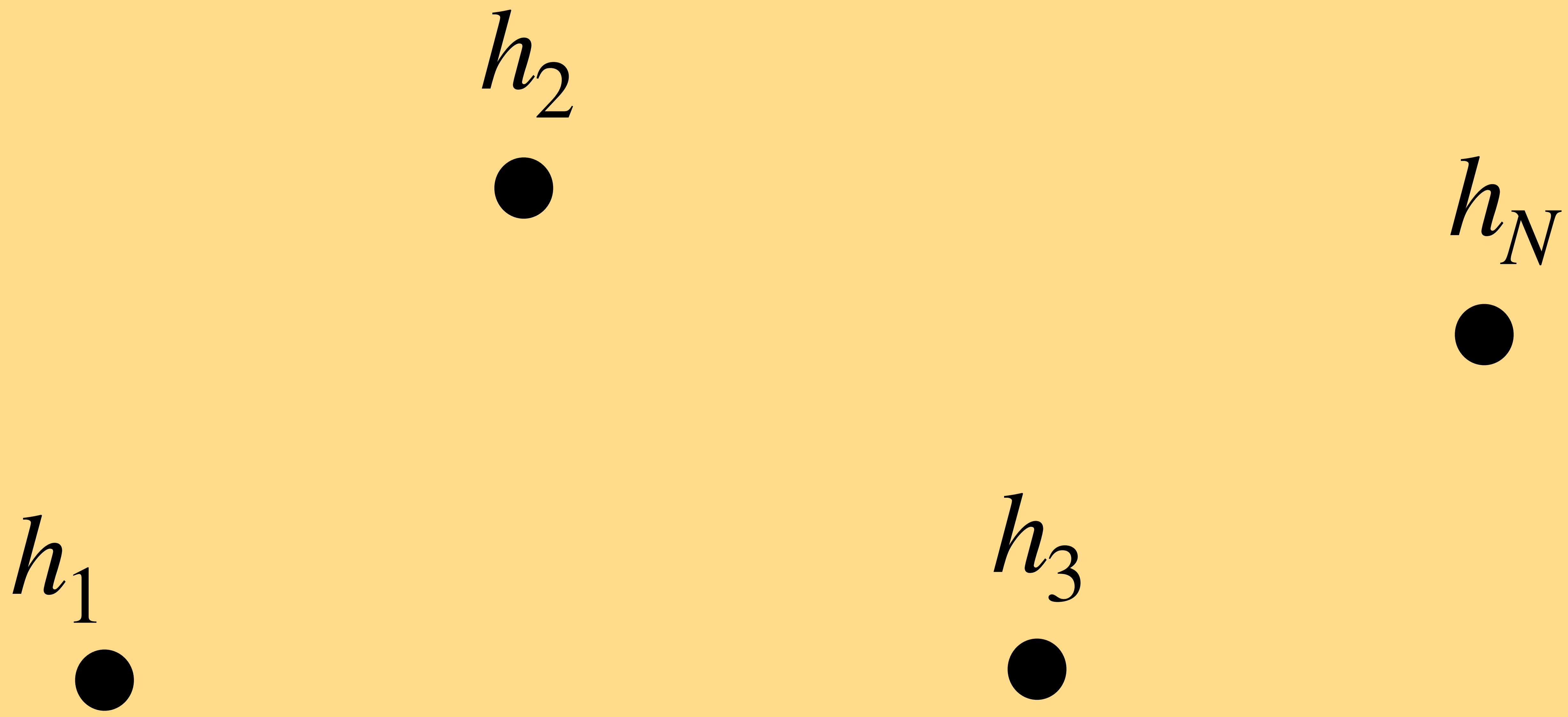

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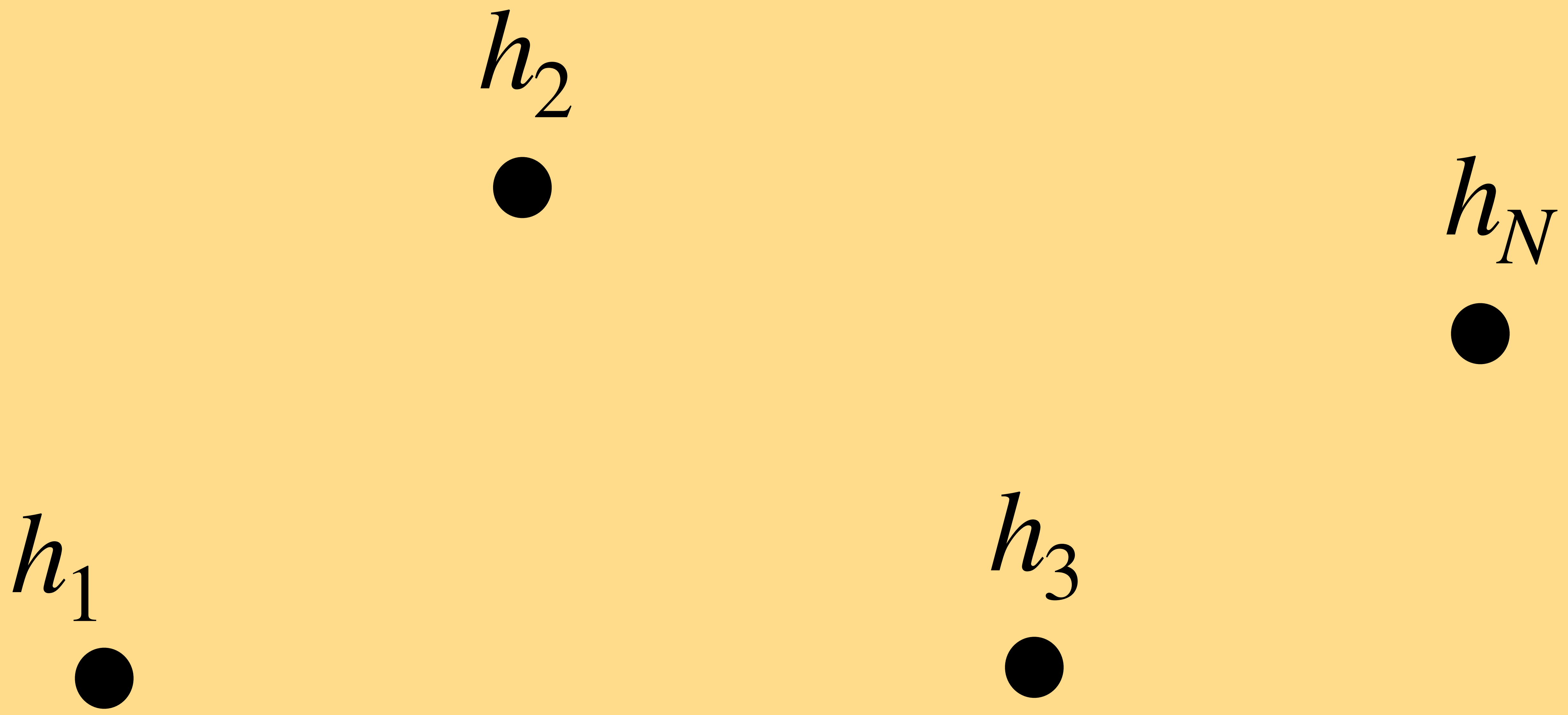

$$[h_1, h_2] = 0$$

$$[h_2, h_3] = 0$$

$$[h_2, h_N] = 0$$

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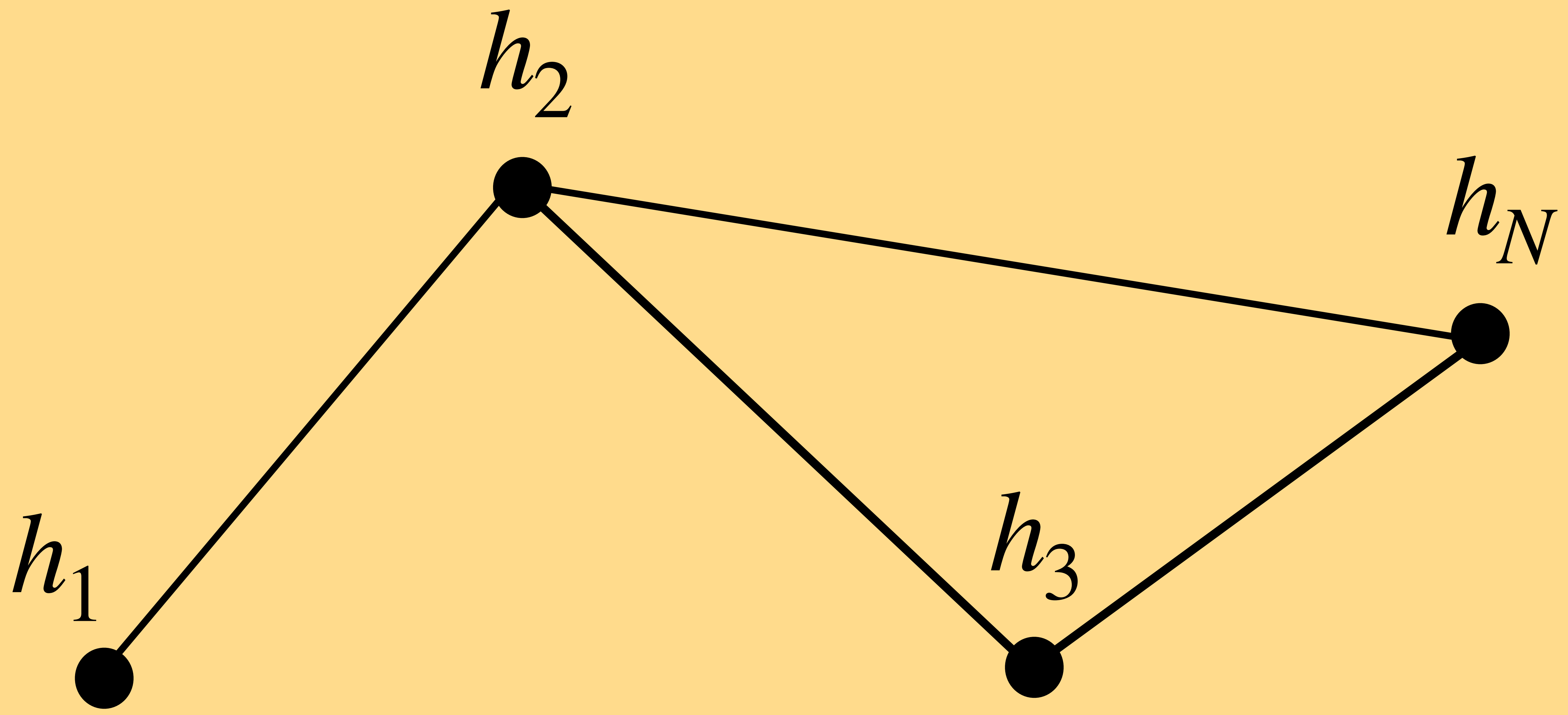

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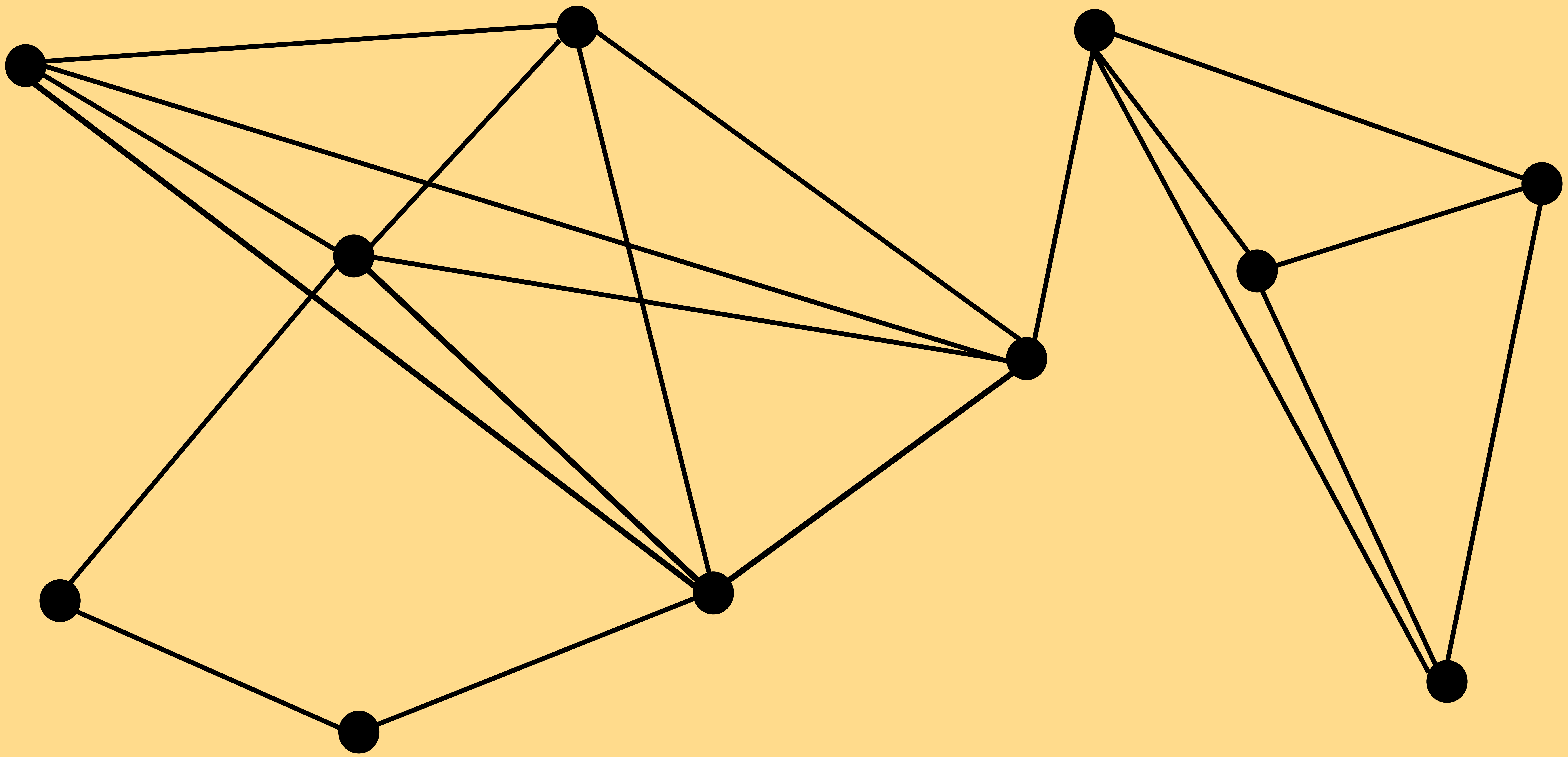

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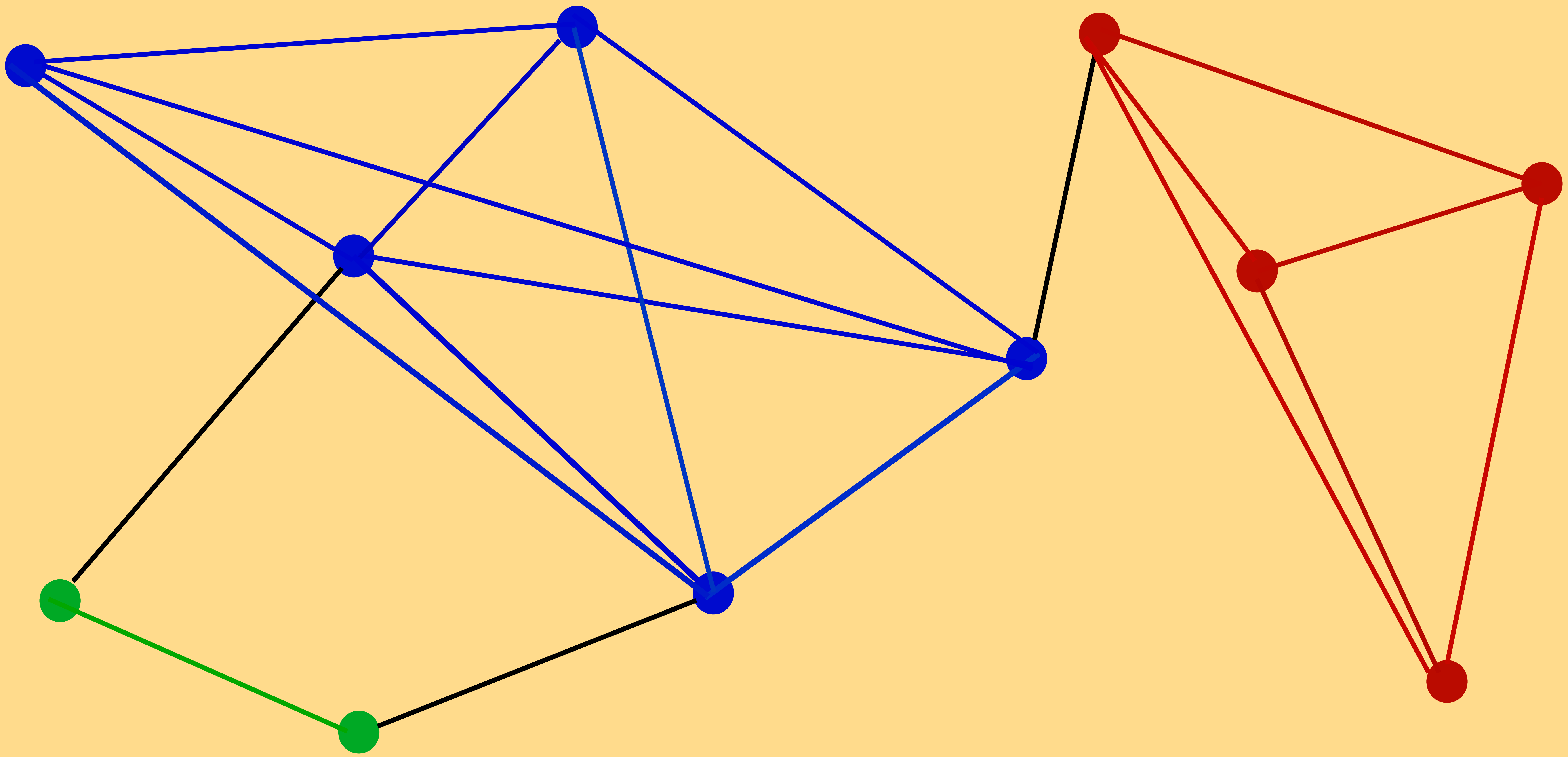

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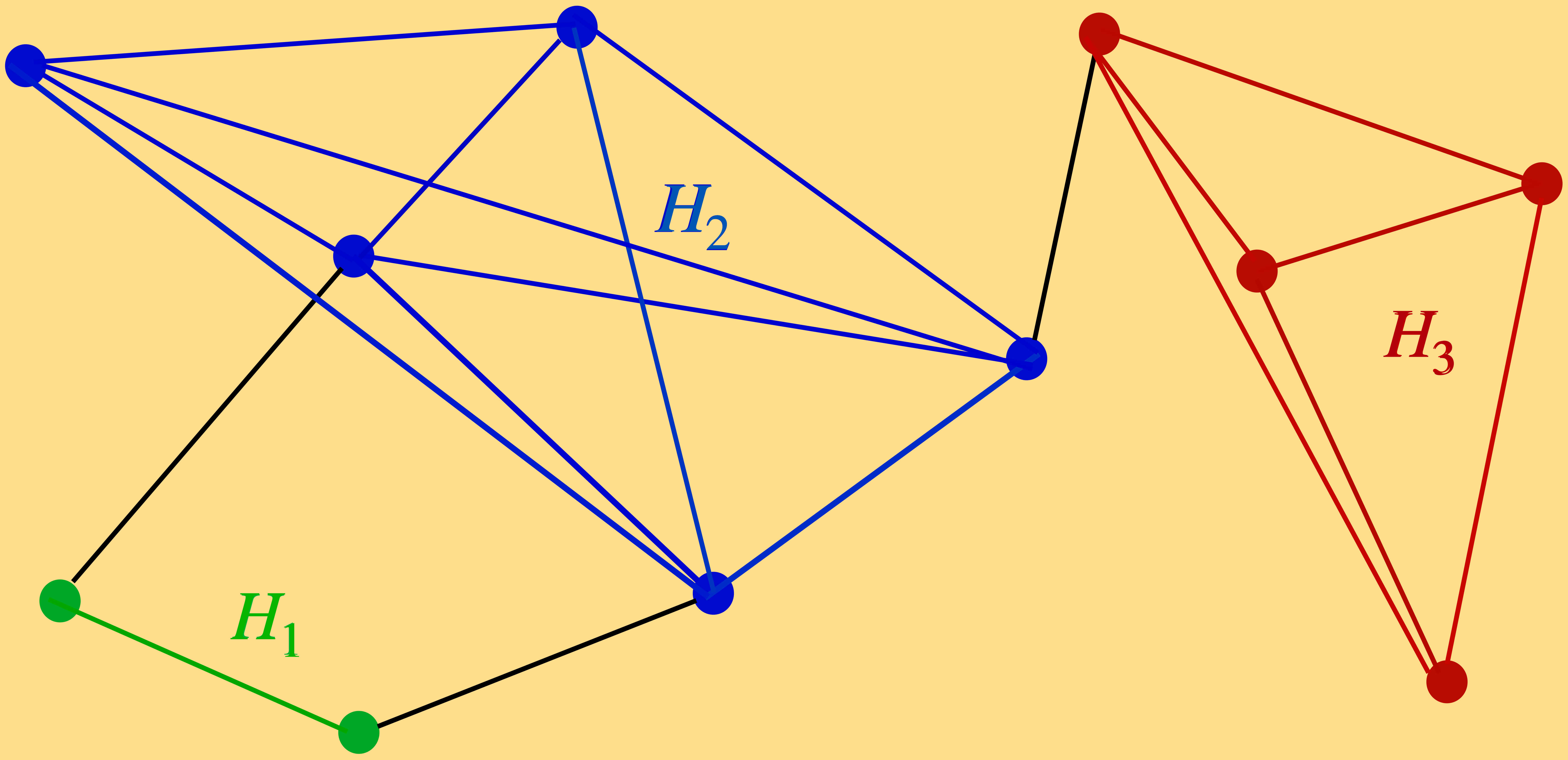

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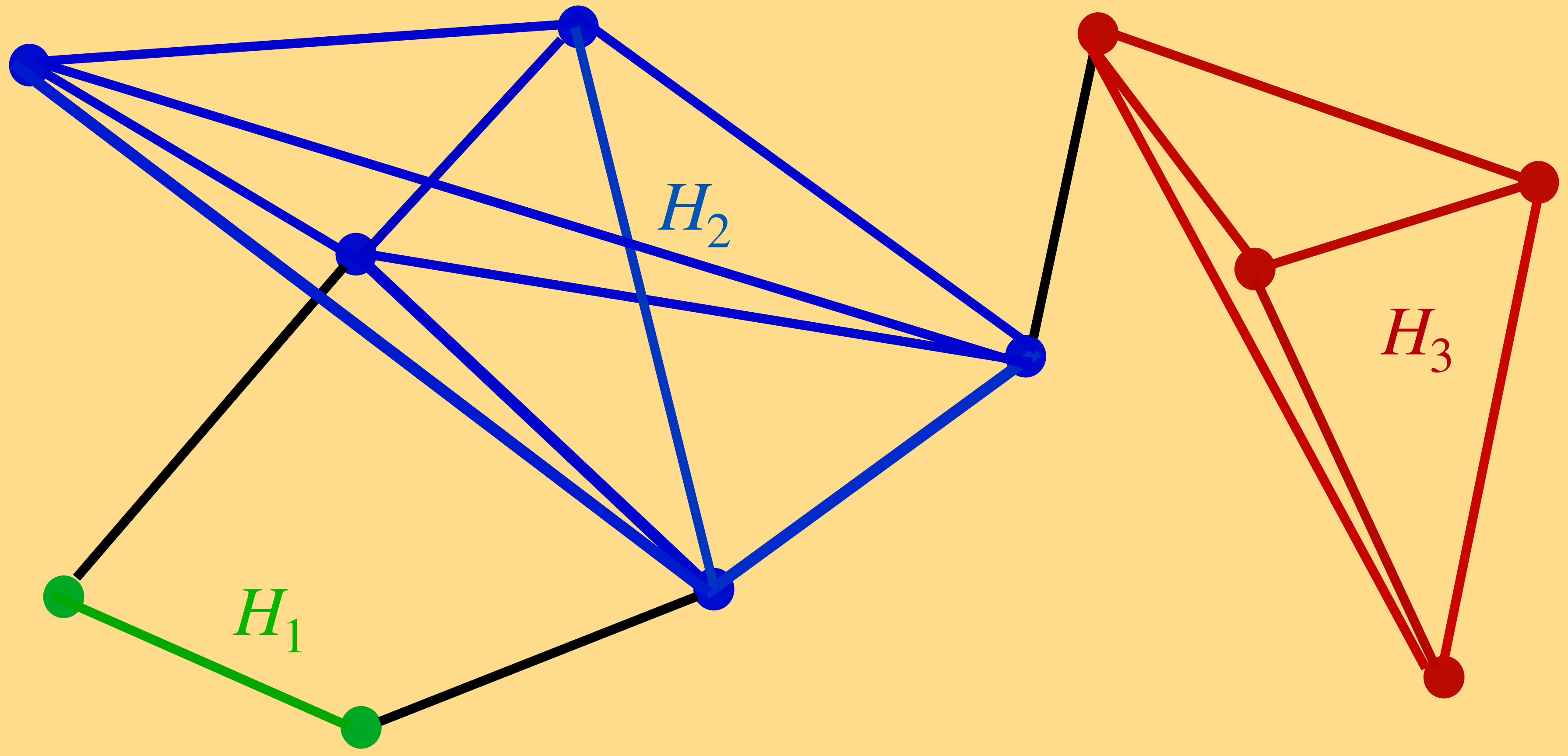

$$\Longrightarrow \qquad e^{-iHt} \approx \left(e^{-iH_1\frac{t}{n}} e^{-iH_2\frac{t}{n}} e^{-iH_3\frac{t}{n}}\right)^n$$

Phase Transitions

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Ground State

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Correlations

Phase Transitions

Ground State

Correlations

$$\sigma_z(i,j) = \langle \sigma_z^i \sigma_z^j \rangle - \langle \sigma_z^i \rangle \langle \sigma_z^j \rangle$$

Correlations

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Max Correlation

Correlations

Max Correlation

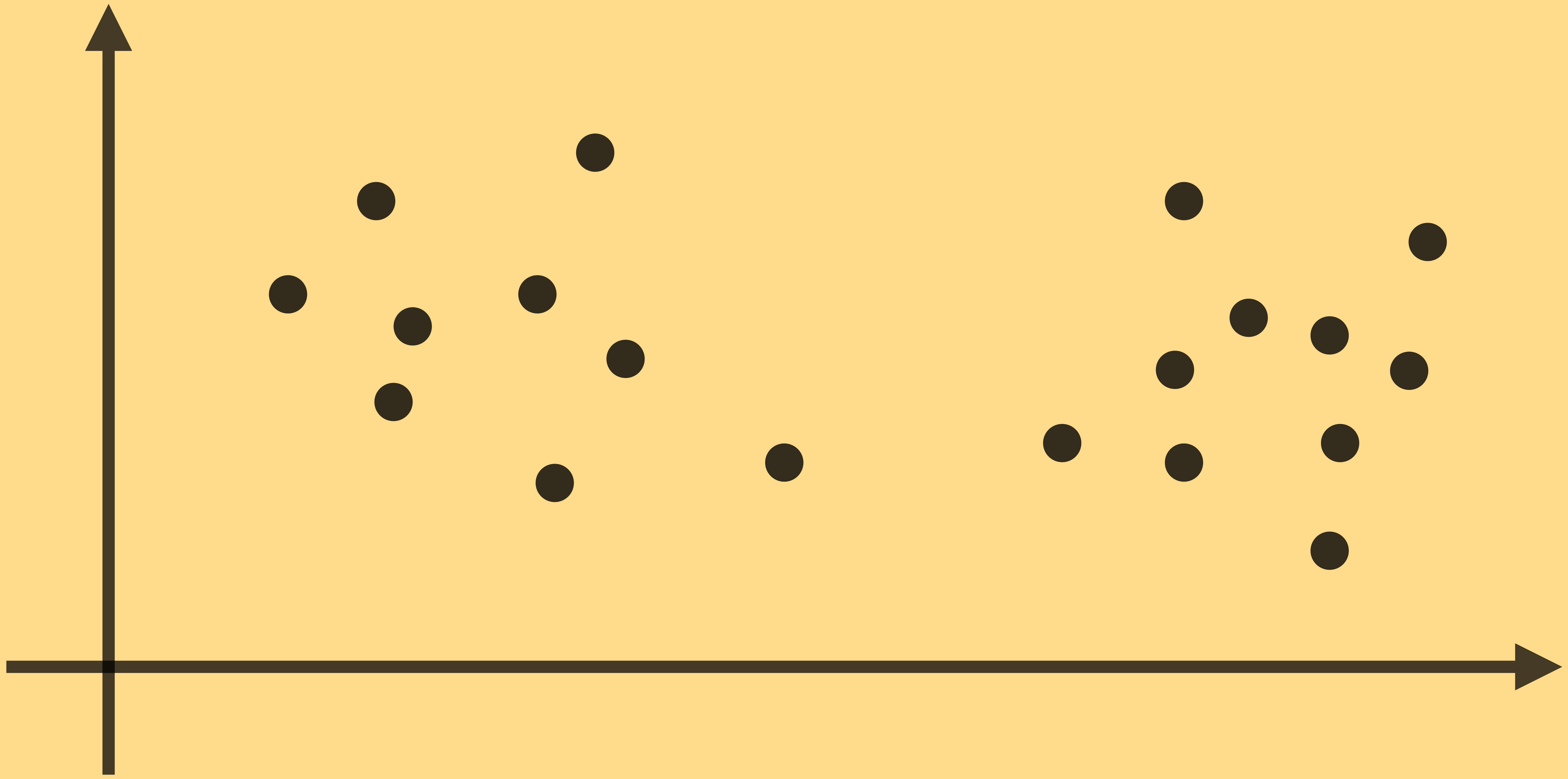
Supervised Learning

Correlations

Max Correlation

Unsupervised Learning

Clustering



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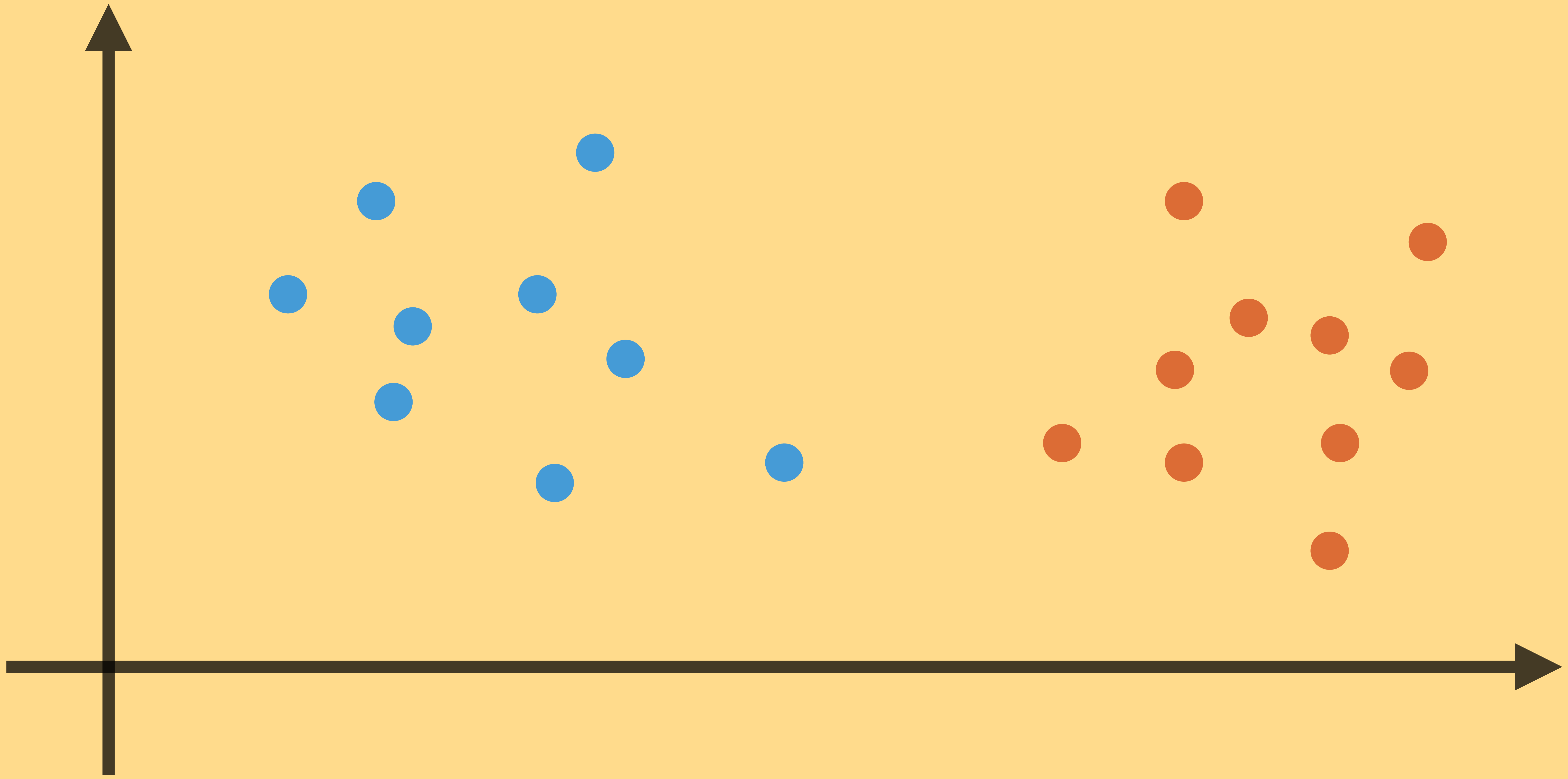


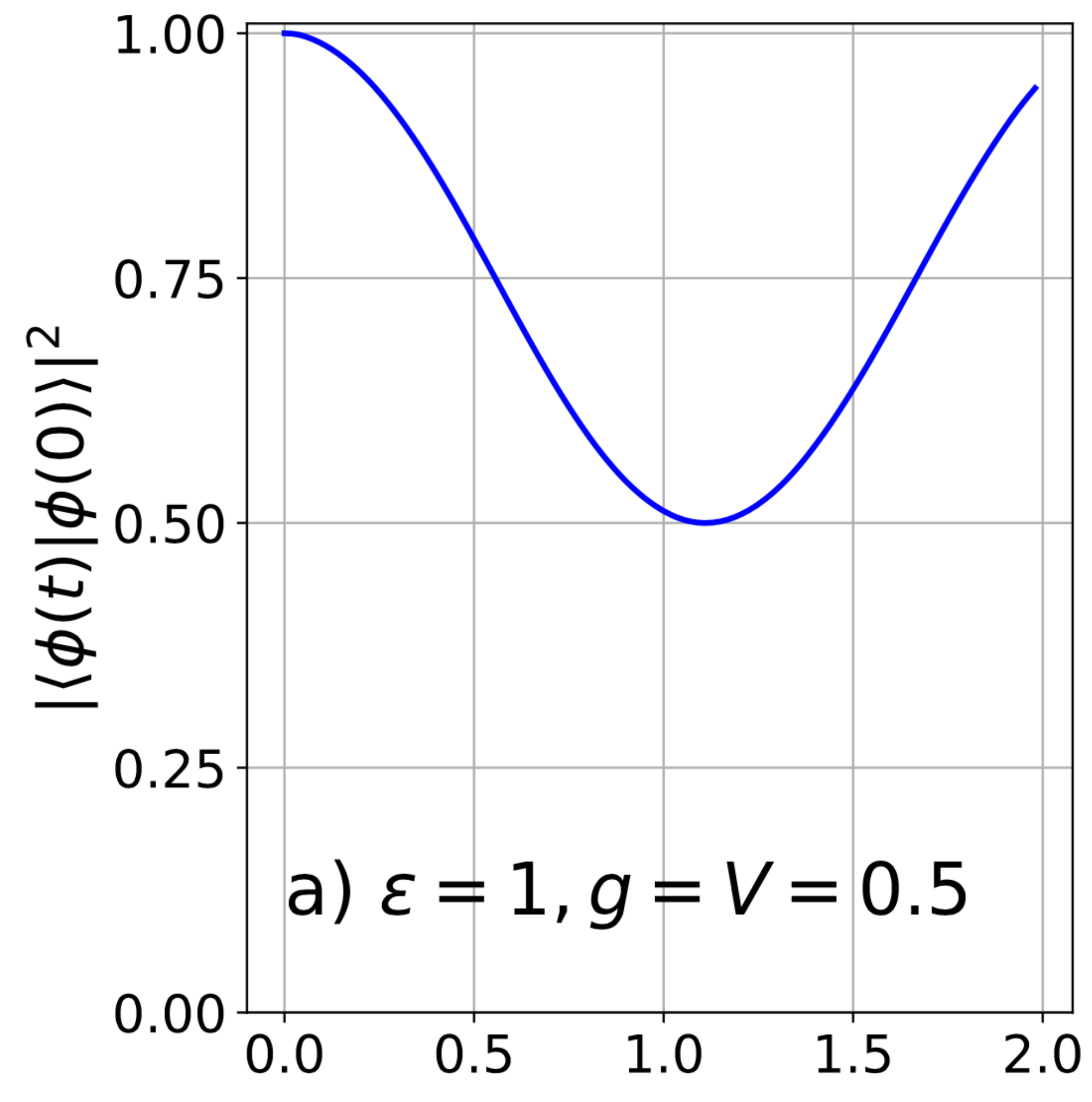
Simulation for j=1

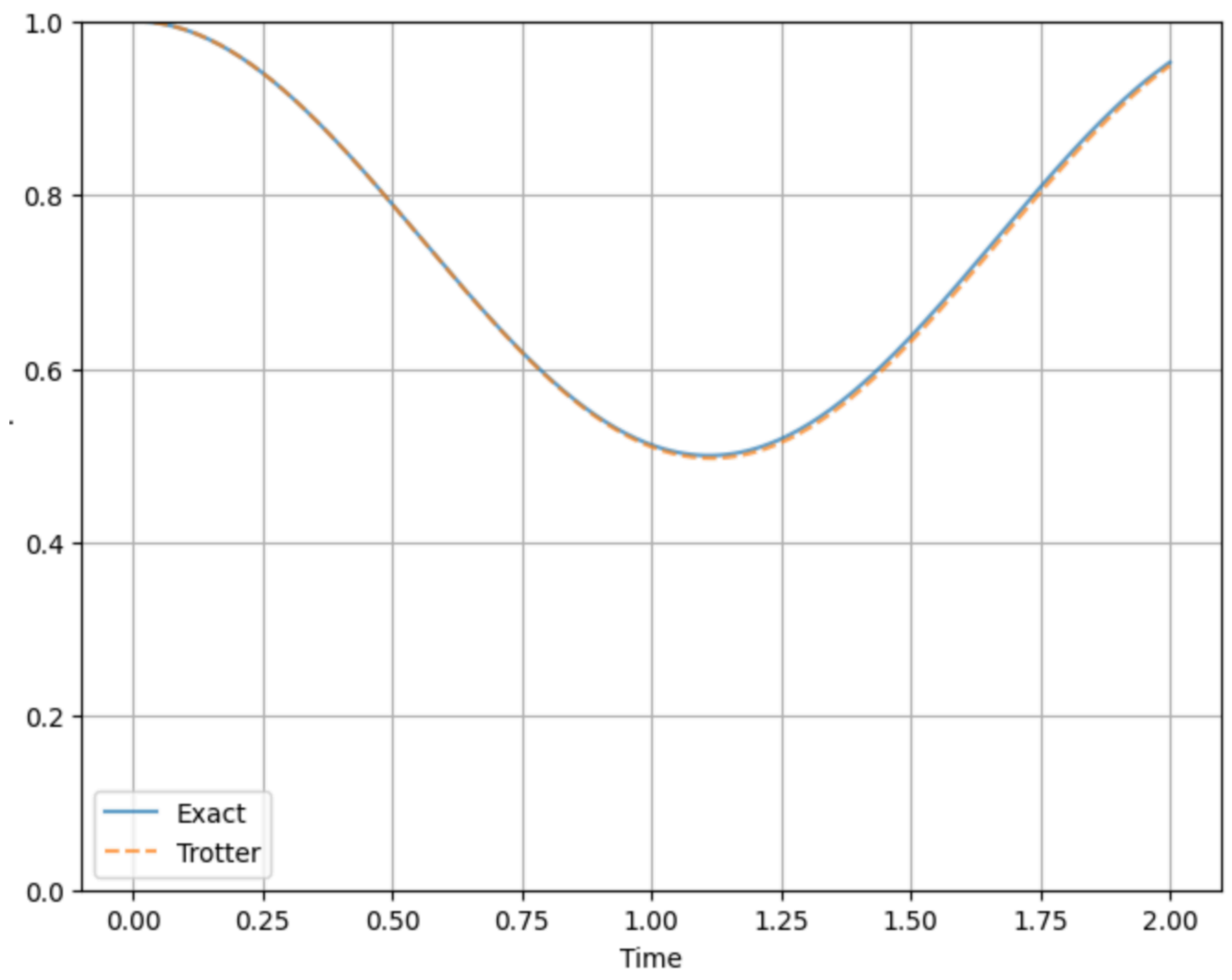
A digital quantum simulation of the Agassi model

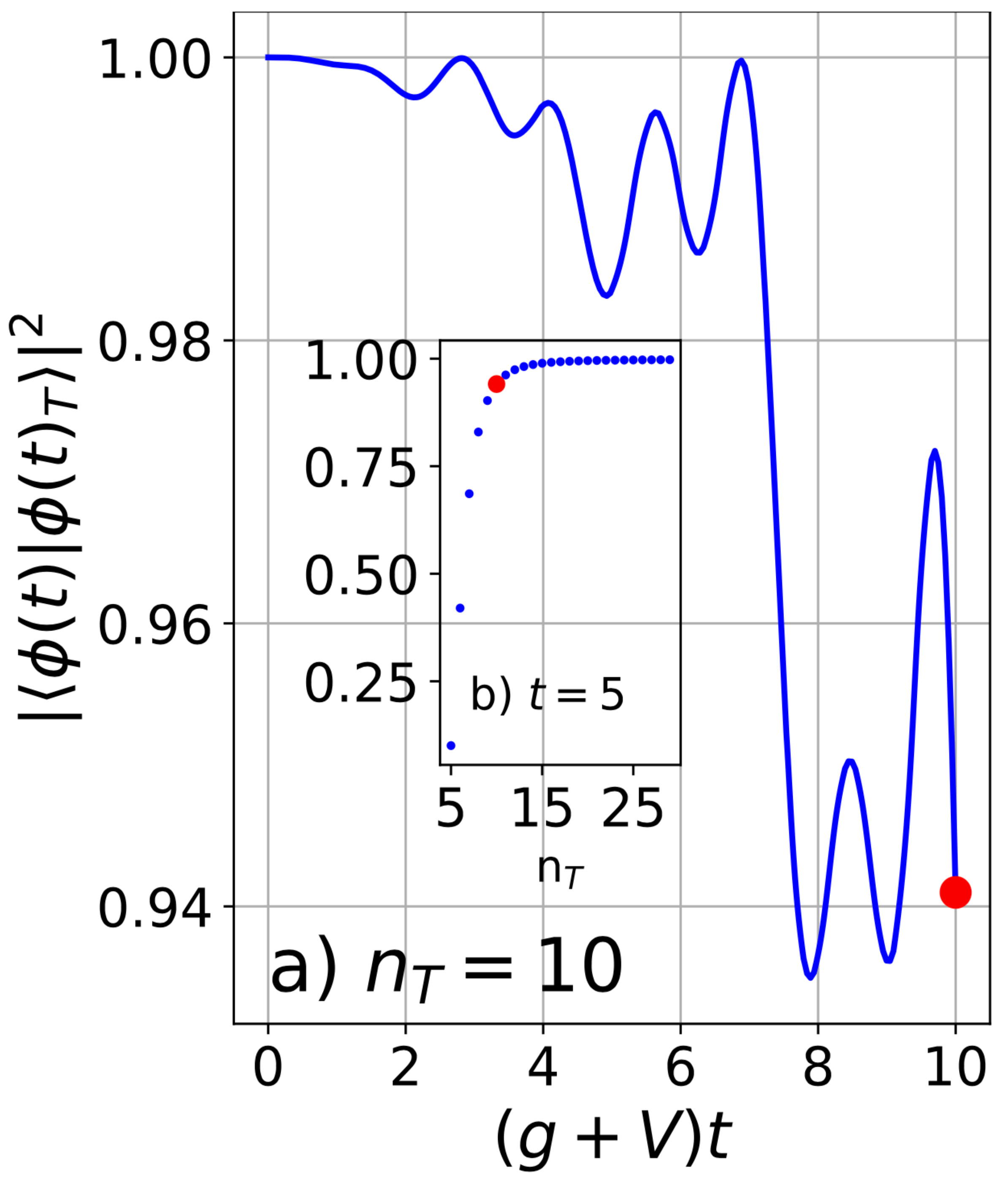
Pedro Pérez-Fernández, José Miguel Arias, José Enrique García-Ramos, Lucas Lamata

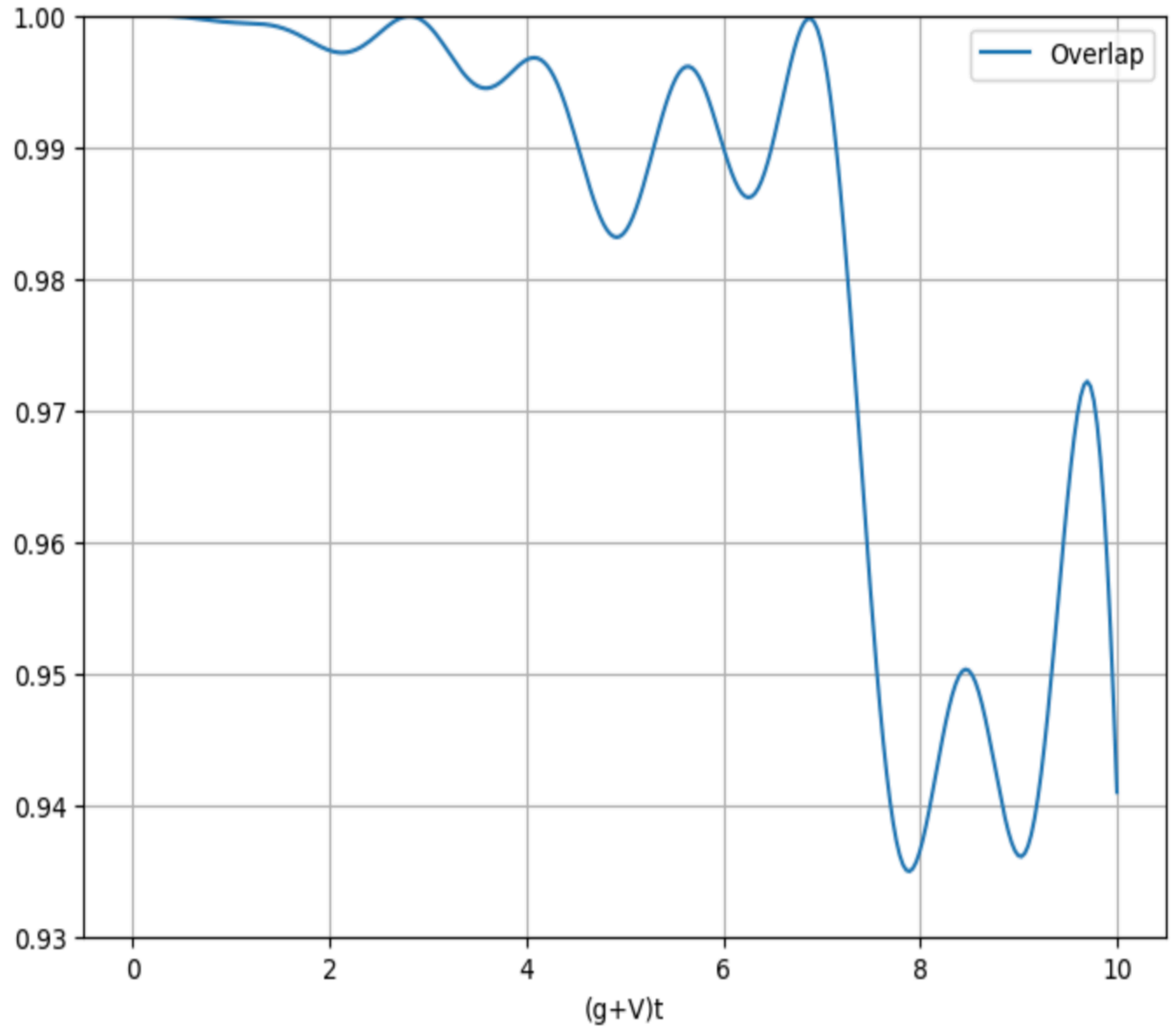
Digital quantum simulation of an extended Agassi model: Using machine learning to disentangle its phase-diagram

Álvaro Sáiz, José-Enrique García-Ramos, José Miguel Arias, Lucas Lamata, and Pedro Pérez-Fernández









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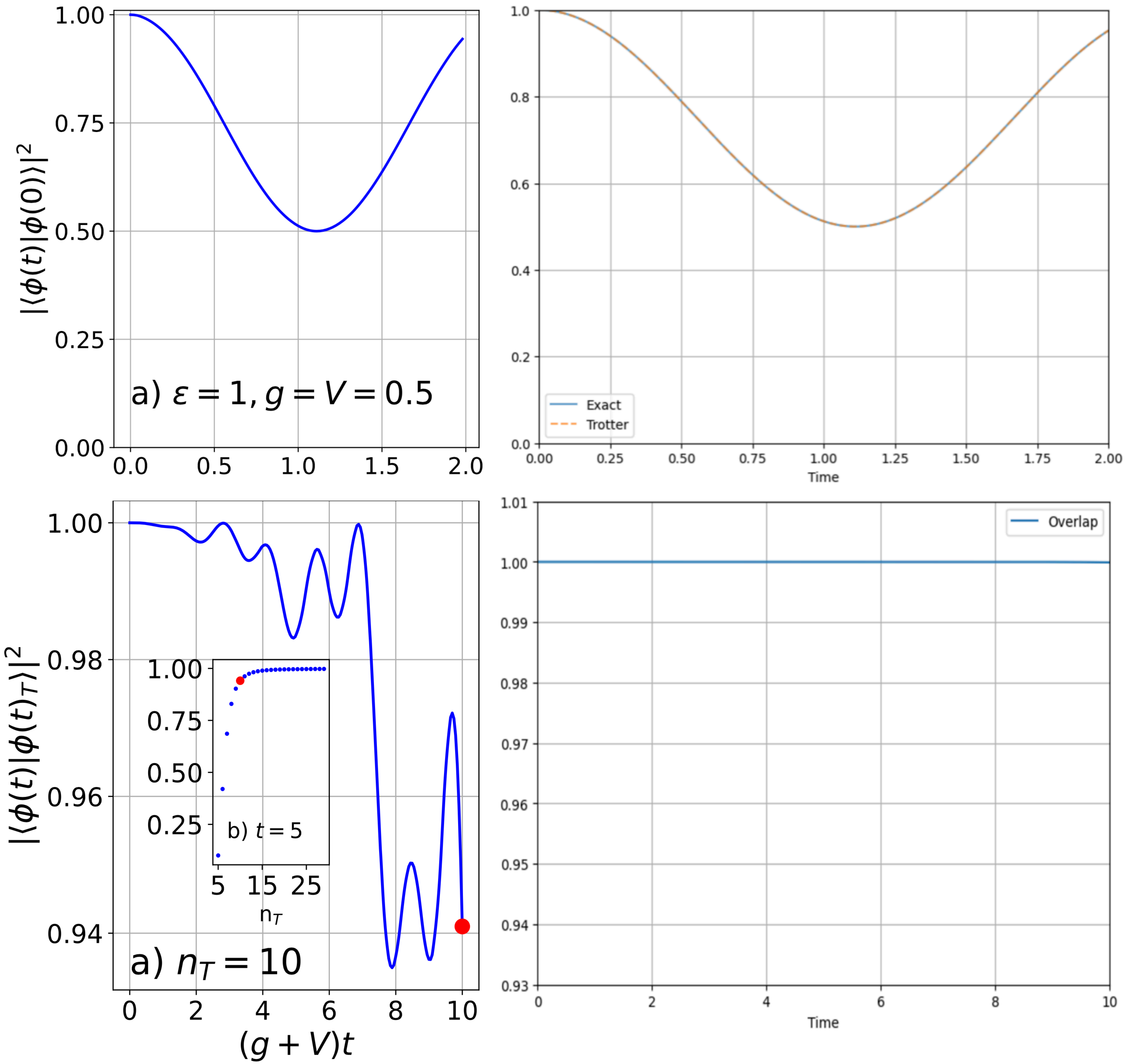
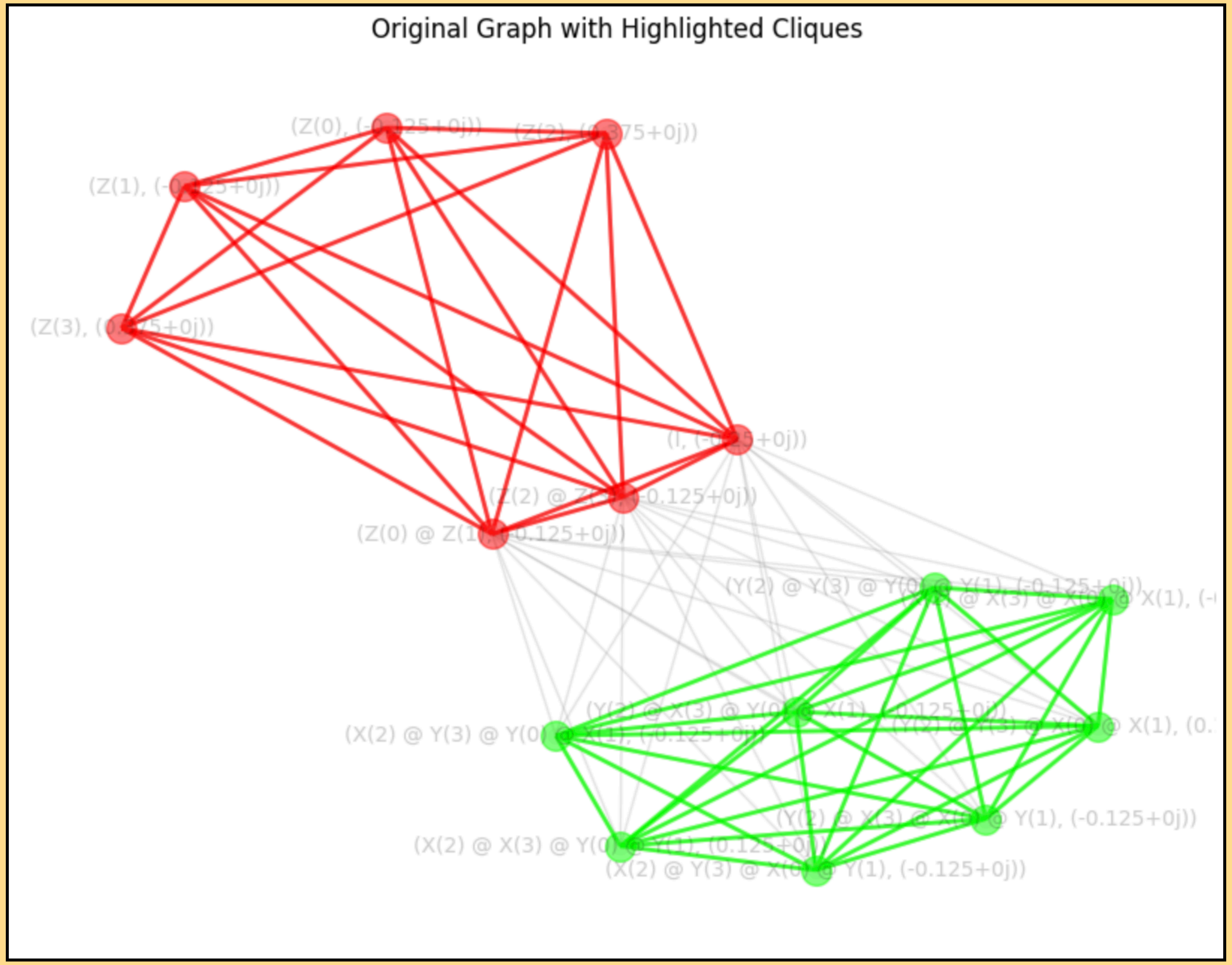


Best Cliques partitioning



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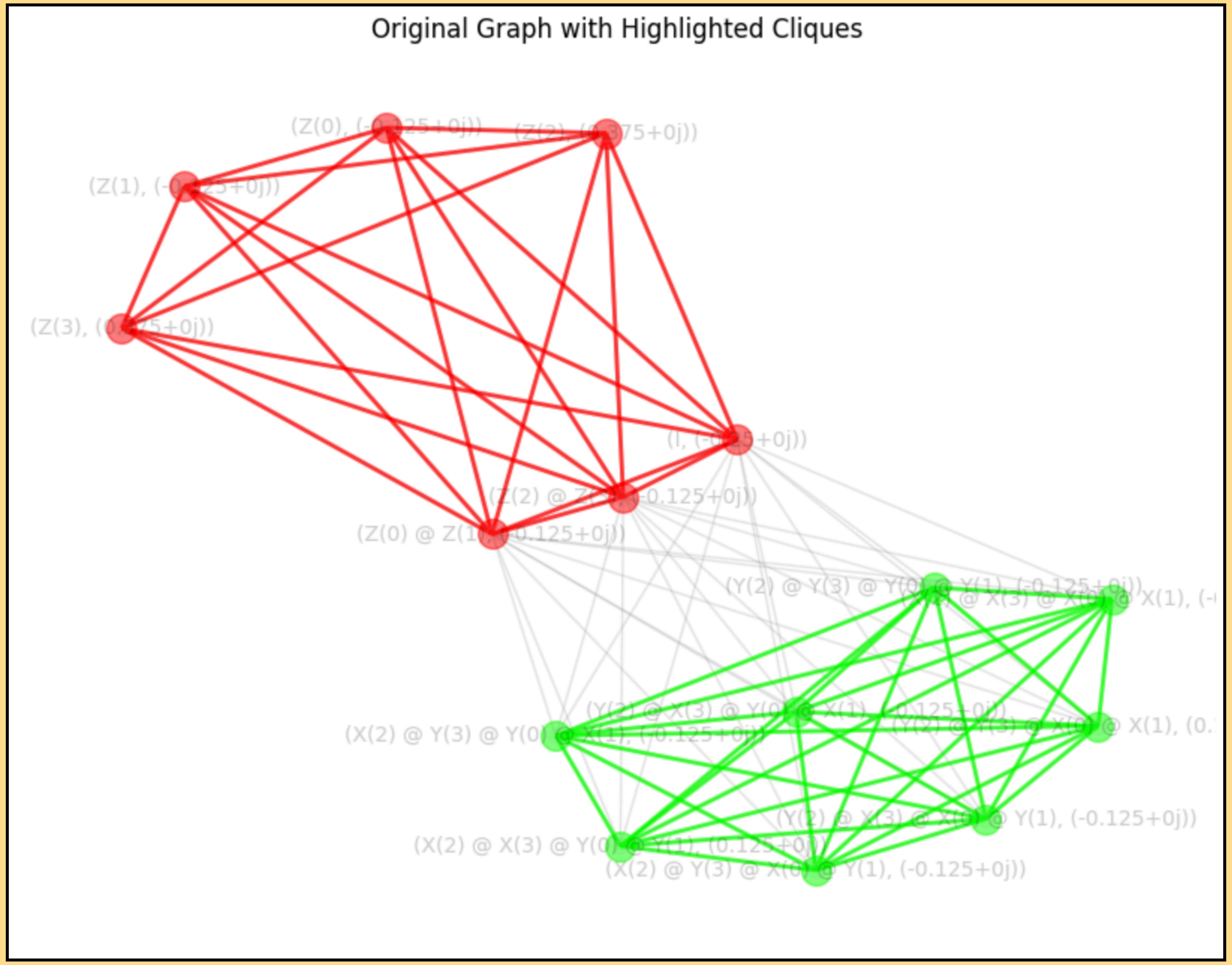
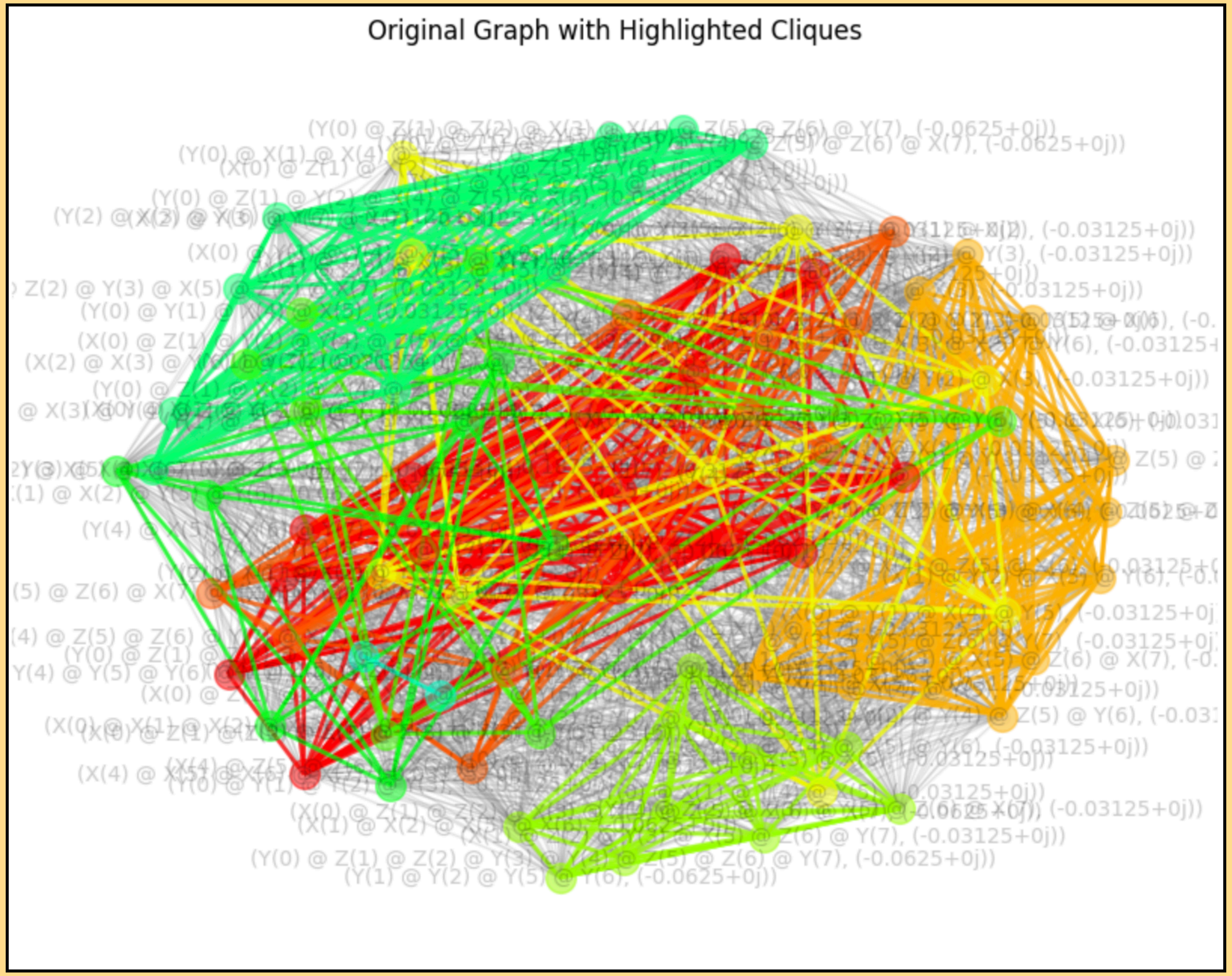
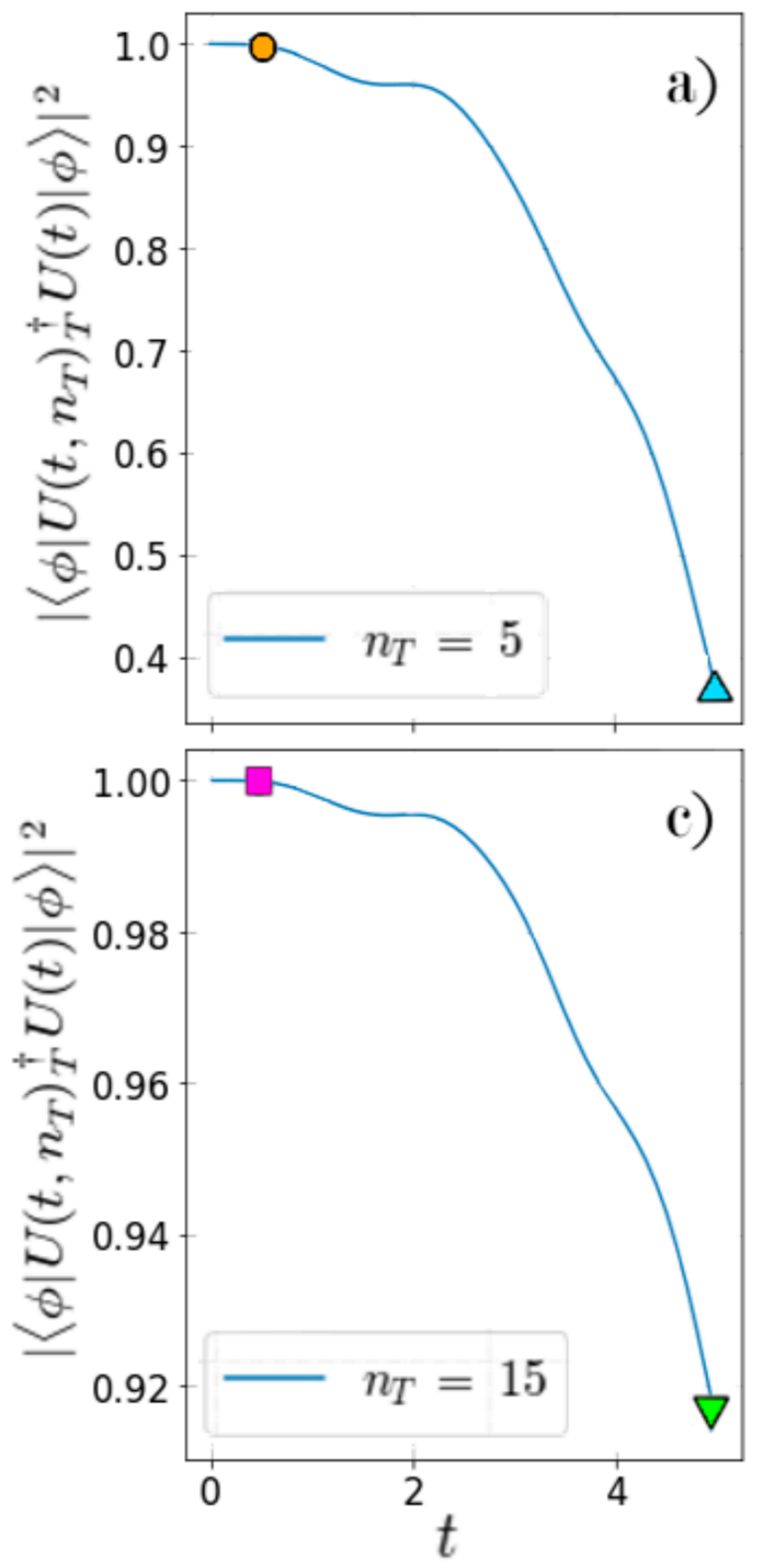


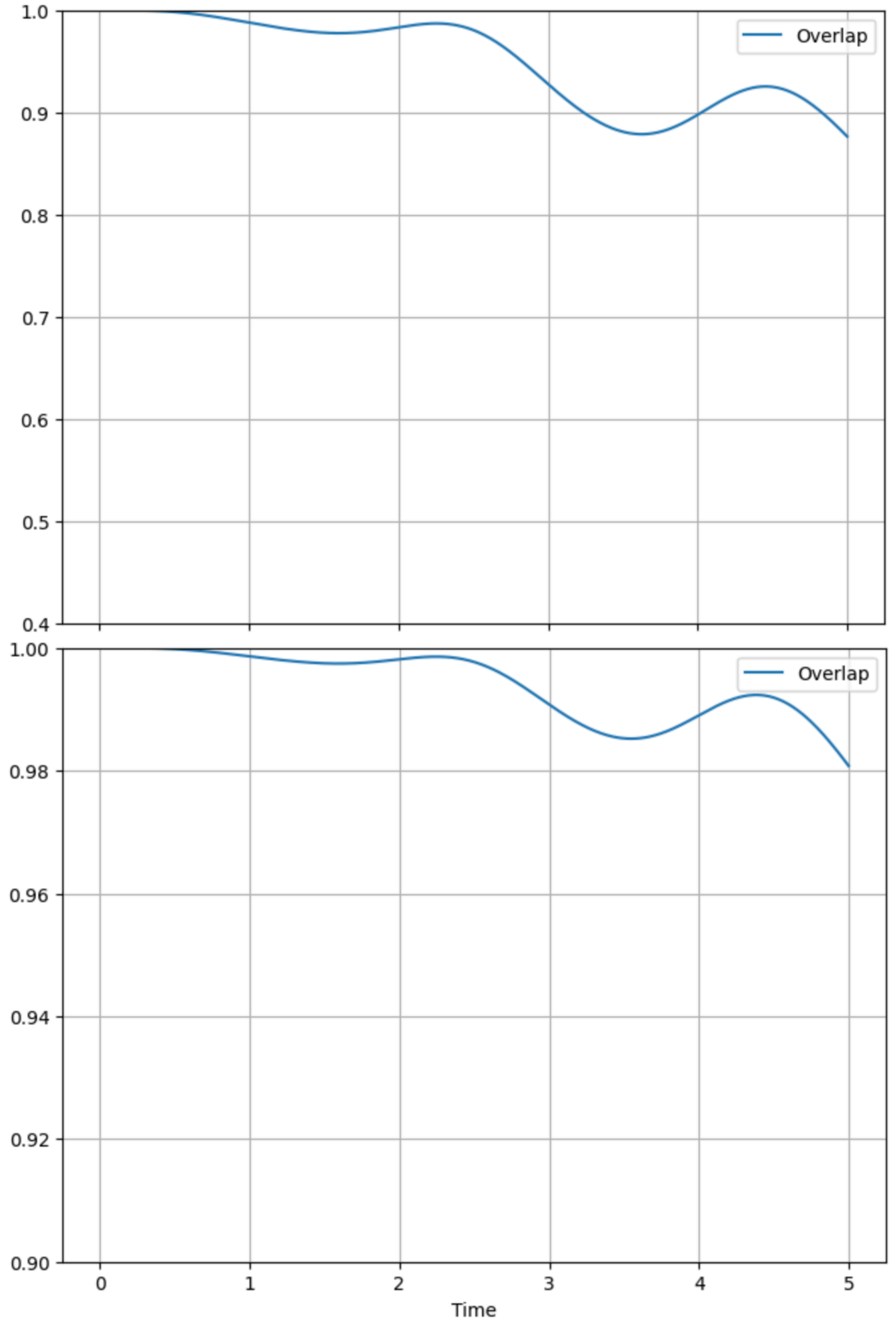
Clique partitioning for j=1



Clique partitioning for j=2

Fidelity Comparison j=2





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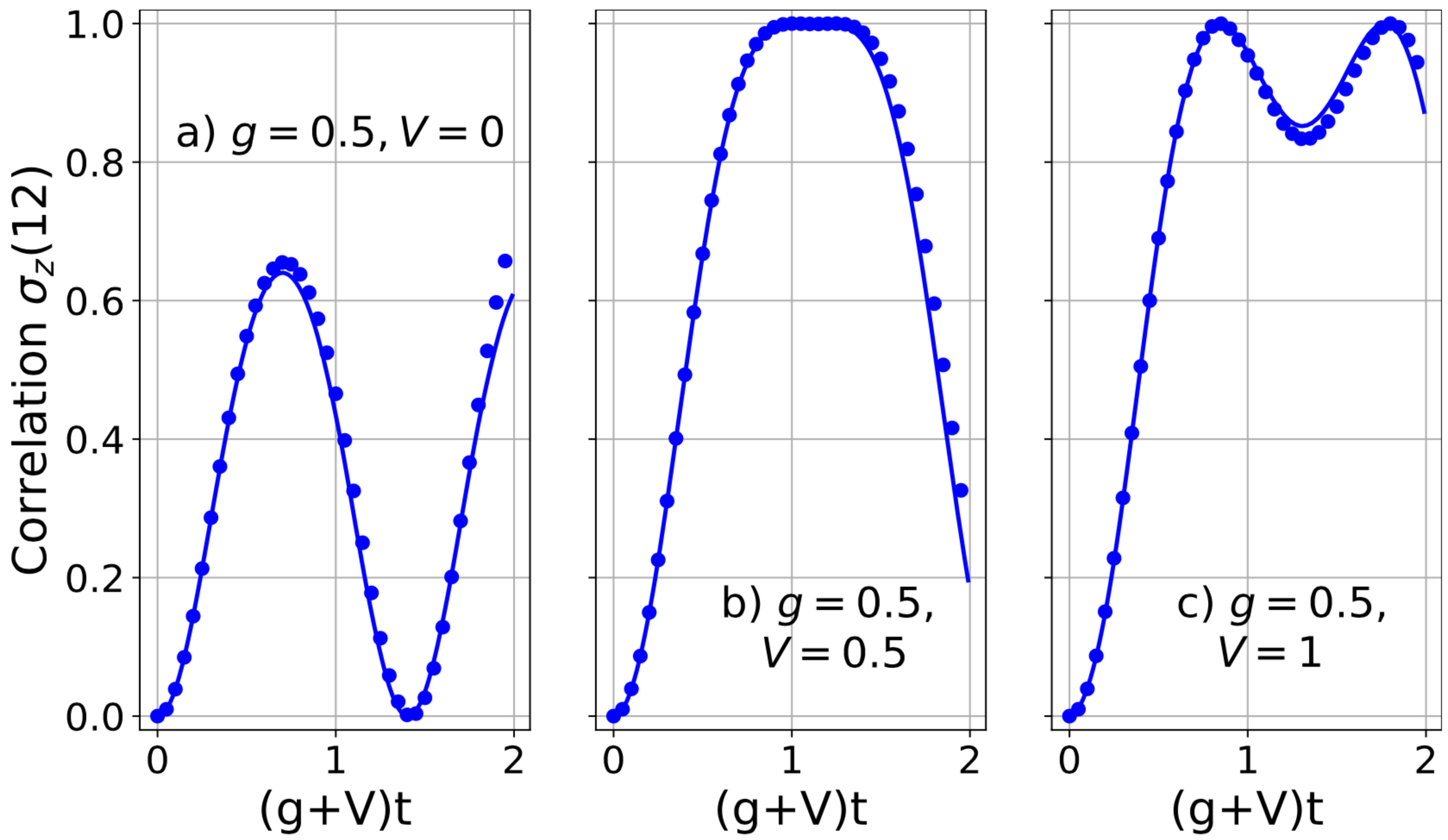
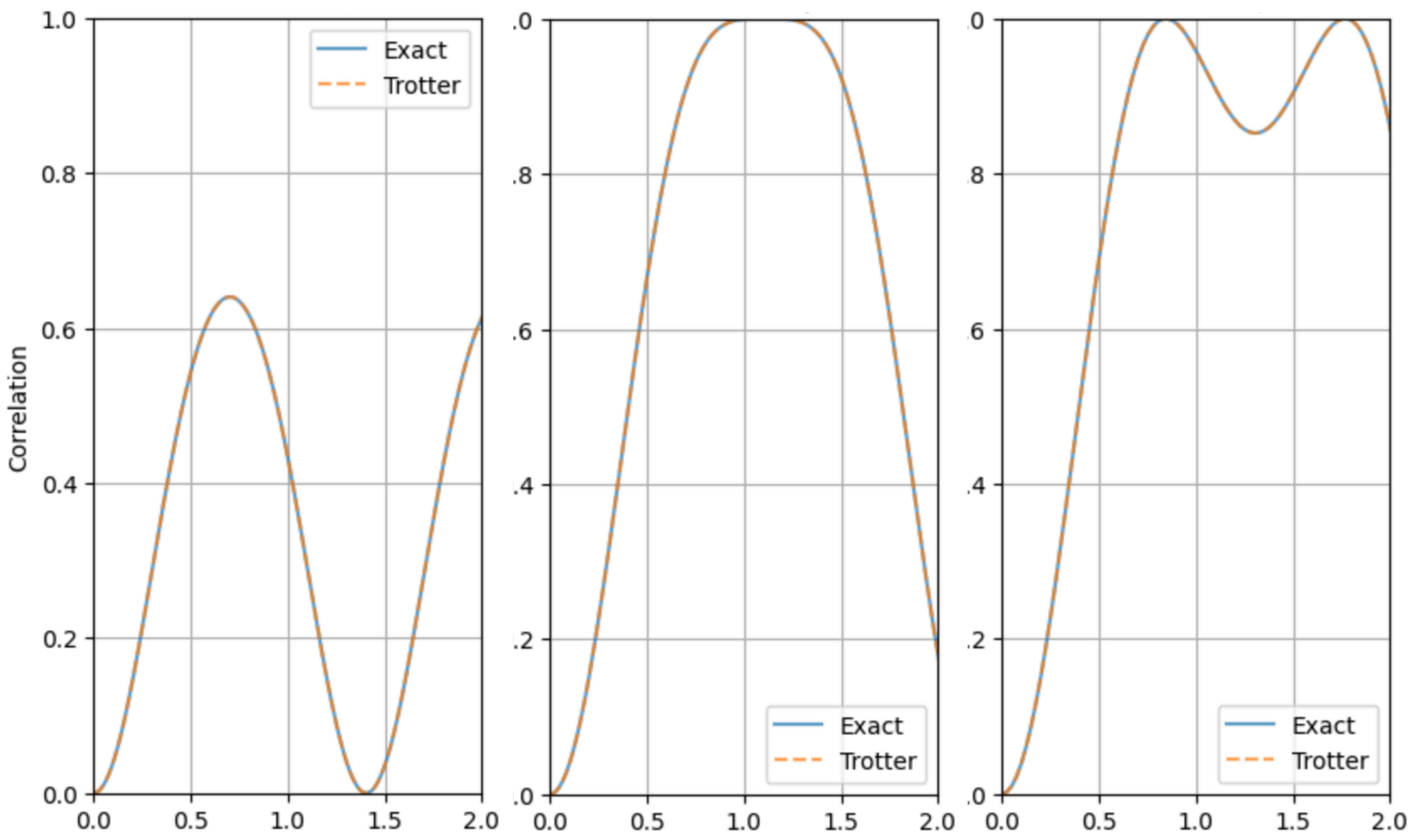
Correlation Comparison





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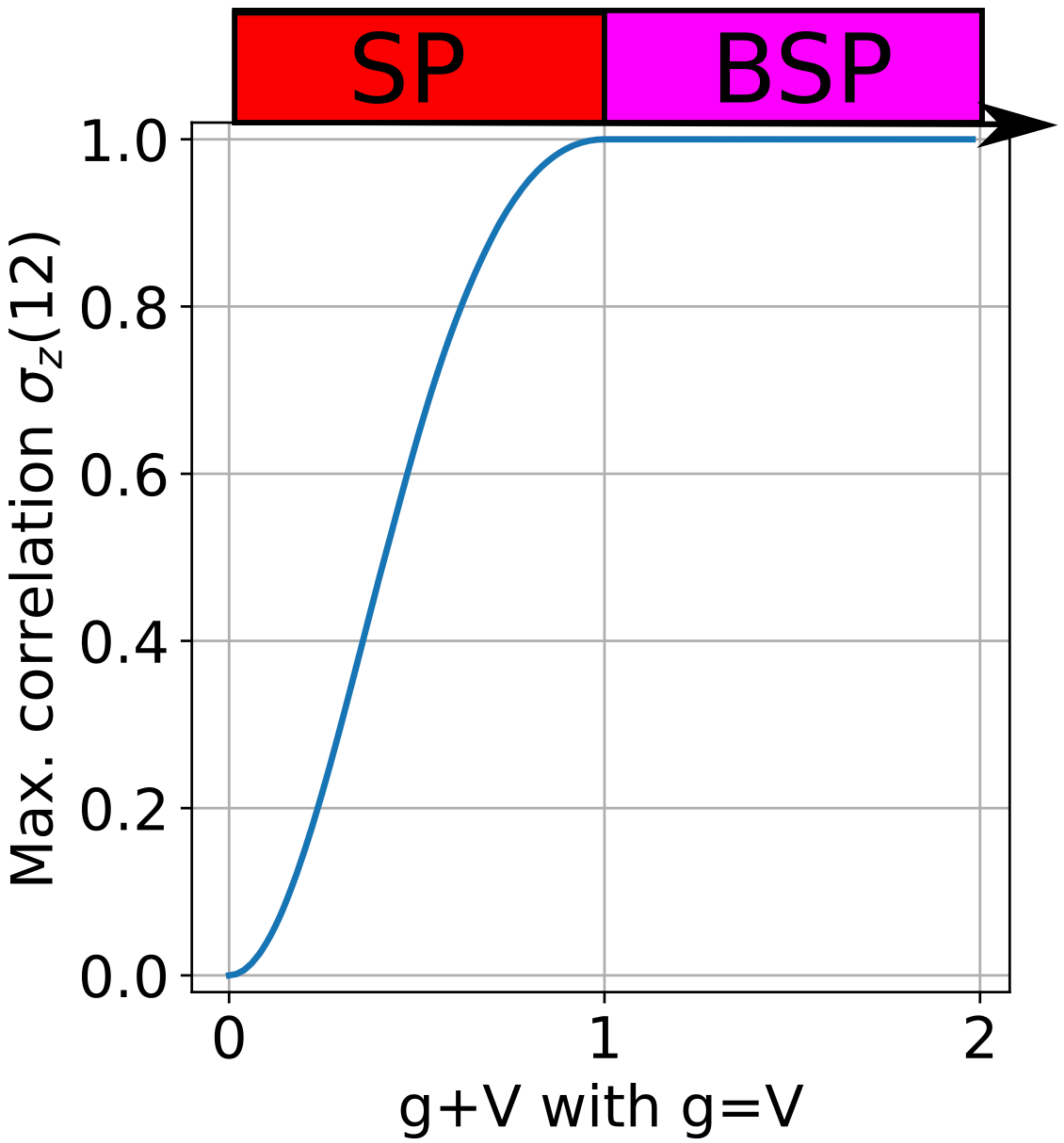

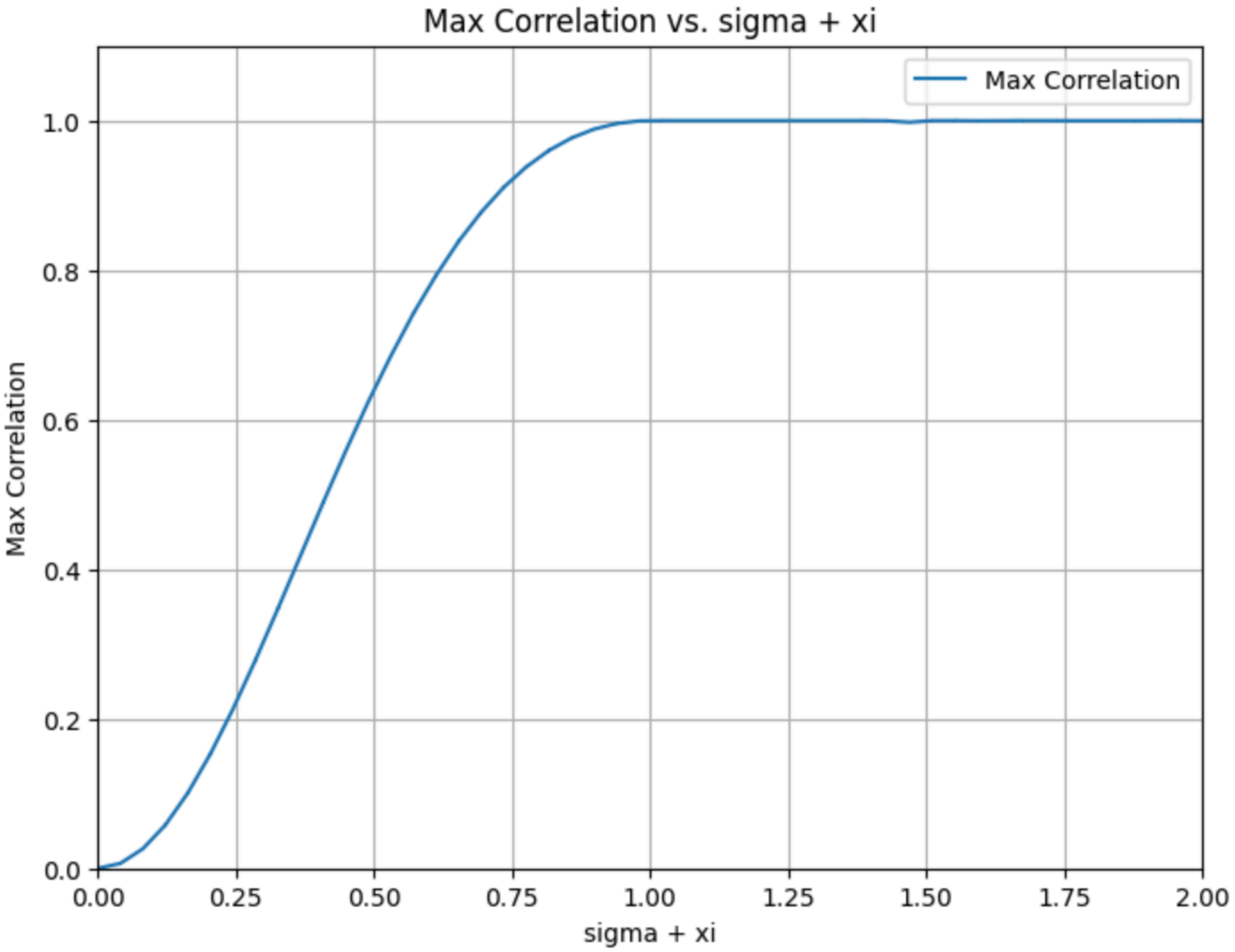

Clustering for $j=2$ and different initial states

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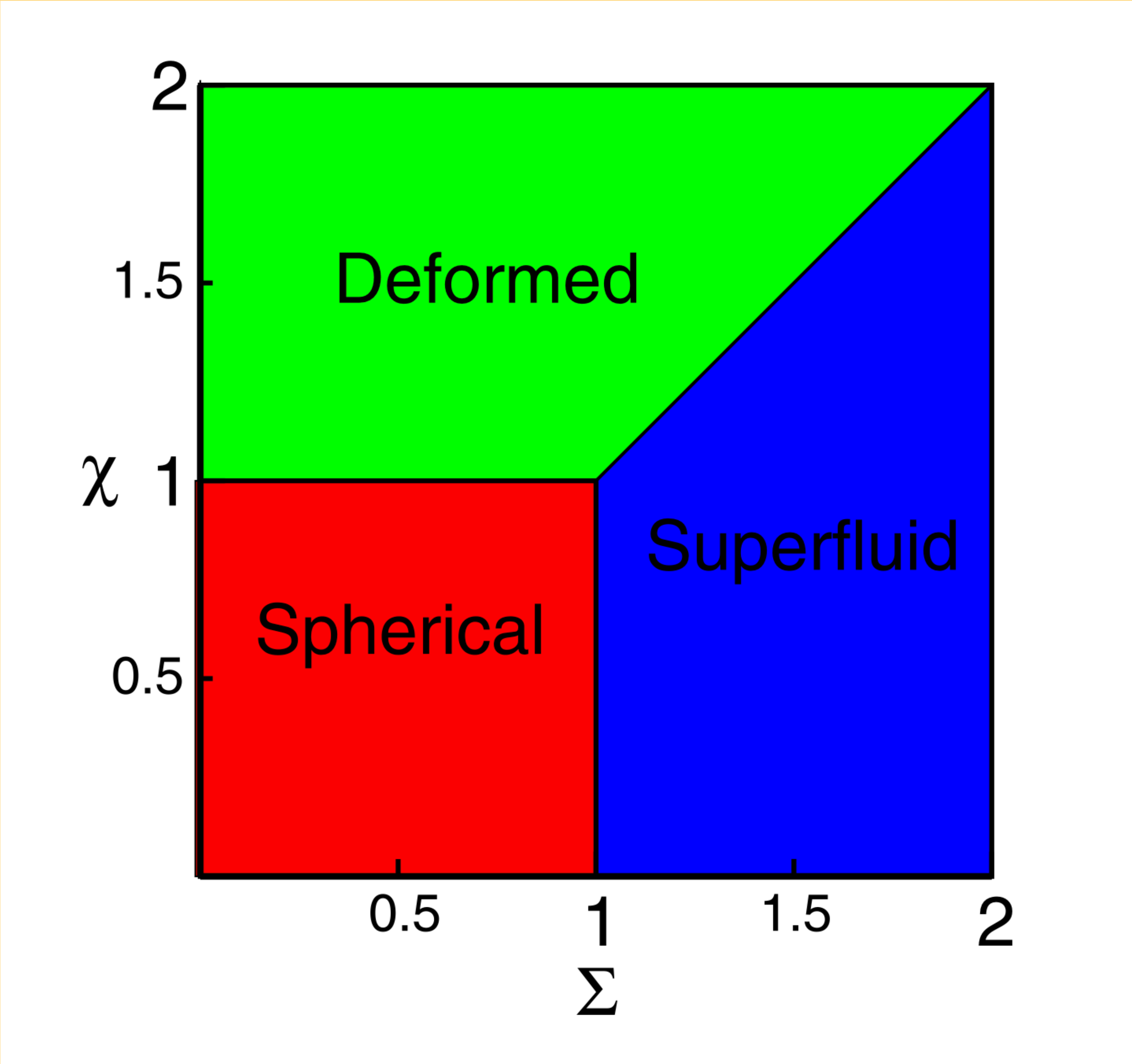

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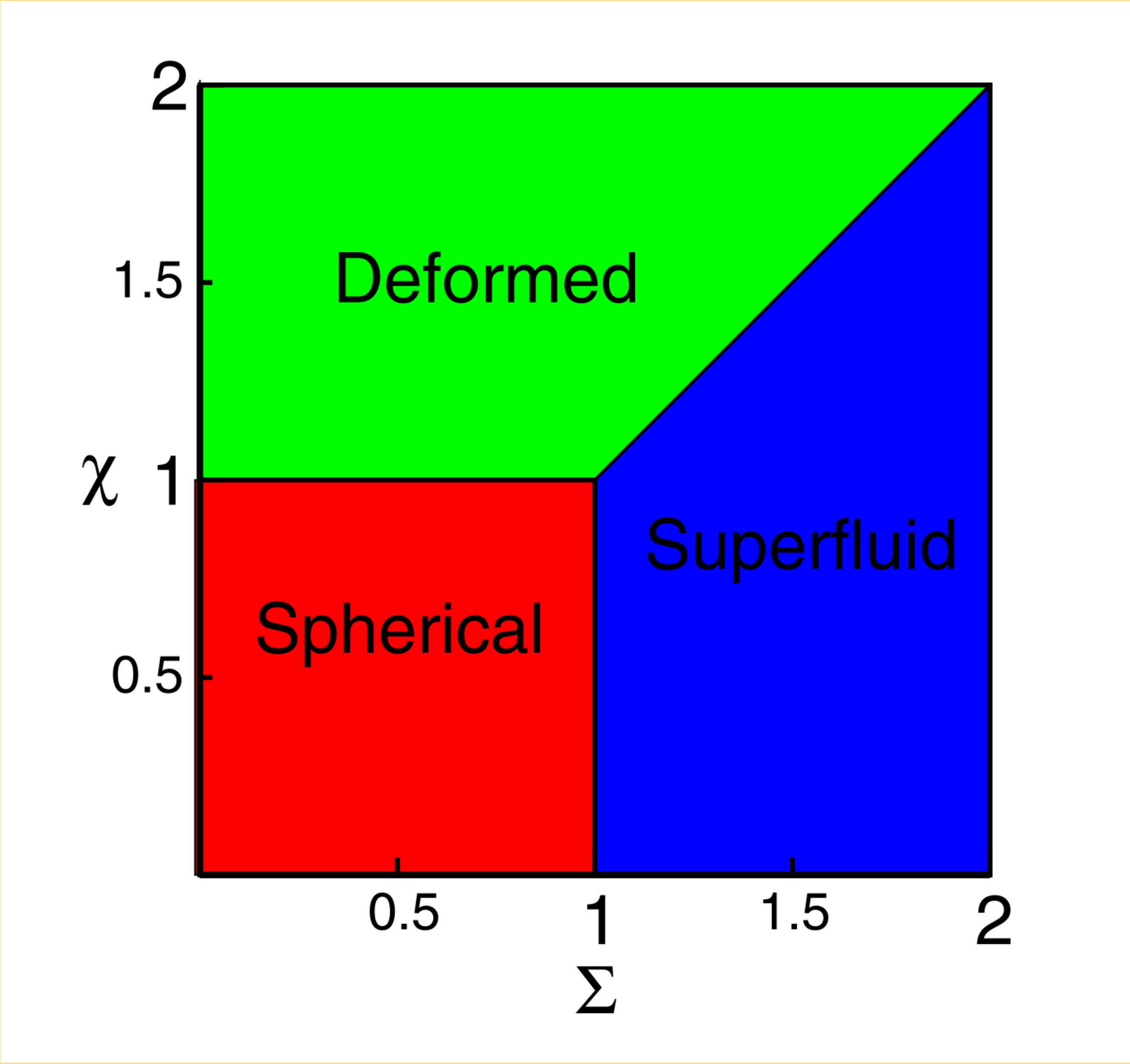

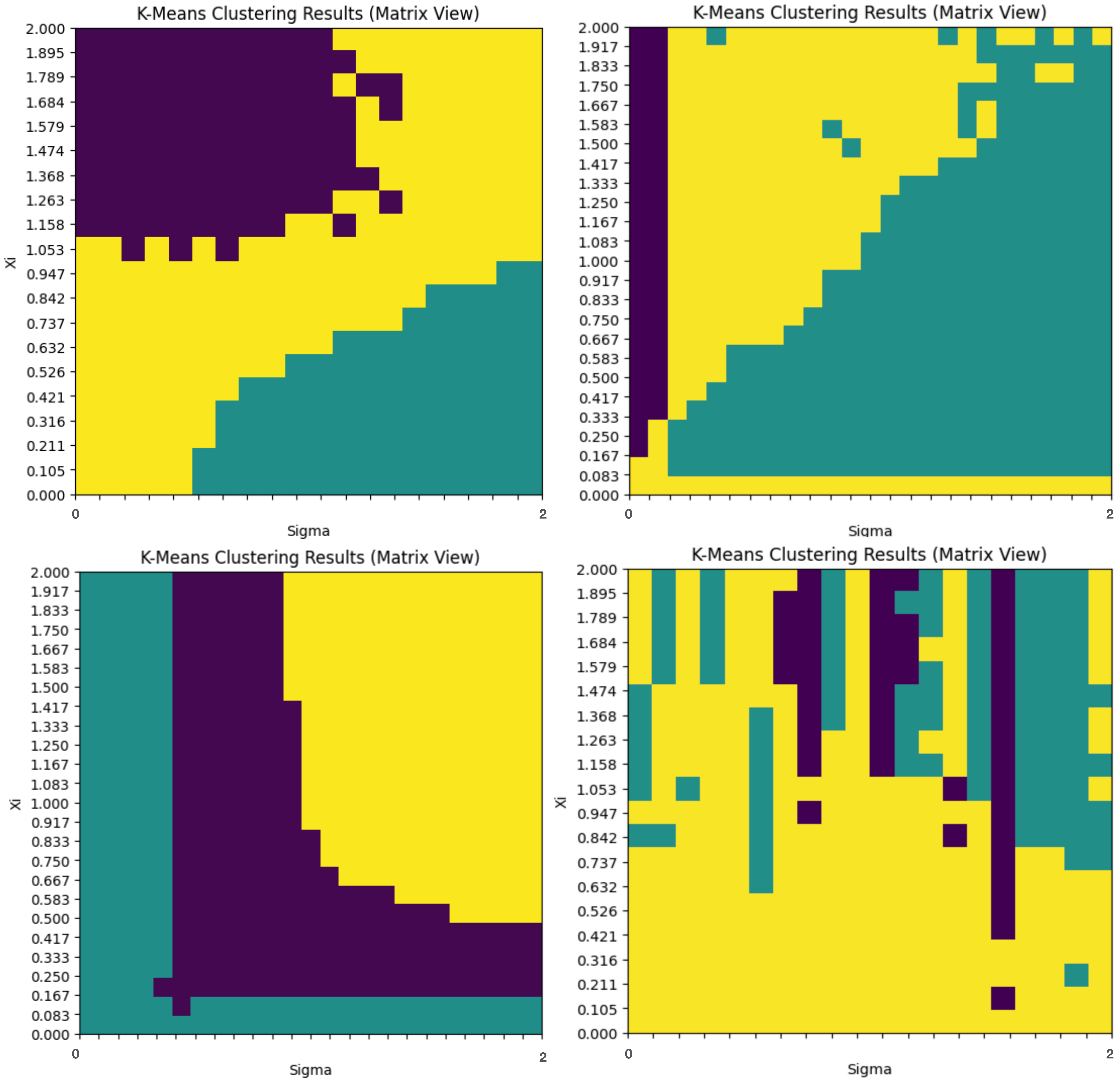

Our Contributions

- Development of an algorithm to perform hamiltonian decomposition for arbitrary values of j
- Use of unsupervised learning for recognizing different phases of the system without introducing any bias

Possible Developments

- Determine the phase transitions for higher values of j
- Autoencoders for unsupervised learning
- Develop better search methods for clique partioning
- Implementation of quantum version of clique partioning

Thank you!