P10.1) $\frac{\partial L}{\partial \theta} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = 0$

$$L = \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} I_2 (\dot{\theta} + \dot{\phi})^2 + M g l_1 \cos\theta$$

$$\frac{\partial L}{\partial \theta} = -M g l_1 \sin\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = I_1 \dot{\theta} + I_2 (\dot{\theta} + \dot{\phi})$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = I_1 \ddot{\theta} + I_2 (\ddot{\theta} + \ddot{\phi})$$

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = -M g l_1 \sin\theta - I_1 \ddot{\theta} - I_2 (\ddot{\theta} + \ddot{\phi}) = 0$$

$$\phi = A + A\cos(\omega_\phi t)$$

$$\dot{\phi} = -A\omega_\phi \sin(\omega_\phi t)$$

$$\ddot{\phi} = -A\omega_\phi^2 \cos(\omega_\phi t)$$

$$\downarrow$$

$$-M g l_1 \sin\theta - I_1 \ddot{\theta} - I_2 (\ddot{\theta} - A\omega_\phi^2 \cos(\omega_\phi t)) = 0$$

$$\ddot{\theta}(I_1 + I_2) = -M g l_1 \sin\theta + I_2 A \omega_\phi^2 \cos(\omega_\phi t)$$

$$\boxed{\ddot{\theta} + \omega_0^2 \sin\theta = \alpha \cos(\omega_\phi t)}$$

$$\omega_0^2 = \frac{M g l_1}{I_1 + I_2} \qquad \alpha = \frac{I_2 A \omega_\phi^2}{I_1 + I_2}$$