Metodos Computacionales-Semana 5

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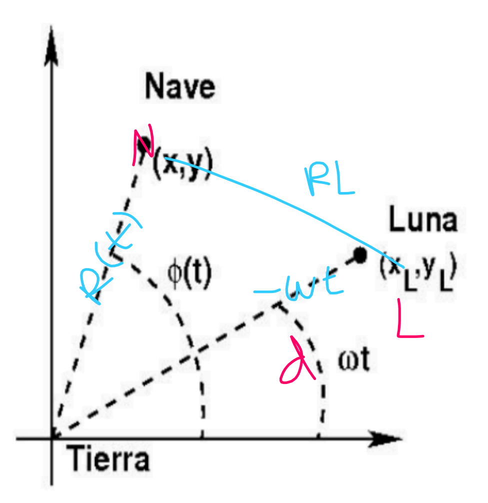

c) Muestre usando la Figura [1] que la distancia Nave-Luna está dada por:

$$r_L(r,\phi,t) = \sqrt{r(t)^2 + d^2 - 2r(t)dcos(\phi - \omega t)}$$

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$$r_L(r,\phi,t) = \sqrt{r(t)^2 + d^2 - 2r(t)d\cos(\phi - \omega t)}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\left.\begin{array}{l} a = R_L \\ b = R(t) \\ c = d \end{array}\right\} \quad R_L^2 = R^2(t) + d^2 - 2R(t)d\cos(\phi(t) - \omega t)$$

$$L = K - V$$

donde $K = \frac{1}{2}mv^2$

$$= \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

$$x(t) = r(t)\cos(\phi(t))$$
$$\dot{x}(t) = \dot{r}\cos\phi - r\sin\phi \cdot \dot{\phi}$$
$$y(t) = r(t)\sin\phi(t)$$
$$\dot{y}(t) = \dot{r}\sin\phi + r\cos\phi \cdot \dot{\phi}$$

Entonces

$$\dot{x}^2(t) + \dot{y}^2(t) = \dot{r}^2\sin^2\phi + r^2\cos^2\phi\dot{\phi}^2 + 2\dot{r}\sin\phi\, r\cos\phi\dot{\phi} + \dot{r}^2\cos^2\phi + r^2\sin^2\phi\dot{\phi}^2 - 2\dot{r}\cos\phi\, r\sin\phi\dot{\phi}$$

$$= \dot{r}^2\sin^2\phi + r^2\cos^2\phi\dot{\phi}^2 + \dot{r}^2\cos^2\phi + r^2\sin^2\phi\dot{\phi}^2$$

$$= \dot{r}^2 + r^2\dot{\phi}$$

d) Usando esta distancia muestre que el Hamiltoniano de la nave está dado por:

$$H = p_r\dot{r} + p_\phi\dot{\phi} - L = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} - G\frac{mm_T}{r} - G\frac{mm_L}{r_L(r,\phi,t)} \tag{3}$$

donde L es la energía cinética menos la energía potencial de la nave en coordenadas polares.

Energia cinetica

$$K = \frac{1}{2}m(\dot{r}^2 + r\dot{\phi}^2)$$

$$V = U_T + U_L$$
$$= -G\frac{mM_T}{r} - G\frac{mM_L}{r_L}$$

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) + Gm\left(\frac{M_T}{r} + \frac{M_L}{r_L}\right)$$

$$H = P_r\dot{r} + P_\phi\dot{\phi} - L$$

$$P_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

$$P_\phi = \frac{\partial L}{\partial \dot{\phi}} = mr^2\dot{\phi}$$

$$H = \frac{P_r^2}{m} + \frac{P_\phi^2}{mr^2} - L$$

$$H = \frac{P_r^2}{m} + \frac{P^2_\phi}{mr^2} - \frac{m\dot{r}^2}{2} - \frac{mr^2\dot{\phi}^2}{2} - G\frac{mM_T}{r} - G\frac{mM_L}{r_L}$$

$$H = \frac{P_r^2}{m} + \frac{P^2_\phi}{mr^2} - \frac{P_r^2}{2m} - \frac{P_\phi^2}{2mr^2} - Gm\left(\frac{M_T}{r} + \frac{M_L}{r_L}\right)$$

$$H = \frac{P_r^2}{2m} + \frac{P^2\phi}{2mr^2} - G\frac{mM_T}{r} - G\frac{mM_L}{r_L}$$

e) Muestre que las ecuaciones de Hamilton, que son las ecuaciones de movimiento están dadas por:

$$\begin{aligned} \dot{r} &= \frac{\partial H}{\partial p_r} = \frac{p_r}{m} \\ \dot{\phi} &= \frac{\partial H}{\partial p_\phi} = \frac{p_\phi}{mr^2} \\ \dot{p}_r &= -\frac{\partial H}{\partial r} = \frac{p_\phi^2}{mr^3} - G\frac{mm_T}{r^2} - G\frac{mm_L}{r_L(r,\phi,t)^3}[r - dcos(\phi - \omega t)] \\ \dot{p}_\phi &= -\frac{\partial H}{\partial \phi} = -G\frac{mm_L}{r_L(r,\phi,t)^3}rdsin(\phi - \omega t) \end{aligned} \tag{4}$$

Note que las dos primeras ecuaciones se refiere al momento lineal y angular de la nave y las segundas a la fuerza. Adicionalmente, este sistema de ecuaciones diferenciales no tiene solución analítica al ser no lineales. Este tipo de sistemas son de gran estudio numérico para establecer órbitas más reales.

$$\dot{P}_r = -\frac{\partial H}{\partial r} = -\left(-\frac{P^2\phi}{mr^3} + G\frac{mM_T}{r^2} + G\frac{mM_L}{r_L^2} \cdot \frac{\partial R_L}{\partial r}\right)$$

$$\frac{\partial R_L}{\partial r} = \frac{r - d\cos(\phi - \omega t)}{R_L}$$

$$\dot{P}_r = \frac{P^2\phi}{mr^3} - G\frac{mM_T}{r^2} - G\frac{mM_L}{r_L^3}\left(r - \cos(\phi - \omega t)\right)$$

$$\dot{P}\phi = -\frac{\partial H}{\partial \phi} = G\frac{mM_L}{r_L^2} \cdot \frac{\partial r_L}{\partial \phi}$$

$$\frac{\partial r_L}{\partial \phi} = \frac{-rd\sin(\phi - \omega t)}{r_L}$$

$$\dot{P}\phi = -\frac{GmM_L rd\sin(\phi - \omega t)}{r_L^3}$$

f) Para reducir el error de redondeo se pueden definir nuevas variables normalizadas a la distancia lunar: $\tilde{r} = r/d$,ϕ,$\tilde{p}_r = p_r/md$ y $\tilde{p}_\phi = p_\phi/md^2$. Muestre que el sistema se puede escribir como sigue:

$$\begin{aligned} \dot{\tilde{r}} &= \tilde{p}_r, \\ \dot{\phi} &= \frac{\tilde{p}_\phi}{\tilde{r}^2}, \\ \dot{\tilde{p}}_r &= \frac{\tilde{p}_\phi^2}{\tilde{r}^3} - \Delta\left\{\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}'^3}[\tilde{r} - cos(\phi - \omega t)]\right\} \\ \dot{\tilde{p}}_\phi &= -\frac{\Delta\mu\tilde{r}}{\tilde{r}'^3}sin(\phi - \omega t) \end{aligned} \tag{5}$$

donde $\Delta \equiv Gm_T/d^3$, $\mu \equiv m_L/m_T$ y $\tilde{r}' \equiv \sqrt{1 + \tilde{r}^2 - 2\tilde{r}cos(\phi - \omega t)}$.

$$\tilde{r} = \frac{r}{d}$$

$$\tilde{P}_r = \frac{P_r}{md}$$

$$\mu = \frac{M_L}{M_T}$$

$$\Delta = G\frac{M_T}{d^3}$$

$$\dot{\tilde{r}} = \tilde{P}_r \qquad \dot{\phi} = \frac{\tilde{P}_\phi}{\tilde{r}^2}$$

$$\frac{\dot{r}}{d} = \frac{P_r}{md} \qquad \dot{\phi} = \frac{P\phi}{mr^2} = \frac{P\phi}{md^2}\frac{r^2}{d^2}$$

$$\frac{\dot{P}_r}{md} = \frac{P^2\phi}{m^2dr^3} - \frac{Gm}{md}\left(\frac{M_T}{r^2} + \frac{M_L}{r_L^3}\left(r - \cos(\phi - \omega t)\right)\right)$$

$$\dot{\tilde{P}}_r = \frac{\tilde{P}^2\phi}{\tilde{r}^3} - \Delta\left(\frac{1}{\tilde{r}^2} + \frac{Md^2}{r_L^3}\left(r - \cos(\phi - \cos(\phi - \omega t))\right)\right)$$

$$\dot{\tilde{P}}_r = \frac{\tilde{P}\phi^2}{\tilde{r}^3} - \Delta\left(\frac{1}{\tilde{r}^2} + \frac{\mu}{\tilde{r}_L^3}\left(\tilde{r} - \cos(\phi - \omega t)\right)\right)$$

$$\dot{P}\phi = -\frac{GmM_L rd\sin(\phi - \omega t)}{r_L^3}$$

$$\frac{\dot{P}\phi}{md^2} = -\frac{GM_L r\sin(\phi - \omega t)}{M_T r_L^3}$$

$$\dot{\tilde{P}} = -\frac{\Delta\mu\tilde{r}}{\tilde{r}_L^3}\sin(\theta - \omega t)$$