ARSCAN
TURKUŠIĆ
1787/18173
LAB 9

1. Langrangeovi multiplikatori

$$f(x,y) = -2x^2 - y^2 + xy + 8x + 3y$$

ovvy max $z = -2x^2 - y^2 + xy + 8x + 3y$

f.o.

$$3000x + 1000y = 10000 \quad /:1000$$

$$3x + y = 10$$

ovvy min $z = 2x^2 + y^2 - xy - 8x - 3y$

$$3x - y - 10 = 0$$

$$L = 2x^2 + y^2 - xy - 8x - 3y + \lambda(3x + y - 10)$$

$$\frac{\partial L}{\partial x} = 4x - y - 8 + 3\lambda = 0$$

$$\frac{\partial L}{\partial y} = 2y - x - 3 + \lambda$$

$$\frac{\partial L}{\partial \lambda} = 3x - y - 10 = 0$$

$\rightarrow 4x - y + 3\lambda - 8 = 0$

$-x + 2y + \lambda - 3 = 0 \quad /(-3)$

$3x + y - 10 = 0$

$$\left.\begin{aligned} 7x - 7y - 1 &= 0 \\ 3x + y - 10 &= 0 \quad /\cdot 7 \end{aligned}\right\} +$$

$28x = 6y \Rightarrow \quad x = \frac{6y}{28}$

$y = \frac{73}{28}$

$\lambda = \frac{1}{4}$

$Z = 15.01$

Hessian: $\frac{\delta^2 f}{\delta x^2} = -4 \cdot \quad \frac{\delta^2 f}{\delta^2 y} = -2 \quad \frac{\delta^2 f}{\delta x \delta y} = \frac{\delta^2 f}{\delta y \delta x} = 1$

$$H = \begin{bmatrix} -4 & 1 \\ 1 & -2 \end{bmatrix} \cdot \left.\begin{array}{l} \Delta_1 = -4 < 0 \\ \Delta_2 = 7 > 0 \end{array}\right\} \text{ negativno definitno}$$

(MAXIMUM)

2. arg max $\cdot z = (30 - x_1)x_1 + (50 - 2x_2)x_2 - 3x_1$

$5x_2 - 10x_3$

$x_1 + x_2 \leq x_3$

$x_3 \leq 17.25$

$\Rightarrow$ arg min $\cdot\ z = -30x_1 + x_1^2 - 50x_2 + 2x_2^2 + 3x_1 +$

$+ 5x_2 + 10x_3$

P.O.

$x_1 + x_2 - x_3 \leq 0 \quad (\mu_1)$

$x_3 - 17.25 \leq 0 \quad (\mu_2)$

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$L = x_1^2 + 2x_2^2 - 27x_1 - 45x_2 + 10x_3 + \mu_1 \cdot (x_1 + x_2 - x_3) +$

$+ \mu_2 (x_3 - 17.25)$

$$\frac{\delta L}{\delta x_1} = 2x_1 - 27 + \mu_1 \qquad \frac{\delta L}{\delta x_2} = 4x_2 - 45 + \mu_1$$

$$\frac{\delta L}{\delta x_3} = 10 - \mu_1 + \mu_2$$

→ KKT услови:

$$2x_1 - 27 + \mu_1 = 0$$

$$10 - \mu_1 + \mu_2 = 0 \qquad \mu_1(-x_1 + x_2 - x_3) = 0$$

$$x_1 + x_2 - x_3 \leq 0 \qquad \mu_2(x_3 - 17.25) = 0$$

$$x_3 - 17.25 \leq 0$$

I. $\mu_1 \neq 0$, $\mu_2 \neq 0 \Rightarrow x_1 + x_2 - x_3 = 0$

$x_3 - 17.25 = 0 \Rightarrow x_3 = 17.25$

$x_1 + x_2 = 17.25 \Rightarrow x_1 = 17.25 - x_2$

$$\left.\begin{array}{l} 2 \cdot (17.25 - x_2) - 27 + \mu_1 = 0 \\ -4x_2 + 45 - \mu_1 = 0 \end{array}\right\} + \quad -6x_2 + 52.5 = 0$$

$\Rightarrow x_2 = 8.75 \qquad \mu_1 = 10$

$x_1 = 8.5 \qquad \mu_2 = 0$

$z = 225.37$

Scanned with CamScanner

$\underline{\text{II}}$ $\mu_1 = 0 \wedge \mu_2 = 0 \Rightarrow 2x_1 - 27 = 0 \Rightarrow x_1 = 13.5$

$\Rightarrow x_2 = 11.25 \Rightarrow 10 - 0 - 0 = 0$

protu-rječno

$\underline{\text{III}}$ $\mu_1 = 0 \wedge \mu_2 \neq 0 \Rightarrow 2x_1 - 27 = 0 \Rightarrow$

$\Rightarrow x_1 = 13.5 \Rightarrow x_2 = 11.25 \Rightarrow 10 \cdot \mu_2 = 0 \Rightarrow$

$\Rightarrow \mu_2 = 0 - 10$

$\underline{\text{IV}}$ $\mu_1 \neq 0 \wedge \mu_2 = 0 \Rightarrow \mu_1 = 10$

$2x_1 - 27 + 10 = 0 \qquad 4x_2 - 45 + 10 = 0$

$x_1 = 8.5 \qquad x_2 = 8.75 \qquad x_3 = 17.25$

$Z = 225.37$

Hessian

$$\frac{\delta^2 \mathcal{L}}{\delta x_1^2} = -2 \qquad \frac{\delta^2 \mathcal{L}}{\delta x_2^2} = -4 \qquad \frac{\delta^2 \mathcal{L}}{\delta x_3} = 0$$

$$\frac{\delta^2 \mathcal{L}}{\delta x_1 \delta x_2} = \frac{\delta^2 \mathcal{L}}{\delta x_1 \delta x_3} = \frac{\delta^2 \mathcal{L}}{\delta x_2 \delta x_3} = \frac{\delta^2 \mathcal{L}}{\delta x_2 \delta x_1} =$$

$$= \frac{\delta^2 \mathcal{L}}{\delta x_3 \delta x_1} = \frac{\delta^2 \mathcal{L}}{\delta x_3 \delta x_2} = 0$$

③ Wolfeov metod:

odg min $z = -x_1 - x_2 + 0.5x_1^2 + x_2^2 - x_1 + x_2$

p.o.

$x_1 + x_2 \leq 3$

$-2x_1 - 3x_2 \leq -6$

$x_1, x_2 \geq 0$

p.o. $x_1 + x + x_3 = 3$

$-2x_1 - 3x_2 + x_4 = -6$

$x_1, x_2, x_3, x_4 \geq 0$

$L(x_1, x_2, x_3, x_4, \lambda_1, \lambda_2, \mu_1, \mu_2) =$

$-x_1 - x_2 + 0.5x_1^2 + x_2^2 - x_1 x_2 + \lambda_1(x_1 + x_2 + x_3 - 3)$
$+ \lambda_2(-2x_1 - 3x_2 + x_4 - 6) - \mu_1 x_1 -$
$\mu_2 x_2 - \mu_3 x_3 - \mu_4 x_4$

$\frac{\partial L}{\partial x_1} = 0$ $\quad \frac{\partial L}{\partial x_2} = 0$ $\quad \cancel{\frac{\partial L}{\partial x_3} = 0}$

$-1 + x_1 - x_2 + \lambda_1 - 2\lambda_2 - \mu_1$ $\quad -1 + 2x_2 - x_1 - \lambda_1 - 3\lambda_2 - \mu_2 = 0$

$$\frac{\partial L}{\partial x_3} = \lambda_1 - \mu_3 = 0 \qquad \frac{\partial L}{\partial x_4} = \lambda_2 - \mu_4 = 0$$

$$\lambda_1 = \mu_3$$

$1^\circ \quad -1 + x_1 - x_2 + \mu_3 - 2\mu_4 - \mu_1 = 0$

$2^\circ \quad -1 + 2x_2 - x_1 + \mu_3 - 3\mu_4 - \mu_2 = 0$

KKT uslovi

$$x_1 - x_2 - \mu_1 + \mu_3 - 2\mu_4 - 1 = 0$$

$$-x_1 + 2x_2 - \mu_2 + \mu_3 - 3\mu_4 - 1 = 0$$

$$x_1 + x_2 + x_3 = 3$$

$$2x_1 + 3x_2 - x_4 = 0$$

$$\forall\, x_{1 \dots 4}, \mu_{1 \dots 4} \geq 0$$

$$\mu_1 x_1 = 0, \; \mu_2 x_2 = 0 \; \dots \; \mu_4 x_4 = 0$$

$$w = -v_1 - v_2 - v_3$$

$$x_1 - x_2 - \mu_1 - \mu_3 - 2\mu_4 + v_1 = 1$$

$$-x_1 + 2x_2 - \mu_2 + \mu_3 - 3\mu_4 + v_2 = 1$$

$$x_1 + x_2 + x_3 = 3$$

$$2x_1 + 3x_2 - x_4 + v_3 = 6$$

$$x_1 \dots x_4, \mu_1 \dots \mu_4, v_1 \dots v_3 \geq 0$$

$$\mu_1 x_1 = \mu_2 x_2 = 0 \dots, \mu_4 x_4 = 0$$

$$v_1 = 1 - x_1 + x_2 + \mu_1 - \mu_3 + 2\mu_4$$

$$v_2 = 1 + x_1 - 2x_2 + \mu_2 - \mu_3 + 3\mu_4$$

$$v_3 = 6 - 2x_1 - 3x_2 + x_4$$

$$w = 2x_1 + 4x_2 - x_4 - \mu_1 - \mu_2 + 2\mu_3 - 5\mu_1 = 2$$

Simpleks

Baza	b_i	x_1	x_2	x_3	x_4	μ_1	μ_2	μ_3	μ_4	v_1	v_2	v_3
v_1	1	1	-1	0	0	-1	0	1	-2	1	0	0
v_2	1	-1	2	0	0	0	-1	1	-3	0	1	0
x_3	3	1	1	1	0	0	0	0	6	0	0	0
v_3	6	2	3	0	-1	0	0	0	0	0	0	1
[illegible]												
z	2	2	4	0	-1	-1	-1	2	-5	0	0	0

$$\boxed{\begin{aligned} &x_1 = 1.8 \quad , x_2 = 1.2 \\ &z = -2.1 \end{aligned}}$$