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Homework 2

CS 250 Discrete Structures 18706

PCC Winter 2017

16 January 2017

1. 2.1) 50-54.

Verify the logical equivalences. Supply a reason for each step.

A number of logical equivalences are summarized in Theorem 2.1.1 for future reference.

a)

(p ∧ q) ∨ p ≡ p ∧ (q ∧ p) by commutative laws

≡ p by absorption laws

b)

p ∧ (~q ∨ p) ≡ (p ∧ ~q) ∨ (p ∧ p) by associative laws

≡ (p ∧ ~q) ∨ p by idempotent laws

≡ p ∨ (p ∧ ~q) by commutative laws

≡ p

c)

~(p ∨ ~q) ∨ (~p∧~q) ≡ (~p ∨ ~~q) ∨ (~p ∧ ~q) by de morgan's laws

≡ (~p ∨ q) ∨ (~p ∧ ~q) by double negative laws

≡ ~p ∨ ~p ∧ ~q ∨ q by associative laws

≡ p ∧ ~q ∨ q by idempotent laws

≡ p ∧ t by negation laws

≡ p

d)

~((~p∧q) ∨ (~p∧~q)) ∨ (p∧q ≡ (~(~p∧q) ∨ ~(~p∧~q)) ∨ (p∧q) by de morgan's laws

≡ (~~p∧~q) ∨ (~~p∧~~q) ∨ (p∧q) by de morgan's laws

≡ (p∧~q) ∨ (p∧q) ∨ (p∧q) by double negative laws

≡ (p∧~q) ∨ (p∧q) by idempotent laws

≡ p

e)

(p∧(~(~p ∨ q))) ∨ (p∧q) ≡ (p∧(~~p ∨ ~q)) ∨ (p∧q) by de morgan's laws

≡ (p∧(p ∨ q)) ∨ (p∧q) by double negative laws

≡ p ∨ (p∧q) by absorption laws

≡ p by absorption laws

2.2 29, 31. If statement forms P and Q are logically equivalent, then P ⇐⇒ Q is a tautology. Conversely, if P ⇐⇒ Q is a tautology, then P and Q are logically equivalent. Use ⇐⇒ to convert each of the logical equivalences in 29-31 to a tautology. Then use a truth table to verify each tautology.

p ⇒ (q ∨ r) ≡ (p ∧ q) ⇒ r

p ⇒ (q ∨ r) ⇔ (p ∧ q) ⇒ r

| p | q | r | (q∨r) | p⇒(q∨r) | (p∧q) | (p∧q)⇒r |
| --- | --- | --- | --- | --- | --- | --- |
| T | T | T | T | T | T | T |
| T | T | F | T | T | T | F |
| T | F | T | T | T | F | T |
| T | F | F | F | F | F | T |
| F | T | T | T | T | F | T |
| F | T | F | T | T | F | T |
| F | F | T | T | T | F | T |
| F | F | F | F | T | F | T |

Since the two statements do **not** have the same values in the truth table above, they are **not** tautologies and therefore **not** logically equivalent.

p ⇒ (q ⇒ r) ≡ (p ∧ q) ⇒ r

p ⇒ (q ⇒ r) ⇔ (p ∧ q) ⇒ r

| p | q | r | (q⇒r) | p⇒(q⇒r) | (p∧q) | (p∧q)⇒r |
| --- | --- | --- | --- | --- | --- | --- |
| T | T | T | T | T | T | T |
| T | T | F | F | F | T | F |
| T | F | T | T | T | F | T |
| T | F | F | T | T | F | T |
| F | T | T | T | T | F | T |
| F | T | F | F | T | F | T |
| F | F | T | T | T | F | T |
| F | F | F | T | T | F | T |

Since the two statements do have the same values in the truth table above, they are tautologies and therefore logically equivalent.

2.2 48, 49. In 48 and 49, (a) use the logical equivalencies p =⇒ q ≡¬p∨q and p ⇐⇒ q ≡ (¬p∨q)∧(¬q ∨p) to rewrite the given statement forms without using the symbols =⇒ or ⇐⇒ , and (b) use the logical equivalence p∨q ≡¬(¬p∧¬q) to rewrite each statement using only ∧ and ¬.

48)

p ∨ ¬q ⇒ r ∨ q

**¬(p ∨ ¬q) ∨ (r ∨ q)**

49)

(p ⇒ r) ⇔ ( q ⇒ r)

¬p ∨ r ⇔ ¬q ∨ r

**(¬(¬p ∨ r) ∨ (¬q ∨ r)) ∧ (¬(¬q ∨ r) ∨ (¬p ∨ r))**

**38. You are visiting the island described in Example 2.3.14, whose natives consist of knights and knaves.**

38c. You encounter natives E and F.  
E says: F is a knave.   
F says: E is a knave.  
How many knaves are there?

There is one knave.  
Let’s suppose E is a knave  
∴what E says is false  
∴F is a knight

And let’s suppose F is a knight  
∴what F says is true   
∴E is a knave

If both E and F were knaves, both would be liars and ∴ both would be knights. This is a contradiction.

If both E and F were knights, both would tell the truth and ∴ both would be knaves. This is a contradiction.

38d. Finally, you meet a group of six natives, U,V,W,X,Y, and Z, who speak to you as follows: U says: None of us is a knight. V says: At least three of us are knights. W says: At most three of us are knights. X says: Exactly ﬁve of us are knights. Y says: Exactly two of us are knights. Z says: Exactly one of us is a knight. Which are knights and which are knaves?

W and Y are the two knights.  
Let’s suppose W is a knight  
∴what E says is false  
∴F is a knight

And let’s suppose Y is a knight  
∴what F says is true   
∴E is a knave

If V is a knight, then V would tell the truth and ∴ none are knights. This is a contradiction.

If X is a knight, then X would tell the truth and ∴ exactly 5 are knights. ∴ Every one of them would express the same thought – that there are 5 knights. No other person does so. This is a contradiction.

If Z is a knight, then Z would tell the truth and ∴ one of them is a knight. This would mean W is telling the truth as well, which would require two knights in total. This is a contradiction.

**40. Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:**

Socko: Lefty killed Sharky.   
Fats: Muscles didn’t kill Sharky.   
Lefty: Muscles was shooting craps with Socko when Sharky was knocked oﬀ.   
Muscles: Lefty didn’t kill Sharky. Who killed Sharky?

Muscles is telling the truth: ∴ Muslces killed Sharky

1a) If Socko is telling the truth, then Lefty killed Sharky  
1b) ∴ Lefty is lying and Muscles was not shooting craps with Socko when Sharky was knocked off  
1c) ∴ Fats is lying and Muscles killed Sharky CONTRADICTION with 1a and 1d  
1d) ∴ Muscles is lying and Lefty did kill Sharky

2a) If Fats is telling the truth, then Muscles didn’t kill Sharky  
2b) ∴ Socko is lying and Lefty did not kill Sharky  
2c) ∴ Lefty is lying and Muscles was not shooting craps with Socko when Sharky was knocked off  
2d) ∴ Muscles is lying and Lefty did kill Sharky CONTRADICTION with 2b

3a) If Lefty is telling the truth, then Muscles was shooting craps with Socko when Sharky was knocked off  
3b) ∴ Socko is lying and Lefty did not kill Sharky  
3c) ∴ Fats is lying and Muscles killed Sharky CONTRADICTION with 3a  
3d) ∴ Muscles is lying and Lefty did kill Sharky

4a) If Muscles is telling the truth, then Lefty didn’t kill Sharky.  
4b) ∴ Socko is lying and Lefty did not kill Sharky  
4c) ∴ Lefty is lying and Muscles was not shooting craps with Socko when Sharky was knocked off  
4d) ∴ Fats is lying and Muscles killed Sharky

∴ 4d is the only logically true statement  
∴ Muscles killed Sharky

**43, 44. In 41-44, a set of premises and a conclusion are given. Use the valid argument forms listed in Table 2.3.1 to deduce the conclusion from the premises, giving a reason for each step as in Example 2.3.8. Assume all variables are statement variables.**

43.

a. ¬p ⇒ r ∧ ¬s

b. t ⇒ s

c. u ⇒ ¬p

d. ¬w

e. u ∨ w

f. ∴ ¬t

1) u ∨ w by (e)

¬w by (d)

∴ u by elimination

2) u ⇒ ¬p by (c)

u by conclusion of (1)

∴ ¬p by modus ponens

3) ¬p ⇒ r ∧ ¬s by (a)

¬p by conclusion of (2)

∴ r ∧ ¬s by modus ponens

4) t ⇒ s by (b)

¬s by conclusion of (3)

∴ ¬t by modus tollens

44.

a. p ⇒ q

b. r ∨ s

c. ¬s ⇒ ¬t

d. ¬q ∨ s

e. ¬s

f. ¬p ∧ r ⇒ u

g. w ∨ t

h. ∴ u ∧ w

1) ¬q ∨ s by (d)

¬s by (e)

∴ ¬q by elimiation

2) p ⇒ q by (a)

¬q by conclusion of (1)

∴ ¬p by modus tollens

3) r ∨ s by (b)

¬s by (e)

∴ r by elimination

4) ¬p ∧ r ⇒ u by (f)

¬p ∧ r by conclusions of (2) and (3)

∴ u by modus ponens

5) ¬s ⇒ ¬t by (c)

¬s by (e)

∴ ¬t by modus ponnens

6) w ∨ t by (g)

¬t by conclusion of (5)

∴ w by elimintion

7) u by conclusion of (4)

w by conclusion of (6)

∴ u ∧ w by conjunction