

# **Statistical modelling**

## #1.a Hypothesis tests

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# Hypothesis testing

## Trial analogy



Screenshot of the courtroom drama *Twelve Angry Men* (1957)

# Recipe

An hypothesis test is a binary decision rule

Below are the different steps to undertake:

1. Define the variables of interest
2. Formulate the alternative and the null hypotheses,  $\mathcal{H}_a$  and  $\mathcal{H}_0$
3. Choose the test statistic and compute the latter on the sample
4. Compare the numerical value with the null distribution
5. Obtain the  $p$ -value
6. Conclude in the setting of the problem

## Le Tech3Lab est le plus important laboratoire en expérience utilisateur (UX) en Amérique du Nord

Le Tech3Lab est un laboratoire de recherche appliquée en sciences de la gestion qui se spécialise dans l'analyse des interactions entre les interfaces technologiques des organisations et leurs employés ou consommateurs



# Texting while walking: a dangerous habit?

**Le risque de texter en marchant :  
La lenteur du désengagement**

**OBJECTIF : COMPRENDRE LES PROCESSUS COGNITIFS  
SOUS-JACENTS AU MULTITÂCHE MOBILE**

**HYPOTHÈSE**  
L'engagement cognitif ralentit la  
capacité attentionnelle du piéton

**MESURES**  
Électroencéphalographie



The diagram illustrates the cognitive process of multitasking while walking. It features three circular icons representing different stages of engagement:

- MOMENT D'ENGAGEMENT** (Blue circle): The initial stage of cognitive engagement.
- MOMENT DE DÉSANGAGEMENT** (Red circle): The stage of cognitive disengagement, which is highlighted as being slow.
- MOMENT D'ENGAGEMENT** (Blue circle): The final stage of cognitive engagement.

Arrows connect these stages, indicating a sequential process. A small orange circular arrow is positioned below the red circle, suggesting a feedback loop or a return to the initial state.

Arc logo and text: Avec la permission de C/I Radio-Canada

## Study details

- + 35 participants took part in the study.
- + Each person had to walk on a treadmill in front of a screen where obstacles were projected.
- + In one of the sessions, the subjects walked while talking on a cell phone, whereas in another session, they walked while texting.
- + The order of these sessions was determined *at random*.
- + Different obstacles were randomly projected during the session.
- + We are only interested in one kind of scenario: a cyclist riding towards the participant.

# Characteristics

- + Population: adults
- + Sample: 35 individuals
- + Variables:
  - + Time to perceive an obstacle: quantitative
  - + Distraction type (cellphone call or texting): nominal variable
- + Variable of interest: time (in seconds) that it takes for a person to notice the obstacle when walking while texting or talking on a cell phone (measure through an encephalogram)

## #1. Define the variables of interest

- +  $\mu_c$  be the average reaction time (in seconds) during a call (**c**)
- +  $\mu_t$  be the average reaction time (in seconds) while texting (**t**)

## #2. Formulate the null and alternative hypothesis

- + Hypothesis of interest: does texting increases distraction?
  - +  $\mathcal{H}_a : \mu_t > \mu_c$  (one-sided)
- + Null hypothesis (Devil's advocate)
  - +  $\mathcal{H}_0 : \mu_t \leq \mu_c$

Express the hypothesis in terms of the difference of means

$$\mathcal{H}_a : \mu_t - \mu_c > 0.$$



### #3. Choose the test statistic

We compare the difference of the mean reaction time

- + one-sample  $t$ -test for  $\mathbf{t} - \mathbf{c}$  (paired  $t$ -test)

$$T_D = \frac{\bar{D} - \mu_0}{\text{se}(\bar{D})}$$

- +  $\bar{D}$  is the mean difference in the sample.
- + Under  $\mathcal{H}_0 : \mu_0 = \mu_{\mathbf{t}} - \mu_{\mathbf{c}} = 0$ .
- + The standard error of  $\bar{D}$  is  $\text{se}(\bar{D}) = S_D / \sqrt{n}$ , where  $S_D$  is the standard deviation of the variables  $D_i$  and  $n$  the sample size.

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- SAS code + SAS output + R code + R output

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```
proc ttest data=statmod.distraction side=u;  
paired t*c;  
run;
```

## #4. Compare the statistic with the null distribution

The null distribution is Student- $t$  with 34 degrees of freedom,  $\text{St}_{34}$ .

We are only interested in the probability that  $T_D > 2.91$  under  $\mathcal{H}_0$ .

## #5. Obtain a $p$ -value

The  $p$ -value is **0.0032**, which is smaller than  $\alpha = 5\%$ .

## #6. Conclude in the setting of the problem

We reject  $\mathcal{H}_0$ , meaning that the reaction time is significantly higher when texting than talking on the cellphone while walking.

The estimated mean difference is **0.313** seconds.