Statistical modelling

#1.a Hypothesis tests

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Hypothesis testing

Trial analogy

Screenshot of the courtoom drama *Twelve Angry Men* (1957)

Recipe

An hypothesis test is a binary decision rule

Below are the different steps to undertake:

1. Define the variables of interest
2. Formulate the alternative and the null hypotheses, $\mathscr{H}_a$ and $\mathscr{H}_0$
3. Choose the test statistic and compute the latter on the sample
4. Compare the numerical value with the null distribution
5. Obtain the p-value
6. Conclude in the setting of the problem

Tech3Lab

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Fixation oculaire

Texting while walking: a dangerous habit?

Study details

✚ 35 participants took part in the study.

✚ Each person had to walk on a treadmill in front of a screen where obstacles were projected.

✚ In one of the sessions, the subjects walked while talking on a cell phone, whereas in another session, they walked while texting.

✚ The order of these sessions was determined *at random*.

✚ Different obstacles were randomly projected during the session.

✚ We are only interested in one kind of scenario: a cyclist riding towards the participant.

Characteristics

+ Population: adults

+ Sample: 35 individuals

+ Variables:

    + Time to perceive an obstacle: quantitative

    + Distraction type (cellphone call or texting): nominal variable

+ Variable of interest: time (in seconds) that it takes for a person to notice the obstacle when walking while texting or talking on a cell phone (measure through an encephalogram)

#1. Define the variables of interest

✚ μ_c be the average reaction time (in seconds) during a call (c)

✚ μ_t be the average reaction time (in seconds) while texting (t)

#2. Formulate the null and alternative hypothesis

✚ Hypothesis of interest: does texting increases distraction?

   ✚ $\mathcal{H}_a : \mu_t > \mu_c$ (one-sided)

✚ Null hypothesis (Devil's advocate)

   ✚ $\mathcal{H}_0 : \mu_t \leq \mu_c$

Express the hypothesis in terms of the difference of means

$$\mathcal{H}_a : \mu_t - \mu_c > 0.$$

#3.Choose the test statistic

We compare the difference of the mean reaction time

+ one-sample t-test for $\mathbf{t} - \mathbf{c}$ (paired t-test)

$$T_D = \frac{\overline{D} - \mu_0}{\mathsf{se}(\overline{D})}$$

+ $\overline{D}$ is the mean difference in the sample.
+ Under $\mathscr{H}_0 : \mu_0 = \mu_{\mathbf{t}} - \mu_{\mathbf{c}} = 0$.
+ The standard error of $\overline{D}$ is $\mathsf{se}(\overline{D}) = S_D/\sqrt{n}$, where S_D is the standard deviation of the variables D_i and n the sample size.

SAS code ✚ SAS output ✚ R code ✚ R output

```
proc ttest data=statmod.distraction side=u;
paired t*c;
run;
```

#4. Compare the statistic with the null distribution

The null distribution is Student-t with 34 degrees of freedom, St_{34}.

We are only interested in the probability that $T_D > 2.91$ under $\mathscr{H}_0$.

#5. Obtain a p-value

The p-value is 0.0032, which is smaller than $\alpha = 5\%$.

#6. Conclude in the setting of the problem

We reject $\mathscr{H}_0$, meaning that the reaction time is significantly higher when texting than talking on the cellphone while walking.

The estimated mean difference is 0.313 seconds.