MATH 60604A
Statistical modelling
§ 5h - Group heteroscedasticity

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Block covariance structure for group heteroscedasticity

- Assume that observations within groups have the same covariance structure, but the parameters of the latter differ between groups.
- Assuming consecutive grouped measurements, the covariance matrix of all the measurements is

$$\mathrm{Cov}\left(\boldsymbol{Y}\right) = \begin{pmatrix} \boldsymbol{\Sigma}_1 & \mathbf{O} & \cdots & \mathbf{O} \\ \mathbf{O} & \boldsymbol{\Sigma}_2 & \cdots & \mathbf{O} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{O} & \mathbf{O} & \cdots & \boldsymbol{\Sigma}_m \end{pmatrix}.$$

- We assume that $\boldsymbol{\Sigma}_1 \neq \cdots \neq \boldsymbol{\Sigma}_m$.

Group heteroscedasticity

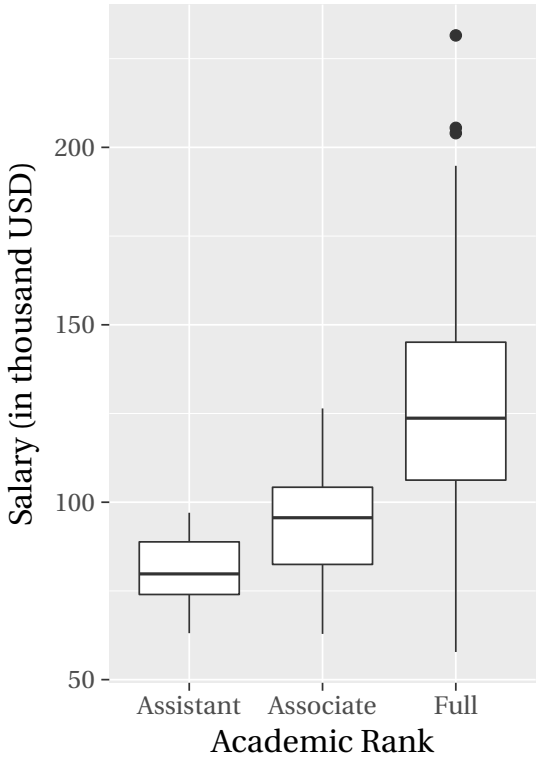
- If the data are independent (within and between group), but heteroscedastic between groups, the matrix $\mathbf{\Sigma}_i = \sigma_i^2 \mathbf{I}$, where $\mathbf{I}$ is the identity matrix with ones on the diagonal and zero for off-diagonal entries.
- In this case, there are m variance parameters to estimate (one per group).
- We could use a different structure for $\mathbf{\Sigma}_i$. SAS allows this, but the blocks cannot share parameters, so we get m times the number of parameters in $\mathbf{\Sigma}_i$. There must be enough observations in each group to reliably estimate the covariance parameters.

Wage discrimination in a US college

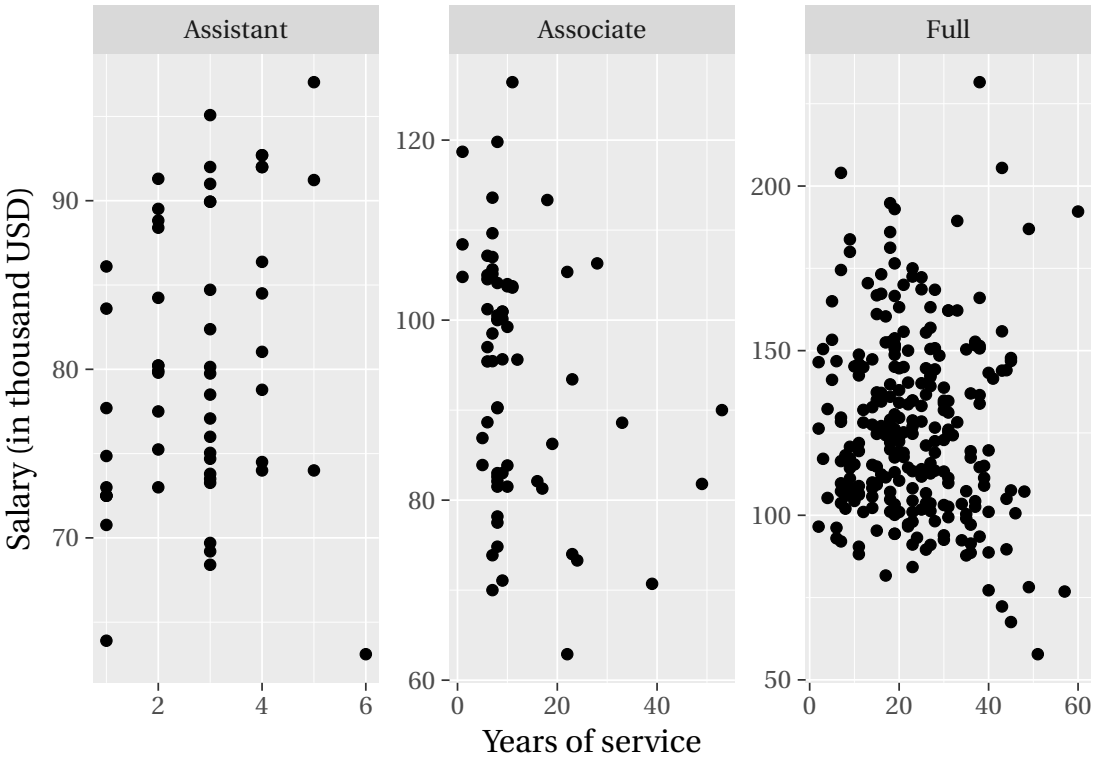
The `college` data set provides the nine-month academic salary (in thousand dollars) in 2008–2009 of professors in a college in the USA.

- `salary`: nine month income (in thousand dollars).
- `rank`: academic rank of the professor (assistant , associate or full).
- `field`: categorical variable indicating whether research field is applied or theoretical.
- `sex`: sex of individual, either man or woman.

Box-and-whiskers plots



Relationship between number of years of service and salary



The explanatory data analysis shows clear heteroscedasticity within academic rank.

Dealing with group heteroscedasticity

SAS code for a different variance per group

```
proc mixed data=statmod.college plots=studentpanel;
class field rank sex;
model salary = sex field rank;
repeated / group = rank;
run;
```

The argument `repeated / group` specifies the group structure.

Variance estimates per group and significance test

Covariance Parameter Estimates

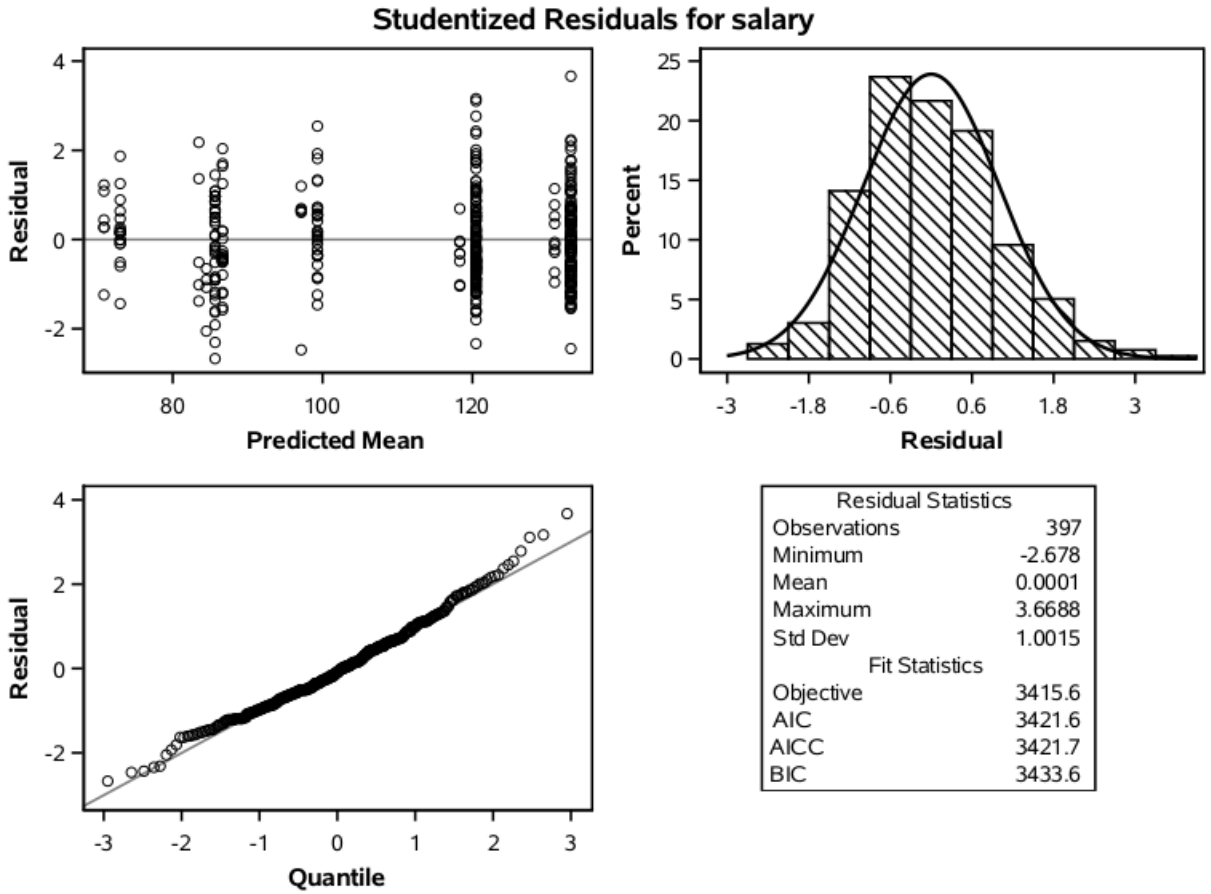
Cov Parm	Group	Estimate
Residual	rank assistant	42.4817
Residual	rank associate	115.29
Residual	rank full	722.44

Null Model Likelihood Ratio Test

DF	Chi-Square	Pr > ChiSq
2	164.78	<.0001

The variance increases with rank. The likelihood ratio test shows that the model with a different group for each rank is significantly better than the linear model which assumes a constant variance for every observation.

Diagnostic plots for the `profsalaries` data



The residual plots show that the model captures most features well. We can be confident in our inference.

Mean parameter estimates

Type 3 Tests of Fixed Effects

Effect	Num DF	Den DF	F Value	Pr > F
sex	1	392	1.55	0.2141
field	1	392	92.85	<.0001
rank	2	392	334.46	<.0001

- Solely comparing the salary of mean and women academics using a two-sample test is wrong, because rank is an important explanatory variable.
- Moreover, the proportion of full professor that are women (7%) is much lower than for assistant or associated professors (16%)
- After accounting for rank and dealing with group heteroscedasticity, there is no evidence of gender gap.