

Analysis of covariance

Session 8

MATH 80667A: Experimental Design and Statistical Methods
HEC Montréal

Analysis of covariance

Covariates

Covariate

Explanatory measured **before** the experiment

Typically, cannot be acted upon.

Example

socioeconomic variables
environmental conditions

IJLR: It's Just a Linear Regression...

All ANOVA models covered so far are linear regression model.

The latter says that

$$\begin{array}{ccc} \mathbf{E}(Y_i) & = & \beta_0 + \beta_1 X_{1i} + \cdots + \beta_p X_{pi} \\ \text{average response} & & \text{linear (i.e., additive) combination of explanatories} \end{array}$$

In an ANOVA, the model matrix \mathbf{X} simply includes columns with -1 , 0 and 1 for group indicators that enforce sum-to-zero constraints.

What's in a model?

In experimental designs, the explanatory variables are

- experimental factors (categorical)
- continuous (dose-response)

**Random assignment implies
no systematic difference between groups.**

ANCOVA = Analysis of covariance

- Analysis of variance with added continuous covariate(s) to reduce experimental error (similar to blocking).
- These continuous covariates are typically concomitant variables (measured alongside response).
- Including them in the mean response (as slopes) can help reduce the experimental error (residual error).

Control to gain power!

Identify external sources of variations

- enhance balance of design (randomization)
- reduce mean squared error of residuals to increase power

These steps should in principle increase power **if** the variables used as control are correlated with the response.

Example

Abstract of [van Stekelenburg et al. \(2021\)](#)

In three experiments with more than 1,500 U.S. adults who held false beliefs, participants first learned the value of scientific consensus and how to identify it. Subsequently, they read a news article with information about a scientific consensus opposing their beliefs. We found strong evidence that in the domain of genetically engineered food, this two-step communication strategy was more successful in correcting misperceptions than merely communicating scientific consensus.

Experiment 2: Genetically Engineered Food

We focus on a single experiment; preregistered exclusion criteria led to $n = 442$ total sample size (unbalanced design).

Three experimental conditions:

Boost

Boost Plus

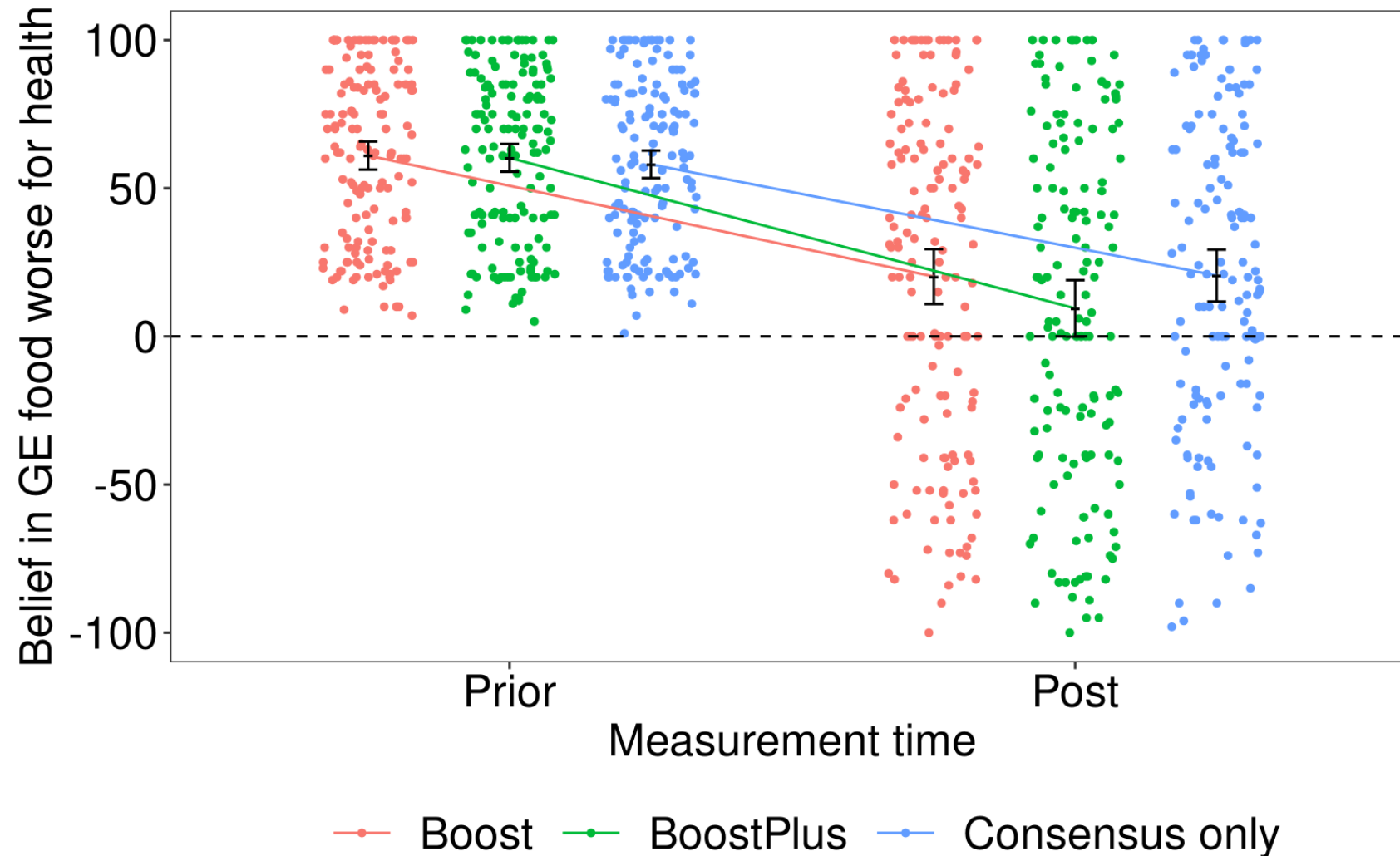
Consensus only (consensus)

Model formulation

Use post as response variable and prior beliefs as a control variable in the analysis of covariance.

their response was measured on a visual analogue scale ranging from -100 (I am 100% certain this is false) to 100 (I am 100% certain this is true) with 0 (I don't know) in the middle.

Plot of post vs prior response



Model formulation

Average for the r th replication of the i th experimental group is

$$\begin{aligned} E(\text{post}_{ir}) &= \mu + \alpha_i \text{condition}_i + \beta \text{prior}_{ir}. \\ \text{Va}(\text{post}_{ir}) &= \sigma^2 \end{aligned}$$

We assume that there is no interaction between `condition` and `prior`

- the slopes for `prior` are the same for each `condition` group.
- the effect of `prior` is linear

Contrasts of interest

1. Difference between average boosts (Boost and BoostPlus) and control (consensus)
2. Difference between Boost and BoostPlus (pairwise)

Inclusion of the prior score leads to increased precision for the mean (reduces variability).

Contrasts with ANCOVA

- The estimated marginal means will be based on detrended values \neq group averages
- In the `emmeans` package, the average of the covariate is used as value.
- the difference between levels of `condition` are the same for any value of `prior` (parallel lines), but the uncertainty changes.

Multiple testing adjustments:

- Methods of Bonferroni (prespecified number of tests) and Scheffé (arbitrary contrasts) still apply
- Can't use Tukey anymore (adjusted means are not independent anymore).

Data analysis - loading data

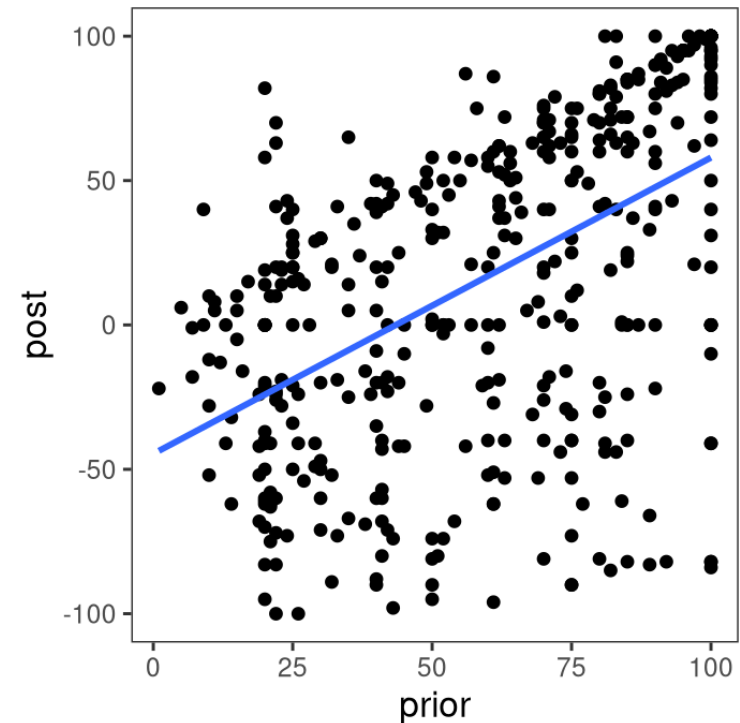
```
library(emmeans)
options(contrasts = c("contr.sum", "contr.poly"))
data(SSVB21_S2, package = "hecedsm")
# Check balance
with(SSVB21_S2, table(condition))
```

```
## condition
##      Boost BoostPlus consensus
##      149      147      146
```

Data analysis - scatterplot

```
library(ggplot2)
ggplot(data = SSVB21_S2,
       aes(x = prior, y = post)) +
  geom_point() +
  geom_smooth(method = "lm",
             se = FALSE)
```

Strong correlation; note responses that achieve max of scale.



Data analysis - model

```
# Check that the data are well randomized
car::Anova(lm(prior ~ condition, data = SSVB21_S2), type = 3)
# Fit linear model with continuous covariate
model1 <- lm(post ~ condition + prior, data = SSVB21_S2)
# Fit model without for comparison
model2 <- lm(post ~ condition, data = SSVB21_S2)
# Global test for differences - of NO INTEREST
car::Anova(model1, type = 3)
car::Anova(model2, type = 3)
```

Data analysis - ANOVA table

term	sum of squares	df	statistic	p-value
(Intercept)	166341	1	71.7	0.00
condition	14107	2	3.0	0.05
prior	385385	1	166.1	0.00
Residuals	1016461	438		

term	sum of squares	df	statistic	p-value
(Intercept)	123377	1	38.64	0.000
condition	11680	2	1.83	0.162
Residuals	1401846	439		

Data analysis - contrasts

```
emm1 <- emmeans(model1, specs = "condition")
# Note order: Boost, BoostPlus, consensus
emm2 <- emmeans(model2, specs = "condition")
# Not comparable: since one is detrended and the other isn't
contrast_list <- list(
  "boost vs control" = c(0.5, 0.5, -1),
  #av. boosts vs consensus
  "Boost vs BoostPlus" = c(1, -1, 0))
contrast(emm1,
         method = contrast_list,
         p.adjust = "holm")
```

Data analysis - t -tests

contrast	estimate	se	df	t stat	p- value
boost vs control	-8.37	4.88	438	-1.72	0.09
Boost vs BoostPlus	9.95	5.60	438	1.78	0.08

Contrasts with ANCOVA with prior (Holm-Bonferroni adjustment with $k = 2$ tests)

contrast	estimate	se	df	t stat	p- value
boost vs control	-5.71	5.71	439	-1.00	0.32
Boost vs BoostPlus	10.74	6.57	439	1.63	0.10

Contrasts for ANOVA (Holm-Bonferroni adjustment with $k = 2$ tests)

Data analysis - assumption checks

```
# Test equality of variance
levene <- car::leveneTest(
  resid(model1) ~ condition,
  data = SSVB21_S2,
  center = 'mean')
# Equality of slopes (interaction)
car::Anova(lm(post ~ condition * prior,
  data = SSVB21_S2),
  model1, type = 3)
```

Levene's test of equality of variance: $F(2, 439) = 2.05$ with a p -value of 0.13.

term	sum of squares	df	statistic	p-value
(Intercept)	165573	1	71.3	0.0
condition	4245	2	0.9	0.4
prior	382596	1	164.9	0.0
condition:prior	3257	2	0.7	0.5
Residuals	1016461	438		

Model with interaction `condition*prior`.
Slopes don't differ between condition.

The kitchen sink approach

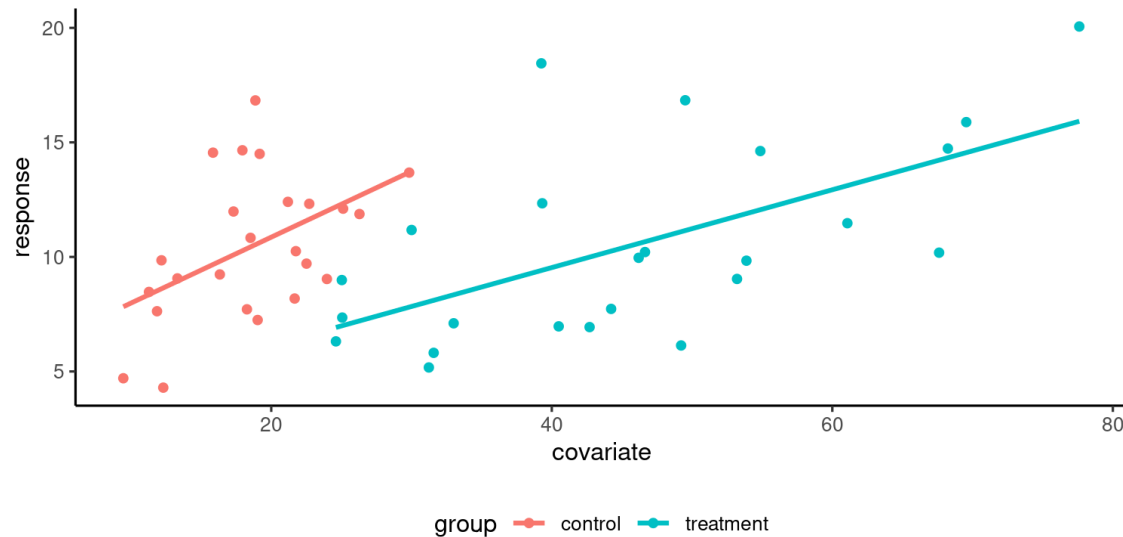
Should we control for more stuff?

NO! ANCOVA is a design to reduce error

- Randomization should ensure that there is no confounding
- No difference (on average) between group given a value of the covariate.
- If it isn't the case, adjustment matters

Equal trends

- If trends are different, meaningful comparisons (?)
- Differences between groups depend on value of the covariate



Due to lack of overlap, comparisons hazardous as they entail extrapolation one way or another.

Testing equal slope

Compare two nested models

- Null \mathcal{H}_0 : model with covariate
- Alternative \mathcal{H}_a : model with interaction covariate * experimental factor

Use `anova` to compare the models in **R**.