Causal mediation

Session 12

MATH 80667A: Experimental Design and Statistical Methods
for Quantitative Research in Management
HEC Montréal

Outline

Directed acyclic graphs

Causal mediation

Linear SEM and mediation

Directed acyclic graphs

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Types of data

Experimental

You have control over which units get treatment

Observational

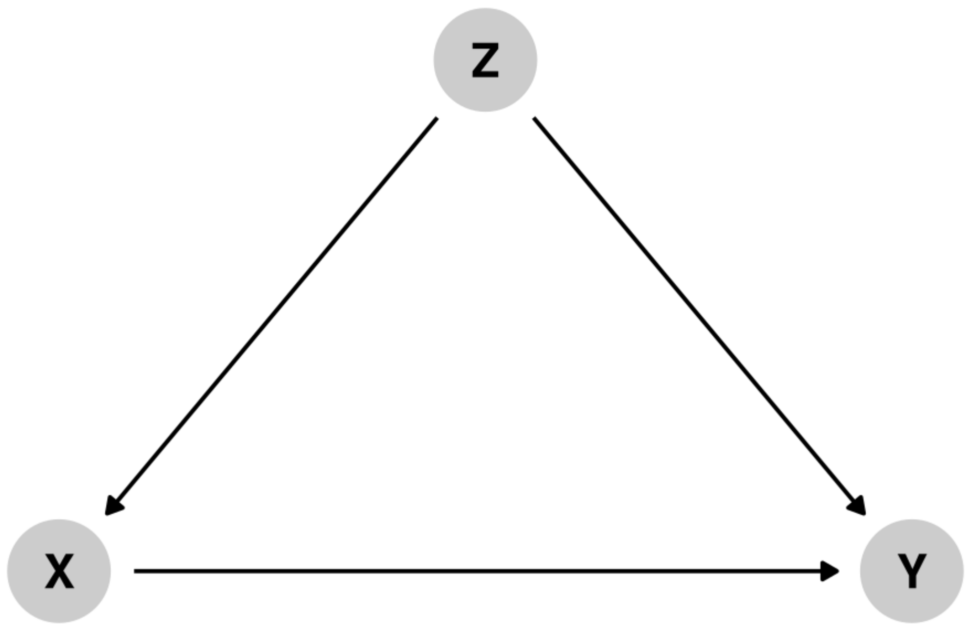
You don't have control over which units get treatment

Causal diagrams

Directed acyclic graphs (DAGs)

Directed: Each node has an arrow that points to another node

Acyclic: You can't cycle back to a node (and arrows only have one direction)



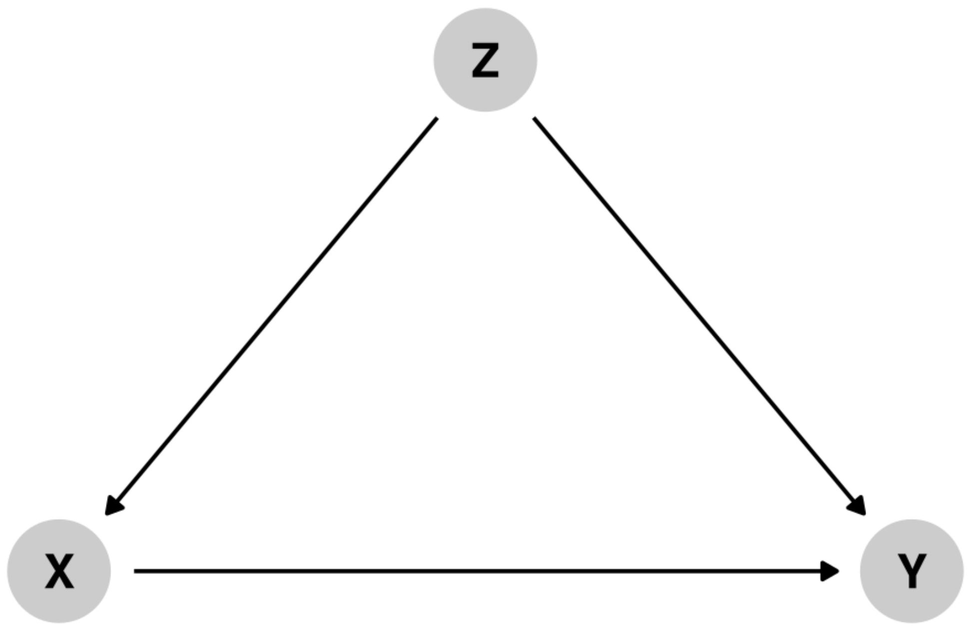
Causal diagrams

Directed acyclic graphs (DAGs)

Graphical model of the process that generates the data

Maps your philosophical model

Fancy math ("*do*-calculus") tells you what to control for to isolate and identify causation



How to draw a DAG

What is the causal effect of an additional year of education on earnings?

Step 1: List variables

Step 2: Simplify

Step 3: Connect arrows

Step 4: Use logic and math to determine which nodes and arrows to measure

1. List variables

Education (treatment) → Earnings (outcome)

Location | Ability | Demographics

Socioeconomic status | Year of birth

Compulsory schooling laws | Job connections

2. Simplify

Education (treatment) → Earnings (outcome)

Location

Ability

Demographics

Socioeonomic status

Year of birth

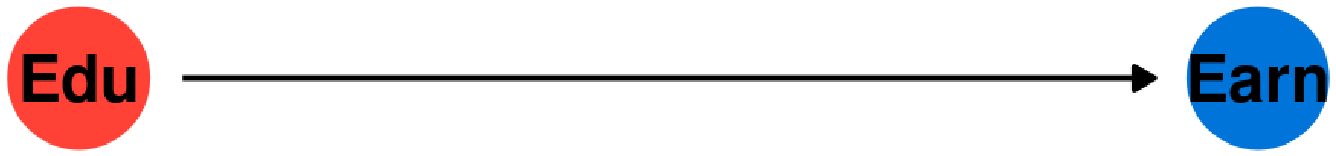
Compulsory schooling laws

Job connections

Background

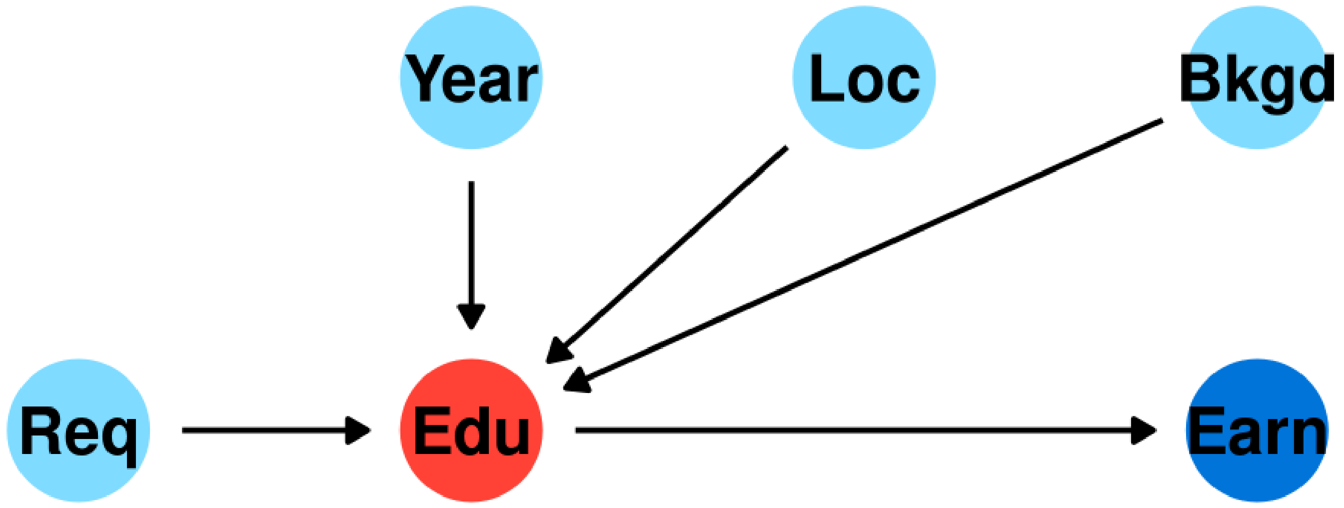
3. Draw arrows

Education causes earnings



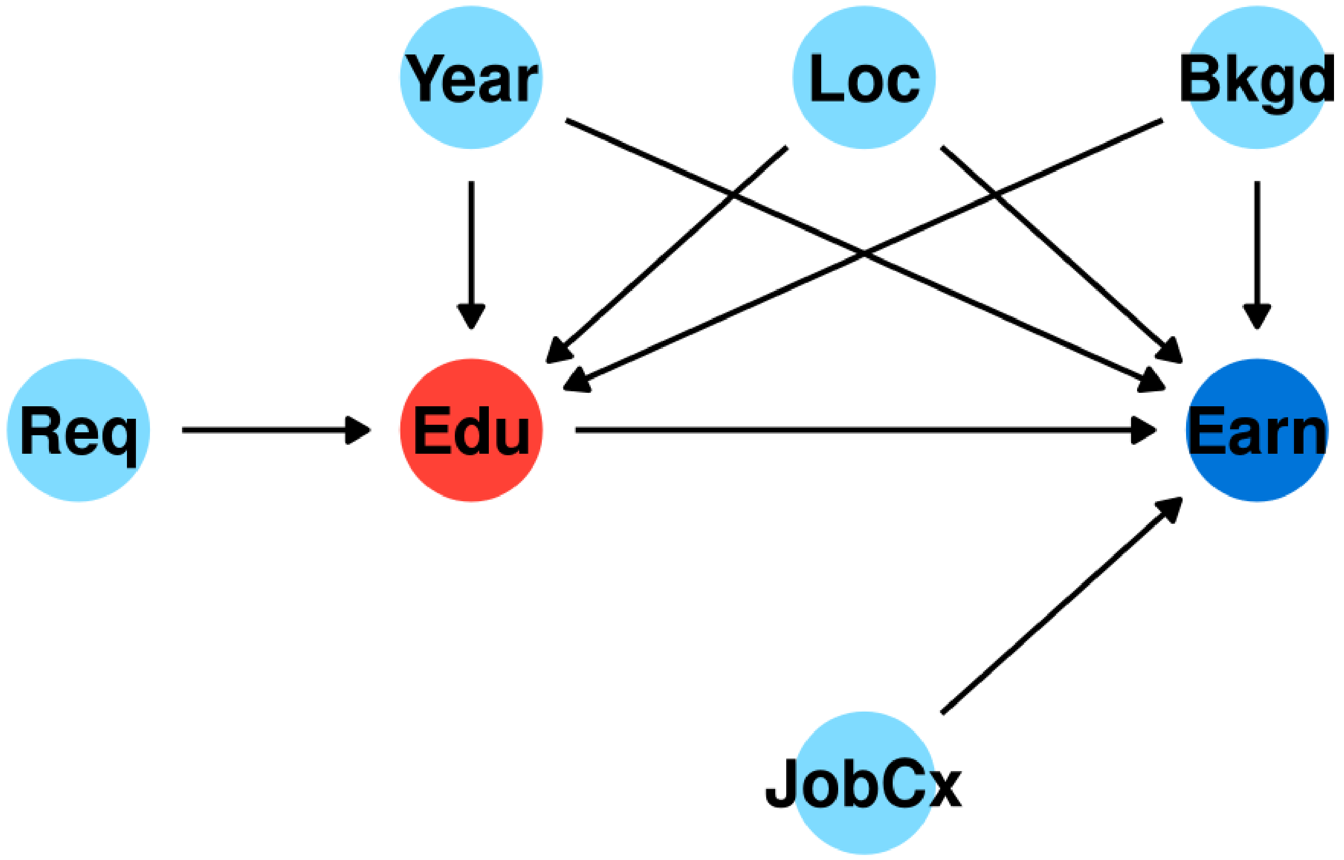
3. Draw arrows

Background, year of birth, location, job connections, and school requirements all cause education



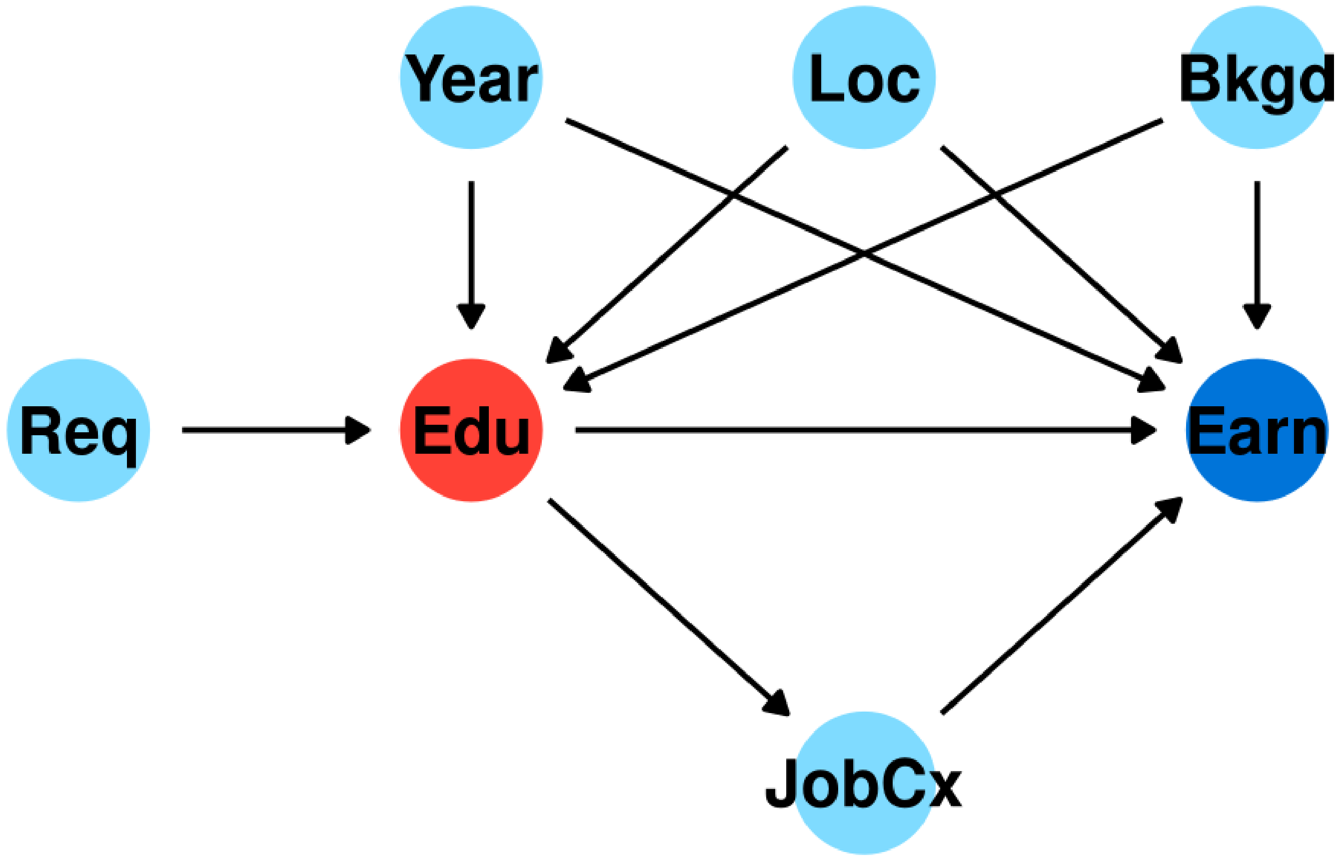
3. Draw arrows

Background, year of birth, and location all cause earnings too



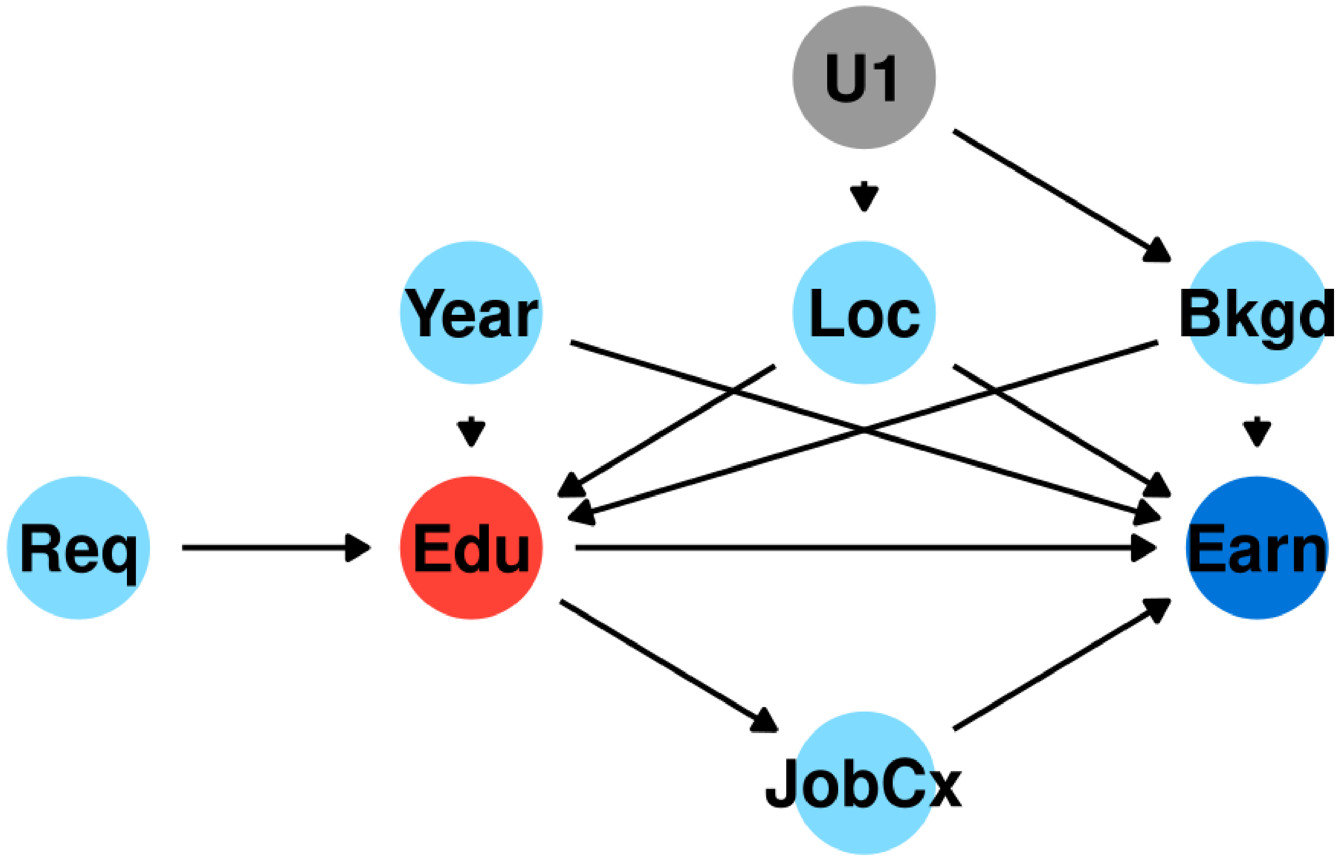
3. Draw arrows

Education causes job earnings



3. Draw arrows

Location and background are probably related, but neither causes the other. Something unobservable (U1) does that.



Causal identification

A causal effect is *identified* if the association between treatment and outcome is propertly stripped and isolated

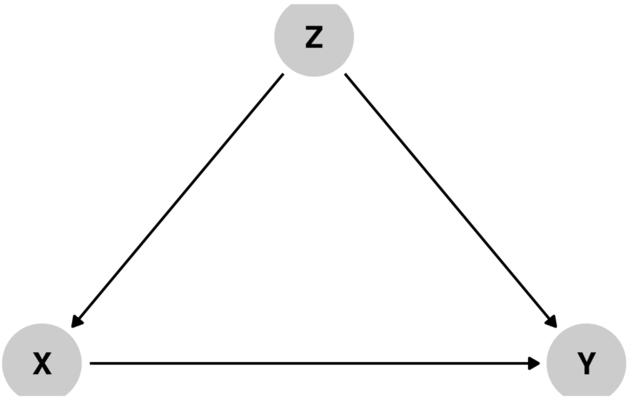
Paths and associations

Arrows in a DAG transmit associations

You can redirect and control those paths by "adjusting" or "conditioning"

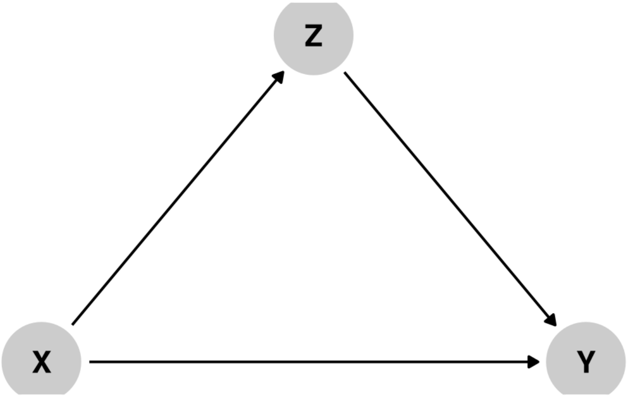
Three types of associations

Confounding



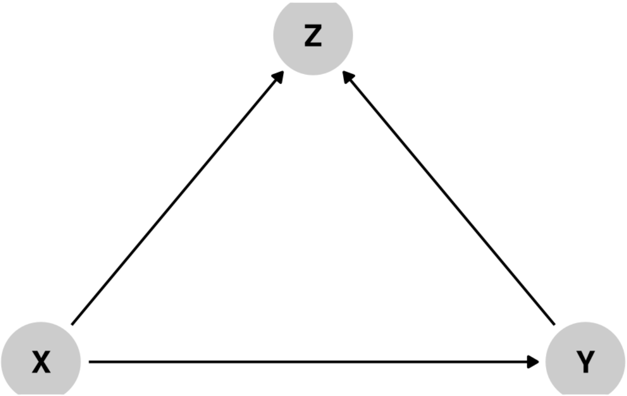
Common cause

Causation



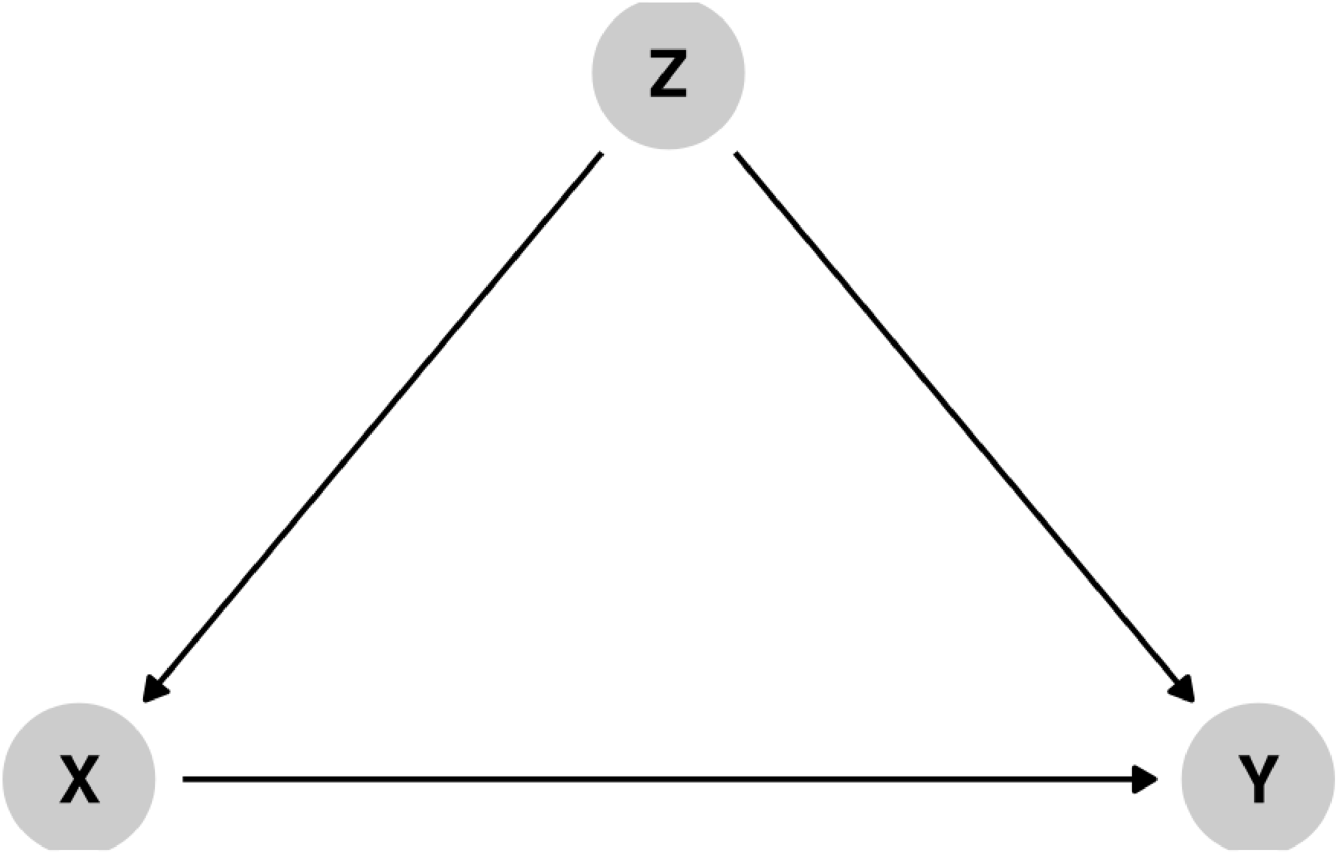
Mediation

Collision



Selection / endogeneity

Confounding

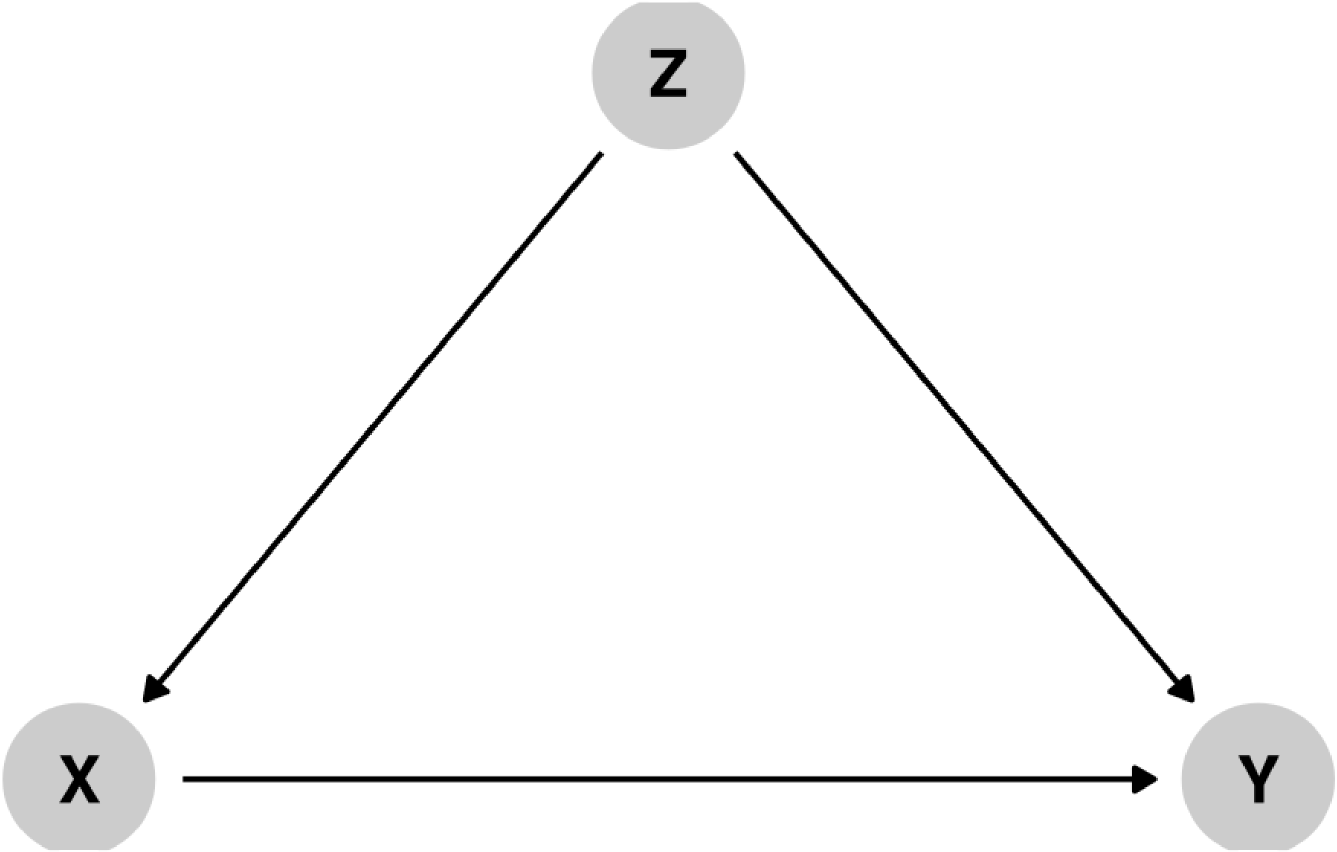


X causes Y

But Z causes both X and Y

Z *confounds* the X → Y association

Paths



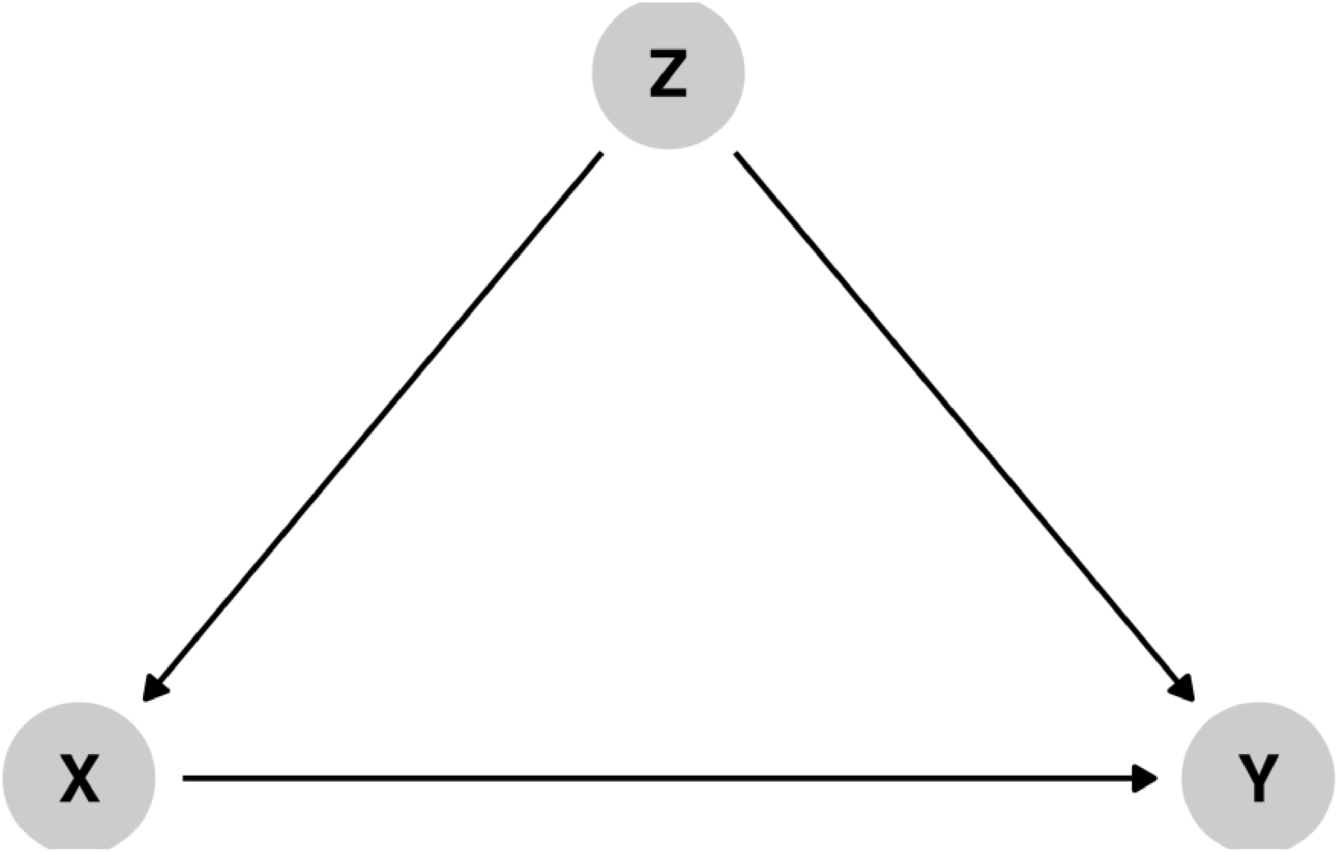
Paths between X and Y?

$X \rightarrow Y$

$X \leftarrow Z \rightarrow Y$

Z is a *backdoor*

d-connection

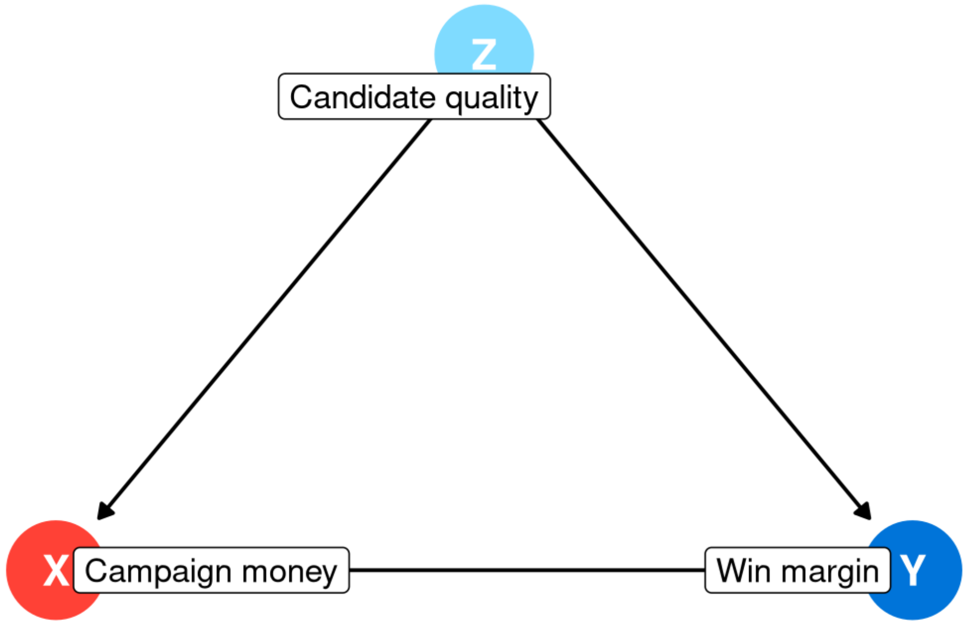


X and Y are "*d*-connected" because associations can pass through Z

The relationship between X and Y is not identified / isolated

Effect of money on elections

What are the paths between money and win margin?

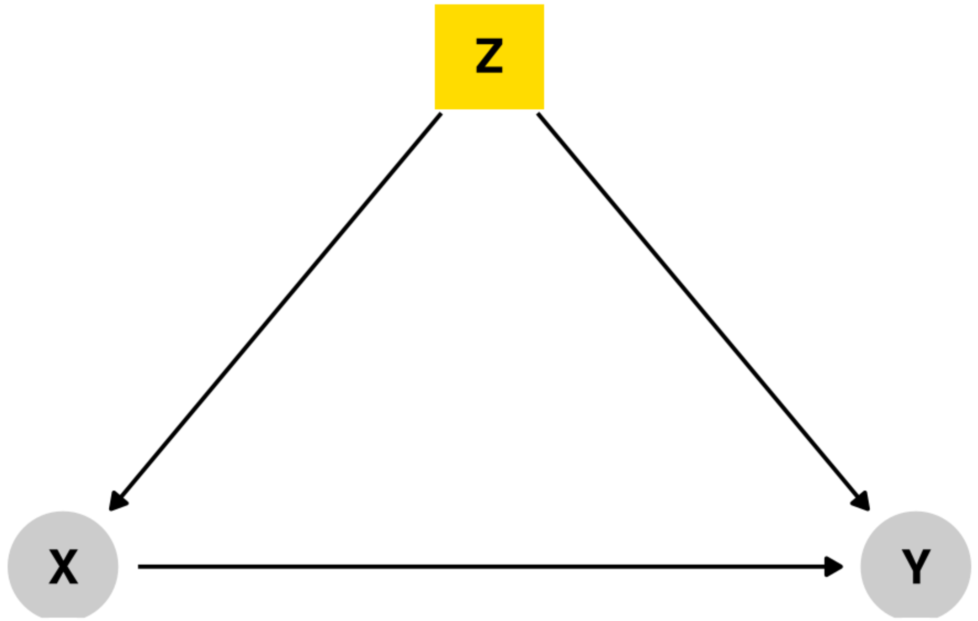


Money → Margin

Money ← Quality → Margin

Quality is a *backdoor*

Closing doors



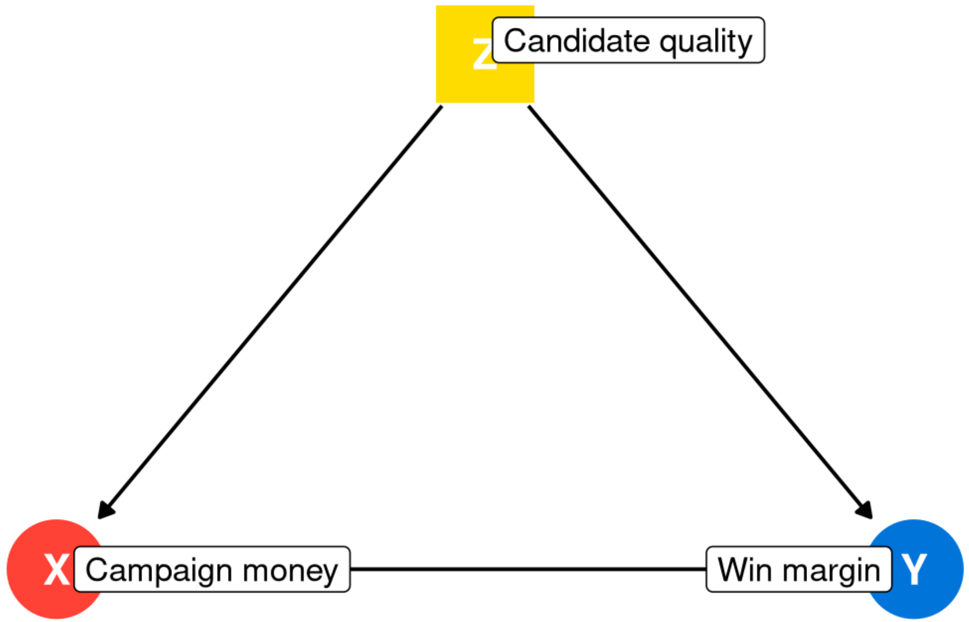
Close the backdoor by adjusting for Z

Closing doors

Find the part of campaign money that is explained by quality, remove it. This is the residual part of money.

Find the part of win margin that is explained by quality, remove it. This is the residual part of win margin.

Find the relationship between the residual part of money and residual part of win margin.
This is the causal effect.

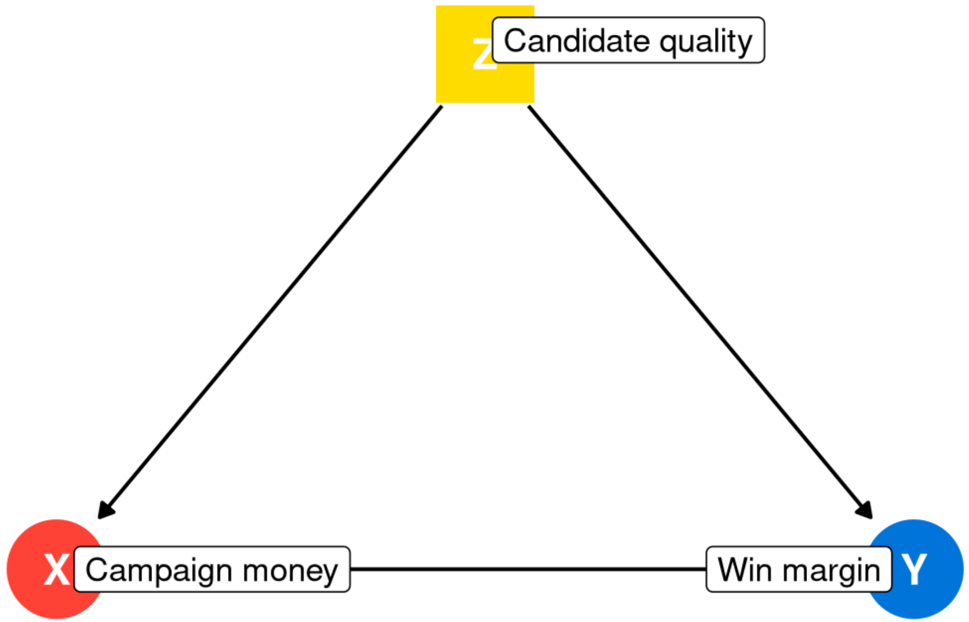


Closing doors

Compare candidates as if they had the same quality

Remove differences that are predicted by quality

Hold quality constant



How to adjust

Include term in regression

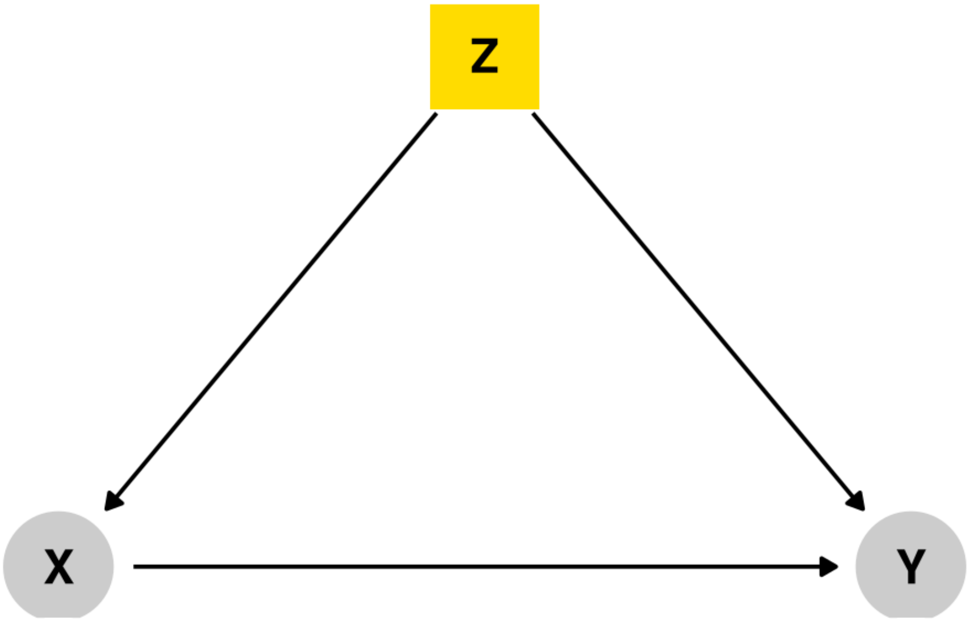
$$\begin{aligned}\text{Win margin} = &\beta_0 + \beta_1 \text{Campaign money} + \\ &\beta_2 \text{Candidate quality} + \varepsilon\end{aligned}$$

Matching

Stratifying

Inverse probability weighting

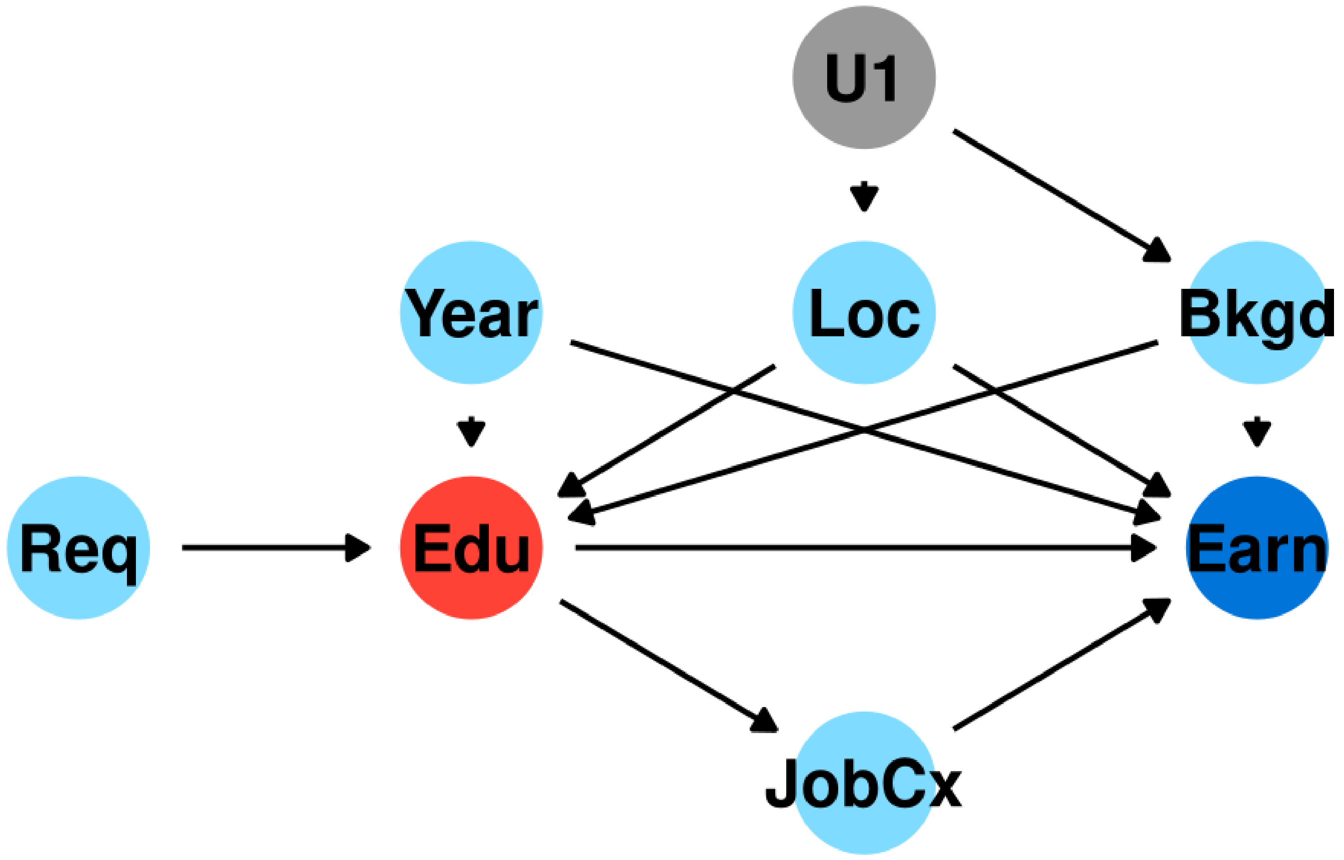
d-separation



If we control for Z, X and Y are now "*d*-separated" and the association is isolated!

Closing backdoors

Block all backdoor paths to identify the main pathway you care about



All paths

Education → Earnings

Education → Job connections → Earnings

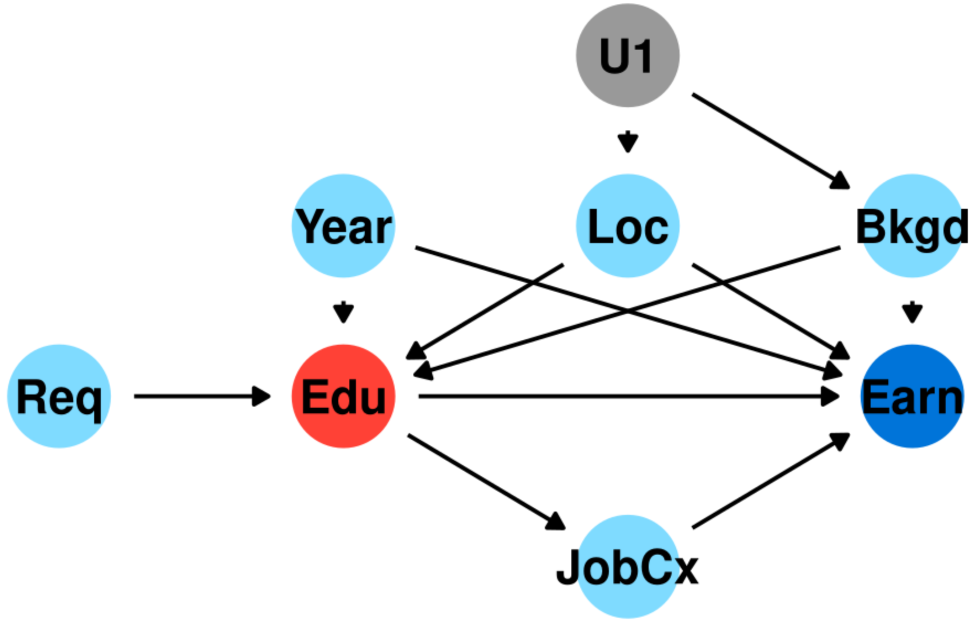
Education ← Background → Earnings

Education ← Background ← U1 → Location → Earnings

Education ← Location → Earnings

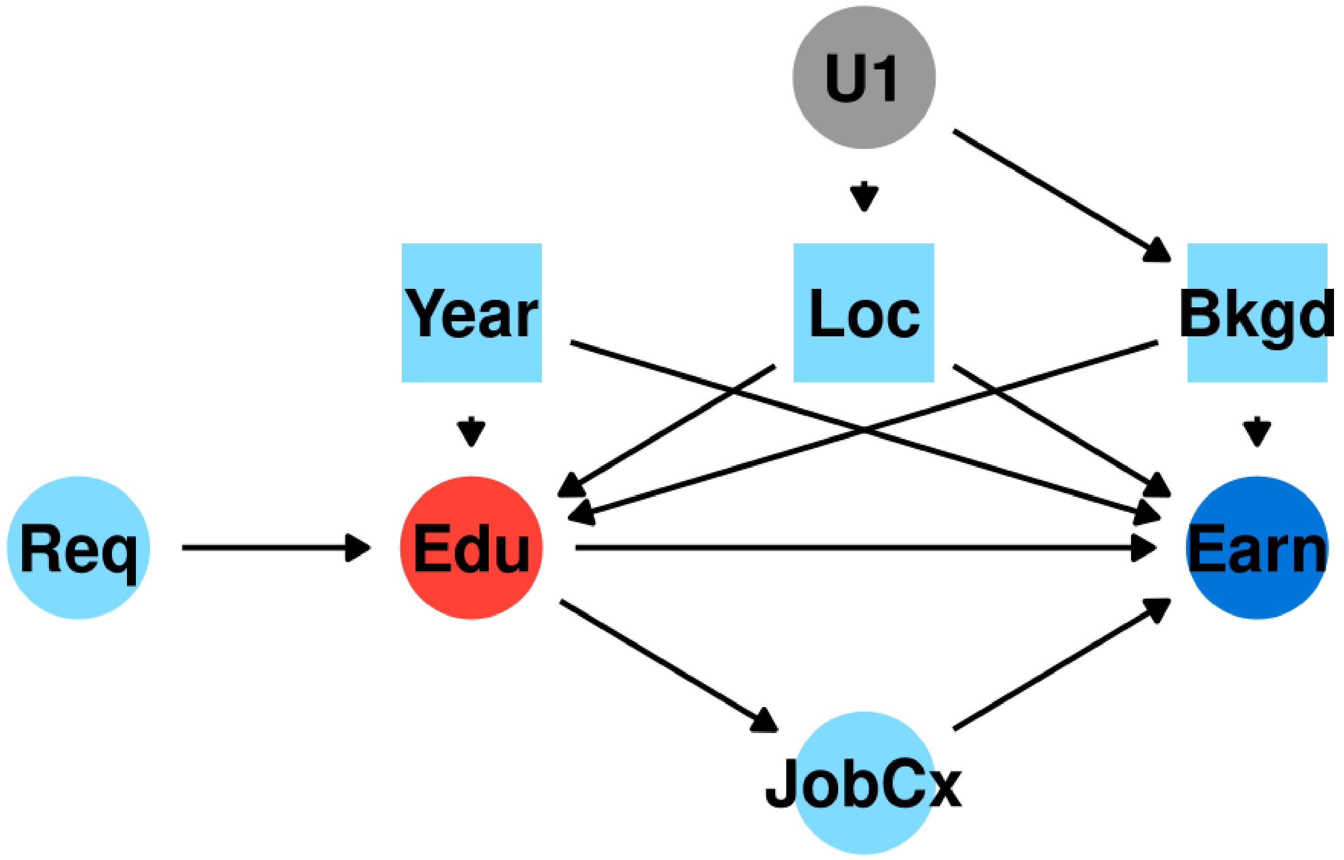
Education ← Location ← U1 → Background → Earnings

Education ← Year → Earnings



All paths

Adjust for Location, Background and Year to isolate the Education → Earnings causal effect

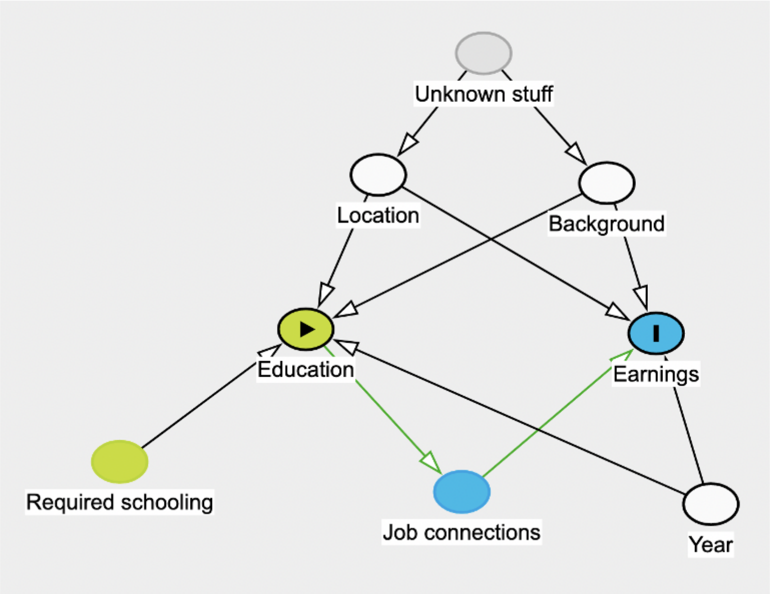


How do you know if this is right?

You can test the implications of the model to see if they're right in your data

$$X \perp Y \mid Z$$

X is independent of Y, given Z

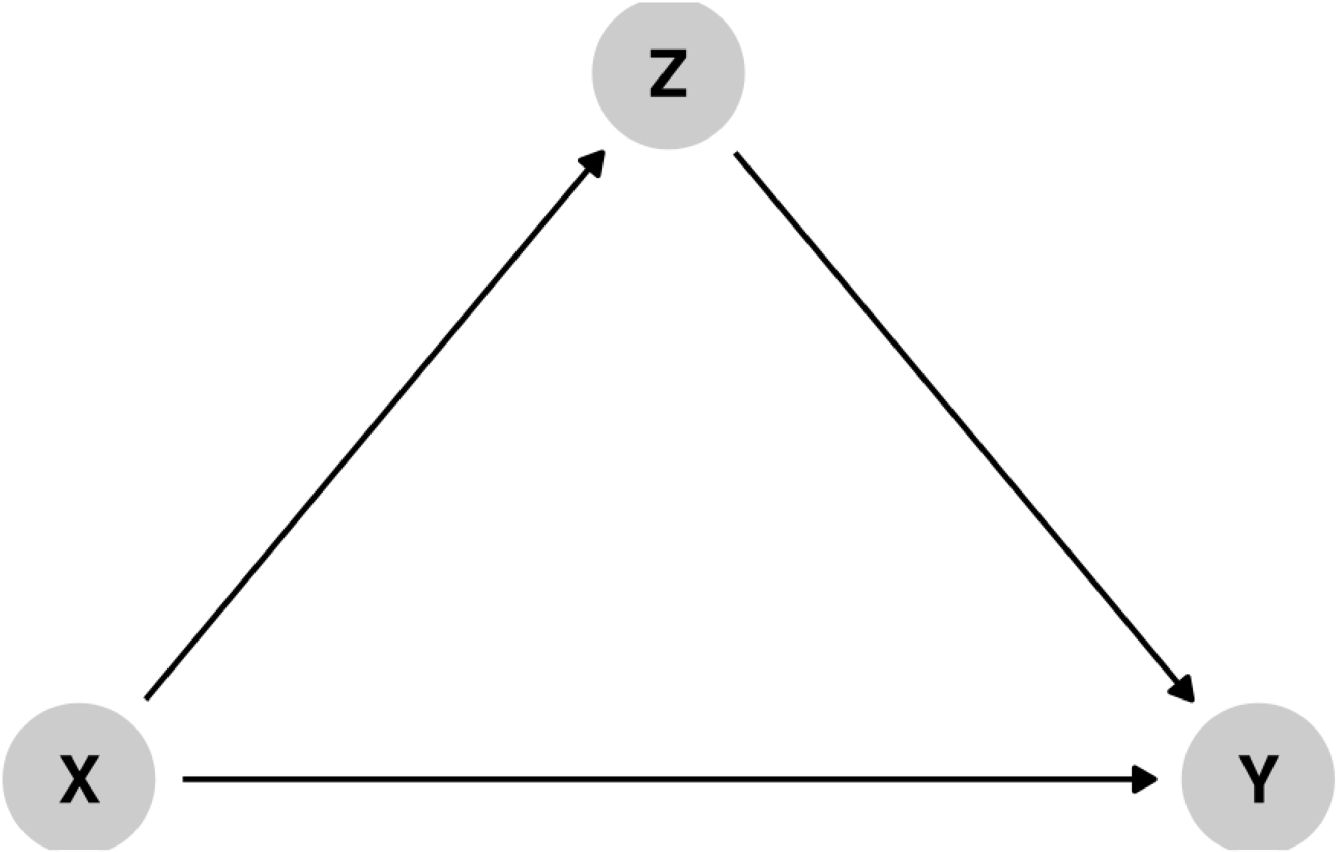


Testable implications

The model implies the following conditional independences:

- Education ⊥ Earnings | Background, Job connections, Location, Year
- Required schooling ⊥ Job connections | Education
- Required schooling ⊥ Year
- Required schooling ⊥ Earnings | Background, Job connections, Location, Year
- Required schooling ⊥ Earnings | Background, Education, Location, Year
- Required schooling ⊥ Background
- Required schooling ⊥ Location
- Job connections ⊥ Year | Education
- Job connections ⊥ Background | Education
- Job connections ⊥ Location | Education
- Year ⊥ Background
- Year ⊥ Location

Causation

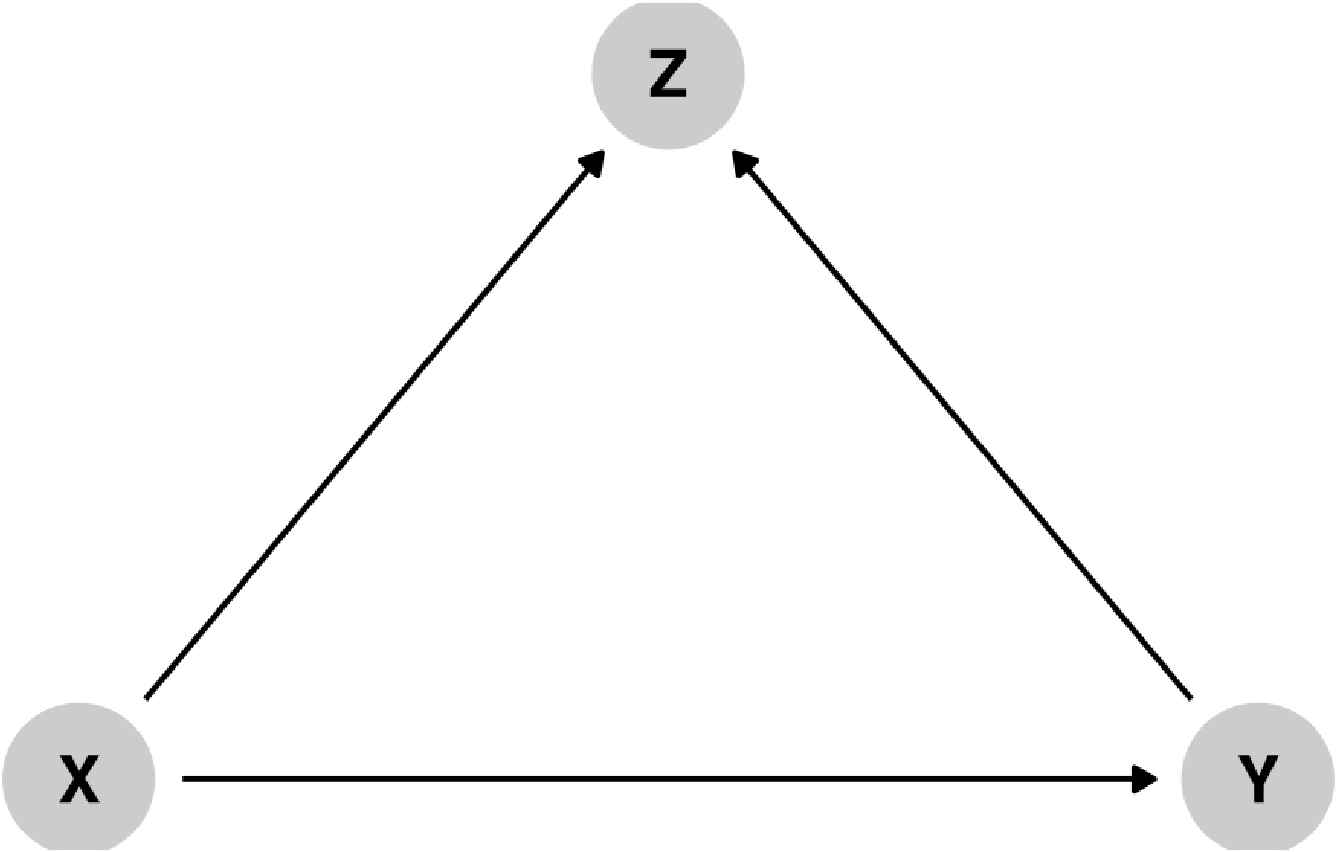


X causes Y

X causes Z which causes Y

Z is a mediator

Colliders



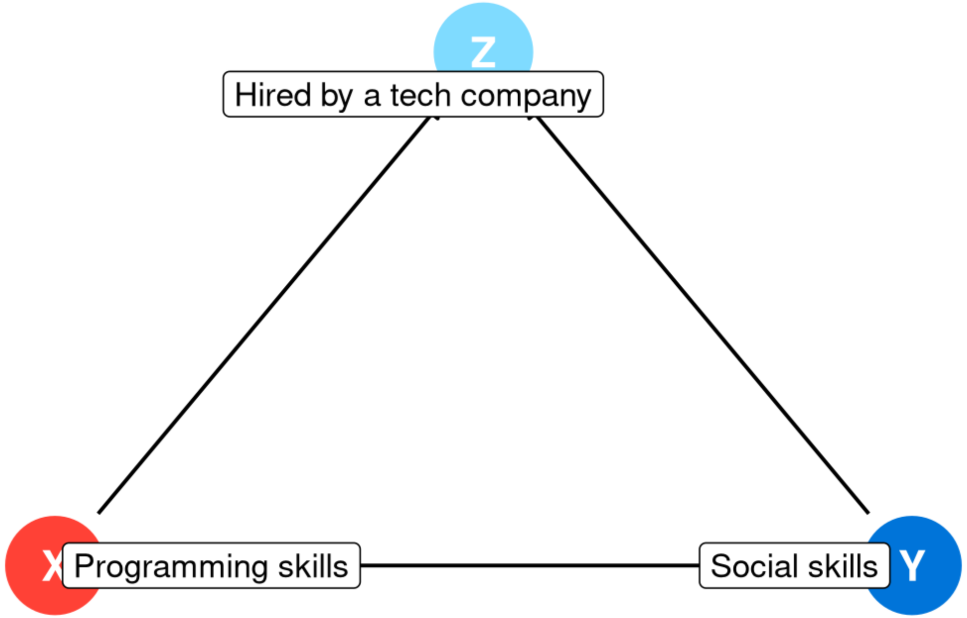
X causes Z

Y causes Z

Should you control for Z?

Programming and social skills

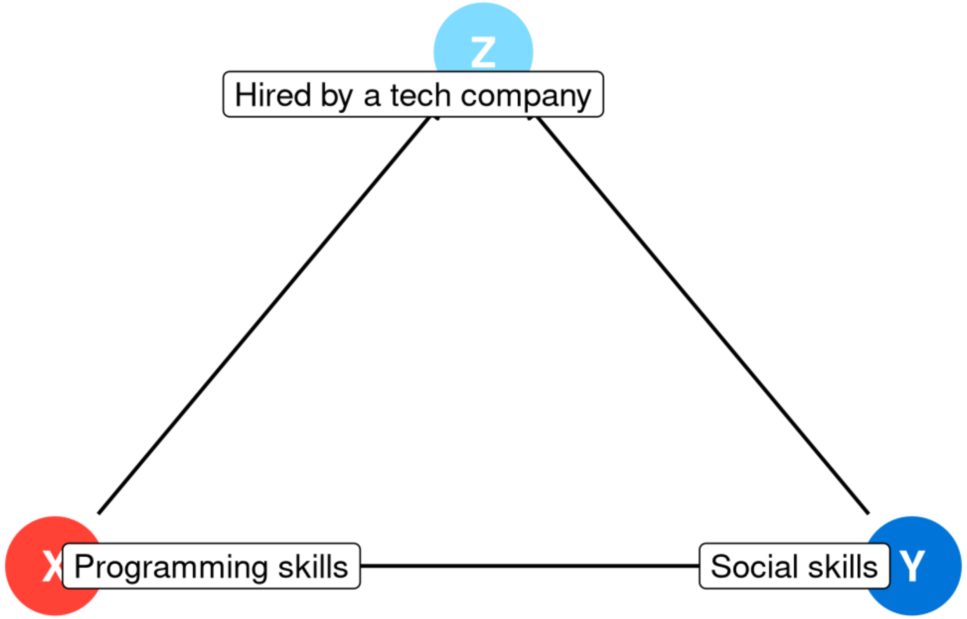
Do programming skills reduce social skills?



You go to a tech company and conduct a survey. You find a negative relationship!
Is it real?

Programming and social skills

Do programming skills reduce social skills?

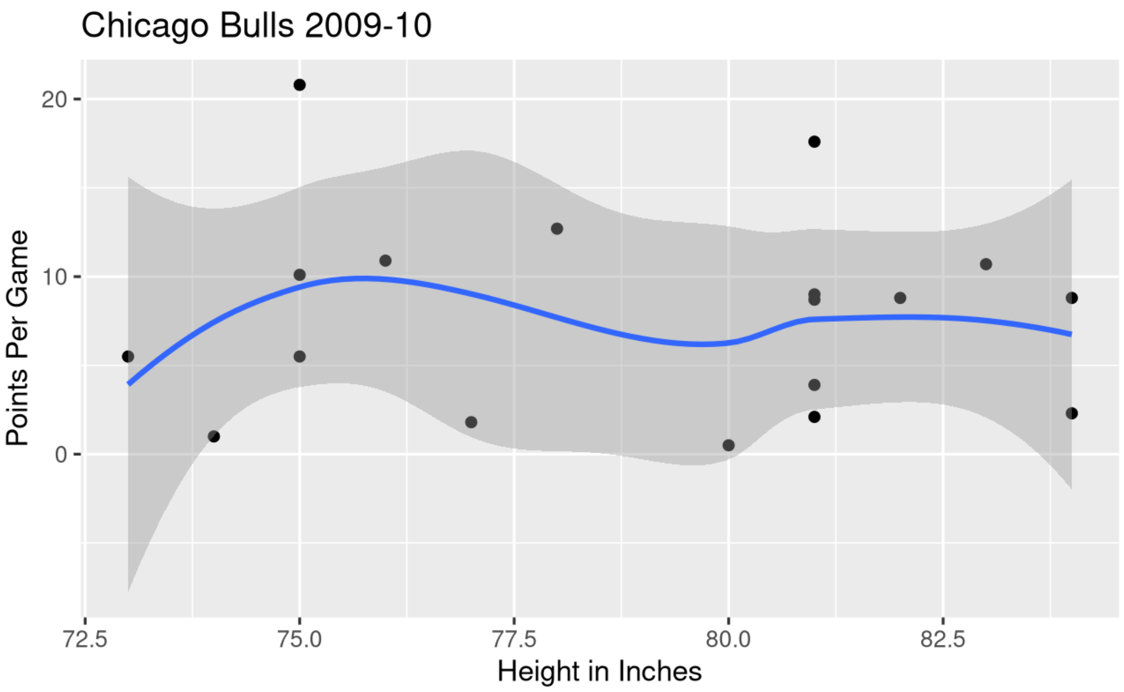


No! **Hired by a tech company** is a collider and we controlled for it.

This inadvertently connected the two.

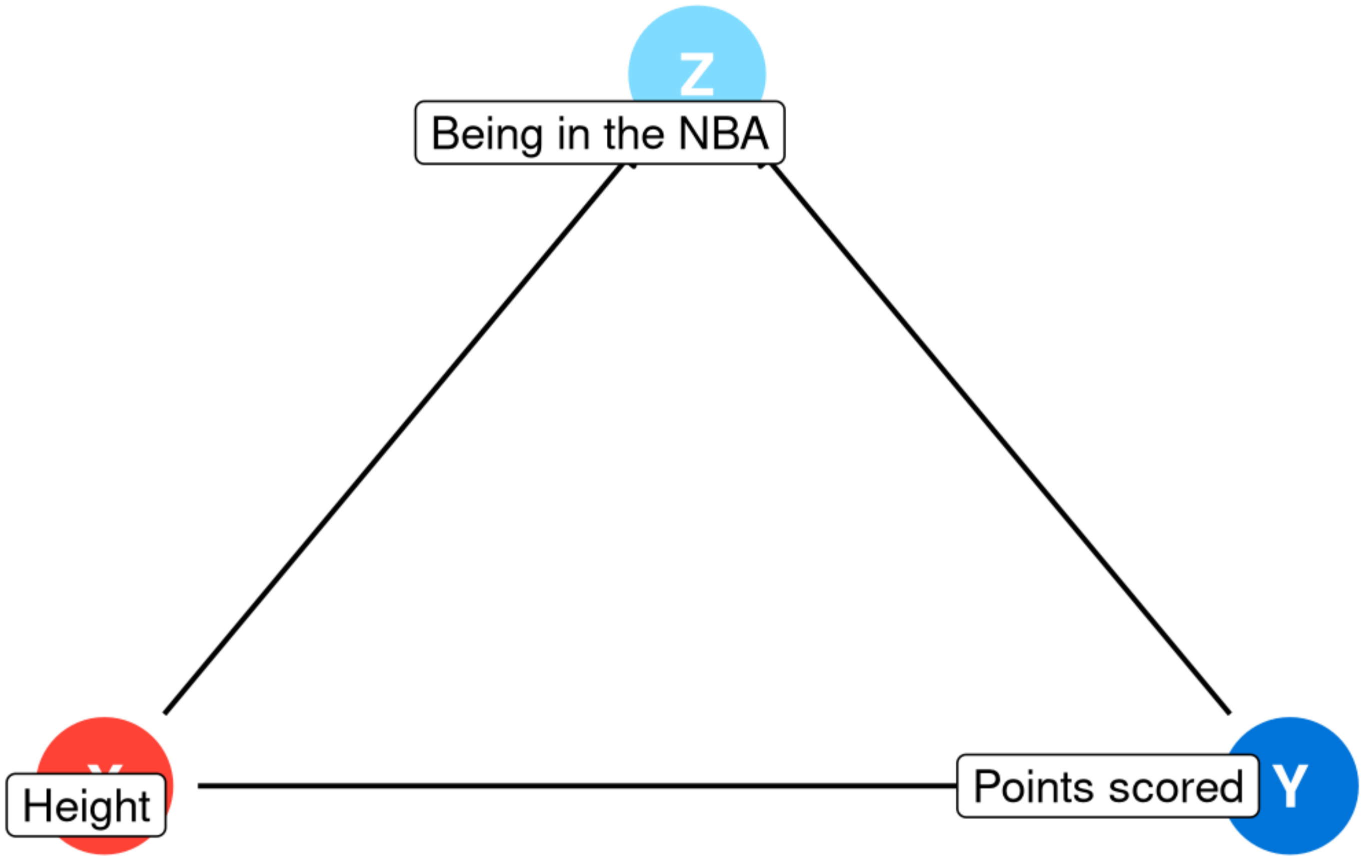
Colliders can create fake causal effects

Colliders can hide real causal effects

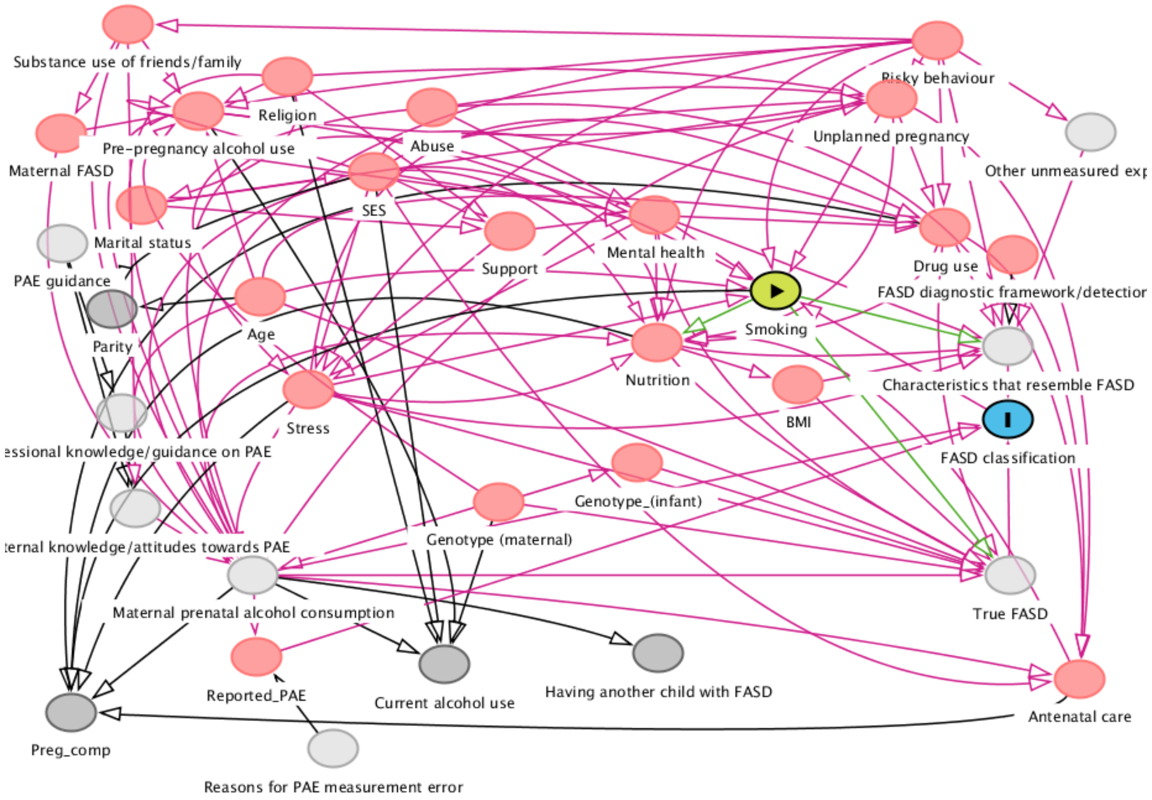


Height is unrelated to basketball skill… among NBA players

Colliders and selection bias



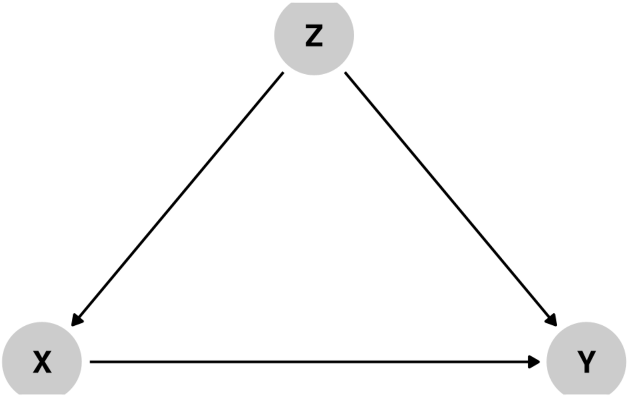
Life is inherently complex



Postulated DAG for the effect of smoking on fetal alcohol spectrum disorders (FASD)

Three types of associations

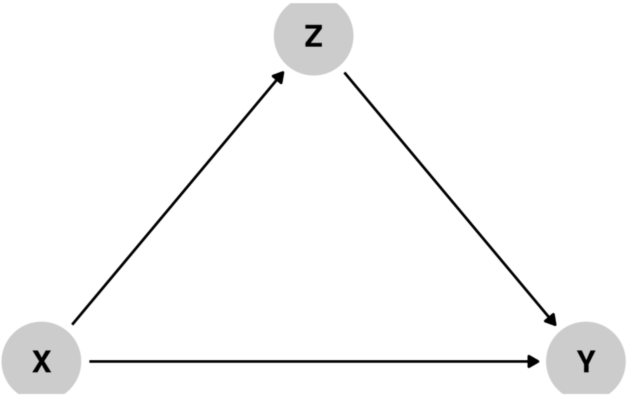
Confounding



Common cause

Causal forks X ← Z → Y

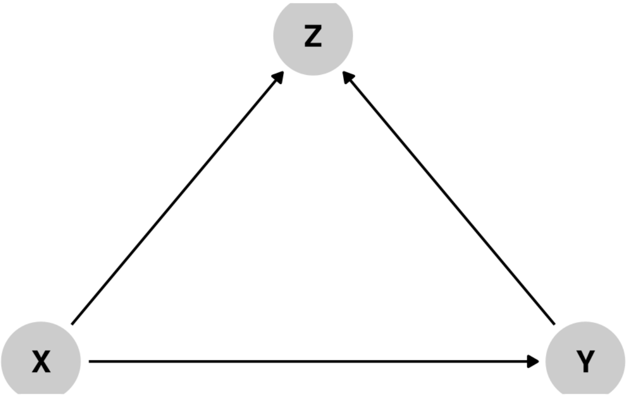
Causation



Mediation

Causal chain X → Z → Y

Collision



Selection / endogeneity

inverted fork X → Z ← Y

Causal mediation

Key references

- Imai, Keele and Tingley (2010), A General Approach to Causal Mediation Analysis, *Psychological Methods*.
- Pearl (2014), Interpretation and Identification of Causal Mediation, *Psychological Methods*.
- Baron and Kenny (1986), The Moderator-Mediator Variable Distinction in Social Psychological Research: Conceptual, Strategic, and Statistical Considerations, *Journal of Personality and Social Psychology*
- Zhao, Lynch and Chen (2010), Reconsidering Baron and Kenny: Myths and Truths about Mediation Analysis, *Journal of Consumer Research*
- David Kenny's website

Fundamental problem of causal inference

Causal effect is a form of average treatment

For binary treatment X_i

$$\mathsf{E}[Y_i \mid \mathrm{do}(X_i = 1)] - \mathsf{E}[Y_i \mid \mathrm{do}(X_i = 0)]$$

Observe outcome for a single treatment

Effect cannot be estimated directly in general, even with randomized experiments.

Examples of mediation

Pearl (2014) discusses an encouragement design with a new education program X (say binary), M the amount of homework and the score on an exam as outcome Y

Data generating mechanism

Assume structural models

$$\begin{aligned} X &= f_X(U_X) \\ M &= f_M(X, U_M) \\ Y &= f_Y(X, M, U_Y) \end{aligned}$$

with U_X, U_M, U_Y representing unexplained variability affecting each of the random variables:

- treatment/experimental factor X
- mediator M
- response Y

Counterfactual framework

$$
\begin{aligned}
M_x &= f_M(x, U), \\
Y_x &= f_Y(x, M_x, U), \\
Y_{x,m} &= f_Y(x, m, U)
\end{aligned}
$$

Potential outcomes: existence of hypothetical values for $Y(X)$

(what-if?)

Assumption of sequential ignorability

Given pre-treatment covariates W, potential outcomes for mediation and treatment are conditionally independent of treatment assignment.

$$Y_i(x', m), M_i(x) \perp\!\!\!\perp X_i \mid W_i = w$$

Given pre-treatment covariates and observed treatment, potential outcomes are independent of mediation.

$$Y_i(x', m) \perp\!\!\!\perp M_i(x) \mid X_i = x, W_i = w$$

The first condition is satisfied in experimental studies due to randomization.

Roadmap

Pearl (2014) defines causal effects in terms of f_T, f_M, f_Y

- Draw a DAG representing the system
- Determine whether we can identify those effects on the basis of data
- Choose a statistical model to estimate the causal effects of interest

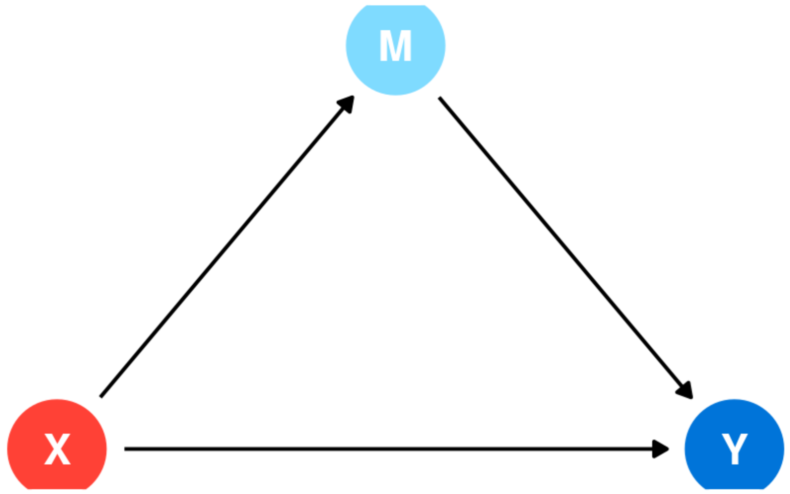
Total effect

Total effect: overall impact of X (both through M and directly)

$$\text{TE}(x, x^*) = \text{E}[Y \mid \mathrm{do}(X = x)] - \text{E}[Y \mid \mathrm{do}(X = x^*)]$$

This can be generalized for continuous X to any pair of values (x_1, x_2).

X → M → Y
plus
X → Y



Average controlled direct effect

$$
\begin{aligned}
\mathsf{CDE}(m, x, x^*) &= \mathsf{E}[Y \mid \mathrm{do}(X = x, m = m)] - \mathsf{E}[Y \mid \mathrm{do}(X = x^*, m = m)] \\
&= \mathsf{E}(Y_{x,m} - Y_{x^*,m})
\end{aligned}
$$

Expected population change in response when the experimental factor changes from x to x^* and the mediator is set to a fixed value m.

Direct and indirect effects

Natural direct effect: $\mathsf{NDE}(x, x^*) = \mathsf{E}(Y_{x,M_{x^*}} - Y_{x^*,M_{x^*}})$

- expected change in Y under treatment x if M is set to whatever value it would take under control x^*

Natural indirect effect: $\mathsf{NIE}(x, x^*) = \mathsf{E}(Y_{x^*,M_x} - Y_{x^*,M_{x^*}})$

- expected change in Y if we set X to its control value and change the mediator value which it would attain under x

Counterfactual conditioning reflects a physical intervention, not mere (probabilistic) conditioning.

Total effect is $\mathsf{TE}(x, x^*) = \mathsf{NDE}(x, x^*) - \mathsf{NIE}(x^*, x)$

Linear structural equation modelling and mediation

The Baron–Kenny model

Assume the following models:

$$
\begin{aligned}
f_M(x) &= c_M + \alpha x + U_M \\
f_Y(x, m) &= c_Y + \beta x + \gamma m + U_Y
\end{aligned}
$$

Plugging the first equation in the second, we get the marginal model for Y given treatment X,

$$
\begin{aligned}
\mathsf{E}_{U_M}(Y \mid x) &= \underbrace{(c_Y + \gamma c_M)}_{\text{intercept}} + \underbrace{(\beta + \alpha\gamma)}_{\text{total effect}} \cdot x + \underbrace{(\gamma U_M + U_Y)}_{\text{error}} \\
&= c_Y' + \tau X + U_Y'
\end{aligned}
$$

The old method

Baron and Kenny recommended running regressions and estimating the three models with

1. whether $\mathscr{H}_0 : \alpha = 0$
2. whether $\mathscr{H}_0 : \tau = 0$ (total effect)
3. whether $\mathscr{H}_0 : \gamma = 0$

Then, test the CDE $\alpha\gamma = 0$ using Sobel's test statistic.

Problems?

Sobel's test

Based on estimators $\widehat{\alpha}$ and $\widehat{\gamma}$, construct a Wald-test

$$S = \frac{\widehat{\alpha}\widehat{\gamma}}{\sqrt{\widehat{\gamma}^2\mathsf{Va}(\widehat{\alpha}) + \widehat{\alpha}^2\mathsf{Va}(\widehat{\gamma}) + \mathsf{Va}(\widehat{\gamma})\mathsf{Va}(\widehat{\alpha})}} \mathrel{\dot{\sim}} \mathsf{No}(0,1)$$

where the point estimate $\widehat{\alpha}$ and its variance $\mathsf{Va}(\widehat{\alpha})$ can be estimated via SEM, or more typically linear regression (ordinary least squares).

Null distribution for the test

The large-sample normal approximation is poor in small samples.

The popular way to estimate the *p*-value and the confidence interval is through the nonparametric **bootstrap** with the percentile method.

Repeat B times, say $B = 10\,000$

1. sample **with replacement** n observations from the database
   - tuples (Y_i, X_i, M_i)
2. recalculate estimates $\widehat{\alpha}^{(b)}\widehat{\gamma}^{(b)}$

Boostrap *p*-values and confidence intervals

Confidence interval

Percentile-based method: for a equi-tailed $1-\alpha$ interval and the collection

$$\{\widehat{\alpha}^{(b)}\widehat{\gamma}^{(b)}\}_{b=1}^{B},$$

compute the $\alpha/2$ and $1-\alpha/2$ empirical quantiles.

Two-sided *p*-value

Compute the sample proportion of bootstrap statistics $S^{(1)},\ldots,S^{(B)}$ that are larger/smaller than zero.

If $S^{(M)} < 0 \leq S^{(M+1)}$ for $1 \leq M \leq B$.

$$p = 2\min\{M/B, 1-M/B\}$$

and zero otherwise

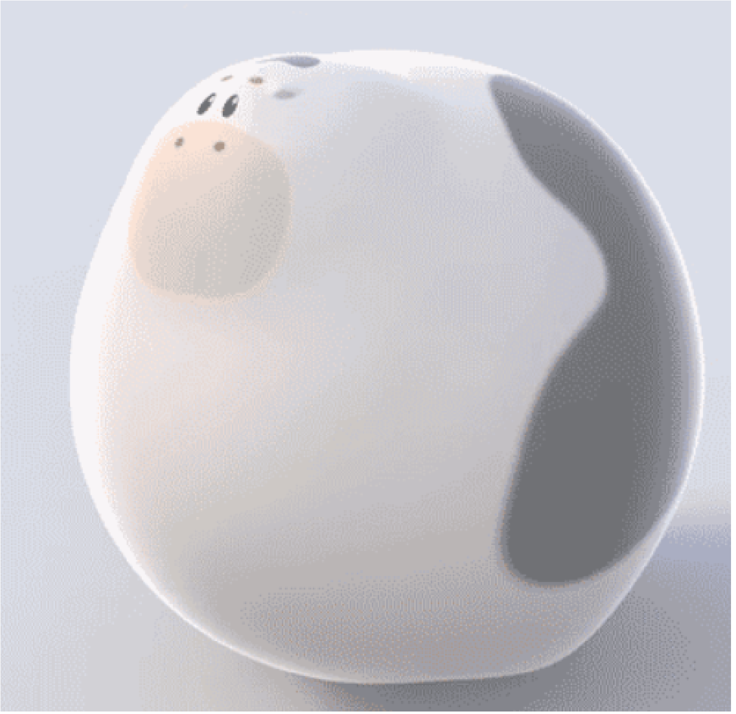
Example from Preacher and Hayes (2004)

> Suppose an investigator is interested in the effects of a new cognitive therapy on life satisfaction after retirement.
>
> Residents of a retirement home diagnosed as clinically depressed are randomly assigned to receive 10 sessions of a new cognitive therapy $(X = 1)$ or 10 sessions of an alternative (standard) therapeutic method $(X = 0)$.
>
> After Session 8, the positivity of the attributions the residents make for a recent failure experience is assessed (M).
>
> Finally, at the end of Session 10, the residents are given a measure of life satisfaction (Y). The question is whether the cognitive therapy's effect on life satisfaction is mediated by the positivity of their causal attributions of negative experiences. ”

Defaults of linear SEM

- Definitions contingent on model
  - (causal quantities have a meaning regardless of estimation method)
- Linearity assumption not generalizable.
  - effect constant over individuals/levels

Additional untestable assumption of uncorrelated disturbances (no unmeasured confounders).



Keenan Crane

Assumptions of causal mediation

Need assumptions to hold (and correct model!) to derive causal statements

- Potential confounding can be accounted for with explanatories.
- Careful with what is included (colliders)!
    - *as-if* randomization assumption
- Generalizations to interactions, multiple mediators, etc. should require careful acknowledgement of confounding.