

PSET 3

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LING227: Language and Computation

October 29, 2017

Problem I.A.

a. Since $Y \subset Z$, let $X \subset Z$ and $\forall x \in X : x \notin Y$. $Z = Y \cup X$ and $P(Z) = P(Y \cup X) = P(Y) + P(X)$. $P(Y) \leq P(Y) + P(X) \implies P(Y) \leq P(Z)$. Q.E.D.

b. To prove $p(X|Z)$ is in the range $[0,1]$, there are three cases that need to be considered based on the definition of $p(X|Z)$: $X \cup Z = \emptyset$, $X \cup Z = Z$, and $X \cup Z \subset Z$. For the first case, $P(\emptyset) = 0$ based on the proof in part c, so $p(X|Z) = 0$. For the second case, $\frac{P(Z)}{P(Z)} = 1$. For the third case, $p(X|Z) < P(Z)$ based on part a and $>$, so $p(X|Z) = (0, 1)$. Since all three cases are satisfied, $p(X|Z)$ is in the range $[0,1]$. Q.E.D.

c. Let $Y \subset Z$, $Y = Z$, and $X \subset Z$ such that $\forall x \in X : x \notin Y$. From part a, $P(Y) \leq P(Z)$ but in this case since $Y = Z$, $P(Y) = P(Z)$. From part a, $P(Y) = P(Y) + P(X)$. X in this case is \emptyset . For the equality to hold, $P(X)$ must equal 0. Q.E.D.

d. $P(X \cap \overline{X}) = p(\mathcal{E})$. $P(X) + P(\overline{X}) = 1$. $P(X) = 1 - P(\overline{X})$ Q.E.D.

e. $p(\text{singing} \cup \text{raining} \mid \text{raining}) = \frac{P(\text{singing} \cap \text{raining} \cap \text{raining})}{P(\text{raining})} = \frac{P(\text{singing} \cap \text{raining})}{P(\text{raining})}$ given that $\text{raining} \cap \text{raining} = \text{raining}$. Also $p(\text{singing} \mid \text{raining}) = \frac{P(\text{singing} \cap \text{raining})}{P(\text{raining})}$ Q.E.D.

f. $P(X|Y \cap \overline{X}|Y) = p(\mathcal{E})$. $P(X|Y) + P(\overline{X}|Y) = 1$. $P(X|Y) = 1 - P(\overline{X}|Y)$ Q.E.D.

g. $(P(X|Y) P(Y) + P(X|\overline{Y})P(\overline{Y})) * \frac{P(Z|X)}{P(Z)}$. $P(X|Y) P(Y) + P(X|\overline{Y})P(\overline{Y}) = P(X \cap Y) + P(X \cap \overline{Y}) = P(Y \cap X) + P(\overline{Y} \cap X)$. From part f, $\frac{P(Y \cap X)}{P(X)} + \frac{P(\overline{Y} \cap X)}{P(X)} = 1$. Therefore $P(Y \cap X) + P(\overline{Y} \cap X) = P(X)$. The current equation is $P(X) \frac{P(Z|X)}{P(Z)}$. $\frac{P(Z|X)}{P(Z)} = \frac{P(Z \cap X)}{P(X)P(Z)}$. The equation is reduced to $\frac{P(Z \cap X)}{P(Z)}$

h. $P(X|Y,Z) = P(X|Y) * Z$. Since $P(X|Y) = 0$, $P(X|Y) * Z = 0$. Q.E.D.

Problem I.B.

a. $\forall y \in \text{situation} \sum_{x \in \text{cry}} p(x|y) = 1$

b.

p(cry, situation)	Predator!	Timber!	I need help!	TOTAL
bwa	0	0	0.64	0.64
bee	0	0	0.08	0.08
kiki	0.2	0	0.08	0.28
TOTAL	0.2	0	.8	1

1. This probability is written as: $p(\text{Predator} \mid \text{kiki})$
2. $\frac{p(\text{Predator}, \text{kiki})}{p(\text{kiki})}$
3. $\frac{.2}{.28} = \frac{2}{2.8}$
4. $\frac{p(\text{kiki}|\text{Predator})p(\text{Predator})}{p(\text{kiki}|\text{Predator})p(\text{Predator})+p(\text{kiki}|\text{Timber})p(\text{Timber})+p(\text{kiki}|\text{Ineedhelp})p(\text{Ineedhelp})}$
5. $\frac{1*0.2}{0.2+0+0.08} = \frac{2}{2.8}$

Problem I.C.

Let $w = \text{"I love New York"}$. $P(w) = P(w_1|w_{bos}, w_{bos}) * P(w_2|w_{bos}, w_1) * P(w_3|w_1, w_2) * P(w_4|w_2, w_3) * P(w_{eos}|w_3, w_4)$.

Using chain rule, $P(A,B,C) = P(A|B,C)P(B,C) = P(A|B,C)P(B|C)P(C)$. Therefore, $P(w) = \frac{P(w_1, w_{bos}, w_{bos})}{P(w_{bos}, w_{bos})} * \frac{P(w_2, w_{bos}, w_1)}{P(w_{bos}, w_1)} * \frac{P(w_3, w_1, w_2)}{P(w_1, w_2)} * \frac{P(w_4, w_2, w_3)}{P(w_2, w_3)} * \frac{P(w_{eos}, w_3, w_4)}{P(w_3, w_4)}$.

$P(w_{reversed}) = P(w_{bos}|w_1, w_2) * P(w_1|w_2, w_3) * P(w_2|w_3, w_4) * P(w_3|w_4, w_{eos}) * P(w_4|w_{eos}, w_{eos})$

Using the chain rule, $P_{reversed}(w) = \frac{P(w_{bos}, w_1, w_2)}{P(w_1, w_2)} * \frac{P(w_1, w_2, w_3)}{P(w_2, w_3)} * \frac{P(w_2, w_3, w_4)}{P(w_3, w_4)} * \frac{P(w_3, w_4, w_{eos})}{P(w_4, w_{eos})} * \frac{P(w_4, w_{eos}, w_{eos})}{P(w_{eos}, w_{eos})}$ Look at terms from the start of $P(w)$ and the terms starting from of the end of $P_{reversed}$, we see that there is congruency between eos and pos because they have the same count. Let $n = 1$, then $P(w) = \frac{P(eos, bos, w)}{P(bos, w)} + \frac{P(bos, bos, w)}{P(bos, bos)}$. For P_{rev} it is the same but the eos and bos are flipped. $P_{rev} = \frac{P(bos, eos, w)}{P(eos, w)} + \frac{P(eos, eos, w)}{P(eos, eos)}$. For the induction step, $P(w)$ will be

$$P(1) * \prod_2^w \frac{P(w_n, w_{n-1}, w_{n-2})}{P(w_{n-1}, w_{n-2})}$$

and $P_{reversed}(w)$ will always be

$$P_{reversed}(1) * \prod_2^{w+1} \frac{P_{reversed}(w_n, w_{n+1}, w_{n+2})}{P(w_{n+1}, w_{n+2})}$$

. Examining the case from $n = 3$ to w for $P(w)$ and $n = 2$ to $w-1$ for $P_{reversed}$, the counts are going to be the same since they do not consider eos and bos.

Problem I.D.

Absolute Discounting

$$\sum_{w_n: r > 0} \frac{r - \sigma}{N} = \frac{N - (V - N_0)\sigma}{N}$$

$$\sum_{w_n:r=0}^{N_0} \frac{(V - N_0)\sigma}{NN_0} = N_0 \frac{(V - N_0)\sigma}{NN_0}$$

$$\sum_{w_n:r=0} \frac{r - \sigma}{N} + \sum_{w_n:r=0}^{N_0} \frac{(V - N_0)\sigma}{NN_0} = \frac{(V - N_0)\sigma}{N} + \frac{N - (V - N_0)\sigma}{N} = 1$$

Linear Discounting

$$\sum_{w_n:r>0} \frac{(1 - \alpha)r}{N} = \frac{(1 - \alpha)r}{N} N = 1 - \alpha$$

$$\sum_{w_n:r=0}^{N_0} \frac{\alpha}{N_0} = N_0 \frac{\alpha}{N_0} = \alpha$$

$$\sum_{w_n:r>0} \frac{(1 - \alpha)r}{N} + \sum_{w_n:r=0}^{N_0} \frac{\alpha}{N_0} = 1$$

Problem 2.A.

The words generated by the bigrams are about the same length with the words generated by the trigram for the KF and aesop corpuses. For the lexicon, trigrams are much longer than the bigrams.

lexicon.trans 2

```
# T R IY #
# V EY DD #
# AE KD #
# K AXR #
# B AXR AX L AE KD T R EY DX IX F AX S T R IX S AXR #
# HH AY T N #
# V EH P AO R DD #
# HH AW N AA IY M N Z #
# CH AXR #
# B OW L #
# W IH R DX IX NG #
# D EH L EH N B AH T R AX N D IH S SH F AO R AY KD #
# R T AA B S EH N IY #
# AX #
# TH AXR B IX K IH L #
# M AX N D UW M P EH R IH N #
# IH DX IX NG #
# S K AX L #
```

HH UW G AE K IY #
 # M #
 # S AE L #
 # D L AA R AY N TD B UH TD #
 # S AX N #
 # W IX NG Z AA JH #
 # Y AX V AY AE K ER F AE S KD #
lexicon.trans 3
 # AX SH ER Z AX KD #
 # IX SH #
 # IX SH #
 # B OW SH AX N OW SH AX N OW SH AX PD TD L AX T OW SH AX PD #
 # HH OW SH AX PD TD L AX SH ER Z AX KD TD L AX SH ER Z AX KD TD L EH
 GD #
 # N OW SH AX #
 # M OW SH AX PD TD L IY OW EH DD TH ER Z AX KD TD L #
 # CH ER TD L AE GD T AE R AX OW K EY OW V ER L Y AX L OW SH AX N Z OW
 Z D AX SH ER Z AX KD TD L AXR OW K OW UH R AX OW K OW UH R OW R AY
 OW D Z EY N OW SH AX N OW SH AX N OW SH AX L OW SH AX N OW SH AX N
 OW SH AX N TD P OW SH AX PD TD L IY B OW SH AX N TD P OW SH AX PD TD
 L IY OW EH DD TH ER Z AX KD TD L AX SH ER Z AX KD TD L AXR OW K Y ER
 K AX SH ER Z AX KD TD L AO R AX OW K OW UH R OW R AY OW D Z EY N OW
 SH AX N OW SH AX N OW SH AX N OW SH AX PD TD L AX SH ER Z AX KD #
 # R Y AX EH N OW SH AX N OW SH AX PD TD L #
 # Y ER K S OW SH AX N Z OW Z D AX SH ER Z AX KD TD L AX SH ER Z AX KD
 TD L AX SH ER Z AX M OW SH AX N Z OW Z D AX SH ER Z AX KD TD #
 # L ER K AX SH ER Z AX KD TD L IY OW EH DD TH ER Z AX KD TD L AO R T
 OW SH AX PD #
 # S OW SH AX PD TD L AX JH ER Z AX KD #
 # B AA SH #
 # S OW SH AX #
 # W ER KD D ER TD L IY OW EH DD TH ER Z AX KD TD L AY SH AW TD P OW
 SH AX N OW SH AX N OW SH AX PD TD L IY OW EH DD TH ER Z AX M OW SH
 AX PD #
 # P OW SH AX N Z OW Z D AX SH ER Z AX TD W AO PD #
 # EY OW V ER L Y AX EH N OW SH AX PD TD L IY OW EH DD TH ER Z AX N
 OW SH AX N OW SH AX PD TD L IY OW EH DD TH ER Z AX M OW SH AX PD #
 # Y ER K AX L OW SH IY OW EH DD TH ER Z #
 # S OW SH AX N OW SH AX N OW SH AX N OW SH AX PD #
 # P OW SH AX N OW SH AX PD TD L IY OW EH DD TH ER Z AX KD TD L AX SH
 ER Z AX KD TD L AX SH ER Z AX KD TD L AXR OW K OW UH R OW R AY OW D
 Z EY N OW SH AX N OW SH AX N OW SH AX N DD #
 # L ER K AX SH ER Z AX B OW SH AX N OW SH AX N IH GD T AE R AX OW K
 OW UH R OW R AY OW D Z EY N OW SH AX N OW SH AX PD TD L IY OW EH DD
 TH ER Z AX KD TD L AX SH ER Z AX KD #

AE GD T AE R AX OW K OW UH R OW R AY OW D Z EY N OW SH AX L OW SH
AX N OW SH AX L OW K OW UH R AX OW K OW UH R IX #
W ER SH AX B OW SH AX PD TD L AX SH ER Z AX KD TD L IX R K OW UH R
AX OW K OW UH R AX OW K OW UH R OW R AY OW D Z EY N OW SH IY OW
EH DD TH ER Z AX KD TD L IX R K OW UH R OW R AY OW D #
L ER K AX SH ER Z AX N OW SH AX PD #
HH OW SH AX PD TD L IY AX SH ER Z AX KD #
IH M P IH JH M AX L IY L

KF-1002.trans 2

M AX N #
D IX NG #
L IY #
M Y UW N #
R #
N AH K L #
M #
N TD #
K AO R AH S TD #
L AY V AX N #
M EH N EH M IH CH AX S T OW #
SH EH SH EH L #
P EY M AO R EH DX IX V ER T IH Z #
F AO R W IH DX AX L #
HH AO R DD #
M #
L IX NG KD #
HH AY M P IH R AE K AX N #
S #
B AX N #
L UH KD #
M T EH R #
B IH TD #
HH AE F #
D #
P AA

KF-1002.trans 3

K ER AX N TD L IY #
T ER N #
K ER AX N TD L IY #
DH OW #
D #
F ER DH #
IX N V AA L V #
S ER F AX #
L ER N

R AX K AX N S ER T AX #
 # K ER AX N TD L IY #
 # S ER F AX #
 # W ER L #
 # IH NG KD #
 # T ER N #
 # D #
 # G ER L #
 # K AX N TD L IY #
 # AO #
 # K ER AX N TD L IY #
 # L ER N #
 # R AX K AX N TD L #
 # DH OW #
 # S EY V #
 # AE N DD #
 # DH EY #
 # P AX N #
aesop.trans 2
 # AE N DD #
 # AE N EH R EH TD #
 # S #
 # JH AXR #
 # W AO R IY DX AXR M #
 # AE N TD #
 # HH AXR S EY M AY DD #
 # S #
 # W UH CH AXR #
 # DH #
 # IH DH AH #
 # AX #
 # T AO L #
 # T EY #
 # HH AA N #
 # AA Z #
 # IH Z #
 # DH EY #
 # AE TD #
 # R EY K AA Z #
 # EH L AX N DD #
 # IH L #
 # IY Z AX Z IH B AX L #
 # AO F T IX K AX L #
 # HH ER K Y UH R #
 # AE TD L AXR L AO NG #

aesop.trans 3

AY Z IX KD T #
AE TD L AXR L AO #
HH ER K Y UH R #
HH ER K Y UH R #
AA #
DH OW Z #
SH #
DH AH S TD L AXR L AO #
S OW #
AH PD T #
S OW #
HH ER K Y UH R #
W ER TH L AX D Z M IX N S ER V IX #
HH ER K Y UH R #
P ER #
DH AH S TD L AXR L AO #
HH ER K Y UH R #
IH L #
AX V ER S TD L AXR L AO #
AE TD L AXR L AO

Problem 2.B.

For aesop, only bigrams can distinguish all the words. For KF-1002, neither the bigrams or the trigrams can distinguish between all the good from the bad words. On average for KF-1002, more words have non-zero probability for the good words. For lexicon, only the bigrams can distinguish between all the good and bad words. To distinguish between the good and bad words, the probability of the words has to be non zero for good words. If all the words are accepted, the perplexity is finite. For the two corpuses that can distinguish between good and bad words aesop and lexicon, bigram lexicon has about half the perplexity of aesop. This suggests that lexicon is a better training file.

aesop 2 good

$P(\# \text{ M IY D AH T } \#) = 4.47849894351\text{e-}11$
 $P(\# \text{ K IY N AH P } \#) = 3.14257904757\text{e-}10$
 $P(\# \text{ HH EY N AH T } \#) = 2.43272709801\text{e-}10$
 $P(\# \text{ S AE SH AH M } \#) = 1.26223683022\text{e-}10$
 $P(\# \text{ S IY N AH N } \#) = 6.58399744881\text{e-}08$
 $P(\# \text{ M AE L AH P } \#) = 1.86754681778\text{e-}10$
 $P(\# \text{ K IH N AH M } \#) = 1.88222679252\text{e-}07$
 $P(\# \text{ HH AE L AH M } \#) = 4.52792530301\text{e-}08$
 $P(\# \text{ HH IH D AH P } \#) = 3.55969710305\text{e-}09$
 $P(\# \text{ S AE N AH P } \#) = 3.98751848147\text{e-}09$
 $P(\# \text{ M EY L AH P } \#) = 7.60575516058\text{e-}10$

$P(\# \text{ K IH D AH T } \#) = 9.98670172858\text{e-}11$
 $P(\# \text{ S IH D AH T } \#) = 6.13789697844\text{e-}11$
 $P(\# \text{ HH EY SH AH M } \#) = 4.43367811153\text{e-}10$
 $P(\# \text{ K EY L AH M } \#) = 4.43666780845\text{e-}08$
 $P(\# \text{ M IY N AH P } \#) = 8.02220083254\text{e-}10$
 $P(\# \text{ HH IH SH AH P } \#) = 7.60364052343\text{e-}10$
 $P(\# \text{ K IY D AH N } \#) = 2.66081878834\text{e-}09$
 $P(\# \text{ M AE N AH N } \#) = 1.2220305472\text{e-}06$
 $P(\# \text{ S IH SH AH P } \#) = 2.62215356211\text{e-}11$
 $P(\# \text{ K EY L AH M } \#) = 4.43666780845\text{e-}08$
 $P(\# \text{ M EY N AH M } \#) = 9.95889703025\text{e-}08$
 $P(\# \text{ S IH L AH P } \#) = 4.8046147492\text{e-}10$
 $P(\# \text{ HH AE L AH N } \#) = 8.44346726585\text{e-}08$
 Perplexity = 27.49827096

aesop 3 good

$P(\# \text{ M IY D AH T } \#) = 0$
 $P(\# \text{ K IY N AH P } \#) = 0$
 $P(\# \text{ HH EY N AH T } \#) = 0$
 $P(\# \text{ S AE SH AH M } \#) = 0$
 $P(\# \text{ S IY N AH N } \#) = 0$
 $P(\# \text{ M AE L AH P } \#) = 0$
 $P(\# \text{ K IH N AH M } \#) = 0$
 $P(\# \text{ HH AE L AH M } \#) = 0$
 $P(\# \text{ HH IH D AH P } \#) = 0$
 $P(\# \text{ S AE N AH P } \#) = 0$
 $P(\# \text{ M EY L AH P } \#) = 0$
 $P(\# \text{ K IH D AH T } \#) = 0$
 $P(\# \text{ S IH D AH T } \#) = 0$
 $P(\# \text{ HH EY SH AH M } \#) = 0$
 $P(\# \text{ K EY L AH M } \#) = 0$
 $P(\# \text{ M IY N AH P } \#) = 0$
 $P(\# \text{ HH IH SH AH P } \#) = 0$
 $P(\# \text{ K IY D AH N } \#) = 0$
 $P(\# \text{ M AE N AH N } \#) = 0$
 $P(\# \text{ S IH SH AH P } \#) = 0$
 $P(\# \text{ K EY L AH M } \#) = 0$
 $P(\# \text{ M EY N AH M } \#) = 0$
 $P(\# \text{ S IH L AH P } \#) = 0$
 $P(\# \text{ HH AE L AH N } \#) = 0$

Perplexity = inf

aesop 2 bad

$P(\# \text{ Z OW ZH AH SH } \#) = 0$
 $P(\# \text{ Y UH G OW L } \#) = 0$
 $P(\# \text{ V AO TH AH SH } \#) = 0$
 $P(\# \text{ G UW DH UH S } \#) = 0$

$P(\# \text{ V UW G UH S } \#) = 0$
 $P(\# \text{ G AO DH AH SH } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ Y AO ZH OW L } \#) = 0$
 $P(\# \text{ G OW G EH D } \#) = 0$
 $P(\# \text{ V UH DH OW L } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ Y OW ZH AH SH } \#) = 0$
 $P(\# \text{ G UW ZH AH D } \#) = 0$
 $P(\# \text{ V AO DH UH S } \#) = 0$
 $P(\# \text{ Y UH TH AH SH } \#) = 0$
 $P(\# \text{ Z OW G OW L } \#) = 0$
 $P(\# \text{ Y AO ZH AH D } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ G AO DH OW L } \#) = 0$
 $P(\# \text{ V UW G EH D } \#) = 0$
 $P(\# \text{ Y OW ZH OW L } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ V UH DH AH SH } \#) = 0$
 $P(\# \text{ G OW G UH S } \#) = 0$
 Perplexity = inf
 aesop 3 bad
 $P(\# \text{ Z OW ZH AH SH } \#) = 0$
 $P(\# \text{ Y UH G OW L } \#) = 0$
 $P(\# \text{ V AO TH AH SH } \#) = 0$
 $P(\# \text{ G UW DH UH S } \#) = 0$
 $P(\# \text{ V UW G UH S } \#) = 0$
 $P(\# \text{ G AO DH AH SH } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ Y AO ZH OW L } \#) = 0$
 $P(\# \text{ G OW G EH D } \#) = 0$
 $P(\# \text{ V UH DH OW L } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ Y OW ZH AH SH } \#) = 0$
 $P(\# \text{ G UW ZH AH D } \#) = 0$
 $P(\# \text{ V AO DH UH S } \#) = 0$
 $P(\# \text{ Y UH TH AH SH } \#) = 0$
 $P(\# \text{ Z OW G OW L } \#) = 0$
 $P(\# \text{ Y AO ZH AH D } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ G AO DH OW L } \#) = 0$
 $P(\# \text{ V UW G EH D } \#) = 0$
 $P(\# \text{ Y OW ZH OW L } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ V UH DH AH SH } \#) = 0$

$P(\# \text{ G OW G UH S } \#) = 0$
 Perplexity = inf
 KF-1002 2 good
 $P(\# \text{ M IY D AH T } \#) = 0$
 $P(\# \text{ K IY N AH P } \#) = 4.87893708618\text{e-}09$
 $P(\# \text{ HH EY N AH T } \#) = 0$
 $P(\# \text{ S AE SH AH M } \#) = 0$
 $P(\# \text{ S IY N AH N } \#) = 6.48209634127\text{e-}07$
 $P(\# \text{ M AE L AH P } \#) = 2.11281606011\text{e-}08$
 $P(\# \text{ K IH N AH M } \#) = 2.06025736405\text{e-}07$
 $P(\# \text{ HH AE L AH M } \#) = 7.51808064706\text{e-}07$
 $P(\# \text{ HH IH D AH P } \#) = 2.33319744219\text{e-}08$
 $P(\# \text{ S AE N AH P } \#) = 2.08024497477\text{e-}08$
 $P(\# \text{ M EY L AH P } \#) = 1.21516597839\text{e-}08$
 $P(\# \text{ K IH D AH T } \#) = 0$
 $P(\# \text{ S IH D AH T } \#) = 0$
 $P(\# \text{ HH EY SH AH M } \#) = 0$
 $P(\# \text{ K EY L AH M } \#) = 9.39295214755\text{e-}08$
 $P(\# \text{ M IY N AH P } \#) = 2.91749104142\text{e-}08$
 $P(\# \text{ HH IH SH AH P } \#) = 0$
 $P(\# \text{ K IY D AH N } \#) = 4.14709652326\text{e-}08$
 $P(\# \text{ M AE N AH N } \#) = 3.30321702045\text{e-}06$
 $P(\# \text{ S IH SH AH P } \#) = 0$
 $P(\# \text{ K EY L AH M } \#) = 9.39295214755\text{e-}08$
 $P(\# \text{ M EY N AH M } \#) = 6.58602090202\text{e-}07$
 $P(\# \text{ S IH L AH P } \#) = 8.94338494473\text{e-}08$
 $P(\# \text{ HH AE L AH N } \#) = 1.21445918145\text{e-}06$
 Perplexity = inf
 KF-1002 3 good
 $P(\# \text{ M IY D AH T } \#) = 0$
 $P(\# \text{ K IY N AH P } \#) = 0$
 $P(\# \text{ HH EY N AH T } \#) = 0$
 $P(\# \text{ S AE SH AH M } \#) = 0$
 $P(\# \text{ S IY N AH N } \#) = 0$
 $P(\# \text{ M AE L AH P } \#) = 0$
 $P(\# \text{ K IH N AH M } \#) = 0$
 $P(\# \text{ HH AE L AH M } \#) = 0$
 $P(\# \text{ HH IH D AH P } \#) = 0$
 $P(\# \text{ S AE N AH P } \#) = 0$
 $P(\# \text{ M EY L AH P } \#) = 0$
 $P(\# \text{ K IH D AH T } \#) = 0$
 $P(\# \text{ S IH D AH T } \#) = 0$
 $P(\# \text{ HH EY SH AH M } \#) = 0$
 $P(\# \text{ K EY L AH M } \#) = 0$
 $P(\# \text{ M IY N AH P } \#) = 0$

$P(\# \text{ HH IH SH AH P } \#) = 0$
 $P(\# \text{ K IY D AH N } \#) = 0$
 $P(\# \text{ M AE N AH N } \#) = 0$
 $P(\# \text{ S IH SH AH P } \#) = 0$
 $P(\# \text{ K EY L AH M } \#) = 0$
 $P(\# \text{ M EY N AH M } \#) = 0$
 $P(\# \text{ S IH L AH P } \#) = 0$
 $P(\# \text{ HH AE L AH N } \#) = 0$
 Perplexity = inf
 KF-1002 2 bad
 $P(\# \text{ Z OW ZH AH SH } \#) = 0$
 $P(\# \text{ Y UH G OW L } \#) = 0$
 $P(\# \text{ V AO TH AH SH } \#) = 0$
 $P(\# \text{ G UW DH UH S } \#) = 0$
 $P(\# \text{ V UW G UH S } \#) = 0$
 $P(\# \text{ G AO DH AH SH } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ Y AO ZH OW L } \#) = 0$
 $P(\# \text{ G OW G EH D } \#) = 1.7832805797\text{e-}08$
 $P(\# \text{ V UH DH OW L } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ Y OW ZH AH SH } \#) = 0$
 $P(\# \text{ G UW ZH AH D } \#) = 0$
 $P(\# \text{ V AO DH UH S } \#) = 0$
 $P(\# \text{ Y UH TH AH SH } \#) = 0$
 $P(\# \text{ Z OW G OW L } \#) = 0$
 $P(\# \text{ Y AO ZH AH D } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ G AO DH OW L } \#) = 0$
 $P(\# \text{ V UW G EH D } \#) = 0$
 $P(\# \text{ Y OW ZH OW L } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ V UH DH AH SH } \#) = 0$
 $P(\# \text{ G OW G UH S } \#) = 0$
 Perplexity = inf
 KF-1002 3 bad
 $P(\# \text{ Z OW ZH AH SH } \#) = 0$
 $P(\# \text{ Y UH G OW L } \#) = 0$
 $P(\# \text{ V AO TH AH SH } \#) = 0$
 $P(\# \text{ G UW DH UH S } \#) = 0$
 $P(\# \text{ V UW G UH S } \#) = 0$
 $P(\# \text{ G AO DH AH SH } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ Y AO ZH OW L } \#) = 0$
 $P(\# \text{ G OW G EH D } \#) = 0$

$P(\# \text{ V UH DH OW L } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ Y OW ZH AH SH } \#) = 0$
 $P(\# \text{ G UW ZH AH D } \#) = 0$
 $P(\# \text{ V AO DH UH S } \#) = 0$
 $P(\# \text{ Y UH TH AH SH } \#) = 0$
 $P(\# \text{ Z OW G OW L } \#) = 0$
 $P(\# \text{ Y AO ZH AH D } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ G AO DH OW L } \#) = 0$
 $P(\# \text{ V UW G EH D } \#) = 0$
 $P(\# \text{ Y OW ZH OW L } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ V UH DH AH SH } \#) = 0$
 $P(\# \text{ G OW G UH S } \#) = 0$
 Perplexity = inf
 lexicon 2 good
 $P(\# \text{ M IY D AH T } \#) = 4.79439255397\text{e-}09$
 $P(\# \text{ K IY N AH P } \#) = 1.88204414788\text{e-}08$
 $P(\# \text{ HH EY N AH T } \#) = 5.31199572838\text{e-}09$
 $P(\# \text{ S AE SH AH M } \#) = 4.95309507415\text{e-}08$
 $P(\# \text{ S IY N AH N } \#) = 2.09488924621\text{e-}07$
 $P(\# \text{ M AE L AH P } \#) = 1.53605337958\text{e-}07$
 $P(\# \text{ K IH N AH M } \#) = 1.08219550361\text{e-}07$
 $P(\# \text{ HH AE L AH M } \#) = 9.36840455715\text{e-}07$
 $P(\# \text{ HH IH D AH P } \#) = 1.84097343074\text{e-}08$
 $P(\# \text{ S AE N AH P } \#) = 3.74686536145\text{e-}08$
 $P(\# \text{ M EY L AH P } \#) = 5.10597887743\text{e-}08$
 $P(\# \text{ K IH D AH T } \#) = 4.27359772134\text{e-}09$
 $P(\# \text{ S IH D AH T } \#) = 8.22782638088\text{e-}09$
 $P(\# \text{ HH EY SH AH M } \#) = 2.4137365234\text{e-}07$
 $P(\# \text{ K EY L AH M } \#) = 3.36398180465\text{e-}07$
 $P(\# \text{ M IY N AH P } \#) = 1.10671287667\text{e-}08$
 $P(\# \text{ HH IH SH AH P } \#) = 3.88083048461\text{e-}08$
 $P(\# \text{ K IY D AH N } \#) = 1.69790522147\text{e-}07$
 $P(\# \text{ M AE N AH N } \#) = 7.47439187096\text{e-}07$
 $P(\# \text{ S IH SH AH P } \#) = 4.29276855912\text{e-}08$
 $P(\# \text{ K EY L AH M } \#) = 3.36398180465\text{e-}07$
 $P(\# \text{ M EY N AH M } \#) = 7.09326203721\text{e-}08$
 $P(\# \text{ S IH L AH P } \#) = 1.57885836623\text{e-}07$
 $P(\# \text{ HH AE L AH N } \#) = 1.96077411963\text{e-}06$
 Perplexity = 15.6152106001
 lexicon 3 good
 $P(\# \text{ M IY D AH T } \#) = 2.50420650131\text{e-}14$
 $P(\# \text{ K IY N AH P } \#) = 1.85706559332\text{e-}13$

$P(\# \text{ HH EY N AH T } \#) = 0$
 $P(\# \text{ S AE SH AH M } \#) = 0$
 $P(\# \text{ S IY N AH N } \#) = 2.8080161639\text{e-}12$
 $P(\# \text{ M AE L AH P } \#) = 0$
 $P(\# \text{ K IH N AH M } \#) = 0$
 $P(\# \text{ HH AE L AH M } \#) = 0$
 $P(\# \text{ HH IH D AH P } \#) = 0$
 $P(\# \text{ S AE N AH P } \#) = 0$
 $P(\# \text{ M EY L AH P } \#) = 0$
 $P(\# \text{ K IH D AH T } \#) = 0$
 $P(\# \text{ S IH D AH T } \#) = 0$
 $P(\# \text{ HH EY SH AH M } \#) = 0$
 $P(\# \text{ K EY L AH M } \#) = 0$
 $P(\# \text{ M IY N AH P } \#) = 7.98194276759\text{e-}13$
 $P(\# \text{ HH IH SH AH P } \#) = 0$
 $P(\# \text{ K IY D AH N } \#) = 0$
 $P(\# \text{ M AE N AH N } \#) = 0$
 $P(\# \text{ S IH SH AH P } \#) = 0$
 $P(\# \text{ K EY L AH M } \#) = 0$
 $P(\# \text{ M EY N AH M } \#) = 0$
 $P(\# \text{ S IH L AH P } \#) = 0$
 $P(\# \text{ HH AE L AH N } \#) = 0$
 Perplexity = inf
 lexicon 2 bad
 $P(\# \text{ Z OW ZH AH SH } \#) = 0$
 $P(\# \text{ Y UH G OW L } \#) = 2.68307030201\text{e-}08$
 $P(\# \text{ V AO TH AH SH } \#) = 3.50318116772\text{e-}10$
 $P(\# \text{ G UW DH UH S } \#) = 0$
 $P(\# \text{ V UW G UH S } \#) = 8.32344206621\text{e-}12$
 $P(\# \text{ G AO DH AH SH } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 1.25669990038\text{e-}10$
 $P(\# \text{ Y AO ZH OW L } \#) = 8.29870151775\text{e-}12$
 $P(\# \text{ G OW G EH D } \#) = 7.59181243887\text{e-}09$
 $P(\# \text{ V UH DH OW L } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ Y OW ZH AH SH } \#) = 0$
 $P(\# \text{ G UW ZH AH D } \#) = 0$
 $P(\# \text{ V AO DH UH S } \#) = 0$
 $P(\# \text{ Y UH TH AH SH } \#) = 0$
 $P(\# \text{ Z OW G OW L } \#) = 2.28527492015\text{e-}09$
 $P(\# \text{ Y AO ZH AH D } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ G AO DH OW L } \#) = 0$
 $P(\# \text{ V UW G EH D } \#) = 7.2688189025\text{e-}11$
 $P(\# \text{ Y OW ZH OW L } \#) = 3.00297581885\text{e-}10$

$P(\# \text{ Z UW TH EH D } \#) = 1.25669990038\text{e-}10$
 $P(\# \text{ V UH DH AH SH } \#) = 0$
 $P(\# \text{ G OW G UH S } \#) = 8.6932983006\text{e-}10$
 Perplexity = inf
 lexicon 3 bad
 $P(\# \text{ Z OW ZH AH SH } \#) = 0$
 $P(\# \text{ Y UH G OW L } \#) = 0$
 $P(\# \text{ V AO TH AH SH } \#) = 0$
 $P(\# \text{ G UW DH UH S } \#) = 0$
 $P(\# \text{ V UW G UH S } \#) = 0$
 $P(\# \text{ G AO DH AH SH } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ Y AO ZH OW L } \#) = 0$
 $P(\# \text{ G OW G EH D } \#) = 1.37509430504\text{e-}14$
 $P(\# \text{ V UH DH OW L } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ Y OW ZH AH SH } \#) = 0$
 $P(\# \text{ G UW ZH AH D } \#) = 0$
 $P(\# \text{ V AO DH UH S } \#) = 0$
 $P(\# \text{ Y UH TH AH SH } \#) = 0$
 $P(\# \text{ Z OW G OW L } \#) = 0$
 $P(\# \text{ Y AO ZH AH D } \#) = 0$
 $P(\# \text{ Z UH TH UH S } \#) = 0$
 $P(\# \text{ G AO DH OW L } \#) = 0$
 $P(\# \text{ V UW G EH D } \#) = 0$
 $P(\# \text{ Y OW ZH OW L } \#) = 0$
 $P(\# \text{ Z UW TH EH D } \#) = 0$
 $P(\# \text{ V UH DH AH SH } \#) = 0$
 $P(\# \text{ G OW G UH S } \#) = 0$
 Perplexity = inf

Problem 2.C.

When add one smothing is added, the perplexities that were infinite are now finite but very big. The add1 smoothing causes the probably of the words that have been seen to significantly decrease. For the generation part, the trigrams are signifincanntly longer than their bigram counterparts for lexicon and aesop. For KF, the bigrams are a lot longer.