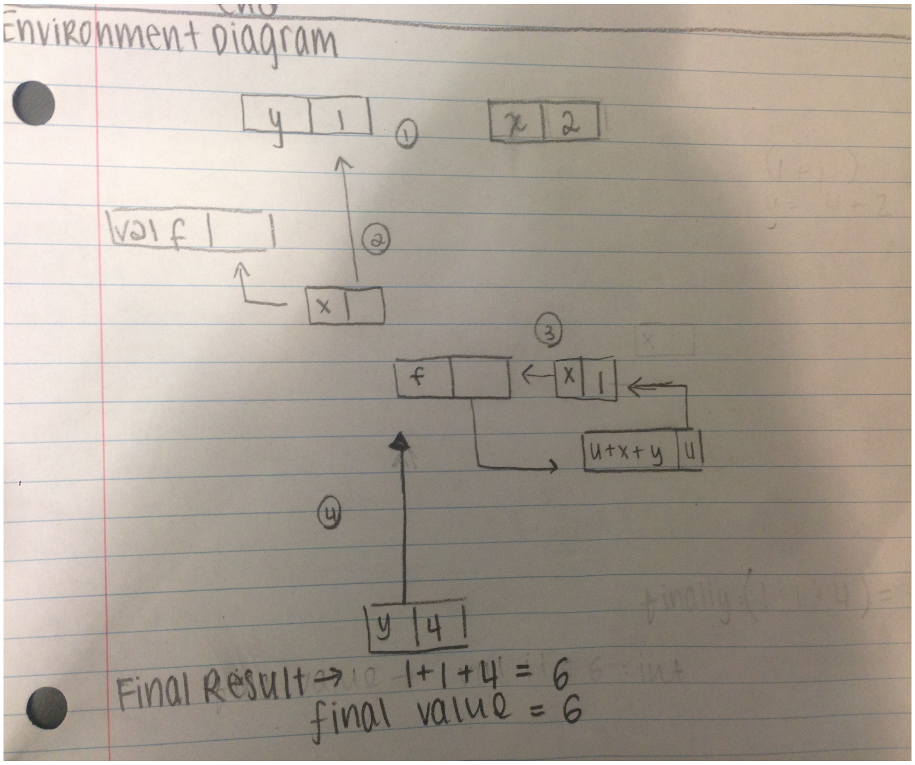
COMP302 - Assignment 4

Lucia Eve Berger — 260473661

Sourced from Stackoverflow and Wikipedia, Slides, Bridgette Pienka's book

1 Question 3



2 Question 4

Specification In this proof, we will consider:

```
(* reduce:  'a list * 'a * ('a * 'a -> 'a) -> 'a *)
fun reduce (nil, base, op) = base
| reduce (h::t, base, op) = op (h, reduce(t, base, op)
(* reduce tr:  'a list * 'a * ('a * 'a -> 'a) -> 'a *)
fun reducetr (nil, base, op) = base
| reduce tr (h::t, base, op) = reduce$_t r(t, op(h, base), op)$
```

Proof We will prove that the two below programs return the same result if applied to the same arguments. That is we

```
reduce(h::t,base,op)==reduce tr(h::t,base,op)
```

Conditions We will complete the proof using the lemma and the below conditions:

```
(a) op(base, m) = m
(b) op(n,m) = op(m,n)
(c) op(n, op(m,k))) = op(op(n,m), k)
```

Lemma `op(h, reduce(l, n, op)) = reduce tr(l, op(h, n), op)`

Initial Inductive Hypothesis To prove the two give the same result with the same arguments, we need to consider the properties of a,b,c.

Induction Step `Take k::  t = k::  h ::  t`

```
op(k, reduce(h::k, base,op)) = reduce tr (t,op,h,n,op)
reduce tr (h::t,op(k,base,op)) -> reduce tr (t,op(h,op(k,base),op))
```

Induction Expansion By the induction hypothesis, we can further expand while considering the conditions:

```
op (h, reduce(t,op(k,base),op))
reduce(h::t, op(k,base),op)
op(h,op(t,op(k,base)))
op(k,op(h,op,(t,base))
op (h,op(t,base)) x reduce(h:t,base,op)
op(k,reduce(h::t,base,op))
```

Final Step `reduce(h::t, base,op) = reduce tr (t,op(h,base),op)`
`op(h,reduce(t,base,op)) = reduce tr(t,op(h,base),op)`

by the lemma this is true, completing the proof