COMP251 Assignment 5

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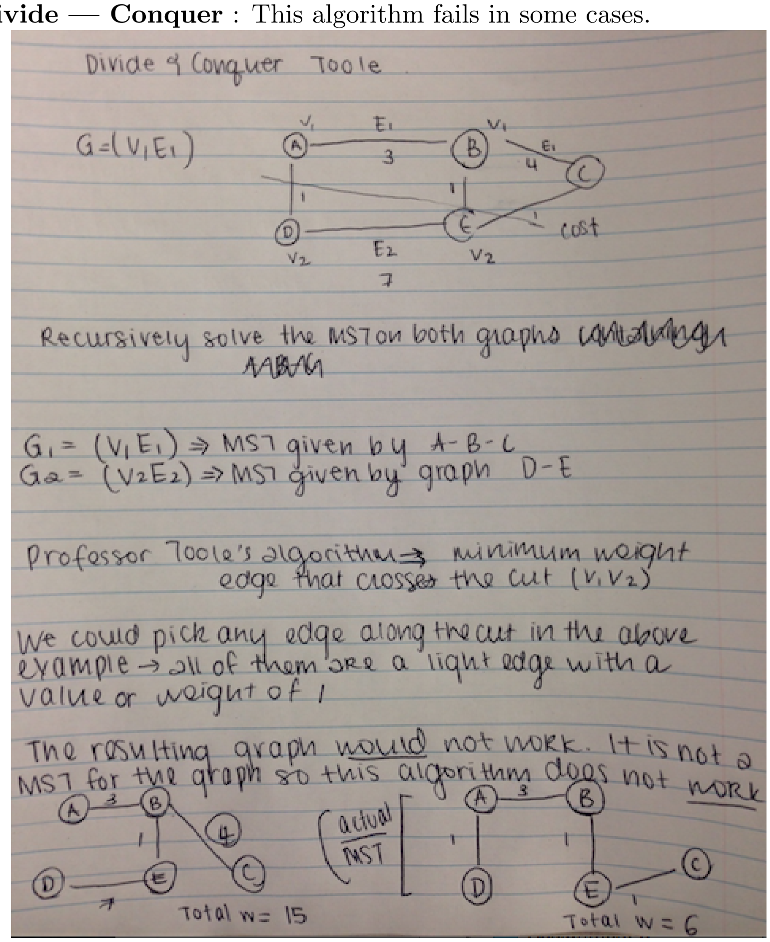
Sources: Stack Overflow, Wikipedia, Class Slides

1 Problem One

Divide — Conquer : This algorithm fails in some cases.



Divide & Conquer Toole

$G=(V_1 E_1)$

Recursively solve the MST on both graphs

$G_1 = (V_1 E_1) \Rightarrow$ MST given by A-B-C
$G_2 = (V_2 E_2) \Rightarrow$ MST given by graph D-E

Professor Toole's algorithm $\Rightarrow$ minimum weight edge that crosses the cut $(V_1 V_2)$

We could pick any edge along the cut in the above example → all of them are a light edge with a value or weight of 1

The resulting graph would not work. It is not a MST for the graph so this algorithm does not work

2 Problem Two

Randomized Quicksort: Used to improve the performance of Quicksort by randomly picking one element in the sequence, swaps with the first element and then calls Partition. For Randomized Quicksort, 0(n) calls are made to RANDOM in both cases. Effectively, Randomized Quicksort improves Quicksorts against its worst-case.

The worst case:

$$T(n) = T(n-1) + T(0) + (1)$$

$$T(0) = 0$$

randomized-partition is not called on a subproblem of size 0, so

$$T(n) = T(n-1) + (1)$$

$$T(n) = (n)$$

The best case: By the Master Theorem:

$$T(n)2T(n/2) + (1)$$

By case 1 of the Master Theorem, T(n) has the solution

$$T(n) = (n)$$

3 Problem Three

Let Eij be the event that $i < j$ and $A[i] > A[j]$ AND Let
$Xij = IEij = 1if(i,j)$ is an inversion of A $0if(i,j)$ is not an inversion of A

$$LetX = (i = 1 \Rightarrow n)(j = 1 \Rightarrow n)(Xij) \Rightarrow$$

Number of A inversions

$$E[X] = E[(i = 1 \Rightarrow n)(j = 1 \Rightarrow n)(Xij)]$$

$$= (i = 1 \Rightarrow n)(j = 1 \Rightarrow n)(E[Xij])$$

$$= (i = 1 \Rightarrow n)(j = 1 \Rightarrow n)(P(Eij))$$

$$= (i = 1 \Rightarrow n)(j = i + 1 \Rightarrow n)(P(Eij))$$

as we must have $i < j$

$$= (i = 1 \Rightarrow n)(j = i + 1 \Rightarrow n)(1/2)$$

we can choose the two numbers in $C(n, 2)$ ways and arrange them as required. So $P(Eij) = C(n, 2)/n(n-1))$ Thus

$$= (i = 1 \Rightarrow n)((n-i)/2)$$

$$= n(n-1)/4$$

4 Problem Four

probability — code — length

0.32	00	2
0.25	10	2
0.2	011	3
0.18	01	2
0.05	111	3

$$l = 0.32(2) + 0.25(2) + 0.2(3) + 0.18(2) + 0.05(3)$$

$$0.64 + 0.5 + 0.6 + 0.36 + 0.15$$

$$2.25 bit/symbol$$

5 Problem Five

MERCI!