COMP251 - Assignment 3

Lucia Eve Berger — 260473661

Sourced from Stackoverflow and Wikipedia.

1 Question 3

Longest monotonically increasing sub-sequence of a sequence of n numbers

Algorithm: let $S[pos]$ be defined as the smallest integer that ends an increasing sequence of length pos.

Iteration: Iterate through every integer i of the input set and do the following:

if $i >$ last element in S, then append i to the end of S. Until we have found a new largest LIS.

Otherwise find the smallest element in S, which is >= than i, and change it to i. Because S is sorted at any time, the element can be found using binary search in log(N).

Total RUNTIME - N integers and a binary search for each of them - N * log(N) = O(N log N)

Example:

$$Set : 263412958$$

$$Steps :$$

$$0.S = -InitializeStotheemptyset$$

$$1.S = 2 - NewlargestLIS$$

$$2.S = 2, 6 - NewlargestLIS$$

$$3.S = 2, 3 - Changed6to3$$

$$4.S = 2, 3, 4 - NewlargestLIS$$

$$5.S = 1, 3, 4 - Changed2to1$$

$$6.S = 1, 2, 4 - Changed3to2$$

$$7.S = 1, 2, 4, 9 - NewlargestLIS$$

$$8.S = 1, 2, 4, 5 - Changed9to5$$

$$9.S = 1, 2, 4, 5, 8 - NewlargestLIS$$

So the length of the LIS is 5 (the size of S).

To reconstruct the actual LIS we will again use a parent array. Let parent[i] be the predecessor of element with index i in the LIS ending at element with index i.

We can keep in the array S, not the actual integers, but their indices in the set. We do not keep 1, 2, 4, 5, 8, but keep 4, 5, 3, 7, 8.

That is input[4] = 1, input[5] = 2, input[3] = 4, input[7] = 5, input[8] = 8.

Updates to the the parent array, the actual LIS is:

input[S[lastElementOfS]], input[parent[S[lastElementOfS]]], input[parent[parent[S[lastElementOfS]]]],

Updates to the parent area: T

i. If i >last element in S, then parent[indexi] = indexLastElement. This means the parent of the newest element is the last element. We just prepend i to the end of S.

ii. Otherwise find the index of the smallest element in S, which is>= than i, and change it to i. Here parent[index i] = S[index - 1].

2 Question 4