SIR Model Overview

An Example of Infectious Disease Modeling

SIR Model Basics

In the SIR model, each individual in the population resides in one of three categories: Susceptible, Infected, or Recovered.

Individuals move strictly from the Susceptible group to the Infected group, and strictly from the Infected group to the Recovered group.

We can also call the Infected group the Infectious group, as these individuals are the only ones who can pass the disease to those in the Susceptible group.

Once an individual is in the Recovered group, they stay there permanently. We assume that re-infection is not possible.

Key Notation

Susceptible

$S(t)$ is the number of people at time t that are susceptible to the disease

$s(t) = S(t)/N$, the proportion of the population at time t that is susceptible to the disease

Infected

$I(t)$ is the number of people at time t that are infected with the disease (infectious)

$i(t) = I(t)/N$, the proportion of the population at time t that is infected with the disease

Recovered

$R(t)$ is the number of people at time t that have recovered from the disease

$r(t) = R(t)/N$, the proportion of the population at time t that have recovered from the disease

Key Assumptions

Population Size

Ignore births, deaths, immigration, and emigration - no one is added to the total population.

$S(t) + I(t) + R(t) = N$, where N denotes the constant total population.

Changes in S(t)

The size of S(t) can only decrease (or stay the same), because no one is added to the Susceptible group.

The only way that a person leaves the susceptible group is if they become infected.

Changes in R(t)

The size of R(t) can only increase (or stay the same), because once an individual enters the Recovered group, they do not leave.

Individuals that die from the disease are included in the Recovered group.

Equation Coefficients

b - likelihood an infected person will pass the disease onto a susceptible individual

k - rate of transition from the infected group to the recovered group (1 over the amount of time any individual is infectious)

Equations

ds / dt =

- b * s(t) * i(t)

Must be negative because s(t) can only decrease. Dependent on b, number of susceptible individuals remaining, and number of infectious individuals spreading the disease.

di / dt =

b * s(t) * i(t) - k * i(t)

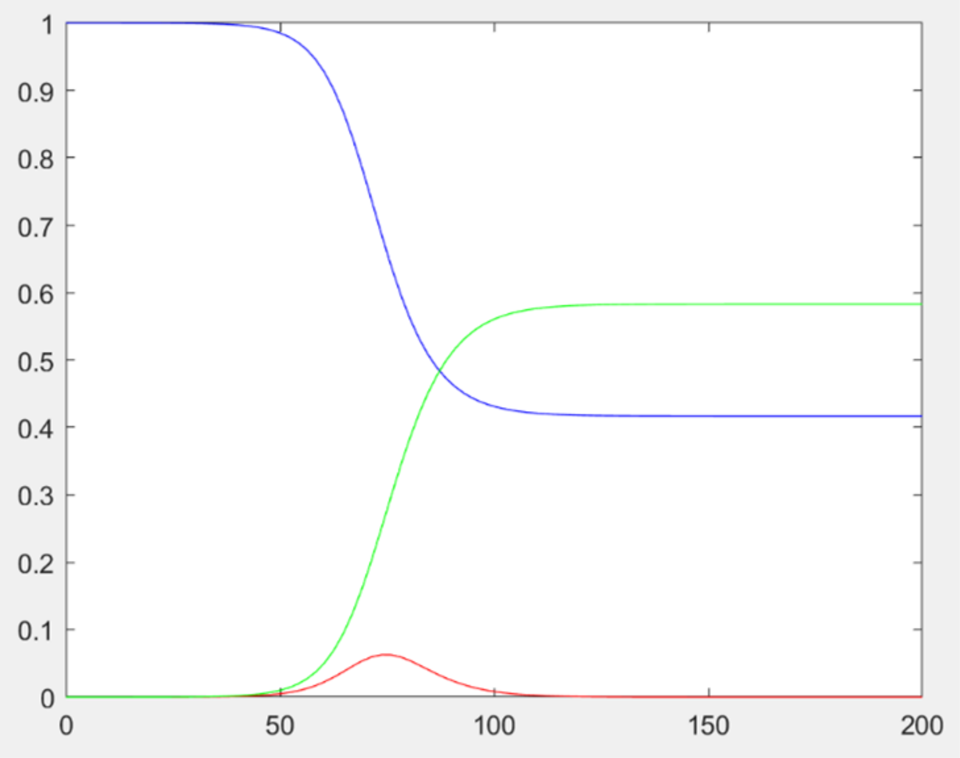
The infectious group can only gain members from the susceptible group, and can only lose members to the recovered group.

dr / dt =

k * i(t)

K is the rate of transition from the infected group to the recovered group, so k is how quickly infectious individuals recover from the disease.

Preliminary Example (b = ½, k = ⅓)



Max infected: 6.28% on day 73.76

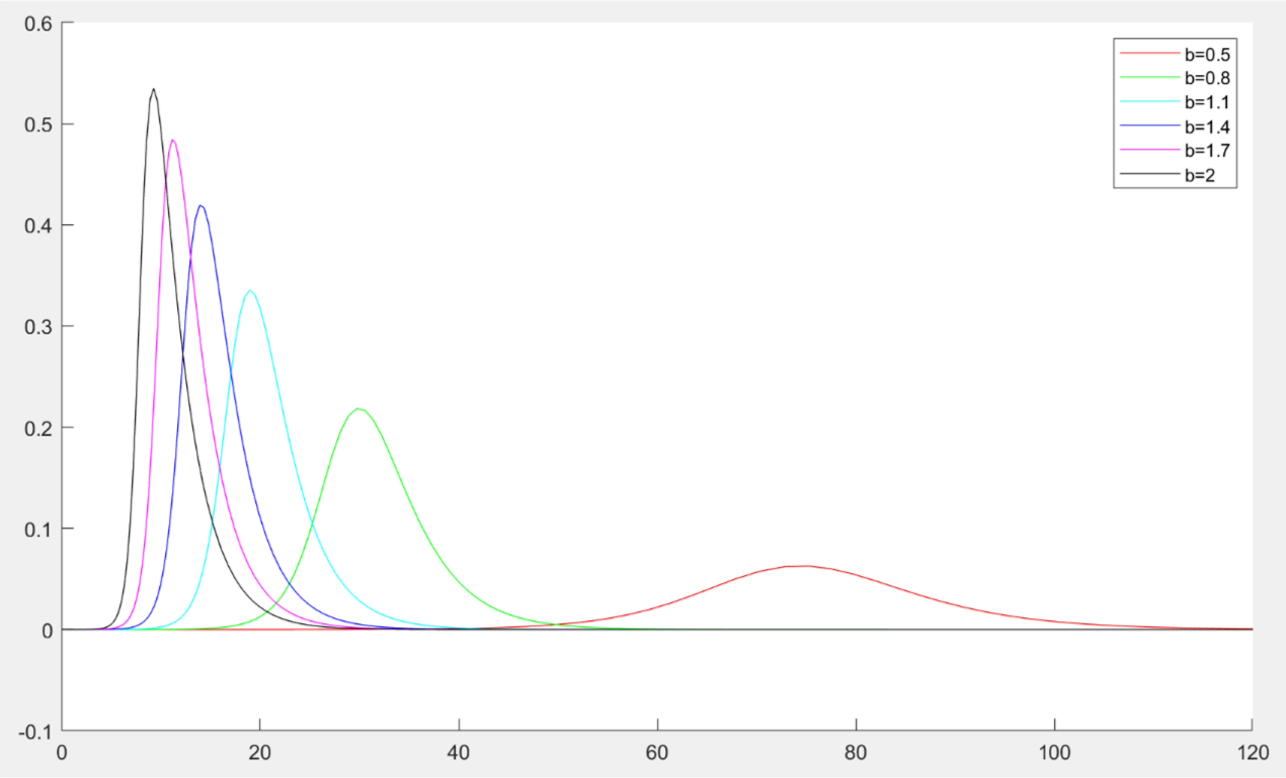
Time required for < 1% of the population to be infected: 98.71 days

Note: sum of lines is always 1

All lines stabilize around day 110 - the disease is gone, there is no one left to infect the remaining susceptible individuals

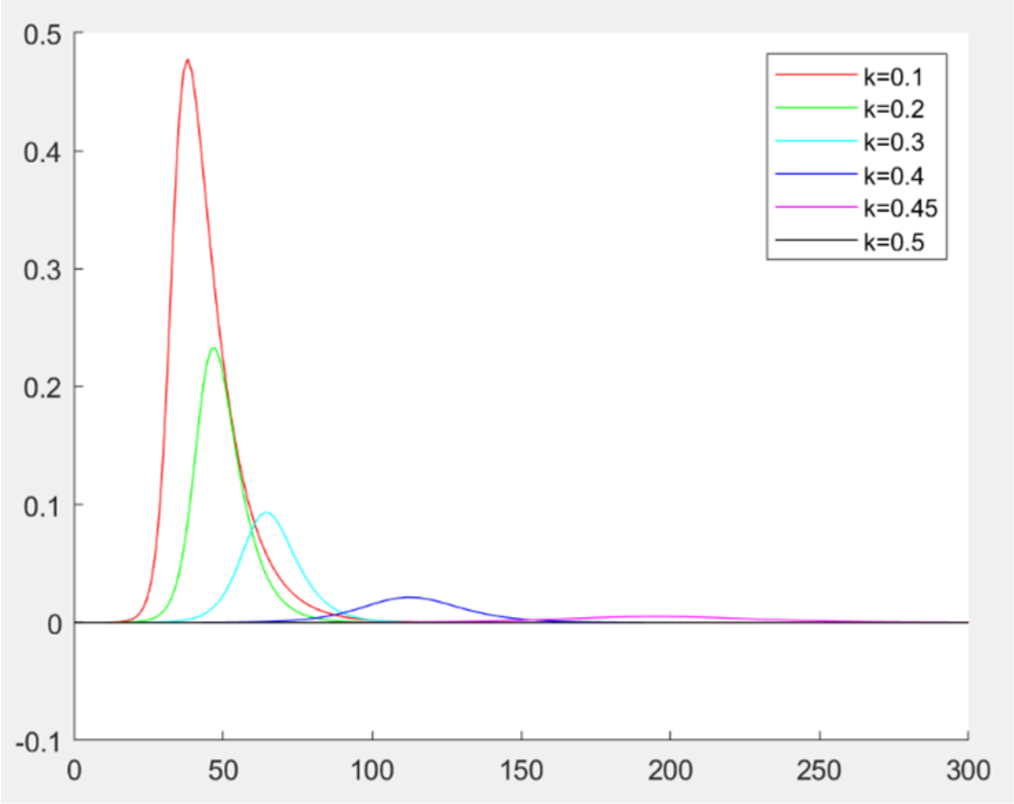
Varying b (holding k = ⅓ constant)

Graph of i(t):



Varying k (holding b = ½ constant)

Graph of i(t):



Contact Number

c = b / k

b / k = number of close contacts per day between an infected and a susceptible x length of infection (in days)

c = b / k = total number of close contacts between an infected and a susceptible

Vaccination

And Herd Immunity

Vaccination provides a direct route from the Susceptible group to the Recovered group.

Vaccinating a large enough proportion of the Susceptible population can avoid an epidemic.

In particular, if $s(0) < 1 / c$, then no epidemic will develop.

Example:
Hong Kong Flu

Assumptions

- The number of excess deaths in a week was proportional to the number of new cases of flu in some earlier week
- Total population N = 7,900,000
- Very few people were immune in the beginning, so almost everyone was susceptible
    - **$s(0) = 1$**
- Trace level of infection in the population ~ 10 people
    - **$i(0) = 1.27 \times 10^{-6}$**
- Hold all previous model assumptions

Excess Pneumonia-Influenza Deaths (‘68-‘69)

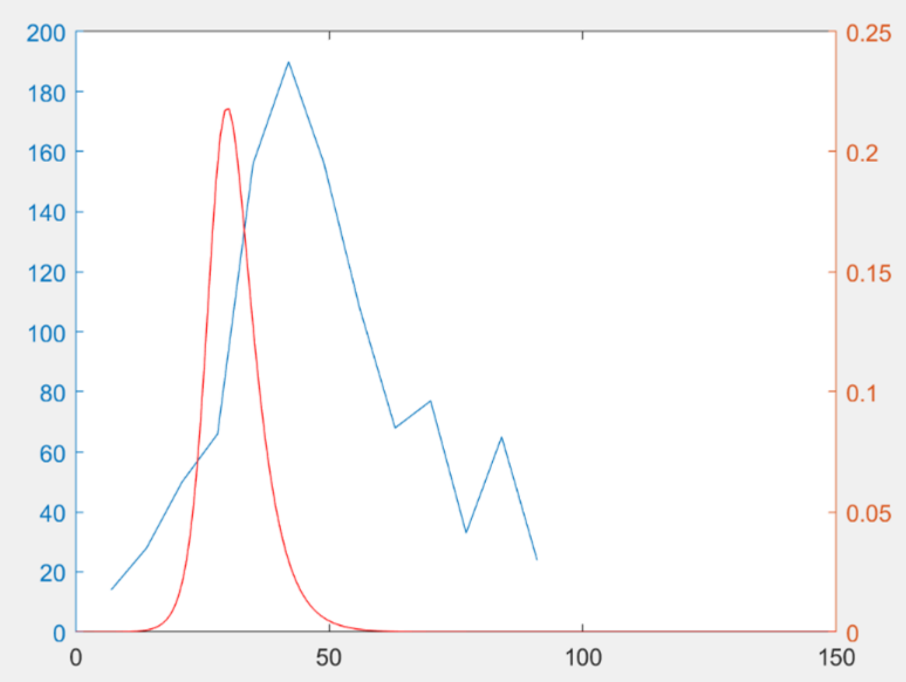
Week	Number of flu-related deaths	Week	Number of flu-related deaths
1	14	8	108
2	28	9	68
3	50	10	77
4	66	11	33
5	156	12	65
6	190	13	24
7	156		

Source: CDC

b = 0.8, k = ⅓

The peak in deaths occurs about 10 days after the peak of infected individuals.

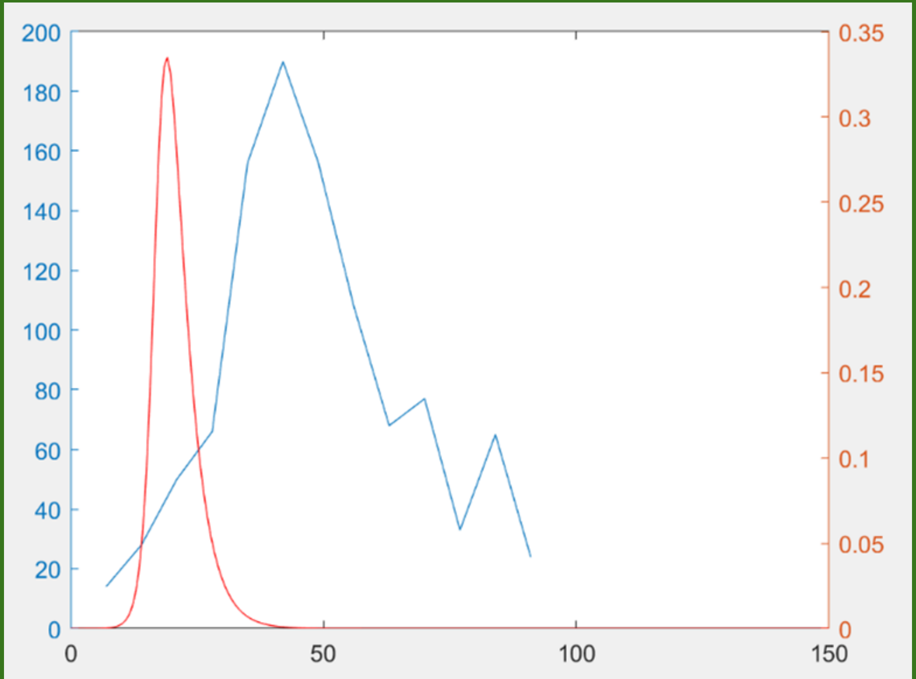
Graph of i(t) & # of deaths



b = 1.1, k = ⅓

The peak in deaths occurs about 3.5 weeks after the peak of infected individuals.

Graph of i(t) & # of deaths



Conclusion

If we assume that the excess number of deaths in a week was proportional to the number of new cases of the flu approximately 3 weeks earlier, an appropriate estimate for b would be somewhere around 1 (assuming we are holding $k = \frac{1}{3}$ constant).