LINEAR ALGEBRA

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SCIPY

- An extension of **NumPy** (this is why we push over, say, math)
- Documentation: docs.scipy.org
  - Assumes (this is standard practice):

```
>>> import numpy as np
>>> import matplotlib as mpl
>>> import matplotlib.pyplot as plt
```

- How you can extend from today. Today we're using **NumPy**

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SYSTEMS OF LINEAR EQUATIONS

- Solve for a number of unknowns from the same number of equations, i.e.:

$$x + 4y = -8$$
$$2x + 6y = 0$$

- In matrix notation:

$$\begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 0 \end{pmatrix}$$

- Equivalent to: $\mathbf{A}\vec{x} = \vec{b}$ (math notation), $\mathbf{A}\vec{v} = \vec{r}$ (notation in the notebook)
- What are each of A, v, and r? Which one(s) is/are matrices? Which one(s) is/are vectors?

SOLVING SYSTEMS VIA THE INVERSE

- If it were just algebra, we would see $Ax = b$ → $x = b/A$ We cannot do that with a matrix. We have to figure out another way to get Ax → $1 * x$ besides division
- We do this by defining a matrix that when multiplied by A, it always gives a matrix filled with 1s. This is the **inverse that gives the identity matrix**.

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}, A^{-1} = \frac{1}{2}\begin{pmatrix} -6 & 4 \\ 2 & -1 \end{pmatrix}$$

- Let's do the matrix multiplication to prove this gives us an identity matrix

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SOLVING SYSTEMS OF LINEAR EQUATIONS (INVERSES)

- Returning to our original example and with our inverse:

$$\begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 0 \end{pmatrix}, \quad A^{-1} = \frac{1}{2} \begin{pmatrix} -6 & 4 \\ 2 & -1 \end{pmatrix}$$

- How do we rewrite $\mathbf{A}\vec{v} = \vec{r}$ to solve for $\vec{v}$?

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SOLVING SYSTEMS OF LINEAR EQUATIONS (INVERSES)

- Returning to our original example and with our inverse:

$$\begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ 0 \end{pmatrix}, \quad A^{-1} = \frac{1}{2}\begin{pmatrix} -6 & 4 \\ 2 & -1 \end{pmatrix}$$

- Applying $\vec{v} = A^{-1}r$:

$$\frac{1}{2}\begin{pmatrix} -6 & 4 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} -8 \\ 0 \end{pmatrix} = \begin{pmatrix} 24 \\ -8 \end{pmatrix}, \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 24 \\ -8 \end{pmatrix}$$

- Can you think of some pitfalls of this process?

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SOLVING SYSTEMS OF LINEAR EQUATIONS (INVERSES)

Tedious! Switch to the notebook to work through the code. Make sure to read it, you'll be explaining it line by line to each other in a second

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SOLVING SYSTEMS OF LINEAR EQUATIONS (WITHOUT INVERSES)

- There's a lot of math to explain how to do this without inverses, but long story short:
  - It factorizes A using LU decomposition and then solves for x using forward/backward substitution
  - This makes things that aren't invertible or not easily invertible work. It's also a more efficient process and we like efficiency!!

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SOLVING SYSTEMS OF LINEAR EQUATIONS (WITHOUT INVERSES)

Work through the example and explain it to your neighbor

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EXAMPLE: STATICS

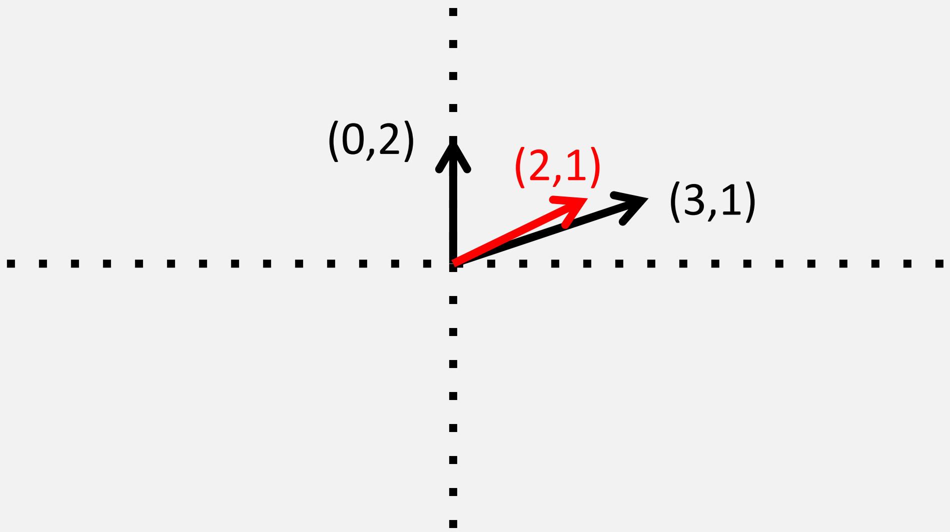
Work through the code. We're going to go for 10 min (there's one other example I want to get to)

EIGENVECTORS

- This is important for QM!! We're going to take a step back and think about matrices/vectors:

$$\overleftrightarrow{M}$$

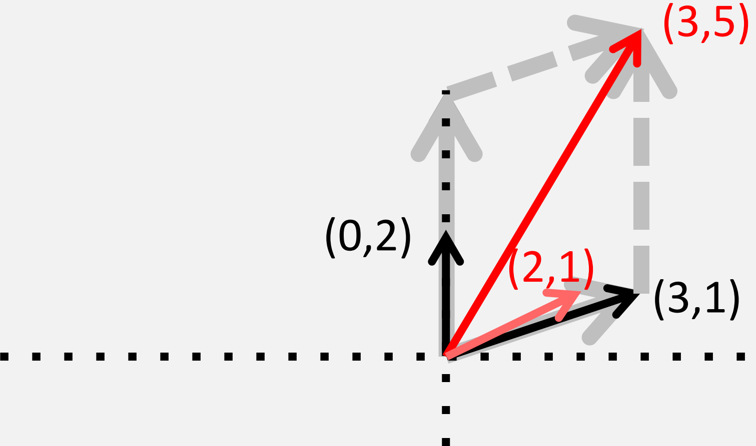
$$\begin{pmatrix} 0 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} =$$



EIGENVECTORS

$\overleftrightarrow{M}$

$$\begin{pmatrix} 0 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2\begin{pmatrix} 0 \\ 2 \end{pmatrix} + 1\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$



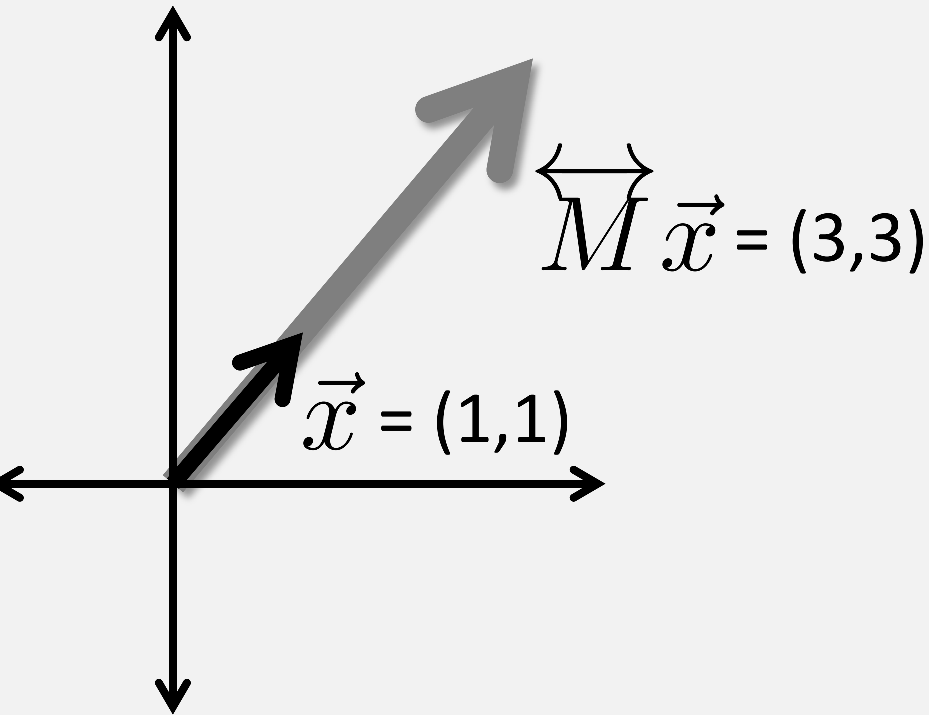
The new vector is:

1) **rotated**

2) **scaled**

EIGENVECTORS

- If a given vector **ONLY** gets scaled when multiplied by a matrix, it is an **eigenvector** of that matrix. The scaling factor is the **eigenvalue**:



$$\overset{\overleftrightarrow{M}}{\begin{pmatrix} 0 & 3 \\ 2 & 1 \end{pmatrix}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

EIGENVECTORS

- To formalize:

$$\boldsymbol{M}\,\vec{e} = \lambda\vec{e}$$

$$(\boldsymbol{M} - \lambda\,\boldsymbol{I})\vec{e} = \vec{0}$$

- And we usually say we can solve this with the **determinant** of $(\boldsymbol{M} - \lambda\,\boldsymbol{I})$

$$\left| det \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right| = |ad - bc|$$

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EIGENVECTORS

Work through the provided code to look at what's available, mimic the provided code to give the second set of eigenvectors and eigenvalues, and then we'll talk about it

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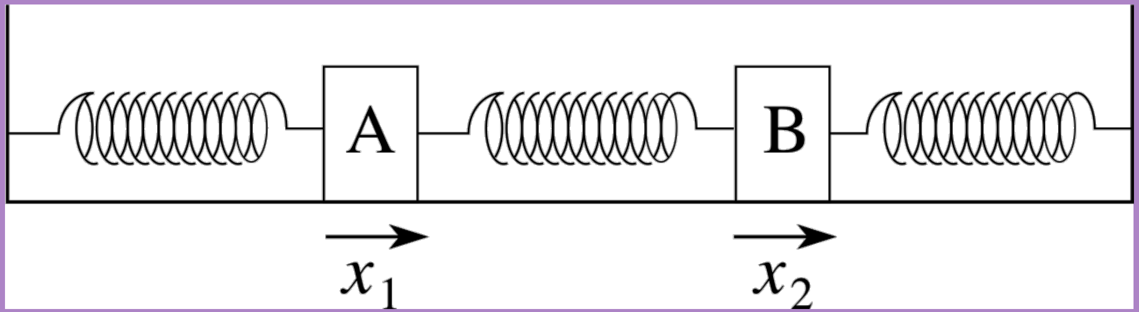
EXAMPLE: SMALL OSCILLATIONS OF A COUPLED OSCILLATOR

- No way you're going to get through all of this – this is a homework assignment that probably people would have a week to do. This is to get you to think through how to approach. And see how much can get done. No stress



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