Modern Physics

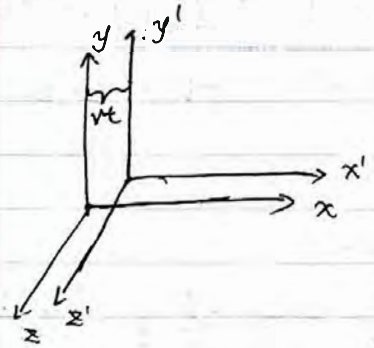
Chapter 1

1. Classical Relativity

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

Newton's Laws of motion only work correctly in inertial reference frames, that is, reference frames in which the Law of inertia holds

invariant, unchanged in any reference frames that moves with constant velocity relative to an inertial frame.



v — relative velocity of x' to x (constant)
u_x — velocity of x
u_x' — velocity of x'

$$x = x' + vt \qquad y = y' \qquad z = z \qquad t = t'$$

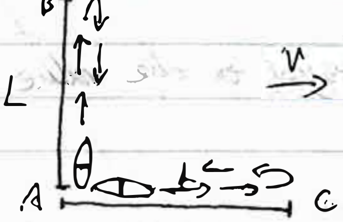
$$u_x = u_x' + v \qquad u_y = u_y' \qquad u_z = u_z'$$

$$\frac{du_x}{dt} = \frac{d}{dt}(u_x' + v) \Rightarrow a_x = a_x' \quad (\because \frac{dv}{dt} = 0)$$

$$\therefore \vec{F} = m\vec{a} = \vec{F}'$$

Theorem: Any reference frame that moves at constant velocity with respect to an inertial frame is also an inertial frame. Newton's laws of mechanics are invariant in all reference systems connected by a Galilean transformation.

Ex 1: Two equally matched rowers race each other over courses. Each oarsman rows at speed c in still water; the current in the river moves at speed v. Boat 1 goes from A to B, a distance L, and back. Boat 2 goes from A to C, also a distance L, and back. Which boat wins the race?



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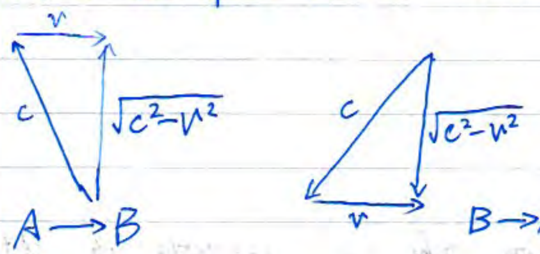
Boat 1: the speed relative to the ground: $\sqrt{c^2-v^2}$



Thus, the round-trip time t_1 for boat 1:

$$t_1 = t_{A\to B} + t_{B\to A} = \frac{L}{\sqrt{c^2-v^2}} + \frac{L}{\sqrt{c^2-v^2}} = \frac{2L}{\sqrt{c^2-v^2}} = \frac{2L}{c\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{2L}{c}\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} \simeq \frac{2L}{c}\left(1+\frac{1}{2}\frac{v^2}{c^2}\right) \text{ (binomial expansion)}$$

Boat 2: the speed relative to the ground, $c+v$ $\quad c-v$

$$\xrightarrow{c}\xrightarrow{v} \qquad \xrightarrow{c}\xleftarrow{v}$$

$$A \to C \qquad\qquad C \to A$$

Thus the round-trip time t_2 for boat 2:

$$t_2 = t_{A\to C} + t_{C\to A} = \frac{L}{c+v} + \frac{L}{c-v} = \frac{L(c-v)}{c^2-v^2} + \frac{L(c+v)}{c^2-v^2}$$

$$= \frac{2Lc}{c^2-v^2} = \frac{2Lc}{c^2}\frac{1}{1-\frac{v^2}{c^2}} = \frac{2L}{c}\left(1-\frac{v^2}{c^2}\right)^{-1}$$

$$\simeq \frac{2L}{c}\left(1+\frac{v^2}{c^2}\right) \text{ (binomial expansion)}$$

The difference Δt between the round-trip times of the boats:

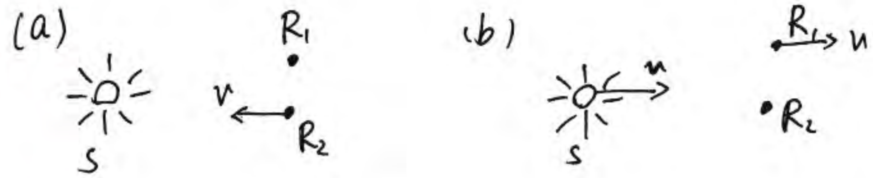
$$\Delta t = t_2 - t_1 = \frac{2L}{c}\left(1+\frac{v^2}{c^2}\right) - \frac{2L}{c}\left(1+\frac{1}{2}\frac{v^2}{c^2}\right) = \frac{Lv^2}{c^3}$$

The quantity Lv^2/c^3 is always positive; therefore, $t_2 > t_1$ and rower 1 has the faster average speed and wins the race.

2. Einstein's Postulates

Postulate 1: The Laws of physics are the same in all inertial reference frames. (no inertial system is preferred over any other; absolute motion cannot be detected.)

Postulate 2: The speed of light in a vacuum is equal to the value c, independent of the motion of the source.



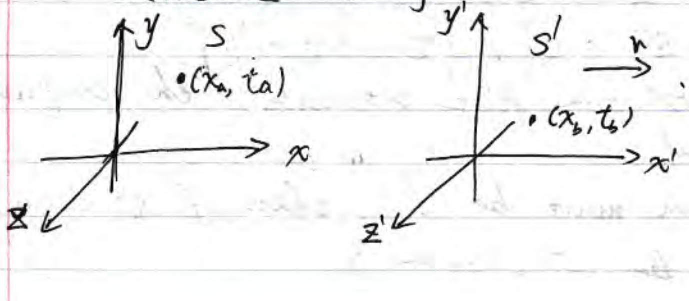
A light source S and two observers R_1, at rest relative to S, and R_2, moving toward S with speed v. The speed of light measured by R_1 is $c = 3\times10^8$ m/s. The speed measured by R_2 is NOT $c+v$.
By postulate 1, (a) is equivalent to (b), in which R_2 is at rest and the source S and R_1 are moving with speed v. That is, since absolute motion cannot be detected, it is not possible to say which is really moving and which is at rest.
By postulate 2, the speed of light from a moving source is independent of the motion of the source.
Thus, at (b) R_2 measures the speed of light to be c, just as R_1 does.

Event — something that happens,
independent of the particular inertial reference frame in order to describe the event.
Events don't "belong" to any reference frame.

Observer — an array of recording clocks located throughout the inertial reference system.

Relativity of Simultaneity:
Two spatially separated events simultaneous in one reference frame are not, in general, simultaneous in another inertial frame moving relative to the first.
Clocks synchronized in one reference frame are not, in general, synchronized in another inertial frame moving relative to the first.

3. The Lorentz Transformation



In Galilean transformation:
for S', $x' = x - vt$, $y' = y$, $z' = z'$, $t' = t$
for S, $x = x' + vt'$, $y = y'$, $z = z'$, $t = t'$

This classic transformation is not consistent with the Einstein postulates of the special relativity: the speed in S' should be $u_x' = c$ rather than $c - v$.

Lorentz Transformation:

$$x' = \gamma(x - vt)$$

where γ is a constant that can depend upon v and c but not on the coordinates.

$$x = \gamma(x' + vt')$$

time t':

$$x' = \gamma(x - vt) \quad ; \quad x = \gamma(x' + vt')$$

$$x = \gamma[\gamma(x - vt) + vt']$$

$$= \gamma^2 x - \gamma^2 vt + \gamma vt'$$

$$\therefore t' = \frac{x - \gamma^2 x + \gamma^2 vt}{\gamma v} = \frac{(1-\gamma^2)x}{\gamma v} + \gamma t$$

$$\therefore t' = \gamma\left(t + \frac{1-\gamma^2}{\gamma^2}\frac{x}{v}\right)$$

Now let a flash of light start from the origin of S at $t=0$. Since the origins coincide at $t = t' = 0$, the flash also starts at the origin of S' at $t'=0$. The flash expands from both origins as a spherical wave.

eqn for wave front according to an observer in S:

$$x^2 + y^2 + z^2 = c^2 t^2$$

in S':

$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

Relativistic Transformation Multiplier γ

$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

$$\gamma^2(x - vt)^2 + y^2 + z^2 = c^2\gamma^2\left(t + \frac{1-\gamma^2}{\gamma^2}\frac{x}{v}\right)^2$$

To be consistent with the first postulate, the coefficients of two eqns must be identical.

(coefficient of x^2 term must be 1, that of xt term must be 0, that of t^2 term must be c^2, ...)

for example, using the coefficient of x^2:

$$\gamma^2 - c^2\gamma^2 \frac{(1-\gamma^2)^2}{\gamma^4 v^2} = 1 \Rightarrow -c^2\frac{(1-\gamma^2)^2}{\gamma^2 v^2} = (1-\gamma^2) \Rightarrow$$

$$-c^2\frac{(1-\gamma^2)}{v^2} = \gamma^2 \Rightarrow \frac{c^2}{v^2}\gamma^2 - \frac{c^2}{v^2} = \gamma^2 \Rightarrow \left(\frac{c^2}{v^2}-1\right)\gamma^2 = \frac{c^2}{v^2} \Rightarrow$$

$$\gamma^2 = \frac{c^2}{v^2}\cdot\frac{v^2}{c^2} \Big/ \left(\frac{c^2}{v^2}-1\right)\cdot\frac{v^2}{c^2}$$

$$\therefore \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x' = \gamma(x - vt) \quad y' = y \quad z' = z \quad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

and inverse

$$x = \gamma(x' + vt') \quad y = y' \quad z = z' \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

with

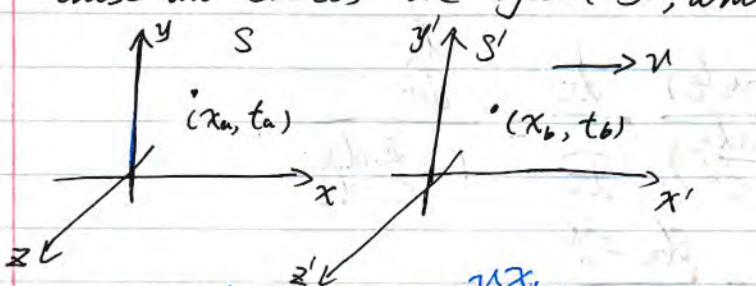
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

where $\beta = \frac{v}{c}$.

$\gamma = 1$ for $v = 0$ and $\gamma \to \infty$ for $v = c$

Transformation of Time Intervals.

The arrivals of two cosmic-ray μ leptons are recorded by detectors, one at time t_a at location x_a and the second at time t_b at location x_b in the laboratory reference frame. Find the time interval between those two events in System S', which moves relative to S at speed v?



recall: $t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$$t_b' - t_a' = \gamma\left(t_b - \frac{vx_b}{c^2}\right) - \gamma\left(t_a - \frac{vx_a}{c^2}\right)$$

$$= \gamma(t_b - t_a) - \frac{\gamma v}{c^2}(x_b - x_a)$$

Although the clocks in S are synchronized with each other, they are not, in general, synchronized for observers in other inertial frames

Special Case 1: $x_a = x_b$, the time $(t_b - t_a)$ is called proper time interval. Since $\gamma > 1$ for all frames moving relative to S, the ~~pp~~ proper time interval is the minimum time interval that can be measured between those events.

Special Case 2: $x_a = x_b$ and the two events will be simultaneous in a system S'' for which $t_b'' - t_a'' = 0$, when

$$\gamma(t_b - t_a) = \frac{\gamma v}{c^2}(x_b - x_a)$$

or

$$\beta = \frac{v}{c} = \left(\frac{t_b - t_a}{x_b - x_a}\right) c$$

Note, the time $(x_b - x_a)/c$ is for a light beam to travel from x_a to x_b

Relativistic Velocity Transformations

Suppose a particle moves in S with velocity $\vec{u}$ whose components are
$u_x = dx/dt$, $u_y = dy/dt$, $u_z = dz/dt$.
An observer in S' would measure the components:
$u_x' = dx'/dt'$, $u_y' = dy'/dt'$, $u_z' = dz'/dt'$.
Using the transformation equations, we obtain:

$$dx' = \gamma(dx - v\,dt) \qquad dy' = dy$$

$$dt' = \gamma\left(dt - \frac{v\,dx}{c^2}\right) \qquad dz' = dz$$

So,

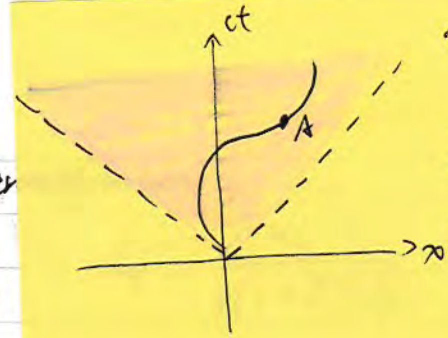
$$u_x' = \frac{dx'}{dt'} = \frac{\gamma(dx - v\,dt)}{\gamma\left(dt - \frac{v\,dx}{c^2}\right)} \cdot \frac{\frac{1}{dt}}{\frac{1}{dt}} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}}$$

$$\therefore \quad u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}$$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{v u_x}{c^2}\right)} \qquad u_z' = \frac{u_z}{\gamma\left(1 - \frac{v u_x}{c^2}\right)}$$

Inverse velocity tr

$$u_x = \frac{u_x' + v}{\left(1 + \frac{v u_x'}{c^2}\right)}$$



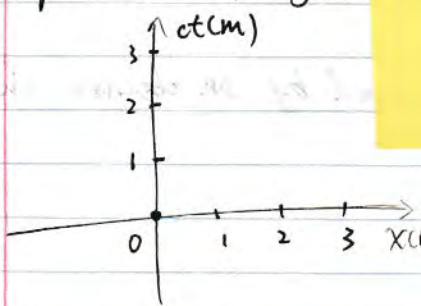
$-1 < \text{slope} < 1$

relativity limits the speed of particles with mass to less than c.

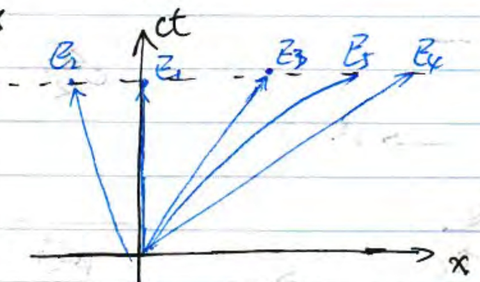
Spacetime Diagram

required for

10^{-9} s.



worldlines



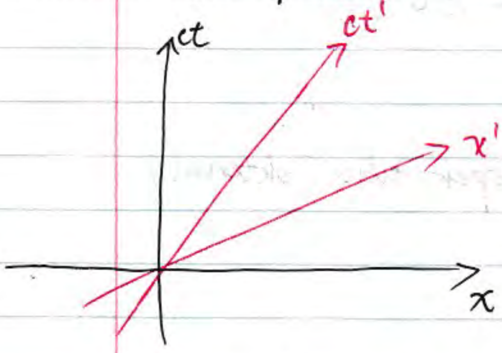
E_1 $(ct, 0)$ $v = 0$

E_2, E_3, E_4 v constant

E_5 $a > 0$

The worldlines is the record of the particle's travel through spacetime, giving its speed (= 1/slope) and acceleration (= 1/rate at which the slope changes) at every instant. $u = \Delta x / \Delta t = 1/(\Delta t / \Delta x) = 1/\text{slope}$

Two inertial frames S' and S with S' moving in the +x direction of S at speed v. Draw spacetime diagram.



$t = t' = 0$ as two origins $x = 0$ and $x' = 0$ coincide.

Lorentz transformation:

$x' = \gamma(x - vt) = 0$ for $x' = 0$

or $x = vt = \left(\frac{v}{c}\right) ct = \beta ct$

and $ct = \frac{x}{\beta}$

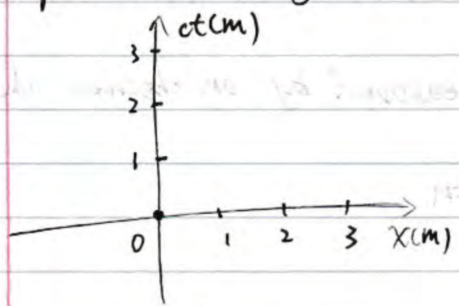
which says that the slope (in S) of the worldline of the point $x' = 0$, the ct' axis, is $\frac{1}{\beta}$

As the origin of S' (the point where $x' = 0$) moves in S, the worldline is the ct' axis since the ct' axis is the locus of all points with $x' = 0$

inverse velocity transformations:

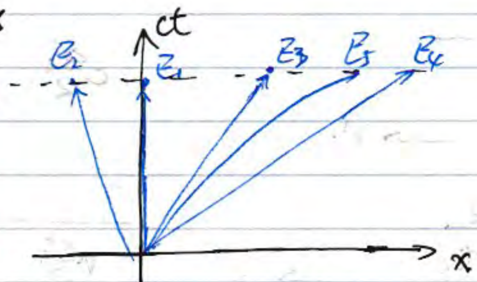
$$u_x = \frac{u_x'+v}{(1+\frac{vu_x'}{c^2})} \qquad u_y = \frac{u_y'}{\gamma(1+\frac{vu_x'}{c^2})} \qquad u_z = \frac{u_z'}{\gamma(1+\frac{vu_x'}{c^2})}$$

Spacetime Diagrams



a meter of time means the time required for light to travel one meter i.e., 3.3×10^{-9} s.

worldlines



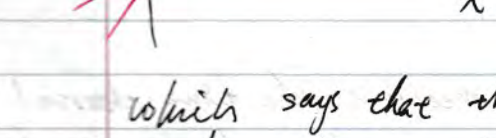
E_1 $(ct, 0)$ $v = 0$

E_2, E_3, E_4 v constant

E_5 $a>0$

The worldlines is the record of the particle's travel through spacetime, giving its speed (= 1/slope) and acceleration (= 1/rate at which the slope changes) at every instant. $u = \Delta x/\Delta t = 1/(\Delta t/\Delta x) = 1/\text{slope}$

Two inertial frames S' and S with S' moving in the +x direction of S at speed v. Draw spacetime diagram.



$t = t' = 0$ as two origins $x=0$ and $x'=0$ coincide.

Lorentz transformation:

$x' = \gamma(x - vt) = 0$ for $x' = 0$

or $x = vt = \left(\frac{v}{c}\right)ct = \beta ct$

and $ct = \frac{x}{\beta}$

which says that the slope (in S) of the worldline of the point $x'=0$, the ct' axis, is $\frac{1}{\beta}$

As the origin of S' (the point where $x'=0$) moves in S, the worldline is the ct' axis since the ct' axis is the locus of all points with $x'=0$

In the same manner, the x' axis can be located using the fact that it is the locus of points for which $ct' = 0$.

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = 0$$

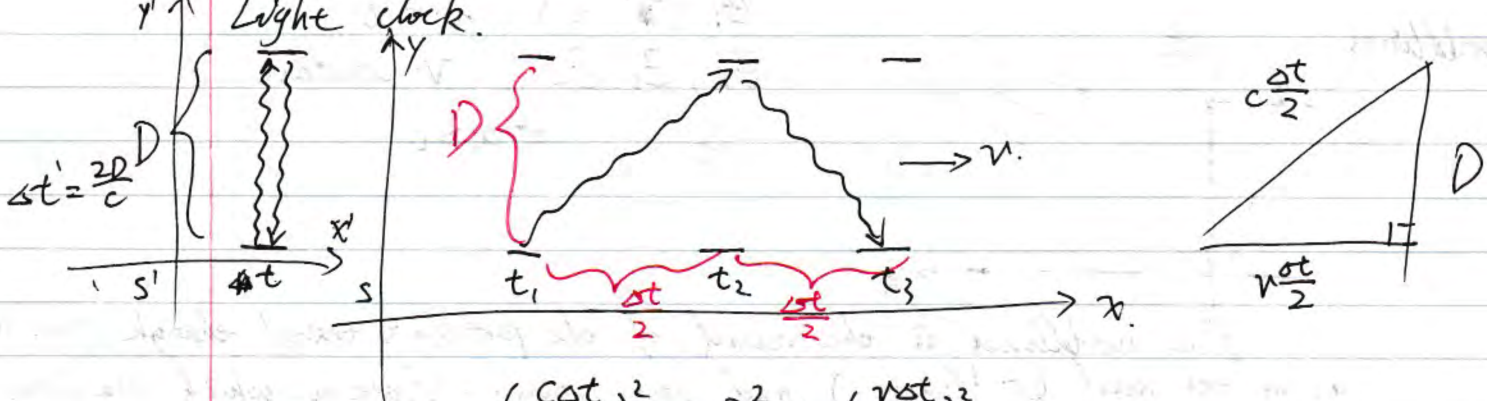
or $t = \frac{vx}{c^2} \Rightarrow ct = \frac{vx}{c} = \beta x$

Thus the slope of the x' axis as measured by an observer in S is β.

1.4. Time Dilation and Length Contraction

Time Dilation

Light clock.



$$\left(\frac{c\Delta t}{2}\right)^2 = D^2 + \left(\frac{v\Delta t}{2}\right)^2$$

$t_3 - t_1 = \Delta t$.

total path length $= 2\cdot\sqrt{D^2 + \frac{v^2\Delta t^2}{4}}$

$$\Delta t^2 = \left(D^2 + \frac{v^2\Delta t^2}{4}\right)\cdot\frac{4}{c^2} = \left(\frac{2D}{c}\right)^2 + \Delta t^2\left(\frac{v}{c}\right)^2$$

$$\Delta t^2\left(1 - \frac{v^2}{c^2}\right) = \left(\frac{2D}{c}\right)^2$$

since $\Delta t' = \frac{2D}{c} \equiv \Delta t_p$ (t_p — proper time interval)

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\,\Delta t_p$$

The observer in frame S always measures the time interval between two events to be longer (since $\gamma > 1$) than the corresponding interval measured on the clock.

Lorentz transformation : $x' = \gamma(x - vt) = \gamma(x - \beta ct)$

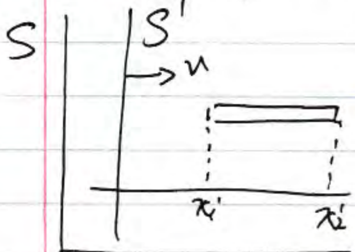
$$ct' = \gamma(ct - \beta x) \quad \beta = \frac{v}{c}.$$

$$y' = y \qquad z' = z$$

Length Contraction.

proper length L_p — the length of an object measured in the reference frame in which the object is at rest.

assume $L_p = x_2' - x_1'$



~~t_2~~

$t_1 = t_2 = t_0$

$t_1' \neq t_2'$

$E_1 = (ct_0, x_1)$

$E_2 = (ct_0, x_2)$

$x_1' = \gamma(x_1 - vt_0)$

$x_2' = \gamma(x_2 - vt_0)$

$$\Delta x = x_2' - x_1' = \gamma(x_2 - x_1)$$

$$\therefore x_2 - x_1 = \frac{1}{\gamma}(x_2' - x_1') = \frac{1}{\gamma}L_p = L.$$

$$\therefore L = \frac{1}{\gamma}L_p = \sqrt{1 - \frac{v^2}{c^2}}\,L_p$$

$E_A = (0, L)$

$E_B = (0, 0)$

$ct_A' = \gamma(ct_A - \beta x_A) = \gamma(\beta L)$

$x_A' = \gamma(x_A - \beta ct) = \gamma L$

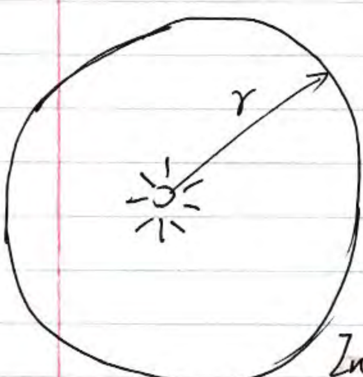
$E_A' = (-\gamma\beta L, \gamma L)$

$E_B' = (0, 0)$

Spacetime Interval

$$\Delta s^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$$

$[\text{interval}]^2 = [\text{separation in time}]^2 - [\text{separation in space}]^2$



flash light at $t = 0$

light front is at $r = ct$ for any observer

or

$$c^2\Delta t^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2) = 0$$

two events separated by Δt, Δx, Δy & Δz.

Invariant interval between two events

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z$$

for 1-D : $\Delta s^2 = c^2\Delta t^2 - \Delta x^2$

In S : $\Delta t = 0 \quad \Delta x = -L \quad (x_B - x_A)$

$$\Delta s^2 = 0 - L^2 = -L^2$$

invariant: observers in both S and S' measure the same number for the separation of the two events in spacetime

In S' : $c\Delta t = \gamma\beta L \quad \Delta x = \gamma L$

$$\Delta s^2 = c^2\gamma^2\beta^2L^2 - \gamma^2L^2 = \gamma^2L^2(c^2\beta^2-1) = \gamma^2L^2\left(-\frac{1}{\gamma^2}\right) = -L^2$$

$(\Delta s)^2$ may be positive, negative, or zero depending on the relative sizes of the time and space separation.

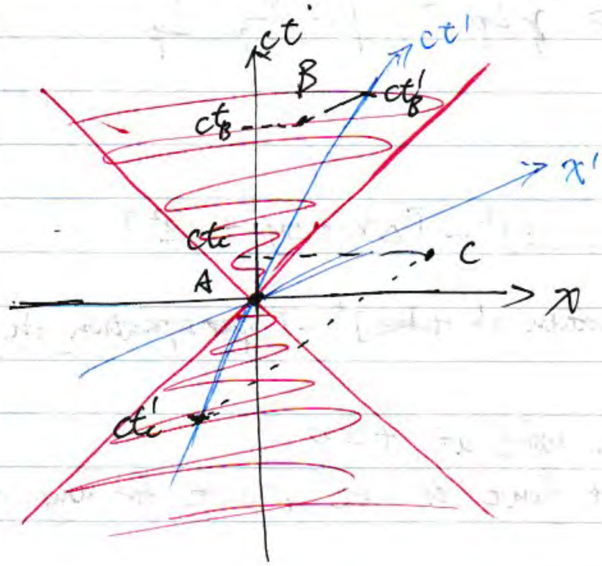
$(\Delta s)^2 > 0$ Timelike

$$\Delta t_p = \frac{\Delta s}{c} = \sqrt{(\Delta t)^2 - \left(\frac{\Delta x}{c}\right)^2}$$

since $\Delta t = \Delta t_p$ when $\Delta x = 0$

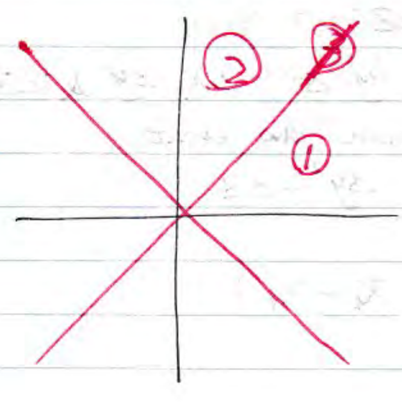
$(\Delta s)^2 < 0$ Spacelike

$\Delta s^2 = 0$ Lightlike

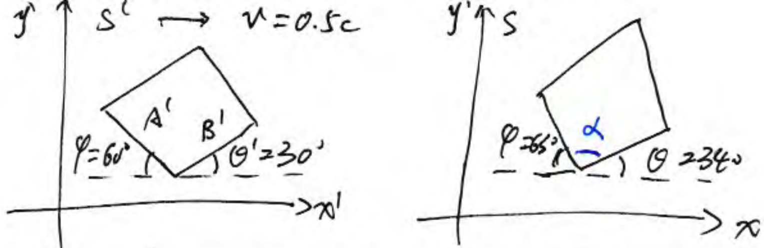


upper shaded area — A's future
lower shaded area — A's past.

C occurs after A in S but before A in S'



① spacelike
② Timelike
③ lightlike



Ex: Consider the square in the $x'y'$ plane of S' with one side making 30° angle with the x' axis. If S' moves with $\beta=0.5$ relative to S. What's the shape and orientation of the figure in S?

$$A=\frac{1}{\gamma}A'=\left(\cos^2 30^\circ+\frac{\sin^2 30^\circ}{\gamma^2}\right)^{\frac{1}{2}}A'=0.968\,A'$$

$$B=\frac{1}{\gamma}B'=\left(\sin^2 30+\frac{\cos^2 30^\circ}{\gamma^2}\right)^{\frac{1}{2}}B'=0.901\,B'$$

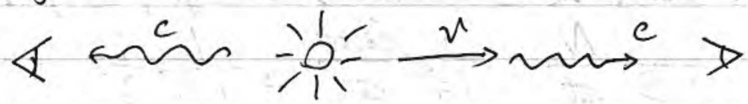
$\because$ square, $A'=B'$ (in S')

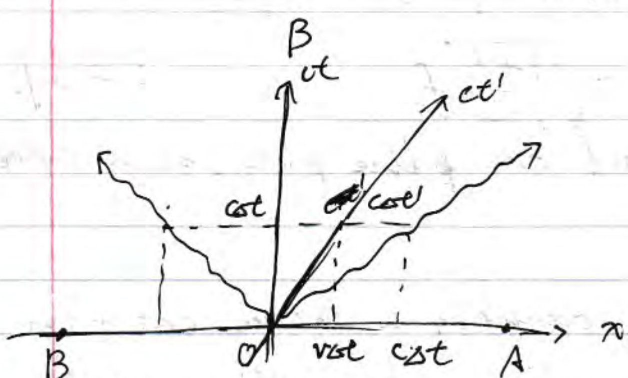
$$\theta=\tan^{-1}\left[\frac{B'\sin 30^\circ}{B'\cos 30^\circ/\gamma}\right]=\tan^{-1}\left[\gamma\frac{\sin 30^\circ}{\cos 30^\circ}\right]=33.7^\circ$$

$$\varphi=\tan^{-1}\left[\frac{A'\cos 30^\circ}{A'\sin 30^\circ/\gamma}\right]=\tan^{-1}\left[\gamma\frac{\cos 30^\circ}{\sin 30^\circ}\right]=63.4^\circ$$

$\alpha=180^\circ-(63.4^\circ+33.7^\circ)=82.9^\circ$, the geometry is not a square (in S)

1.5 The Doppler Effect





A, B at rest. The source is located at $x'=0$, and its worldline is the ct' axis. Let the source emit a train of N electromagnetic waves in each direction at $x=x'=0$, $t=t'=0$.

For A: the source emits N waves over Δt.
the first wave emitted will have traveled a distance $c\Delta t$,
the source itself a distance $v\Delta t$ in S.
Thus, the N waves are seen by the observer at A to occupy a distance $c\Delta t-v\Delta t$ and their wavelength λ.

$$\lambda=\frac{c\Delta t-v\Delta t}{N}\qquad\frac{(\text{total length})}{\#\text{ of waves}}$$

and frequency $f = c/\lambda$

$$f = \frac{c}{\lambda} = \frac{cN}{c\Delta t - v\Delta t} = \frac{cN}{(c-v)\Delta t} = \frac{1}{1-\frac{v}{c}}\frac{N}{\Delta t} = \frac{1}{1-\beta}\frac{N}{\Delta t}$$

$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$$= \frac{1}{1-\beta}\frac{N}{\Delta t}$$

The frequency of the source in S', called the proper frequency.

$$f_0 = \frac{c}{\lambda'} = \frac{N}{\Delta t'},$$ where $\Delta t'$ is measured in S', the rest system of the source.

$\Delta t' = t_p$ the proper time interval since the light waves, in particular the first and the Nth, are all emitted at $x'=0$; hence $\Delta x' = 0$ between the first and the Nth in S'.

Thus $\Delta t = \gamma \Delta t'$

When the source and receiver are moving toward each other, the observer ~~A~~ A in S measures the frequency,

$$f = \frac{1}{1-\beta}\frac{N}{\Delta t} = \frac{1}{1-\beta}\frac{f_0\Delta t'}{\Delta t} = \frac{f_0}{1-\beta}\frac{1}{\gamma} = \frac{\sqrt{1-\beta^2}}{1-\beta}f_0$$

approaching

$$= \frac{\sqrt{1-\beta}\sqrt{1+\beta}}{\sqrt{1-\beta}\sqrt{1-\beta}}f_0 = \sqrt{\frac{1+\beta}{1-\beta}}f_0 \qquad (f > f_0)$$

a shift of visble light toward the ~~p~~ blue part, it is called a Blueshift.

For B: in S see N waves occupying a distance $c\Delta t + v\Delta t$.

$$f = \frac{c}{\lambda} = \frac{cN}{c\Delta t + v\Delta t} = \frac{1}{1+\beta}\frac{N}{\Delta t} = \frac{1}{1+\beta}\frac{f_0\Delta t'}{\Delta t} = \frac{1}{1+\beta}\frac{f_0}{\gamma}$$

receding

$$= \frac{\sqrt{1-\beta^2}}{1+\beta}f_0 = \sqrt{\frac{1-\beta}{1+\beta}}\, f_0 \qquad (f < f_0)$$

Redshift

binomial theorem, $(1+x)^n \approx 1+nx \quad x \ll 1$

$$f=\sqrt{\frac{1+\beta}{1-\beta}}f_0 = f_0(1+\beta)^{\frac{1}{2}}(1-\beta)^{-\frac{1}{2}} \approx f_0(1+\tfrac{1}{2}\beta)(1+\tfrac{1}{2}\beta)$$

$$=f_0(1+\beta+\tfrac{1}{4}\beta) \approx f_0(1+\beta)$$

For approaching: $\frac{f}{f_0}=1+\beta$

For receding: $\frac{f}{f_0}=1-\beta$

$$\Delta f = f-f_0 = f_0(1+\beta)-f_0 = f_0\beta \Rightarrow \left|\frac{\Delta f}{f_0}\right|=\beta$$

$$=f_0(1-\beta)-f_0 = -f_0\beta$$

Ex: The Sun rotates at the equator once in about 25.4 days. The Sun's radius is 7.0×10^8 m. Compute the Doppler effect that you would expect to observe at the left and right limbs (edges) of the Sun near the equator for light of wavelength $\lambda = 550$ nm $= 550\times10^{-9}$ m (yellow light). Is this a redshift or blueshift.

the speed of limbs:

$$v=\frac{2\pi R}{T}=\frac{2\pi(7.0\times10^8)\,\text{m}}{25.4\,\text{d}\cdot 3600\,\text{s/hr}\cdot 24\,\text{hr/d}} = 2000\ \text{m/s}$$

$\because v \ll c$

$\therefore$ may use $\frac{\Delta f}{f_0}\approx\beta \Rightarrow \Delta f \approx \beta f_0 = \beta\frac{c}{\lambda}$.

$$\Delta f = \beta\frac{c}{\lambda} = \frac{v}{\lambda} = \frac{2000\ \text{m/s}}{550\times10^{-9}\,\text{m}} = 5.45\times10^{14}\ \text{Hz}$$

redshift $\rightarrow$ receding limb blueshift $\rightarrow$ approaching limb