

# List of errata

## The Lattice Boltzmann Method: Principles and Practice

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*Have you found anything else that should be on this list? If so, please send us an email about it to [authors@lbmbook.com](mailto:authors@lbmbook.com). The most recent version of this document can be found at <https://github.com/lbm-principles-practice/errata>.*

### Chapter 1

**Page 8:** Equation (1.20) should be  $u_x(y) = -\frac{1}{2\eta} \frac{dp}{dx} y(d-y)$ . (*Thanks to Hiran Wijesinghe.*)

**Page 20:** The function arguments on the left-hand side of (1.44c) should be  $v_x^2$ ,  $v_y^2$ , and  $v_z^2$ , and the function argument on the left-hand side of (1.44d) should be  $|\mathbf{v}|^2$ . (*Thanks to Bert Rubash.*)

**Page 20:** In the right-hand expression on the first line of (1.45),  $p$  and  $\rho$  are flipped in the exponential. The correct equilibrium distribution, as expressed through  $p$  and  $\rho$ , is  $\rho (\rho/2\pi p)^{3/2} e^{-\rho|\mathbf{v}|^2/(2p)}$ . (*Thanks to Hiran Wijesinghe.*)

**Page 24:** The sentence above (1.57) should start with “Thus, the first moment of the Boltzmann equation. . .” (*Thanks to Lei Zhou.*)

## Chapter 3

**Page 72:** In the third term on the left-hand side of (3.14), the force  $F_\alpha$  is mistyped as  $f_\alpha$ . (Thanks to Hiran Wijesinghe.)

**Page 72:** Below (3.17), the non-dimensionalisation should be  $\Omega^\star = \Omega \ell V^{d-1} / \rho_0$ . The other non-dimensionalisation expressions are correct as long as the density  $\rho$  is taken as mass per length to the power of  $d$ . (Thanks to Hiran Wijesinghe.)

**Page 79:** The factor  $1/2$  is missing in the quadratic term. (Thanks to Rohan Vernekar.)

**Page 82:** The physical velocity  $\mathbf{u}$  is also scaled along with lattice velocities  $\mathbf{c}$  with the factor  $\sqrt{3}$ . (Thanks to Rohan Vernekar.)

## Chapter 5

**Page 198:** Equation (5.46), at the very last line, should be  $\rho_w u_{w,x} = c (f_1 - f_3) + \frac{1}{3} \rho_w u_{w,x} + 2c N_x$ . (Thanks to Zhang Chuangde.)

**Page 198:** Equation (5.52) should be  $f_8 = f_6 + \frac{1}{2} (f_2 - f_4) - \frac{1}{2c} \rho_w u_{w,y} + \frac{1}{6c} \rho_w u_{w,x}$ . (Thanks to Tang Jun.)

## Chapter 6

**Page 233:** The equation at the end of the line below (6.5) should be  $S_i = \left(1 - \frac{\Delta t}{2\tau}\right) F_i$ . (Thanks to Yongsoo Park.)

**Page 239:** Equation (6.26c) contains an extra  $\frac{1}{\tau}$  in the pre-factor of the force contribution. The correct form should be  $\Pi = \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i \tilde{f}_i \mathbf{c}_i \mathbf{c}_i + \frac{\Delta t}{2\tau} \sum_i f_i^{\text{eq}} \mathbf{c}_i \mathbf{c}_i + \frac{\Delta t}{2} \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i F_i \mathbf{c}_i \mathbf{c}_i$ .

**Page 241:** In Table 6.1,  $A = \tau / \Delta t$  for the Shan and Chen method.

**Page 246:** The definition of  $\Pi_{\alpha\beta}^{(1)}$  is faulty along this page, missing the inclusion of the discrete lattice  $\left(1 - \frac{\Delta t}{2\tau}\right)$ . The correct form should be  $\Pi_{\alpha\beta}^{(1)} = \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i \left(f_i^{(1)} + \frac{\Delta t}{2} F_i^{(1)}\right) c_{i\alpha} c_{i\beta}$ . Due to this mistake, several equations on this page require correction. They are:

- Equation (6.37b) should be  $\partial_t^{(2)}(\rho u_\alpha) + \partial_\beta^{(1)} \Pi_{\alpha\beta}^{(1)} = 0$ .
- Equation (6.38b) should be  $\left(\epsilon \partial_t^{(1)} + \epsilon^2 \partial_t^{(2)}\right)(\rho u_\alpha) + \epsilon \partial_\beta^{(1)} \Pi_{\alpha\beta}^{\text{eq}} = \epsilon F_\alpha^{(1)} - \epsilon^2 \partial_\beta^{(1)} \Pi_{\alpha\beta}^{(1)}$ .

- Equation (6.39) should be  $\partial_t^{(1)} \Pi_{\alpha\beta}^{\text{eq}} + \partial_\gamma^{(1)} \Pi_{\alpha\beta\gamma}^{\text{eq}} - \sum_i F_i^{(1)} c_{i\alpha} c_{i\beta} = -\left(\frac{2}{2\tau - \Delta t}\right) \Pi_{\alpha\beta}^{(1)}$ .
- Equation (6.40) should be  $\Pi_{\alpha\beta}^{(1)} = \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i f_i^{(1)} c_{i\alpha} c_{i\beta} + \frac{\Delta t}{2} \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i F_i^{(1)} c_{i\alpha} c_{i\beta}$ .
- Equation (6.41) should be  $\Pi_{\alpha\beta}^{(1)} = -\rho c_s^2 \left(\tau - \frac{\Delta t}{2}\right) \left(\partial_\beta^{(1)} u_\alpha + \partial_\alpha^{(1)} u_\beta\right) + \mathcal{O}(u^3)$ .
- The sentence below equation (6.41) should read: “Therefore, the viscous stress is given by  $\sigma_{\alpha\beta} = -\Pi_{\alpha\beta}^{(1)}$ .”<sup>1</sup>

(Thanks to Bart Postma.)

## Chapter 7

**Page 283:** After (7.26) it is written that  $\bar{u} = \hat{u}/2$  for 2D Poiseuille flow, but it must be  $\bar{u} = 2\hat{u}/3$ . (The wrong equation in the book is correct for 3D Poiseuille flow.) (Thanks to Christoph Rettinger.)

## Chapter 8

**Page 311:** The first line should read “. . . larger than unity lead to reduced accuracy.” (Thanks to Mohammed Boraie.)

## Chapter 10

**Page 421:** The equilibrium moment  $e^{\text{eq}}$  in (10.32) should be  $-2\rho + 3\rho(u_x^2 + u_y^2)$ . (Thanks to Mohammed Boraie.)

**Page 421:** The equilibrium moment  $q_x^{\text{eq}}$  in (10.32) should be  $\rho u_x(3u_y^2 - 1)$ . Accordingly,  $q_y^{\text{eq}}$  in (10.32) should be  $\rho u_y(3u_x^2 - 1)$ . (Thanks to Christophe Guy Coreixas.)

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<sup>1</sup> Still concerning this sentence, two notes on the relation between  $\Pi_{\alpha\beta}^{(1)}$  and the deviatoric stress  $\sigma_{\alpha\beta}$  are in order:

*Note 1.* By introducing the correct form of  $\Pi_{\alpha\beta}^{(1)}$ , as given by Eq. (6.40), into  $\sigma_{\alpha\beta} = -\Pi_{\alpha\beta}^{(1)}$  we obtain the  $\sigma_{\alpha\beta}$  given by Eq. (6.4), where  $\sum_i f_i^{\text{neq}} c_{i\alpha} c_{i\beta} \simeq \sum_i f_i^{(1)} c_{i\alpha} c_{i\beta}$  and  $\sum_i F_i^{(1)} c_{i\alpha} c_{i\beta} = F_\alpha u_\beta + u_\alpha F_\beta$  according to Eq. (6.15).

*Note 2.* The relation between  $\sigma_{\alpha\beta}$  and  $\Pi_{\alpha\beta}^{(1)}$  given in Chapter 6 differs from that given in Chapter 4. Namely,  $\sigma_{\alpha\beta} = -\Pi_{\alpha\beta}^{(1)}$  in Chapter 6 and  $\sigma_{\alpha\beta} = -\left(1 - \frac{\Delta t}{2\tau}\right) \Pi_{\alpha\beta}^{(1)}$  in Chapter 4 (cf. Eq. (4.14)). The difference lies in the discrete lattice pre-factor  $\left(1 - \frac{\Delta t}{2\tau}\right)$  and it is due to the following reason. While the LBE analysed in Chapter 4 is based on the rectangular discretisation of the collision operator, in Chapter 6 it is based on the trapezoidal discretisation, which uses  $\bar{f}$  and  $\bar{\tau}$ , instead. We have just hidden the difference by dropping the bars in the notation, as we state in the greybox on page 239, but the difference is still there, and it comes out in the second order velocity moment. For more details on these discretisation choices we refer to Section 3.5.2 and Section 6.3.2.

## Chapter 11

**Page 445:** The minus sign in (11.8) should be a plus. Compare this with the correct equation in (5.79) or (11.2). (*Thanks to Rohan Vernekar.*)

## Chapter 13

**Page 537:** The first paragraph should end with (line 7) “. . . from one to two orders of magnitude”. (*Thanks to Mohammed Boraei.*)