List of errata

The Lattice Boltzmann Method:
Principles and Practice

Timm Krüger, Halim Kusumaatmaja, Alexandr Kuzmin, Orest Shardt,
Gonçalo Silva, Erlend Magnus Viggen

Errata last updated: August 8, 2017

Have you found anything else that should be on this list? If so, please send us an email about it to `authors@lbmbook.com`. *The most recent version of this document can be found at* `https://github.com/lbm-principles-practice/errata`.

Chapter 1

Page 8: Equation (1.20) should be $u_x(y) = -\frac{1}{2\eta} \frac{dp}{dx} y(d - y)$. *(Thanks to Hiran Wijesinghe.)*

Page 20: The function arguments on the left-hand side of (1.44c) should be v_x^2, v_y^2, and v_z^2, and the function argument on the left-hand side of (1.44d) should be $|\mathbf{v}|^2$. *(Thanks to Bert Rubash.)*

Chapter 5

Page 198: Equation (5.52) should be $f_8 = f_6 + \frac{1}{2} (f_2 - f_4) - \frac{1}{2c} \rho_w u_{w,y} + \frac{1}{6c} \rho_w u_{w,x}$. *(Thanks to Tang Jun.)*

Chapter 6

Page 233: The equation at the end of the line below (6.5) should be $S_i = \left(1 - \frac{\Delta t}{2\tau}\right) F_i$. *(Thanks to Yongsoo Park.)*

Page 239: Equation (6.26c) contains an extra $\frac{1}{\tau}$ in the pre-factor of the force contribution. The correct form should be $\boldsymbol{\Pi} = \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i \bar{f}_i \boldsymbol{c}_i \boldsymbol{c}_i + \frac{\Delta t}{2\tau} \sum_i f_i^{\text{eq}} \boldsymbol{c}_i \boldsymbol{c}_i + \frac{\Delta t}{2} \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i F_i \boldsymbol{c}_i \boldsymbol{c}_i$.

Page 246: The definition of $\Pi_{\alpha\beta}^{(1)}$ is faulty along this page, missing the inclusion of the discrete lattice $\left(1 - \frac{\Delta t}{2\tau}\right)$. The correct form should be $\Pi_{\alpha\beta}^{(1)} = \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i \left(f_i^{(1)} + \frac{\Delta t}{2} F_i^{(1)}\right) c_{i\alpha} c_{i\beta}$. Due to this mistake, several equations on this page require correction. They are:

- Equation (6.37b) should be $\partial_t^{(2)}(\rho u_\alpha) + \partial_\beta^{(1)} \Pi_{\alpha\beta}^{(1)} = 0$.
- Equation (6.38b) should be $\left(\epsilon \partial_t^{(1)} + \epsilon^2 \partial_t^{(2)}\right)(\rho u_\alpha) + \epsilon \partial_\beta^{(1)} \Pi_{\alpha\beta}^{\text{eq}} = \epsilon F_\alpha^{(1)} - \epsilon^2 \partial_\beta^{(1)} \Pi_{\alpha\beta}^{(1)}$.
- Equation (6.39) should be $\partial_t^{(1)} \Pi_{\alpha\beta}^{\text{eq}} + \partial_\gamma^{(1)} \Pi_{\alpha\beta\gamma}^{\text{eq}} - \sum_i F_i^{(1)} c_{i\alpha} c_{i\beta} = -\left(\frac{2}{2\tau - \Delta t}\right) \Pi_{\alpha\beta}^{(1)}$.
- Equation (6.40) should be $\Pi_{\alpha\beta}^{(1)} = \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i f_i^{(1)} c_{i\alpha} c_{i\beta} + \frac{\Delta t}{2} \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i F_i^{(1)} c_{i\alpha} c_{i\beta}$.
- Equation (6.41) should be $\Pi_{\alpha\beta}^{(1)} = -\rho c_s^2 \left(\tau - \frac{\Delta t}{2}\right) \left(\partial_\beta^{(1)} u_\alpha + \partial_\alpha^{(1)} u_\beta\right) + O(u^3)$.
- The sentence below equation (6.41) should read: "Therefore, the viscous stress is given by $\sigma_{\alpha\beta} = -\Pi_{\alpha\beta}^{(1)}$."[1]

(Thanks to Bart Postma.)

Chapter 10

Page 421: The equilibrium moment e^{eq} in (10.32) should be $-2\rho + 3\rho(u_x^2 + u_y^2)$. *(Thanks to Mohammed Boraei.)*

---

[1] Still concerning this sentence, two notes on the relation between $\Pi_{\alpha\beta}^{(1)}$ and the deviatoric stress $\sigma_{\alpha\beta}$ are in order:

Note 1. By introducing the correct form of $\Pi_{\alpha\beta}^{(1)}$, as given by Eq. (6.40), into $\sigma_{\alpha\beta} = -\Pi_{\alpha\beta}^{(1)}$ we obtain the $\sigma_{\alpha\beta}$ given by Eq. (6.4), where $\sum_i f_i^{\text{neq}} c_{i\alpha} c_{i\beta} \simeq \sum_i f_i^{(1)} c_{i\alpha} c_{i\beta}$ and $\sum_i F_i^{(1)} c_{i\alpha} c_{i\beta} = F_\alpha u_\beta + u_\alpha F_\beta$ according to Eq. (6.15).

Note 2. The relation between $\sigma_{\alpha\beta}$ and $\Pi_{\alpha\beta}^{(1)}$ given in Chapter 6 differs from that given in Chapter 4. Namely, $\sigma_{\alpha\beta} = -\Pi_{\alpha\beta}^{(1)}$ in Chapter 6 and $\sigma_{\alpha\beta} = -\left(1 - \frac{\Delta t}{2\tau}\right) \Pi_{\alpha\beta}^{(1)}$ in Chapter 4 (cf. Eq. (4.14)). The difference lies in the discrete lattice pre-factor $\left(1 - \frac{\Delta t}{2\tau}\right)$ and it is due to the following reason. While the LBE analysed in Chapter 4 is based on the rectangular discretisation of the collision operator, in Chapter 6 it is based on the trapezoidal discretisation, which uses $\bar{f}$ and $\bar{\tau}$, instead. We have just hidden the difference by dropping the bars in the notation, as we state in the greybox on page 239, but the difference is still there, and it comes out in the second order velocity moment. For more details on these discretisation choices we refer to Section 3.5.2 and Section 6.3.2.