List of errata

The Lattice Boltzmann Method:
Principles and Practice

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Springer International Publishing Switzerland, 2017
1st ed., ISBN 978-3-319-44647-9 (print) / ISBN 978-3-319-44649-3 (eBook)

Errata last updated: January 24, 2025

Have you found anything else that should be on this list? If so, please send us an email about it to `authors@lbmbook.com`*. The most recent version of this document can be found at* `https://github.com/lbm-principles-practice/errata`*.*

Chapter 1

Page 8: Equation (1.20) should be $u_x(y) = -\frac{1}{2\eta}\frac{\mathrm{d}p}{\mathrm{d}x}y(d-y)$. *(Thanks to Hiran Wijesinghe.)*

Page 20: Equation (1.44b) should sum the logarithms of $f_{\mathrm{1D}}^{\mathrm{eq}}$ instead of those of f^{eq}, the function arguments on the left-hand side of (1.44c) should be v_x^2, v_y^2, and v_z^2, and the function argument on the left-hand side of (1.44d) should be $|\boldsymbol{v}|^2$. *(Thanks to Bert Rubash and Alexandre Mechineau.)*

Page 20: In the right-hand expression on the first line of (1.45), p and ρ are flipped in the exponential. The correct equilibrium distribution, as expressed through p and ρ, is $\rho\left(\rho/2\pi p\right)^{3/2}\mathrm{e}^{-\rho|\boldsymbol{v}|^2/(2p)}$. *(Thanks to Hiran Wijesinghe.)*

Pages 23 and 27: The references to (1.46) should actually be references to (1.47). *(Thanks to Florian Rohm.)*

Page 24: The sentence above (1.57) should start with "Thus, the first moment of the Boltzmann equation. . . " *(Thanks to Lei Zhou.)*

Page 27: The units of entropy density should be $[\rho s] = \mathrm{J/K\,m^3}$. *(Thanks to Vlad Levenfeld.)*

Page 27: Equation (1.67) takes the logarithm of a dimensioned quantity, $[f] = \mathrm{kg\,s/m^6}$, which is improper[1] — logarithms should have dimensionless arguments. To correct this, we should therefore normalise by another quantity with the same units, *e.g.* $\log(f/f_0)$ with $f_0 = 1\,\mathrm{kg\,s/m^6}$. However, there is no unique choice for this normalisation quantity; the different $\mathcal{H}$ values that we get from choosing different normalisations f_0 are related by offsets that are linear in $\ln(f_0)$. Whatever we choose for the normalisation, $\mathcal{H}$ still retains the same quintessential property: It always decreases until it reaches its minimum value as f reaches equilibrium. *(Thanks to Vlad Levenfeld.)*

Page 28: The link in (1.71) between Boltzmann's $\mathcal{H}$ quantity and the entropy density is somewhat controversial. While several sources support this link, other sources disagree.[2] *(Thanks to Vlad Levenfeld.)*

Chapter 3

Page 72: In the third term on the left-hand side of (3.14), the force F_α is mistyped as f_α. *(Thanks to Hiran Wijesinghe.)*

Page 72: Below (3.17), the non-dimensionalisation should be $\Omega^\star = \Omega \ell V^{d-1}/\rho_0$. The other non-dimensionalisation expressions are correct as long as the density ρ is taken as mass per length to the power of d. *(Thanks to Hiran Wijesinghe.)*

Page 79: The factor $1/2$ is missing in the quadratic term. *(Thanks to Rohan Vernekar.)*

Page 82: The physical velocity $\boldsymbol{u}$ is also scaled along with lattice velocities $\boldsymbol{c}$ with the factor $\sqrt{3}$. *(Thanks to Rohan Vernekar.)*

Chapter 5

Page 198: Equation (5.46), at the very last line, should be $\rho_\mathrm{w} u_{\mathrm{w},x} = c\left(f_1 - f_3\right) + \frac{1}{3}\rho_\mathrm{w} u_{\mathrm{w},x} + 2cN_x$. *(Thanks to Zhang Chuangde.)*

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[1] See Don Koks' article *"Can you take the logarithm of a dimensioned quantity?"* in the Original Usenet Physics FAQ for more details.

[2] For an opposing view, see for example Arieh Ben-Naim, *"Entropy, Shannon's Measure of Information and Boltzmann's H-Theorem"*, Entropy **19**(2) (2017).

Page 198: Equation (5.52) should be $f_8 = f_6 + \frac{1}{2}\left(f_2 - f_4\right) - \frac{1}{2c}\rho_\mathrm{w} u_{\mathrm{w},y} + \frac{1}{6c}\rho_\mathrm{w} u_{\mathrm{w},x}$. *(Thanks to Tang Jun.)*

Chapter 6

Page 233: The equation at the end of the line below (6.5) should be $S_i = \left(1 - \frac{\Delta t}{2\tau}\right) F_i$. *(Thanks to Yongsoo Park.)*

Page 239: Equation (6.26c) contains an extra $\frac{1}{\bar{\tau}}$ in the pre-factor of the force contribution. The correct form should be $\boldsymbol{\Pi} = \left(1 - \frac{\Delta t}{2\bar{\tau}}\right) \sum_i \bar{f}_i \boldsymbol{c}_i \boldsymbol{c}_i + \frac{\Delta t}{2\bar{\tau}} \sum_i f_i^\mathrm{eq} \boldsymbol{c}_i \boldsymbol{c}_i + \frac{\Delta t}{2}\left(1 - \frac{\Delta t}{2\bar{\tau}}\right) \sum_i F_i \boldsymbol{c}_i \boldsymbol{c}_i$.

Page 241: In Table 6.1, $A = \tau/\Delta t$ for the Shan and Chen method.

Page 246: The definition of $\Pi^{(1)}_{\alpha\beta}$ is faulty along this page, missing the inclusion of the discrete lattice $\left(1 - \frac{\Delta t}{2\tau}\right)$. The correct form should be $\Pi^{(1)}_{\alpha\beta} = \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i \left(f_i^{(1)} + \frac{\Delta t}{2} F_i^{(1)}\right) c_{i\alpha} c_{i\beta}$. Due to this mistake, several equations on this page require correction. They are:

- Equation (6.37b) should be $\partial_t^{(2)}(\rho u_\alpha) + \partial_\beta^{(1)} \Pi^{(1)}_{\alpha\beta} = 0$.
- Equation (6.38b) should be $\left(\epsilon \partial_t^{(1)} + \epsilon^2 \partial_t^{(2)}\right)(\rho u_\alpha) + \epsilon \partial_\beta^{(1)} \Pi^\mathrm{eq}_{\alpha\beta} = \epsilon F_\alpha^{(1)} - \epsilon^2 \partial_\beta^{(1)} \Pi^{(1)}_{\alpha\beta}$.
- Equation (6.39) should be $\partial_t^{(1)} \Pi^\mathrm{eq}_{\alpha\beta} + \partial_\gamma^{(1)} \Pi^\mathrm{eq}_{\alpha\beta\gamma} - \sum_i F_i^{(1)} c_{i\alpha} c_{i\beta} = -\left(\frac{2}{2\tau - \Delta t}\right) \Pi^{(1)}_{\alpha\beta}$.
- Equation (6.40) should be $\Pi^{(1)}_{\alpha\beta} = \left(1 - \frac{\Delta t}{2\tau}\right) \sum_i f_i^{(1)} c_{i\alpha} c_{i\beta} + \frac{\Delta t}{2}\left(1 - \frac{\Delta t}{2\tau}\right) \sum_i F_i^{(1)} c_{i\alpha} c_{i\beta}$.
- Equation (6.41) should be $\Pi^{(1)}_{\alpha\beta} = -\rho c_\mathrm{s}^2 \left(\tau - \frac{\Delta t}{2}\right)\left(\partial_\beta^{(1)} u_\alpha + \partial_\alpha^{(1)} u_\beta\right) + O(u^3)$.
- The sentence below equation (6.41) should read: "Therefore, the viscous stress is given by $\sigma_{\alpha\beta} = -\Pi^{(1)}_{\alpha\beta}$."[3]

(Thanks to Bart Postma.)

[3] Still concerning this sentence, two notes on the relation between $\Pi^{(1)}_{\alpha\beta}$ and the deviatoric stress $\sigma_{\alpha\beta}$ are in order: *Note 1.* By introducing the correct form of $\Pi^{(1)}_{\alpha\beta}$, as given by Eq. (6.40), into $\sigma_{\alpha\beta} = -\Pi^{(1)}_{\alpha\beta}$ we obtain the $\sigma_{\alpha\beta}$ given by Eq. (6.4), where $\sum_i f_i^\mathrm{neq} c_{i\alpha} c_{i\beta} \simeq \sum_i f_i^{(1)} c_{i\alpha} c_{i\beta}$ and $\sum_i F_i^{(1)} c_{i\alpha} c_{i\beta} = F_\alpha u_\beta + u_\alpha F_\beta$ according to Eq. (6.15).
Note 2. The relation between $\sigma_{\alpha\beta}$ and $\Pi^{(1)}_{\alpha\beta}$ given in Chapter 6 differs from that given in Chapter 4. Namely, $\sigma_{\alpha\beta} = -\Pi^{(1)}_{\alpha\beta}$ in Chapter 6 and $\sigma_{\alpha\beta} = -\left(1 - \frac{\Delta t}{2\tau}\right) \Pi^{(1)}_{\alpha\beta}$ in Chapter 4 (cf. Eq. (4.14)). The difference lies in the discrete lattice pre-factor $\left(1 - \frac{\Delta t}{2\tau}\right)$ and it is due to the following reason. While the LBE analysed in Chapter 4 is based on the rectangular discretisation of the collision operator, in Chapter 6 it is based on the trapezoidal discretisation, which uses $\bar{f}$ and $\bar{\tau}$, instead. We have just hidden the difference by dropping the bars in the notation, as we state in the greybox on page 239, but the difference is still there, and it comes out in the second order velocity moment. For more details on these discretisation choices we refer to Section 3.5.2 and Section 6.3.2.

Chapter 7

Page 283: After (7.26) it is written that $\bar{u} = \hat{u}/2$ for 2D Poiseuille flow, but it must be $\bar{u} = 2\hat{u}/3$. (The wrong equation in the book is correct for 3D Poiseuille flow.) *(Thanks to Christoph Rettinger.)*

Chapter 8

Pages 298 and 303: The right-hand-sides of (8.2) and (8.15) should be multiplied by Δt since the dimension of the collision operator and the source term equals the dimension of the populations divided by the dimension of time. Compare (8.3) and (8.16), which are correct. *(Thanks to Jamie Nance.)*

Page 310: The correct equation in (8.40) should be $C = \sum_i g_i + \frac{q\Delta t}{2}, Q_i = \left(1 - \frac{\Delta t}{2\tau_g}\right) w_i q$. *(Thanks to Rohan Vernekar.)*

Page 311: The first line should read "... larger than unity lead to reduced accuracy." *(Thanks to Mohammed Boraei.)*

Chapter 10

Page 421: The equilibrium moment e^{eq} in (10.32) should be $-2\rho + 3\rho(u_x^2 + u_y^2)$. *(Thanks to Mohammed Boraei.)*

Page 421: The equilibrium moment q_x^{eq} in (10.32) should be $\rho u_x(3u_y^2 - 1)$. Accordingly, q_y^{eq} in (10.32) should be $\rho u_y(3u_x^2 - 1)$. *(Thanks to Christophe Guy Coreixas.)*

Page 427: The right hand side of (10.47) should be multiplied by Δt, which yields $\nu = c_s^2\left(\frac{1}{\omega^+} - \frac{\Delta t}{2}\right)$. *(Thanks to Rohan Vernekar.)*

Chapter 11

Page 445: The minus sign in (11.8) should be a plus. Compare this with the correct equation in (5.79) or (11.2). *(Thanks to Rohan Vernekar.)*

Page 457: The unit of the first term on the right-hand-side of (11.19) differs from the unit of the last three terms. This is because the density ρ has been assumed to be constant and unity (in lattice units). Strictly, the last three terms should be multiplied by ρ. *(Thanks to Luyu Wang.)*

Chapter 13

Page 537: The first paragraph should end with (line 7) "... from one to two orders of magnitude". *(Thanks to Mohammed Boraei.)*

Appendix A

Page 657: The line preceding (A.15c) should refer to (A.13) instead of (A.12). *(Thanks to Rohan Vernekar.)*