Measure of the center of gravity and inertia parameters for a bicycle part

This file aims to describe the method used for identifying the center of gravity and some of the inertial parameters for different bicycle's parts. This protocol can be used for all parts of a bicycle, with certain specificities. The principal axes of a 2-wheeler are illustrated in the figure 1, to show that we do not need to measure all of the inertial parameters, only few of them matter for modelling the vehicle dynamics.

Materials:

- Something to hang the parts and make them oscillating
- Tools for dismantle the bicycles
- An Inertial Measurement Units
- A camera
- Some ropes

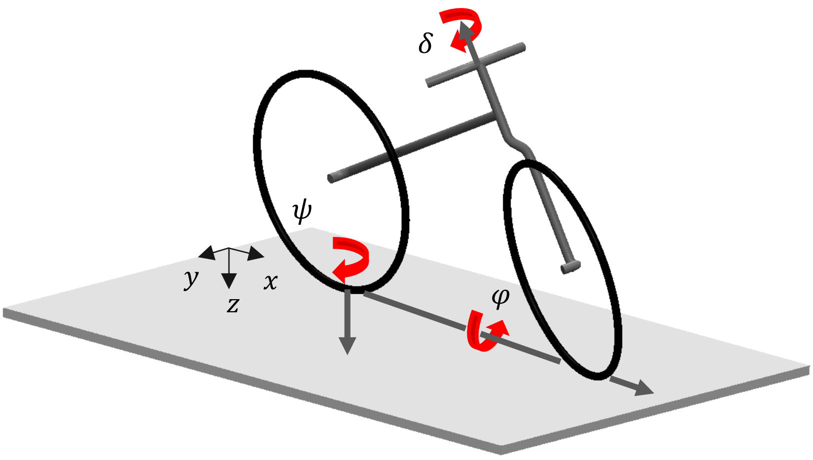

Figure 1: principal degrees of mobility for a bicycle

Center of gravity measurement:

The center of gravity measurement will be done by identifying the intersection of multiple equilibrium positions of the part.

We first hang the part with a rope. This rope will allow the part to take a stable equilibrium position. We take photo of this equilibrium position, and then plot on the image the vertical line formed by the rope (as showed on the figure 2). By superposition of multiple configurations, the intersection of all the lines will give the position of the center of gravity.

Step for each position:

- Place the part in the stable equilibrium position
- Take a photo of the configuration

After each photo:

- Plot the vertical line (represented by the rope)
- Superpose all the lines on a single image
- Determine the position of the center of gravity in the world reference frame (same as figure 1 but centered in the rear wheel contact point).

Specificity for a cargo bike:

For some cargo bikes, the steering assembly is divided in 2 parts, so we have to determine the parameters of all this part independently.

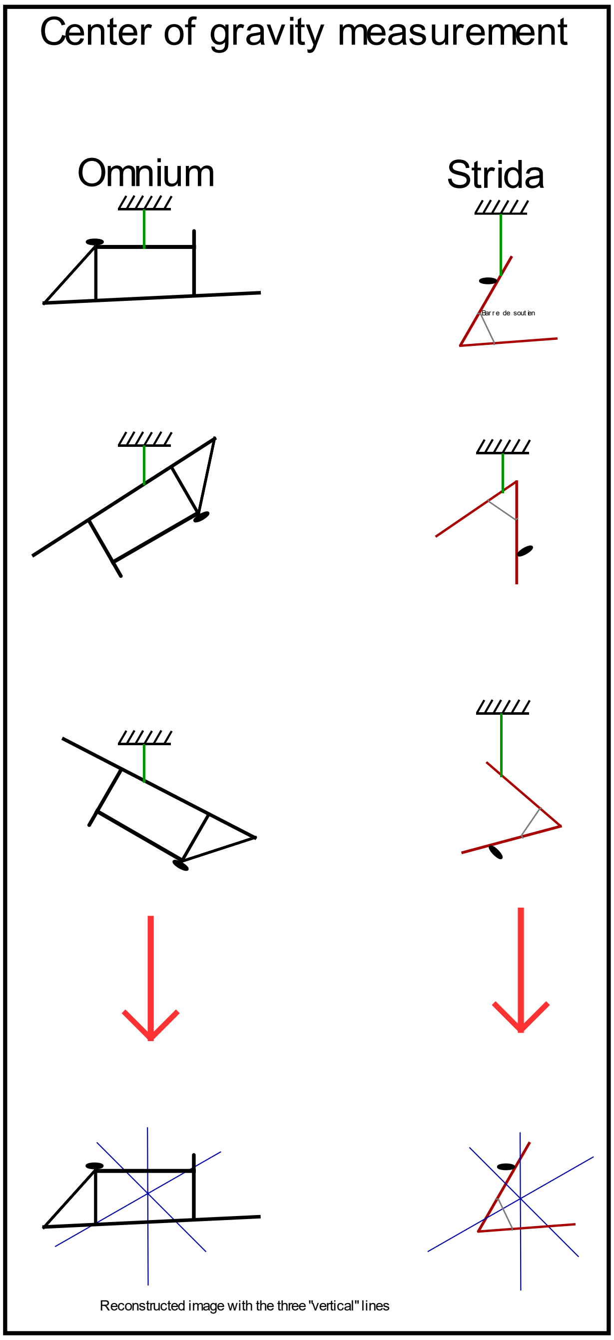

Figure 2: Scheme to illustrate the method used for measuring the center of gravity of 2 different frames

Specificity for the method chosen:

The fact that we use photography to determine the position of a point relative to a solid requires us to be sure that the photography is aligned with the symmetric plan of the part we are measuring. First, we need to define a reference frame linked to the part, to do so, we used 2 different methods depending on the size of the part.

For smaller parts: Method 1

We use a QR code (https://github.com/sushil-bharati/pyzbar.git), with which we are able to automatically identify the position of its summits and make the calibration of the image. But this method does not give us the coordinates in the world reference frame. So we have to select two points, one for which we know the coordinates (usually the wheel center), and an other to create an axis that we know the angle relative to the horizontal axis.

For the frame of a cargo bike: Method 2

Usually, the frame of a cargo bike is a big part (wheelbase ~1.50m). And sometimes, it has multiple tubes that creates quadrangle, which can be used for image calibration. This is why we chose, for one specific part, to calibrate the image based on the points we pointed on the image. This method requires us to first measure the right position of all the summits of the quadrangle in the world reference frame.

Then, we just have to select the vertical line formed by the rope on which is hanged the part.

After that, we are able to superpose all of the lines in one image (calibrated) and know the position of the intersection in the world reference frame.

Estimation of the uncertainty linked to the pointing:

The intersection of the lines is never perfect, so we chose to use the 3 farthest intersection points, compute the mean of these 3 points, and the distance of this mean point and the farthest point gives us the uncertainty by the radius of the circle (as illustrated in the Figure 3).

Reference frame transformation:

Generally, the reference frame transformation we did was done following these steps:

1. Finding a point and an axis of interest, measure the position and the orientation of the axis
2. Finding the translation to do from the QR code (or frame pointing) to the known point
3. Finding the angle that the horizontal axis might have in the normal position of the bike
4. Compute the rotation matrix
5. Find the translation

Then we can compute the coordinates of the center of gravity in the world reference frame.

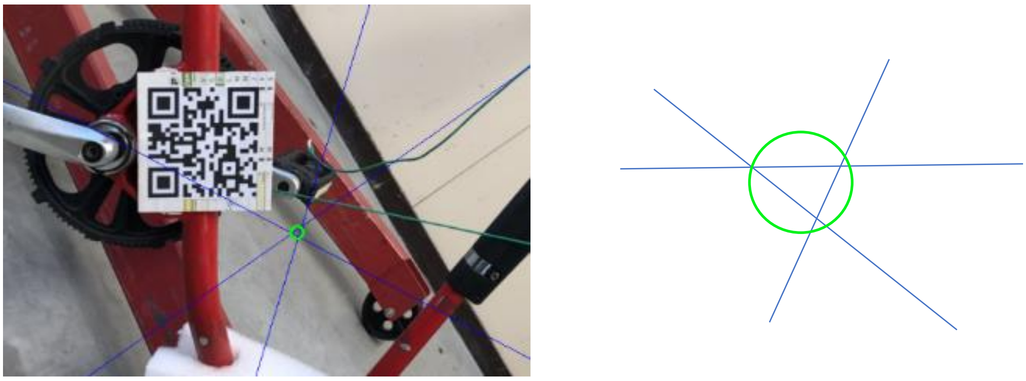

Figure 3: A. Calibrated image showing the intersection of multiple lines. B. More precision of the line's intersection on which the circle radius gives the uncertainty of the measurement.

Estimation of the uncertainties:

To estimate the total uncertainties, we have to take into account 2 types:

- The first one that is directly linked to the measured variables

- The second one which permits to make the uncertainty propagation through an equation

Measured uncertainties:

Pointing: $\sigma = 5\ \mathrm{mm}$

Direct measurement: $\sigma = 5$ mm

Wheelbase measurement: $\sigma = $ 5 mm

Wheel radius measurement: $\sigma = $ 2 mm

Angle measurement: $\sigma = 1$ °

Uncertainties propagation through equations:

The formula used are inspired by the work from [1].

$$\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}$$

$$\sigma_{x*y} = (x * y)\sqrt{\left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2}$$

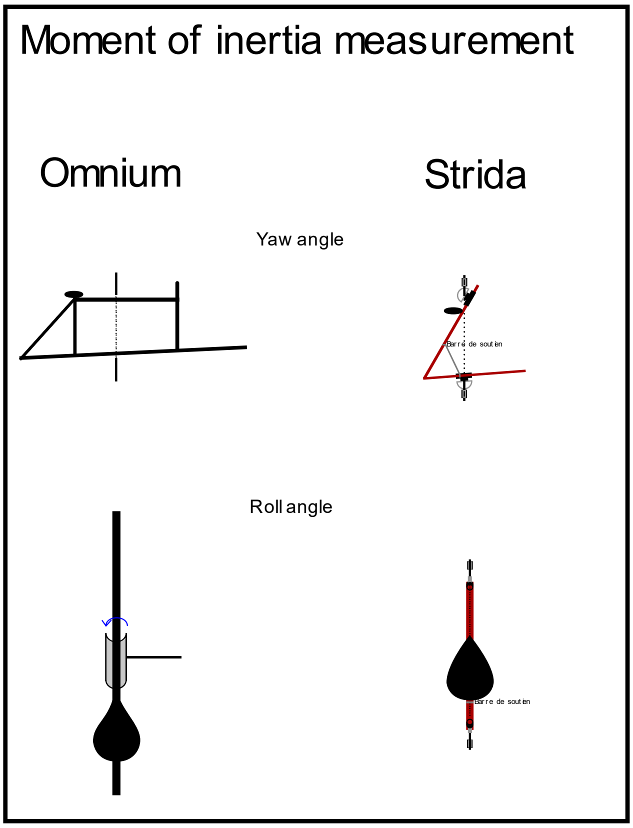

Figure 4: Scheme to illustrate the method used for measuring the 2 moments of inertia of 2 bicycle's frames

Moment of inertia measurement:

To measure the moments of inertia, we hang the parts and let it oscillates around an equilibrium position.

For our model, we need 3 inertia parameters, 2 moments and one product of inertia (Ixx, Izz, Ixz). We directly measure the 2 moments of inertia, and for the product we use a specific method.

For the moments of inertia, we let the part oscillates around an equilibrium position, and measure the distance between the axis of oscillation and the center of gravity, and the pulsation of the oscillations. Then we use the Second law of Newton to compute the inertia of the part, around this axis:

$$I = \frac{mgL}{\omega^2} - mL^2$$

With the following parameters:

- m : weight of the part (kg)
- g : gravity (m/s^2)
- L : distance between the center of gravity and the oscillation axis (m)
- ω : pulsation of the oscillations (rad/s)

For each moment of inertia, we do multiple measures to minimize the uncertainties linked to the method.

For the product of inertia, we measure the inertia around a third axis for which we know the orientation angle (as represented in the figure 5). And then compute the 3 parameters we need by using these relations:

$$J_{\beta_i} = cos(\beta_i)^2 I_{Bxx} - 2sin(\beta_i)cos(\beta_i)I_{Bxz} + sin(\beta_i)^2 I_{Bzz}$$

Where J_{β_i} are the measured inertia, I_{Bxx}, I_{Bxz}, I_{Bzz} are the parameters we want to measure, and β_i the angle of the oscillation axis, expressed in the world reference frame taken when the bicycle is upright. The 3 configurations gives us a linear system with 3 variables and 3 equations.

Specificities for a cargo bike's steering system:

First, we evaluate the moments of inertia of the fork and the handlebar. To compute the moment of inertia of the all steering system, we consider that the 2 parts are moving at the same speed, so we can add the two moments of inertia. For now, we consider that the crank has a negligeable influence on the system.

For the center of gravity of the all system, we translate the handlebar along the horizontal axis to align his axis with the fork's axis. So we take into account the fact that the center of gravity of the front assembly is not aligned with the steering axis, and we take into account the vertical position of all the system correctly.

For the moments and product of inertia, we compute them using the following formula:

$$Iss_{xx} = IH_{xx} + IF_{xx} + mH(xH - x_{ss})^2 + mF(xF - x_{ss})^2$$

$$Iss_{zz} = IH_{zz} + IF_{zz} + mH(zH - z_{zz})^2 + mF(zF - z_{zz})^2$$

$$Iss_{xz} = IH_{xz} + IF_{xz} + mH(z_{zz} - zH)(x_{ss} - xH) + mF(z_{ss} - zF)(x_{ss} - xF)$$

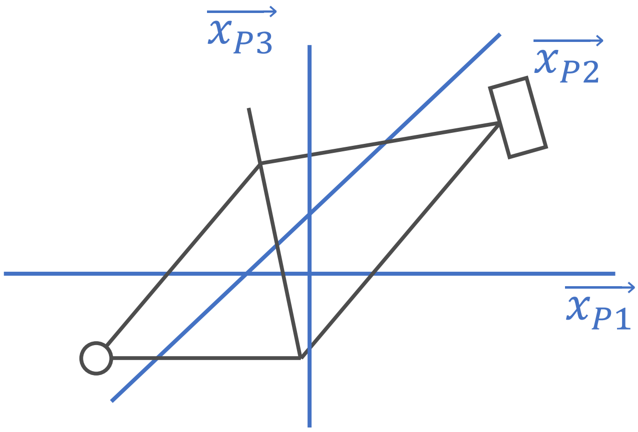

Figure 5: Example of different axes for the inertia measurement

Method used for parts oscillations:

Given the fact that all the parts are completely different, we cannot give one single method for all the parts, but we describe the idea we used during the measurements.

We used the « Knife-edge » method, which consists to use 2 points to construct a revolute joint and make the part oscillates along this joint. You can observe on the figure 6 and 7 an example used for the frame of the Omnium cargo bike.

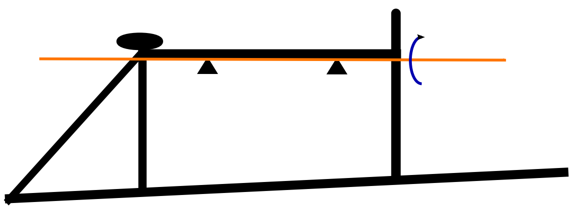

Figure 6: Scheme to demonstrates the "Knife-edge" method

Figure 7: Photography of the configuration used for measuring the moment of inertia of the Omnium cargo bicycle along the roll axis

Method for the wheels:

For the wheels, we had to measure only the moments of inertia along 2 axis, and we neglected the product of inertia. So we simply reused the previous method and make the wheels oscillates along 2 directions (as shown in the figure 8). The first axis is one parallel to the rotation axis, and the second one is any perpendicular to the previous.

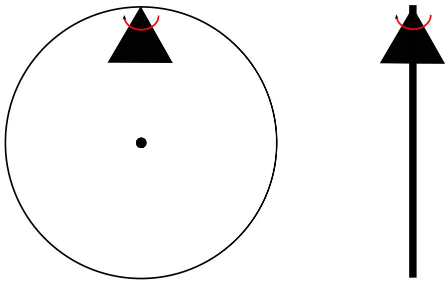

Figure 8: Oscillation axis for the wheels

Bibliographie:

[1] I. Farrance et R. Frenkel, « Uncertainty of Measurement: A Review of the Rules for Calculating Uncertainty Components through Functional Relationships ».