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Robust Distributed Volt/var Control of Distribution Systems

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***Abstract*— Volt/var control (VVC) is essential to achieve a reliable and efficient operation of modern and future distribution systems with high penetration of distributed energy resources (DERs). In real time operation, uncertainties cause by variations of loads and fluctuations of other system components may result in a deviation from the desired voltage profile. In this paper, we propose a robust distributed control framework to formally address these uncertainties. The objective of the proposed controller is to optimize the closed-loop performance with stability guarantee while optimally rejecting disturbances arising from uncertain loads and DERs. In the proposed scheme, the structural constraint due to the underlying communication network makes the controller synthesis problem nonconvex and hence difficult to solve in general. An effective convex approximation is proposed for any communication structure to address this issue. Case studies are presented to illustrate the contribution of the proposed robust distributed VVC control scheme.**

I. INTRODUCTION

Any modern power system is experiencing an increasing penetration of distributed energy resources (DERs) that gradually replaces conventional energy resources that are exhaustible and limited in supply, and helps improving power system operations. However, in the absence of carefully designed voltage control schemes, the integration of such DERs may result in voltage fluctuations and even instability [1], [2].

Generally speaking, reactive power injections into a distribution system are often considered as the primary source of voltage control. Traditional voltage control schemes mainly focused on using shunt capacitors/reactors [3], [4], [5] or on-load tap changers [6], [7] installed at certain buses in the distribution system to compensate reactive power flows through the network and hence tp regulate the voltage profile. More recently, due to technological advances in power electronics, inverter based VVC schemes have been proposed and adopted widely [8] to complement and replace conventional equipment.

In general, the Volt/var regulation problem in distribution systems usually consists of two layers: an (upper) optimization layer and a (lower) control layer. Assuming perfect and complete information on the distribution system, the optimization layer solves an optimal power flow (OPF) problem [9], [10] to determine the optimal nominal operating condition of the distribution system. However, during real time operation, the actual distribution system is likely to deviate from the model considered in the OPF formulation due to uncertainties and disturbances. In order to maintain admissible voltage levels, local or distributed Volt/var control schemes need to be devised in the control layer to regulate the voltage profile in real time. This voltage control problem has been extensively studied in the literature [11], [12], [13], [14]. In [15], the authors studied the stability property of a local nonlinear non-incremental control scheme and derived a sufficient condition to ensure voltage stability. More recently, several incremental control schemes have been developed [16], [17], [18], [19] to improve stability conditions of the local control strategies. Furthermore, a distributed voltage control framework has also been proposed recently in [20].

Despite this rich literature regarding local and distributed VVC control problems, most existing works do not consider uncertainties in the real time operation of the distribution system, which primarily come from the variability of active and reactive power consumption of the loads and the active power injections of PV units due to the volatile solar irradiation. Representing these uncertainties is essential in designing reliable controllers. In [21], the authors analyzed a local droop-like controller with a robust design procedure to cope with uncertainties. Analysis on decentralized schemes under uncertainties is reported in [22] as well. In this paper, we focus on the control layer problem and leverage the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ formulation proposed in the robust control literature [23], [24], [25], [26] to systematically represent uncertainties. In particular, this technique minimizes the $\mathcal{H}_2$ performance of the closed-loop system while simultaneously guaranteeing closed-loop system performance under the worst case scenario. Furthermore, due to the limited information exchange within the distribution system, an additional structural constraint on the controller structure is imposed. It is well-known that the controller design problem subject structural constraints is in general nonconvex [27], [28], [29], [30]. In [29], the authors presented a necessary and sufficient condition which characterizes a class of convex structured robust control problems. Unfortunately, our problem does not belong to this particular class due to the arbitrarily prescribed communication network. To address this issue, an effective convex approximation of the controller synthesis problem with arbitrary communication structure is proposed.

The rest of this paper unfolds as follows. Section II specifies the network model and formulates the distributed VVC problem under uncertainty. In Section III, we first provide a rigorous mathematical formulation of the distributed VVC problem under uncertainty adopting the robust control theory and then describe how to convexify the controller synthesis problem with structural constraints. Case studies illustrating the effectiveness of the proposed solution are described in Section IV. Finally, concluding remarks and future work are provided in Section V.

II. NETWORK MODEL AND PROBLEM FORMULATION

A. Network Model

Consider an N-bus distribution system as shown in Fig. 1 with n photovoltaic (PV) units installed at certain buses whose reactive power injections can be controlled to help operating this distribution system. To begin with, we first introduce the power flow model describing the distribution system. A typical distribution system can be abstracted by a tree graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where

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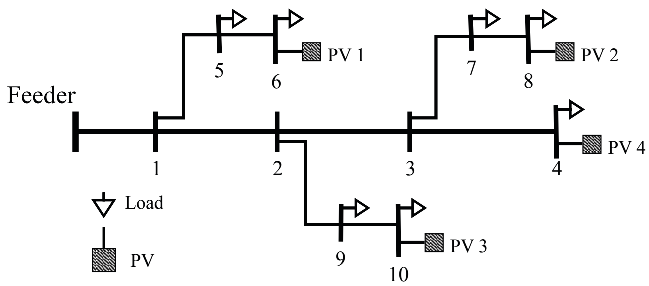


Fig. 1. A Distribution System Example

$\mathcal{N} = \{0, 1, \ldots, N\}$ is the set of buses and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ is the set of distribution lines. For each bus $i \in \mathcal{N}$, let v_i be the voltage magnitude in per unit (p.u.), and p_i and q_i be its net active and reactive power injections, respectively. For each distribution line $(i, j) \in \mathcal{E}$, let r_{ij} and x_{ij} be its resistance and reactance, respectively, and P_{ij} and Q_{ij} be its active and reactive power flows, respectively. Let bus 0 be the feeder of this distribution network and for each line $(i, j) \in \mathcal{E}$, the following set of equations (1) describe the linearized power flow model extensively used in the Volt/var control literature [15], [17], [18], [19].

$$P_{ij} = \sum_{k:(j,k)\in\mathcal{E}} P_{jk} - p_j, \tag{1a}$$

$$Q_{ij} = \sum_{k:(j,k)\in\mathcal{E}} Q_{jk} - q_j, \tag{1b}$$

$$v_i - v_j = r_{ij}P_{ij} + x_{ij}Q_{ij}, \tag{1c}$$

Defining $v = [v_1, v_2, \ldots, v_N]^T$, $p = [p_1, p_2, \ldots, p_N]^T$ and $q = [q_1, q_2, \ldots, q_N]^T$, the above power flow mode can be compactly written as

$$v = \hat{R}p + \hat{X}q + v_0, \tag{2}$$

where $v_0 \in \mathbb{R}^N$ is a including all nodal voltages, and $\hat{R} \in \mathbb{R}^{N\times N}$ and $\hat{X} \in \mathbb{R}^{N\times N}$ are two matrices determining the relationship between the voltage profile and the power injections.

Lemma 1 ([15], [19]): Both $\hat{R} \in \mathbb{R}^{N\times N}$ and $\hat{X} \in \mathbb{R}^{N\times N}$ are symmetric and strictly positive definite matrices and $R_{ij} > 0$ and $X_{ij} > 0$ for all $i, j \in \mathcal{N}$.

The readers are referred to [15], [19] for detailed derivations, proofs and discussions of this linearized model. Furthermore, in [31], the authors derived an approximation error of the aforementioned linearized model.

B. Problem Formulation

Generally speaking, the Volt/var control problem has a hierarchical structure, which consists of an optimization layer and a control layer. In the upper optimization layer, an optimal power flow (OPF) problem is solved and the optimal solution provides references (set points) for the lower control layer. With this reference information, the lower control layer determines the reactive power injections of the PV units to regulate the real time operation of the distribution system.

In this paper, we focus on the control layer problem. Let $v^r \in \mathbb{R}^N$ be the voltage reference from the upper optimization layer encoding the reference voltage profile of all buses in the distribution system and let $q^G \in \mathbb{R}^n$ be the reactive power injections of the PV units which are the control decisions in our control scheme. Let $X \in \mathbb{R}^{N\times n}$ be a matrix extracted from $\hat{X}$ which captures the relationship between nodal voltages and the reactive power injections. In this case, the system model (2) can be rewritten as

$$v = Xq^G + w, \tag{3}$$

where w is the nodal voltage vector of the distribution system determined by all other participants, including the active and reactive power consumption of all loads and active power injections of all PV units

In most of the existing literature, it is assumed that w is an unknown but fixed constant. However this might not the case in real time operation of distribution systems. Uncertainties of loads and other components need to be carefully represented. In particular, the uncertainties of power generations/consumptions to uncertainties in w. We adopt the following model to characterize the volatility of w:

$$w^+ = A_w w + d + \overline{w}, \tag{4}$$

which is proposed and analyzed in [32], where A_w is a Schur matrix, i.e. all eigenvalues of A_w lie in the open unit circle, d is a zero-mean Gaussian noise and $\overline{w}$ is a constant establishing the static offset.

In this paper, we consider a distributed VVC problem where each PV bus can exchange information with several other buses by using a pre-defined communication structure. Specifically, a fully decentralized scenario is included in this setting as well. We use two graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ to describe the communication network among PV units and the communication network between PV units and all other buses, respectively. To be specific, the two graphs are defined as follows: $(i, j) \in \mathcal{G}_1$ if PV i can communicate with PV j and $(i, j) \in \mathcal{G}_2$ if PV i can obtain information from general bus j.

The distributed Volt/var control problem under uncertainty studied in the paper is described as follows.

Problem 1: Suppose that the system model (3), the uncertainty model (4) and the communication structure $(\mathcal{G}_1, \mathcal{G}_2)$ are given. Then, given the voltage reference $v^r \in \mathbb{R}^N$ from the upper optimization layer, find a controller subject to the communication structure to determine the reactive power injections q^G of the PV units that minimizes the voltage mismatch error between the voltage profile and the given reference.

In the following section, we adopt the robust control theory and present a rigorous mathematical formulation.

III. A Robust Control Framework with Structural Constraints

The goal of our distributed VVC problem under uncertainty is to optimally reject disturbances caused by the changing components in the distribution system. In this section, we propose a robust distributed control framework to design the distributed voltage controller that formally incorporates uncertainties.

Robust control theory is an important class of the control theory that aims at rejecting disturbances entering the system. In particular, the specific goal of robust control problem is to minimize a certain norm of the transfer function from some exogenous disturbance d to the output of the system y. Without loss of generality, let us consider the system described by the diagram in Fig. III, where $\mathcal{P}$ is the system model given by (3), $\mathcal{K}$ is the controller to be designed and $\mathcal{W}$ is the uncertainty model.

Let $\mathcal{T}_{dy}$ be the transfer function from the random noise d to the system output y. The$\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of this transfer function are defined as follows.

Definition 1: Suppose that $\mathcal{T}_{dy}$ is a given transfer function, then the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of this transfer function are given,

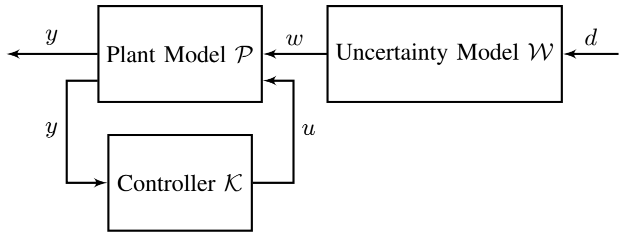


Fig. 2. System Diagram

respectively, by

$$\|\mathcal{T}_{dy}\|_2 = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} |\mathcal{T}_{dy}(\mathbf{i}\omega)| d\omega\right)^{\frac{1}{2}} \tag{5a}$$

$$\|\mathcal{T}_{dy}\|_\infty = \sup_{\omega} |\mathcal{T}_{dy}(\mathbf{i}\omega)| \tag{5b}$$

In this paper, we adopt the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control framework proposed in the robust control literature, whose objective is to find a controller $\mathcal{K}$, possibly dynamic, to minimize the $\mathcal{H}_2$ norm of the closed-loop system subject to the constraint that the $\mathcal{H}_\infty$ norm of the system is bounded by a pre-specified value. In this framework, the controller not only minimizes the $\mathcal{H}_2$ performance of the system but also guarantees worst-case performance.

In addition, given the communication structure $\mathcal{G}_1$ and $\mathcal{G}_2$, we define two matrices $G_1 \in \{0,1\}^{n\times n}$ and $G_2 \in \{0,1\}^{n\times N}$ to mathematically formalize the communication constraints. These matrices are defined as follows:

$$G_1(i,j) = \begin{cases} 0, & \text{if } (i,j) \in \mathcal{G}_1 \\ 1, & \text{otherwise} \end{cases}, \quad G_2(i,j) = \begin{cases} 0, & \text{if } (i,j) \in \mathcal{G}_2 \\ 1, & \text{otherwise} \end{cases}$$

Definition 2: We say that a dynamic controller $\mathcal{K} = (A_\xi, B_\xi, C_\xi, D_\xi)$ satisfies the communication constraints G_1, G_2, denoted by $\mathcal{K} \sim (G_1, G_2)$, if $G_1 \circ A_\xi = \mathbf{0}$, $G_1 \circ C_\xi = \mathbf{0}$ and $G_2 \circ C_\xi = \mathbf{0}$, $G_2 \circ D_\xi = \mathbf{0}$ where $\mathbf{0}$ is the zero matrix with appropriate dimension.

Given the system model (3) and the uncertainty model (4), and given a voltage reference v^r, the mathematical Volt/var model with a dynamic controller of the system described by Figure III is given as follows.

$$\mathcal{P}: \qquad y(k) = Xq(k) + w(k) - v^r \tag{6a}$$

$$\mathcal{W}: \qquad w(k+1) = A_w w(k) + d(k) + \overline{w} \tag{6b}$$

$$\mathcal{K}: \qquad \begin{cases} \xi(k+1) & = A_\xi \xi(k) + B_\xi y(k) \\ q(k) & = C_\xi \xi(k) + D_\xi y(k) \end{cases} \tag{6c}$$

In this paper, we aim at solving the following structured $\mathcal{H}_2/\mathcal{H}_\infty$ problem.

Problem 2 (Structured Mixed $\mathcal{H}_2/\mathcal{H}_\infty$): Given the communication matrices G_1, G_2 and a constant γ, find $\mathcal{K}$ such that $\mathcal{K} \sim (G_1, G_2)$ which minimizes $\|\mathcal{T}_{dy}\|_2$ subject to $\|\mathcal{T}_{dy}\|_\infty < \gamma$.

Despite the fact that the classical mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem has been extensively studied in the robust control literature, the additional structural constraints introduce significant difficulties to solve Problem 2. In the following section, we first review how the centralized mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem is solved. Then, a sufficient condition for the structured version is presented.

$\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems constitute the core of classical robust control theory that has been studied extensively during the past several decades. In this section, we first review the classical centralized approach in solving this problem, mainly based on [24], [25].

Considering the system described in (6), we obtain the following closed-loop system by plugging (6c) into (6a) and (6b):

$$\begin{bmatrix} w^+ \\ \xi^+ \\ q^+ \\ y \end{bmatrix} = \begin{bmatrix} A_w & \mathbf{0} & \mathbf{0} & I \\ B_\xi & A_\xi & B_\xi X & \mathbf{0} \\ D_\xi & C_\xi & D_\xi X & \mathbf{0} \\ I & \mathbf{0} & X & \mathbf{0} \end{bmatrix} \begin{bmatrix} w \\ \xi \\ q \\ d \end{bmatrix} \tag{7}$$

In the sequel, for notational simplicity, we define

$$\left[\begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array}\right] = \left[\begin{array}{ccc|c} A_w & \mathbf{0} & \mathbf{0} & I \\ B_\xi & A_\xi & B_\xi X & \mathbf{0} \\ D_\xi & C_\xi & D_\xi X & \mathbf{0} \\ \hline I & \mathbf{0} & X & \mathbf{0} \end{array}\right] \tag{8}$$

The following theorem from [26] provides a full characterization of the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem.

Theorem 1: Given the above closed-loop system (7) and two positive constant γ_1 and γ_2, $\sigma(A_o) \in \mathbb{C}^o$ and $\|\mathcal{T}_{dy}\|_2 < \gamma_1$ and $\|\mathcal{T}_{dy}\|_\infty < \gamma_2$ if and only if there exist symmetric matrices P_1, P_2 and Z with $\operatorname{trace}\{Z\} < \gamma_1$ and

$$\begin{bmatrix} P_1 & P_1 A_o & P_1 B_o \\ A_o^T P_1 & P_1 & \mathbf{0} \\ B_o^T P_1 & \mathbf{0} & \gamma_1 I \end{bmatrix} \succ 0 \tag{9a}$$

$$\begin{bmatrix} P_1 & \mathbf{0} & C_o^T \\ \mathbf{0} & I & D_o \\ C_o & D_o & Z \end{bmatrix} \succ 0 \tag{9b}$$

$$\begin{bmatrix} P_2 & \mathbf{0} & A_o^T P_2 & C_o^T \\ \mathbf{0} & \gamma_2 I & B_o^T P_2 & D_o^T \\ P_2 A_o & P_2 B_o & P_2 & \mathbf{0} \\ C_o & D_o & \mathbf{0} & \gamma_2 I \end{bmatrix} \succ 0 \tag{9c}$$

In the above theorem, inequalities (9a) and (9b) ensure that the closed-loop $\mathcal{H}_2$ performance is smaller than γ_2 and (9c) enforces $\mathcal{H}_\infty$ performance. The readers are referred to [24], [25], [26] for detailed discussions. In addition, in the absence of disturbance, the stability of the closed-loop system is guaranteed by the above inequalities which can be seen by setting d and w to zero and adopting a Lyapunov analysis. Therefore, we focus on (9) in the sequel.

Note that in the above theorem, the decision variables are P_1, P_2 and $A_\xi, B_\xi, C_\xi, D_\xi$. Hence, the matrix inequalities (9) are bi-linear in nature. It is well known that bi-linear matrix inequalities (BMIs) are in general nonconvex and thus $\mathcal{NP}$-hard to solve. Effective change of variable technique has been proposed in the literature [25] to transform this problem into a convex one, which can be efficiently solved using diverse methods.

However, despite these powerful results in generic $\mathcal{H}_2/\mathcal{H}_\infty$ problems, the change of variable technique is no longer applicable to our case with dynamics (7) due to the additional structural constraints $\mathcal{K} \sim (\mathcal{G}_1, \mathcal{G}_2)$. To address this issue, we provide a sufficient convex approximation to the original BMIs (9).

To begin with, we note the following lemma for positive definiteness of product of matrices.

Lemma 2 ([33]): Suppose $A \succ 0, B \succ 0, C \succ 0$, then $ABC \succ 0$ if ABC is symmetric, i.e. $ABC = (ABC)^T$.

Proof: We refer the readers to [33] for detailed proof of this lemma. ■

The main idea of our technique is to decouple the decision variables A_o, B_o and P_1 in (9a) without introducing new variables. With the above lemma, we are ready to present our main result which is summarized in the following theorem.

Theorem 2: If P_1 and A_o, B_o, C_o, D_o satisfy the following linear matrix inequality for some $\eta > 0$, then the matrix inequality (9a) holds.

$$\begin{bmatrix} \eta I & A_o & B_o \\ A_o^T & P_1 & \mathbf{0} \\ B_o^T & \mathbf{0} & \gamma_1 I \end{bmatrix} \succ 0 \tag{10a}$$

$$0 \prec P_1 \prec \frac{1}{\eta} I \tag{10b}$$

$$K \sim (\mathcal{G}_1, \mathcal{G}_2) \tag{10c}$$

Proof: We start from the inequality (9a). Applying the Schur complement lemma, we obtain the following equivalence

$$\begin{bmatrix} P_1 & P_1 A_o & P_1 B_o \\ A_o^T P_1 & P_1 & \mathbf{0} \\ B_o^T P_1 & \mathbf{0} & \gamma_1 I \end{bmatrix} \succ 0$$
$$\Leftrightarrow \begin{cases} P_1 \succ 0 \\ P_1^{-1} - A_o P_1^{-1} A_o^T - \frac{1}{\gamma} B_o B_o^T \succ 0 \end{cases}$$

In the sequel, we show that (10a) and (10b) are sufficient for the above inequality to hold.

To see this, we perform congruence transformation to (10a) by multiplying $\operatorname{diag}\{P_1, I, I\}$ from both sides, which renders

$$\begin{bmatrix} P_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I \end{bmatrix} \begin{bmatrix} \eta I & A_o & B_o \\ A_o^T & P_1 & \mathbf{0} \\ B_o^T & \mathbf{0} & \gamma_1 I \end{bmatrix} \begin{bmatrix} P_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I \end{bmatrix}$$
$$= \begin{bmatrix} \eta P_1^2 & P_1 A_o & P_1 B_o \\ A_o^T P_1 & P_1 & \mathbf{0} \\ B_o^T P_1 & \mathbf{0} & \gamma I \end{bmatrix} \succ 0$$

due to Lemma 2 and (10b). Applying Schur complement lemma, it follows that

$$\begin{cases} \begin{bmatrix} P_1 & \mathbf{0} \\ \mathbf{0} & \gamma I \end{bmatrix} \succ 0 \\ \eta P_1^2 - \begin{bmatrix} P_1 A_o & P_1 B_o \end{bmatrix} \begin{bmatrix} P_1^{-1} & \mathbf{0} \\ \mathbf{0} & \frac{1}{\gamma} I \end{bmatrix} \begin{bmatrix} A_o^T P_1 \\ B_o^T P_1 \end{bmatrix} \succ 0 \end{cases}$$
$$\Leftrightarrow \begin{cases} P_1 \succ 0 \\ \eta I - A_o P_1^{-1} A_o^T - \frac{1}{\gamma} B_o B_o^T \succ 0 \end{cases}$$

Add $P_1^{-1} - \eta I$ to both sides of the last inequality, and we obtain

$$P_1^{-1} - A_o P_1^{-1} A_o^T - \frac{1}{\gamma} B_o B_o^T \succ P_1^{-1} - \eta I$$

Considering (10b), it directly follows that

$$P_1^{-1} - A_o P_1^{-1} A_o^T - \frac{1}{\gamma} B_o B_o^T \succ 0$$

which is the desired result. ■

Theorem 2 shows that (10) is a set of sufficient conditions for (9a) to hold. In addition, it is clear that (10) is in fact a set of linear matrix inequalities (LMIs) that can be efficiently solved.

Based on the main result presented above, we are ready to synthesize a controller with the structural constraint for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ problem. This result is summarized in the following proposition.

Proposition 1: Given weighting factors α_1, α_2, system matrix X, disturbance model A_w and communication graphs $\mathcal{G}_1$ and $\mathcal{G}_2$, if the convex optimization problem below has a solution, then we have $\|\mathcal{T}_{dy}\|_2 < \gamma$ and $\|\mathcal{T}_{dy}\|_\infty < \sqrt{\eta}$ with the structural constraint $K \sim (\mathcal{G}_1, \mathcal{G}_2)$ satisfied.

$$\underset{\Omega}{\text{Minimize}}\ \alpha_1 \gamma + \alpha_2 \eta \tag{11a}$$

$$\text{subject to } \gamma > 0, \eta > 0, \operatorname{trace}\{Z\} < \gamma \tag{11b}$$

$$\begin{bmatrix} \eta I & A_o & B_o \\ A_o^T & Q_1 & \mathbf{0} \\ B_o^T & \mathbf{0} & \gamma I \end{bmatrix} \succ 0 \tag{11c}$$

$$\begin{bmatrix} Q_1 & \mathbf{0} & C_o^T \\ \mathbf{0} & I & D_o^T \\ C_o & D_o & Z \end{bmatrix} \succ 0 \tag{11d}$$

$$\begin{bmatrix} Q_2 & \mathbf{0} & A_o^T & C_o^T \\ \mathbf{0} & I & B_o^T & D_o^T \\ A_o & B_o & \eta I & \mathbf{0} \\ C_o & D_o & \mathbf{0} & \eta I \end{bmatrix} \succ 0 \tag{11e}$$

$$Q_1 \succ \eta I, Q_2 \succ \eta I \tag{11f}$$

$$(A_\xi, B_\xi, C_\xi, D_\xi) \sim (\mathcal{G}_1, \mathcal{G}_2) \tag{11g}$$

where $\Omega = (\gamma, Z, Q_1, Q_2, \eta, A_\xi, B_\xi, C_\xi, D_\xi)$ and A_o, B_o, C_o, D_o are defined by (8).

Proof: The proof of the above proposition directly follows from Theorem 1 and Theorem 2. The original Lyapunov matrices P_1 and P_2 can be recovered directly by letting $P_1 = Q_1^{-1}$ and $P_2 = Q_2^{-1}$. ■

In this section, we have focused on the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control problem with structural constraints. The centralized solution of this problem has been extensively studied in the literature; however, incorporating the structural constraints imposed on the controller is a nontrivial task due to the bilinear nature of the coupling between the decision variables. To address this issue, we present a sufficient condition for transforming a bilinear matrix inequality into a linear matrix inequality in order to incorporate the structural constraints which is linear in the decision variable. A convex optimization is then derived for the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ controller synthesis problem with structural constraints.

IV. Simulation Result

In this section, we implement the proposed solution to a real-world distribution system. The system model we use is a 42 bus distribution feeder of South California Edison with a high penetration of photovoltaic (PV) generation. The diagram of this test system is shown in Fig. 3. The detailed configuration of this system can be found in [17], [18].

In this system, there are 42 buses in total and only 5 buses $(2, 12, 26, 29, 31)$ include PV units. The reactive power injections of these 5 PV buses are governed by a certain Volt/var control scheme. The operation of this distribution system can be described as follows. At time 0, an optimal power flow problem is solved given predicted load consumption and solar irradiation levels over a given time horizon T. The optimal solution to the OPF problem is set to be the reference signal and is broadcast to each bus of the distribution system. The initial reactive power injection is q^r from the OPF solution. At each discrete time instant $k\Delta t \in [0, T]$ for $k = 1, 2, \ldots, \lceil \frac{T}{\Delta t} \rceil$ the voltage level at each bus is measured and transmitted to neighboring buses according to the underlying communication structure. The reactive power injection at each PV bus is then updated based on different VVC schemes.

Typically, the OPF problem is solved every $T = 5$ minutes. During every $T = 5$ minutes, for synchronous VVC schemes, the time step Δt is usually selected according to the time needed for taking measurements and communicating. In this paper, we assume that $\Delta t = 1$ second. Furthermore, albeit the proposed framework

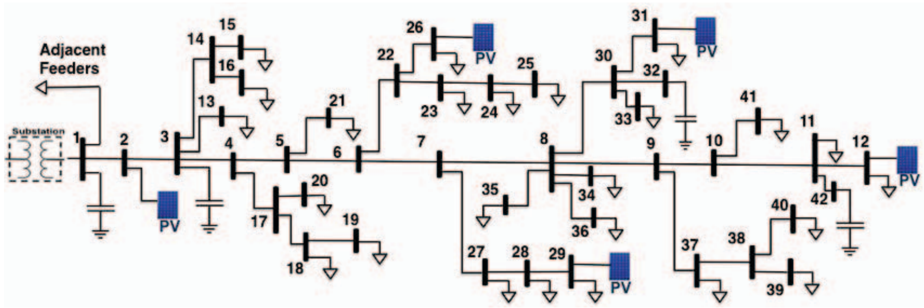
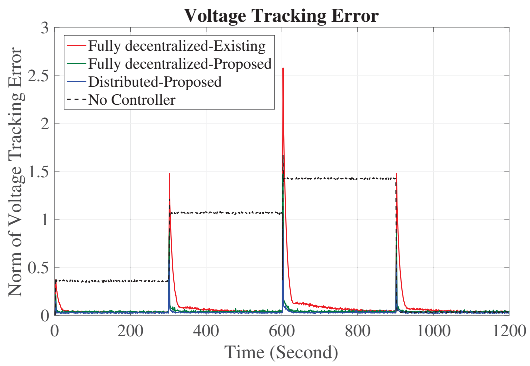
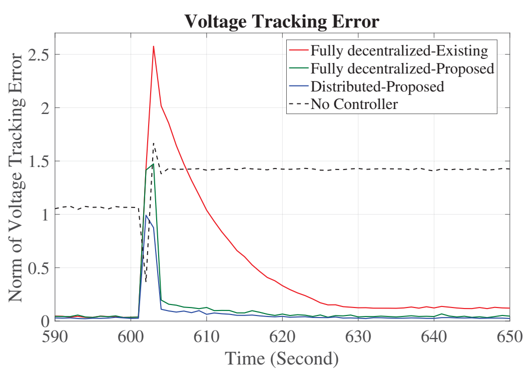


Fig. 3. Circuit Diagram of SCE Distribution System [17]

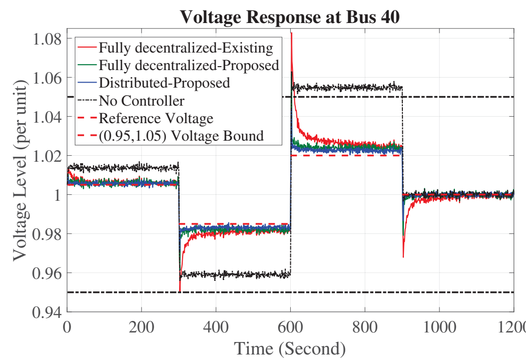


(a) Norm of Voltage Tracking Error

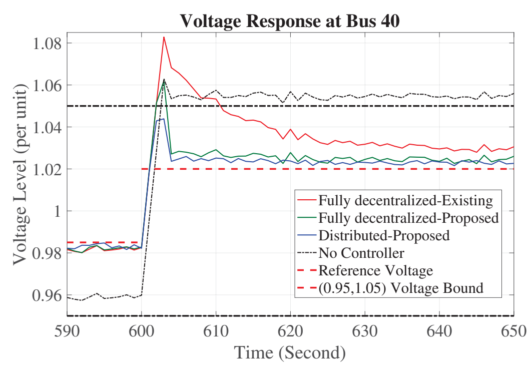


(b) Norm of Voltage Tracking Error -Zoomed

Fig. 4. Norm of Voltage Tracking Error



(a) Voltage Tracking Performance at Bus 40



(b) Voltage Tracking Performance at Bus 40-Zoomed

Fig. 5. Voltage Tracking Performance at bus 40

is built on a linear model, in this simulation section the voltage values are computed via a exact nonlinear power flow model using MATPOWER toolbox [34].

In all the simulations in this section, the parameters of the uncertainty model are $A_w = \text{diag}\{0.1\}$, and stochastic noise d is assumed to be a zero mean Gaussian noise with variance 0.001. We simulate the performance of different controllers for 4 OPF cycles, i.e. 1200 seconds with changing voltage references to illustrate not only the steady states under different controllers but also the corresponding transient performance. In particular, we compare a local controller whose gain satisfies the convergence condition proposed in the literature, a fully decentralized controller synthesized using the proposed technique and a distributed controller with additional communication with neighboring buses. Specifically, in the following case studies, the PV generator installed at bus 2 in Fig. 3 can communicate with buses 1 and 3 and the PV generator installed at bus 26 can communicate with bus 22.

The comparisons of performances of different controllers are illustrated in Fig. 4 and Fig. 5. Specifically, we show both the norm of the voltage tracking error and the voltage profile at bus 40 to demonstrate both system-wide and local tracking performance. It can be seen in Fig. 4 that the controllers synthesized using the proposed framework result in much faster convergence rates as compared to the existing controller due to the fact that the proposed scheme optimizes the transient performance. In addition, by the zoomed-in plots in Fig. 4(b) and Fig. 5(b), it can be observed that the distributed controller exhibits both a smaller overshoot and a smaller steady state tracking error. In particular, from Fig. 5(b), it is clear that with the synthesized distributed controller, the voltage level is always maintained within the typical bounds $(0.95 \text{ p.u.}, 1.05 \text{ p.u.})$ during the transient while all other controllers fail to achieve this.

V. Conclusions and Future Works

In this paper, we consider the control layer of a Volt/var control problem in distribution systems under uncertainty. The uncertainties mainly originate from the variability of the loads of the distribution system and the abrupt changes of solar radiation that affect PV output. In order to incorporate these uncertainties, a mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control framework with structural constraints is developed. With the additional structural constraints, the mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control synthesis problem is generally nonconvex. To address this issue, an effective way to convexify the controller synthesis problem is proposed. A case study is comprehensively analyzed and reported to illustrate the performance of the proposed solution.

In the future, we plan to explicitly incorporate the constraints of the reactive power injections and voltage levels into this framework and to provide a rigorous mathematical analysis considering different communication structures.

References

[1] P. Carvalho, P. F. Correia, and L. A. Ferreira, "Distributed reactive power generation control for voltage rise mitigation in distribution networks," *Power Systems, IEEE transactions on*, vol. 23, no. 2, pp. 766–772, 2008.

[2] K. Turitsyn, P. Šulc, S. Backhaus, and M. Chertkov, "Distributed control of reactive power flow in a radial distribution circuit with high photovoltaic penetration," in *Power and Energy Society General Meeting, 2010 IEEE*. IEEE, 2010, pp. 1–6.

[3] M. E. Baran and M.-Y. Hsu, "Volt/var control at distribution substations," *Power Systems, IEEE Transactions on*, vol. 14, no. 1, pp. 312–318, 1999.

[4] J. Grainger and S. Civanlar, "Volt/var control on distribution systems with lateral branches using shunt capacitors and voltage regulators part i: The overall problem," *Power Apparatus and Systems, IEEE Transactions on*, no. 11, pp. 3278–3283, 1985.

[5] B. A. Robbins, H. Zhu, and A. D. Domínguez-García, "Optimal tap setting of voltage regulation transformers in unbalanced distribution systems," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 256–267, 2016.

[6] Z. Hu, X. Wang, H. Chen, and G. Taylor, "Volt/var control in distribution systems using a time-interval based approach," in *Generation, Transmission and Distribution, IEE Proceedings-*, vol. 150, no. 5. IET, 2003, pp. 548–554.

[7] T. Niknam, A. Ranjbar, and A. Shirani, "Impact of distributed generation on volt/var control in distribution networks," in *Power Tech Conference Proceedings, 2003 IEEE Bologna*, vol. 3. IEEE, 2003, pp. 7–pp.

[8] J. Smith, W. Sunderman, R. Dugan, and B. Seal, "Smart inverter volt/var control functions for high penetration of pv on distribution systems," in *Power Systems Conference and Exposition (PSCE), 2011 IEEE/PES*. IEEE, 2011, pp. 1–6.

[9] M. Farivar, C. R. Clarke, S. H. Low, and K. M. Chandy, "Inverter var control for distribution systems with renewables," in *Smart Grid Communications (SmartGridComm), 2011 IEEE International Conference on*. IEEE, 2011, pp. 457–462.

[10] M. Farivar, R. Neal, C. Clarke, and S. Low, "Optimal inverter var control in distribution systems with high pv penetration," in *Power and Energy Society General Meeting, 2012 IEEE*. IEEE, 2012, pp. 1–7.

[11] K. Turitsyn, P. Sulc, S. Backhaus, and M. Chertkov, "Local control of reactive power by distributed photovoltaic generators," in *Smart Grid Communications (SmartGridComm), 2010 First IEEE International Conference on*, Oct 2010, pp. 79–84.

[12] ——, "Options for control of reactive power by distributed photovoltaic generators," *Proceedings of the IEEE*, vol. 99, no. 6, pp. 1063–1073, June 2011.

[13] P. Jahangiri and D. C. Aliprantis, "Distributed volt/var control by pv inverters," *Power Systems, IEEE Transactions on*, vol. 28, no. 3, pp. 3429–3439, 2013.

[14] K. Kalyanam, S. Darbha, P. Khargonekar, M. Pachter, and P. Chandler, "Optimal cooperative pursuit on a Manhattan grid," in *AIAA Guidance, Navigation, and Control (GNC) Conference*, no. AIAA 2013-4633, 2013.

[15] M. Farivar, L. Chen, and S. Low, "Equilibrium and dynamics of local voltage control in distribution systems," in *Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on*. IEEE, 2013, pp. 4329–4334.

[16] B. Zhang, A. D. Dominguez-Garcia, and D. Tse, "A local control approach to voltage regulation in distribution networks," *arXiv preprint arXiv:1307.0855*, 2013.

[17] M. Farivar, X. Zhou, and L. Chen, "Local voltage control in distribution systems: An incremental control algorithm," in *Proc. IEEE Intl. Conf. on Smart Grid Commun., Miami, FL*, 2015.

[18] X. Zhou, M. Farivar, and L. Chen, "Pseudo-gradient based local voltage control in distribution networks," *arXiv preprint arXiv:1508.02796*, 2015.

[19] H. Zhu and H. J. Liu, "Fast local voltage control under limited reactive power: Optimality and stability analysis," *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 3794–3803, Sept 2016.

[20] S. Bolognani, R. Carli, G. Cavraro, and S. Zampieri, "Distributed reactive power feedback control for voltage regulation and loss minimization," *IEEE Transactions on Automatic Control*, vol. 60, no. 4, pp. 966–981, 2015.

[21] K. Baker, A. Bernstein, E. Dall'Anese, and C. Zhao, "Network-cognizant voltage droop control for distribution grids," *IEEE Transactions on Power Systems*, vol. PP, no. 99, pp. 1–1, 2017.

[22] H. J. Liu, W. Shi, and H. Zhu, "Decentralized dynamic optimization for power network voltage control," *IEEE Transactions on Signal and Information Processing over Networks*, vol. 3, no. 3, pp. 568–579, 2017.

[23] K. Zhou, J. C. Doyle, K. Glover *et al.*, *Robust and optimal control*. Prentice hall New Jersey, 1996, vol. 40.

[24] C. Scherer, P. Gahinet, and M. Chilali, "Multiobjective output-feedback control via lmi optimization," *IEEE Transactions on automatic control*, vol. 42, no. 7, pp. 896–911, 1997.

[25] C. Scherer and S. Weiland, "Linear matrix inequalities in control," *Lecture Notes, Dutch Institute for Systems and Control, Delft, The Netherlands*, vol. 3, 2000.

[26] I. Kaminer, P. P. Khargonekar, and M. A. Rotea, "Mixed h2h control for discrete-time systems via convex optimization," *Automatica*, vol. 29, no. 1, pp. 57–70, 1993.

[27] S.-H. Wang and E. Davison, "On the stabilization of decentralized control systems," *IEEE Transactions on Automatic Control*, vol. 18, no. 5, pp. 473–478, 1973.

[28] N. Sandell, P. Varaiya, M. Athans, and M. Safonov, "Survey of decentralized control methods for large scale systems," *IEEE Transactions on automatic Control*, vol. 23, no. 2, pp. 108–128, 1978.

[29] M. Rotkowitz and S. Lall, "A characterization of convex problems in decentralized control," *IEEE Transactions on Automatic Control*, vol. 51, no. 2, pp. 274–286, 2006.

[30] D. D. Siljak, *Decentralized control of complex systems*. Courier Corporation, 2011.

[31] S. Bolognani and S. Zampieri, "On the existence and linear approximation of the power flow solution in power distribution networks," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 163–172, 2016.

[32] M. Hassanzadeh, C. Y. Evrenosoğlu, and L. Mili, "A short-term nodal voltage phasor forecasting method using temporal and spatial correlation," *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 3881–3890, 2016.

[33] E. P. Wigner, "On weakly positive matrices," in *The Collected Works of Eugene Paul Wigner*. Springer, 1993, pp. 559–563.

[34] R. D. Zimmerman, C. E. Murillo-Sanchez, and R. J. Thomas, "Matpower: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, Feb 2011.