Homework 3

Due: 2017 May 24 (10:30 AM)

- **The analytical solutions can be submitted before the class starts.**
- **Coding assignment can be sent as an email to me (jinkyoo.park@kaist.ac.kr)**

Problem 1 (Replicator Dynamics for Prisoner's Dilemma)

Consider the following prisoner's Dilemma game (player 1's payoff is shown)

	C	D
C	3	0
D	5	1

(1) Write down the replicator dynamic equation for player 1:

(2) What are the steady states?

(3) What are the asymptotically stable steady states?

(4) Compare the steady states and asymptotically stable states to the mixed or pure Nash equilibria

(5) Program the replicator dynamics (complete the blank in the provided code template) and plot the results

Problem 2 (Replicator Dynamics for Rock Paper Scissor game)

Consider the following Rock Paper Scissor game

	Rook	Paper	Scissors
Rook	$1-e$	0	2
Paper	2	$1-e$	0
Scissors	0	2	$1-e$

(1) Write down the replicator dynamic equation for player 1

(2) Find Nash equilibria when $e = 0$

(3) Use the algorithm you coded in problem (1) to simulate the game and plot the results for different range of e and discuss the trends. You can plot the results using the code provided.

Problem 3 (Fictitious Play for zero sum game)

Consider the normal form game given in Table 3. Player 1 is the row player, and player 2 is the column player. The two parts of this question are separate, but both concern fictitious play of this game, with ties broken arbitrarily.

	L	R
U	$2, 1$	$3, 1$
D	$4, 0$	$1, 3$

(1) In this part of the problem, player 1 starts with initial beliefs over (L, R) of $(2.5,1)$ – which means that he initially gives a weight of 2.5 to player 2's action L and a weight of 1 to player 2's action R – and player 2 starts with initial beliefs over (U, D) of $(1,1)$. Show the actions and updated beliefs for the first 6 rounds of fictitious play (starting at round 1) by filling in the following Table. The beliefs of an agent are updated at the end of a round. Thus, the beliefs in round i include the actions that were observed in the i −th round, while the actions in round i are based on the beliefs of round $i - 1$.

Round	Player 1's action	Player 2's action	Player 1's beliefs	Player 2's beliefs
0 (initial)	xxx	xxx	$(2.5,1)$	$(1,1)$
1				
2				
3				
4				
5				
6				

(2) In this part, we do not specify the initial beliefs of the players– we only restrict them. Specifically, in a player's initial beliefs, the weight he assigns to an action of the other player can be any value in the range [1,10]. Of the four action profiles for this game, list all that could possibly be played in the millionth round of fictitious play. Briefly justify your answer.

(3) Implement Fictitious play algorithm for the given problem (Complete the code provided) and show results of a single simulation.