VOLTAGE INSTABILITY IN A POWER SYSTEM WITH SINGLE OLTC

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Abstract - This paper investigates power system voltage stability in the presence of on-load tap-changing (OLTC) transformers. To assess this type of voltage instability, the sensitivity of load voltage to tap position, dV/dn (ΔI), is proposed as a stability criterion. It is shown that: (1) The region of normal control, where inverse control action does not occur, lies inside the stability region when there exists an equilibrium. This region is represented as $0<I<I_{max}$. (2): Even if an equilibrium does not exist, the voltage continues to recover in the region where the above condition holds. (3) The operating point $I=\infty$ corresponds to the point of static bifurcation which is caused by the dynamics of the load at the bus controlled by the OLTC.

1. INTRODUCTION

Recently, due to the tendency of increasing loads on networks with long transmission lines, voltage stability has been one of the major topics among power system engineers. This problem has long been studied by many researchers through various approaches which are roughly divided into two groups: One is the so-called static approach in which algebraic equations such as the load flow equations are used to study this problem [1-6]. The voltage stability limit is treated as the singular point of the load flow Jacobian analogous to the tip of the well known P-V curve. The other approaches consist of investigating dynamic mechanisms by using differential equations [6-25], in which various dynamic models are used. They are, for example, steady state analyses including generator dynamics [7], load dynamics such as induction motor loads [8-10], OLTC transformer dynamics [11-15]; voltage collapse mechanisms and their analysis methods are examined in [16-19]. In [19], it is demonstrated that the stability of slow and fast dynamics can be treated separately and that the stability region is determined as the intersection of the slow and fast stability regions. The static bifurcation has been studied in [19-22], which clarifies the relationship between static and dynamic approaches; a voltage collapse model is proposed based on the idea of a center manifold trajectory [23]. Recent various results in both static and dynamic approaches can be seen in [24].

The voltage stability problem relative to OLTC, which is a main topic in this paper, has been studied as follows: In [11,12], the stability of equilibria has been studied based on eigenvalues by using a continuous model of OLTC. Conditions for voltage stability have been studied for different OLTC models in [13], where a discrete OLTC model has been proposed. In [14], a stability region around the stable equilibrium was derived through a nonlinear analysis for the continuous model. In [15], the adverse effect of the OLTC action for induction motor loads is studied. In [16], mechanisms of voltage collapse have been studied by considering a dynamic load characteristic as well as OLTC dynamics.

Inverse control action of tap-changers is defined as a mechanism through which the voltage under OLTC control behaves as a function of the tap position in a manner opposite to the normal behaviour [25].

This paper investigates the stability of a single OLTC system. Detailed derivation and analysis are performed based on the formulation developed in [19] where a power system has been divided into slow and fast subsystems. That is, OLTC dynamics are examined in the slow subsystem where fast dynamics, such as generators and loads, are treated as static algebraic equations. A discrete OLTC model will be adopted from reference [13]. Alternatively, the OLTC is fixed while the fast variables are examined in the analysis of the fast subsystem. The following assumption will be made: the slow subsystem consists of only OLTC dynamics; the other slow dynamic characteristics such as the mean frequency are neglected. Load dynamics will be treated as fast. This treatment may be justified by the fact that the very slow dynamic characteristics of loads usually come from OLTC actions in the secondary system [26,27]. We will emphasize the case where the equilibrium of the tap position is non-existent in addition to the case where the equilibrium exists. The sensitivity, dV/dn, of a load voltage, V, to a tap position, n, is analyzed to show the following theoretical results:

(1) The region of normal control where inverse control action does not occur lies inside the stability region when there exists an equilibrium. This region is represented as $0<I<I_{max}$.

(2) Even if an equilibrium does not exist, the voltage continues to recover in the region where the above condition holds. (This region does not correspond to a stability region; therefore, the OLTC must be locked before escaping the above region.)

(3) The operating point $I=\infty$ corresponds to the point of the static bifurcation which is caused by the dynamics of the load at the bus controlled by the OLTC.

2. CONDITION OF INVERSE CONTROL ACTION

2.1 System Model

In this paper, a single OLTC system in Figure 1 is analyzed. This system consists of generators, controllers, loads, a network, as well as an OLTC. First, the following OLTC model is cited from reference [13].

$$n(k+1) = n(k) - d \cdot f(v-v_{ref}) \tag{1}$$

where

$$f(v-v_{ref}) = \begin{cases} 1 : v-v_{ref} > \epsilon \\ 0 : -\epsilon < v-v_{ref} < \epsilon \\ -1 : v-v_{ref} < -\epsilon \end{cases} \tag{2}$$

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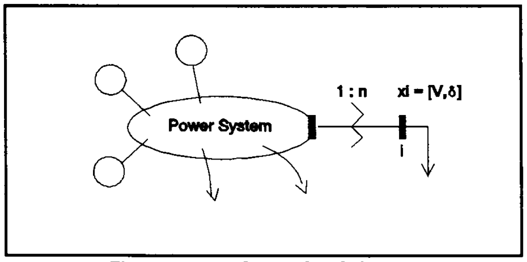

Figure 1 A Single OLTC System

where

n(k)	:	tap position at time k
d	:	step size of tap position
v	:	secondary voltage (= i-th node voltage, which corresponds to one of the element of x in (3) and (4))
v_{ref}	:	reference voltage
e	:	dead band

The fast dynamic characteristics, represented as a set of differential equations with small time constants, are treated as static in this section as mentioned in the INTRODUCTION. That is, the fast derivatives are set to zero to obtain the static characteristics expressed by a set of algebraic equations, which can be simplified to the following two expressions:

$$y = f_{GL}(x, L) \tag{3}$$

$$y = f_{LF}(n, x) \tag{4}$$

where

x	:	node voltage vector (V, δ)
y	:	node injection vector (P, Q)
L	:	parameter vector controlled by consumers

Equation (3) represents the static characteristics of nodes which represents generating units and loads, while (4) expresses the network characteristic corresponding to the load flow equations. In the above equations (3) and (4), the secondary voltage controlled by OLTC, v, corresponds to one of the element of x; therefore, v can be determined when load consumptions, L, and tap position, n, are specified. This means that v can be represented as a function of n and L, which is expressed as in the following form in this paper.

$$v = v(n, L) \tag{5}$$

Note that (5) represents a regular component of the whole manifold (3) and (4); the manifold (5) is bounded by the surface called "impasse point" or "singular point", which can also be understood as a static bifurcation point when the singular perturbed system of (1) to (4), which includes fast dynamics in loads and generators, is considered [19-23]. This condition of the static bifurcation can be represented as follows:

$$\det(J_S) = 0 \tag{6}$$

where

$$\begin{aligned} J_S &= J_{LF} - \partial f_{GL} / \partial x \\ J_{LF} &= \partial f_{LF} / \partial x \ : \ \textit{load flow Jacobian} \end{aligned} \tag{7}$$

2.2 Condition of Inverse Control Action

In this section, the system represented by (1) to (4) is analyzed. In the analysis, we do not assume the existence of the equilibrium.

First, the voltage at time k is denoted as v(n(k),L). Then, the voltage deviation from its reference value is defined as follows:

$$u(k) \triangleq v(n(k), L) - v_{ref} \tag{8}$$

Assuming that L is fixed, u(k+1) can be written as follows:

$$\begin{aligned} u(k+1) &= u(k) + \{ v(n(k+1), L) - v(n(k), L) \} \\ &= u(k) + \{ v(n(k) - d \cdot f(u(k)), L) - v(n(k), L) \} \\ &\approx u(k) - I(k) \cdot d \cdot f(u(k)) \end{aligned} \tag{9}$$

where

$$I(k) = \left[\frac{dv}{dn} \right]_{n=n(k)} \tag{10}$$

where I(k) represents sensitivity dv/dn evaluated at the operating point, n=n(k). Note that, if there exists an equilibrium, it is possible to evaluate dv/dn at the equilibrium, n=0.

Now, we chose a positive definite function, V, as follows:

$$V(u(k)) \triangleq u(k)^2 \tag{11}$$

In the above equation, V(u(k)) becomes a Lyapunov function if there exist the equilibrium, u=0; the equilibrium is stable if the following condition holds along every trajectory of (9) in some region about u=0.

$$\Delta V = V(u(k+1)) - V(u(k)) < 0 \tag{12}$$

On the other hand, if there is no equilibrium points, no stability region may not be defined. However, even in this case, it is understood that the voltage continues to recover in the region where the condition (12) holds.

Now, substituting (9) into (12), we obtain

$$\begin{aligned} \Delta V &= u(k+1)^2 - u(k)^2 \\ &= \{ u(k) - I(k) \cdot d \cdot f(u(k)) \}^2 - u(k)^2 \\ &= -I(k) \cdot \{ d \cdot f(u(k)) \}^2 \cdot \left\{ \frac{2u(k)}{d \cdot f(u(k))} - I(k) \right\} \end{aligned} \tag{13}$$

From the above equation, the equivalent condition to (12) can be written as follows:

$$0 < I(k) < 2 \frac{u(k)}{d \cdot f(u(k))} \tag{14}$$

In the above condition, the lower bound of I(k) corresponds to the boundary of normal control region, where the inverse control action defined in the INTRODUCTION does not occur. The region of the inverse control action is represented as $l<0$. On the other hand, the upper bound represents the stability limit corresponding to the vibration of OLTC around the dead band. It is noted that condition (12) is a sufficient condition for stability. This means that there can exist stability region outside the region of the inverse control action of OLTC.

3 RELATIONSHIP WITH THE STATIC BIFURCATION CONDITION

3.1 Expression of dv/dn

As is analyzed in the previous section, sensitivity l = dv/dn plays an important roll in the voltage phenomena associated with OLTC action. In this section, the characteristic of dv/dn is studied when the system is approaching to the static bifurcation point.

Let the complex voltage at the OLTC bus (bus i) be x_i. The vectors x is divided into two components, one for bus i (subscript i), and another for the remaining buses (subscript R) as $x = [x_i^T, x_R^T]^T$. Assuming that L is fixed in (3), we linearize these equation around the operating point. Then, we obtain

$$\Delta y_l = k_l \, \Delta x_l \tag{15}$$

$$\Delta y_R = k_R \, \Delta x_R \tag{16}$$

Note that matrix k_R is usually the block diagonal matrix having 2x2 dimensional submatrices, each of which represents characteristics of each generating unit or load.
In the same way, the load flow equation (4) becomes as follows:

$$\begin{bmatrix} \Delta y_l \\ \Delta y_R \end{bmatrix} = \begin{bmatrix} J_{ll} & J_{lR} \\ J_{Rl} & J_{RR} \end{bmatrix} \begin{bmatrix} \Delta x_l \\ \Delta x_R \end{bmatrix} + \begin{bmatrix} J_{nl} \\ J_{nR} \end{bmatrix} \Delta n \tag{17}$$

Eliminating Δy_i, Δy_R, Δx_R in (15) to (17) we obtain

$$0 = (J_s - k_l) \, \Delta x_l + J_{sn} \, \Delta n \tag{18}$$

where

$$\Delta x_l = \begin{bmatrix} \Delta\delta \\ \Delta v \end{bmatrix}, \quad k_l = \begin{bmatrix} 0 & \partial P_l/\partial v \\ 0 & \partial Q_l/\partial v \end{bmatrix}$$

$$J_s = J_{ll} - J_{lR}(J_{RR} - k_R)^{-1} J_{Rl} \triangleq \begin{bmatrix} J_{P\delta} & J_{PV} \\ J_{Q\delta} & J_{QV} \end{bmatrix} \tag{19}$$

$$J_{sn} = J_{nl} - J_{lR}(J_{RR} - k_R)^{-1} J_{nR} \triangleq \begin{bmatrix} J_{Pn} \\ J_{Qn} \end{bmatrix}$$

Further, eliminating $\Delta\delta$, sensitivity l (=dv/dn) is obtained as in the following final form.

$$l = \frac{\Delta v}{\Delta n} = \frac{\det(J_m)}{\det(J_s - k_l)} \tag{20}$$

where

$$J_m = \begin{bmatrix} J_{P\delta} & -J_{Pn} \\ J_{Q\delta} & -J_{Qn} \end{bmatrix} \tag{21}$$

3.2 Approximation of The Minimum Eigenvalue

In the previous section, we neglected the fast dynamics existing in generating units and loads, and considered only the static characteristics of these fast components. In this section, taking into account the fast dynamic characteristics, we study the case where the system is approaching to the static bifurcation.

First, we assume the fast subsystem as in the following linearized form:

$$\frac{d}{dt} \begin{bmatrix} \Delta z_l \\ \Delta z_R \end{bmatrix} = \begin{bmatrix} A_{ll} & A_{lR} \\ A_{Rl} & A_{RR} \end{bmatrix} \begin{bmatrix} \Delta z_l \\ \Delta z_R \end{bmatrix} + B_n \Delta n + B_L \Delta L \tag{22}$$

In the above equation, the fast state variable, Δz, is introduced and divided into two components; z_i represents the relatively slow component in z, while Δz_R the remaining faster component. According to the singular perturbation technique, the eigenvalues corresponding to Δz_i, that is the eigenvalues of (22) having the smaller absolute values, can be approximated by the eigenvalues of the following matrix:

$$A_b = A_{ll} - A_{lR} A_{RR}^{-1} A_{Rl} \tag{23}$$

It is noted that the above matrix is obtained as the system matrix when $d\Delta z_R/dt$ is set to zero in the original system, (22).

Now, we consider the static bifurcation situation where one of the eigenvalues of (22) is approaching to zero. This eigenvalue is denoted as λ_b hereafter. Under this condition, where λ_b becomes small enough, it can be approximated as one of the eigenvalue of (23); therefore, the following condition holds.

$$\det(A_b) \to 0 \tag{24}$$

If the operating point is close enough to the static bifurcation point, λ_b will become the minimum eigenvalue of (22). In this situation, it is possible to select only one component in z as z_i corresponding to λ_b. As a result, z_i becomes scalar and A_b directly approximates the minimum eigenvalue, λ_b.

Next, the power system in Figure 1 is analyzed based on the above idea. For this purpose, we assume the situation that z_i corresponds to one of the dynamics variables existing in load bus i. This situation means that some dynamic characteristic in the OLTC load is going to cause the static bifurcation. In order to evaluate matrix A_b in the studied system, only z_i is expressed as the original dynamic form, while z_R is eliminated through setting dz_R/dt to zero. Thus, the characteristic of the load bus i can be represented as in the following form:

$$\Delta y_l = k_{zl}\Delta z_l + k_{xl}\Delta x_l \tag{25}$$

$$\frac{d}{dt}\Delta z_l = C_l\Delta z_l + D_l\Delta x_l \tag{26}$$

Note that the static characteristic of load i, which was used in (15), is obtained by setting $d\Delta z_l/dt=0$ in (25) and (26) as follows:

$$k_l = k_{zl} - k_{xl}C_l^{-1}D_l \tag{27}$$

Equation (16) is used as a expression for the remaining buses since only the static characteristics are needed to evaluate A_b. Now that the system is represented by equations (16), (17), (25), and (26), we eliminate Δy_l, Δy_R, and Δx_R from these equations. Then, we have

$$\frac{d}{dt}\begin{bmatrix}\Delta z_l\\ 0\end{bmatrix} = \begin{bmatrix}C_l & D_l\\ -k_{zl} & J_a - k_{xl}\end{bmatrix}\begin{bmatrix}\Delta z_l\\ \Delta x_l\end{bmatrix} \tag{28}$$

where Δn and ΔL are neglected. Matrix A_b is obtained as the system matrix of (28) as follows:

$$A_b = C_l + D_l(J_a - k_{xl})^{-1}k_{zl} \tag{29}$$

Now, we consider the relationship of the conditions between the inverse control action and the static bifurcation. Denoting the matrix in the right-hand side of (28) as D, the following relationship holds:

$$\begin{aligned}\det(D) &= \det(J_a - k_{xl})\cdot\det(A_b)\\ &= \det(C_l)\cdot\det(J_a - k_l)\end{aligned} \tag{30}$$

The above equation shows that $\det(J_a - k_l)$, which is the denominator of (20), approaches to zero under the static bifurcation condition of (24) if $\det(C_l)\neq 0$. In other words, the condition where dv/dn becomes infinitive corresponds to the static bifurcation occurring at the OLTC node, where some dynamic factor existing in the load collapse, and therefore, voltage collapses. It is noted that the general condition of the static bifurcation, (6), also holds in this situation. This is shown in the following equation:

$$\det(J_S) = \det(J_{RR} - k_R)\cdot\det(J_a - k_l) \tag{31}$$

4. CONCLUSIONS

In this paper, several aspects concerning the sensitivity of voltage to the tap position, $l=dv/dn$, have been studied through theoretical analysis of a single OLTC system. They are,

- The region of normal control action of OLTC, $0<l$, where inverse control action does not occur, lies inside the stability region when the equilibrium of the tap position exists.
- There exists an upper bound of l for stability, $l<l_{max}$.
- In the region of the normal control action, $0<l$, the voltage is in the process of recovering even in the case that an equilibrium does not exist.
- The condition where l becomes infinite corresponds to the static bifurcation condition caused by the fast dynamics at the OLTC bus.

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