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A Fast Scalable Quasi-Static Time Series Analysis Method for PV Impact Studies using Linear Sensitivity Model

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***Abstract*—Understanding the impact of distributed photovoltaic (PV) resources on various elements of the distribution feeder is imperative for their cost effective integration. A year-long Quasi-Static Time Series (QSTS) simulation at 1-second granularity is often necessary to fully study these impacts. However, the significant computational burden associated with running QSTS simulations is a major challenge to their adoption. In this paper, we propose a fast scalable QSTS simulation algorithm that is based on a linear sensitivity model for estimating voltage-related PV impact metrics of a 3-phase unbalanced, non-radial distribution system with various discrete step control elements including tap changing transformers and capacitor banks. The algorithm relies on computing voltage sensitivities while taking into account all the effects of discrete controllable elements in the circuit. Consequently, the proposed sensitivity model can accurately estimate the state of controllers at each time step and the number of control actions throughout the year. For the test case of a real distribution feeder with 2969 buses (5469 nodes), 6 load/PV time series power profiles, and 9 voltage regulating elements including controller delays, the proposed algorithm demonstrates a dramatic time reduction, more than 180 times faster than traditional QSTS techniques.**

***Index Terms*—Quasi-static time series, PV impact studies, multiple linear regression, voltage sensitivity analysis**

I. INTRODUCTION

Over the past decade, distribution networks have seen a significant increase in the penetration of distributed energy resources (DERs). Installations of PV systems alone have seen an average annual growth rate of 68% per year in the United States over the past 10 years [1]. This unprecedented growth has been primarily attributed to the decreasing cost of PV systems as well as federal and state incentives [2]. However, due to high temporal and spatial variability of PV power production, significant penetration of PV systems can have undesired impacts on the distribution circuit elements. Feeders with high PV penetration can suffer from voltage violations, thermal overloading and excessive reverse power flow into the substation [3]. In addition, voltage regulators and capacitors banks in particular, can suffer from an excessive increase in the number of operations and potential oscillatory behavior. These effects can drastically reduce their life expectancy. Currently, scenario-based simulation is commonly used by utilities for screening potential issues while evaluating PV interconnection requests [4]. This process essentially involves solving a small set of static power flows designed to simulate extreme scenarios such as light and peak loading, min/max PV output etc. However, due to the complex locational and temporal interdependence of the PV systems, identifying a limited set of scenarios that capture all their potential impacts is statistically impossible. Furthermore, the impact on discrete step, time dependent controllable elements such as voltage regulators and capacitor banks cannot be accurately captured using static scenario-based simulations [5].

The IEEE standards association defines QSTS as a series of steady state power flows solved chronologically with a time step that ranges from 1-second up to an hour [6]. The input to the QSTS simulation is a time series profile of power injections of various loads, PV, energy storage, and other DERs connected to the feeder. The discrete control elements within the circuit are modeled along with their delays and may change their state from one time step to the next. Since the solution of the power flow equations at each time step serves as the initial condition for the next interval, the complex interactions amongst the time-dependent controllable elements is directly captured. Moreover, by utilizing actual profiles for load and generation, QSTS accurately determines the impact of DERs on the voltage across the feeder. In [7], the authors present a detailed discussion on the advantages of running QSTS over scenario-based simulation and recommend using a year-long, 1-second time step simulation to accurately analyze the PV impacts. However, a QSTS simulation with such granularity corresponds to solving roughly 31.5 million power flows which can take anywhere between 10-120 hours for a realistic feeder using modern computers. Consequently, running multiple QSTS simulations to study locational impacts for various PV sizes becomes an infeasible option. There are numerous challenges in speeding up QSTS simulations which are detailed in [5].

Common approaches widely discussed in literature to reduce the QSTS computation time include reducing the total number of power flows solved, utilizing faster techniques to compute the power flow solution and decreasing the size of

This research was supported by DOE SunShot Initiative, under agreement 30691. Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525

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the modeled distribution feeder using circuit reduction. Vector quantization [8]–[10] groups together scenarios throughout the year that produce similar power flow solutions, thereby limiting the total computations of the power flow equations. Variable time-step [11] on the other hand advances through time at large intervals to drastically reduce the computation time. Circuit reduction [12] relies on decreasing the total number of buses in the distribution feeder to reduce the time taken by the power flow solver. Similarly, A-diakoptics [13] divides the distribution feeder into smaller geographical subnetworks, which can then be solved in parallel on multi-core machines. Computationally efficient power flow analysis techniques based on neural networks, as well as probabilistic algorithms have been proposed in [14]–[17]. The focus of this paper is not on speeding up the individual solution of the power flow, since existing solvers such as OpenDSS and GridLAB-D have already been optimized for this purpose. Our objective is speeding up QSTS by reducing the total number of power flows solved for a year-long simulation.

Recently, a fast QSTS simulation method based on voltage sensitivities of controllable elements has been presented in [18]. The proposed method creates decision boundaries for various controllable elements in the circuit by estimating nodal voltage sensitivity coefficients. Instead of solving power flows at each time instant, it exploits these decision boundaries to predict the controller states for a year-long QSTS simulation. A significant reduction in computation time is reported however, the proposed iterative approach for estimating decision boundaries suffers from scalability issues as the number of unique load/PV power injection profiles increases. In [18], the decision boundaries become more complicated with additional dimensions being added for each profile. Consequently computing boundaries for these higher dimensional spaces requires an exponentially increasing number of power flow solutions, which becomes computationally time intensive. Moreover, the proposed algorithm in [18] is unable to estimate the phase voltages in the feeder, which limits its applicability in PV impact analysis.

The main contribution of this paper is proposing a novel *fast QSTS* simulation algorithm that can perform time series analysis of a 3-phase unbalanced, non-radial distribution system with various discrete step control elements including tap changing transformers and capacitor banks. The algorithm relies on a local linearization of the voltage-power manifold to obtain the voltage sensitivity coefficients. These coefficients are then used to predict the state of time-dependent control elements, as well as the magnitude of phase voltages at each bus. This allows for a fast computation of voltage-related impact metrics, that can easily quantify the adverse effects of installing a PV resource within a distribution feeder. Compared to the existing QSTS methods, the contribution of the proposed algorithm is as follows:

- It significantly reduces the total number of AC power flows solved throughout the year-long QSTS simulation. This directly translates to a significant computational time reduction, when compared to [8], [9], [11].
- It is scalable to any number of times series power injection profiles when compared to [18].
- It provides fast methods to gather all voltage-related PV impact metrics including duration of over voltages, under voltages and feeder max/min voltages.

The remainder of the paper is organized as follows: Section II provides a brief background on sensitivity analysis. In Section III, we formulate the linear sensitivity model. This is followed by Section IV, which details the efficient utilization of multiple linear regression for parameter estimation of the linear sensitivity model. Section V discusses the fast QSTS simulation algorithm. Results of the simulations performed on a realistic feeder with multiple load and PV profiles are presented in Section VI. Section VII concludes the paper with a discussion on future implications of this work.

II. Background

Commercially available distribution system simulators, such as OpenDSS, perform QSTS analysis by solving AC power flow equations at each time step (referred to as the 'brute-force' method henceforth throughout this paper). The non-linear nature of the power flow equations require iterative gradient-type methods, which demand significant computational burden when performing a time series analysis. In contrast, sensitivity analysis projects these complex equations governing the voltage-power relationship into a linear space through sensitivity coefficients. This provides a more intuitive cause-and-effect based correlation between changes in network voltages as a response to changes in load and DER power injections [19]–[24]. Sensitivity analysis has been extensively used in voltage management strategies for distribution networks. In [25], the authors use same-bus sensitivity analysis to mitigate voltage fluctuations caused by the variability of the PV power output. On the other hand, [26] uses sensitivity theory to perform voltage regulation by using the reactive power exchanged between the network and the distributed generators. Furthermore, sensitivity analysis has been used for identifying the optimum location of DERs within the distribution networks to minimize the power losses [27].

Various techniques have been proposed in literature to compute sensitivity coefficients including Gauss-Seidel method [28], Jacobian-based method [29] and adjoint-network method [30]. The inverse of the Jacobian matrix in the Newton-Rapshon load flow technique directly yields the network sensitivity coefficients as a by-product [25]. However, most common distribution load-flow simulation packages either don't provide access to the formed Jacobian or may use alternative load-flow techniques. Consequently, the sensitivity information is rarely available to the user. Efficient analytical approaches to compute voltage and current sensitivities have been presented in [31], [32]. A sensitivity analysis toolkit based on a novel perturb-and-observe algorithm was developed in [33]. In contrast to the analytical methods, it is based on a more empirical approach that relies on introducing small perturbations in the injected power and then measuring the overall impact. One major benefit of this approach is the ease with which network sensitivities can be computed—a working power flow solver is all that is needed. In the subsequent sections, we propose a multiple linear regression based perturb-

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and-observe algorithm to compute voltage sensitivities and consequently utilize them to perform a QSTS analysis.

III. Linear sensitivity Model

In this section, we present a linear sensitivity model that exploits the correlation between the phase voltage magnitudes and power injections in the circuit by using multiple linear regression. The real (P) and reactive power (Q) injections are in the form of a time series profile that represents the per unit value of load or PV at any given time throughout the year. Furthermore, we analyze the impact of controllable elements with discrete steps, such as regulators and capacitor banks, on the bus voltage and extend our model to estimate the state of these elements as well.

A. Basic Model for Bus Voltage

We start by developing a model for the bus voltage in an unbalanced distribution circuit with no controllable elements. Let $v_\phi^{(j)}$ denote the dependent variable which represents the phase-to-ground voltage magnitude of phase $\phi \in \{A, B, C\}$ at bus j. Without loss of generality, we drop the subscript ϕ to keep the formulation concise. Moreover, for q buses, we form the voltage vector $\mathbf{v} = [v^{(1)}, v^{(2)}, ..., v^{(q)}]^\top$. Furthermore, let $[x_1, ..., x_p]$ be the vector of p independent variables with x_p representing the p^{th} load/PV profile of power injections. These power profiles can be only real power injections, only reactive power injections, or a combination scaling both real and reactive power proportionally for fixed-power-factor loads. We assume that there exists a linear relationship between voltage magnitude $v^{(j)}$ and p power injections given by,

$$v^{(j)} = \alpha_0 + \beta_1 x_1 + ... + \beta_p x_p \tag{1}$$

where β_i are the voltage sensitivities defined as,

$$\beta_i = \frac{\partial v^{(j)}}{\partial x_i}, \qquad \forall i \in \{1, 2, ..., p\} \tag{2}$$

In context of geometry, equation (1) represents a hyperplane with coefficients $\mathcal{H}^{(j)} \triangleq [\alpha_0, \beta_1, ..., \beta_p]$ uniquely characterizing a p dimensional flat in $\mathbb{R}^{p+1}$. The plane slopes $\beta_i \in \mathcal{H}^{(j)}$ capture the impact of each profile on the voltage at bus j. A higher magnitude of β_i indicates that the bus voltage is more sensitive to the variation in the profile x_i. Since the power injections in the distribution circuit impact the voltage at each bus differently, we propose (1) for every bus in the circuit.

Fig. 1 shows a voltage plane in $\mathbb{R}^3$ for a single load and a PV profile characterized by $\mathcal{H}^{(j)} = [0.99, 0.024, -0.083]$. Since increasing the load decreases the bus voltage, β_2 has a negative value. Due to the non-linearity of power flow equations, the bus voltages are not strictly linearly correlated to the power injections. However, for small variations in power injections, the linearity assumption produces a small error that is bounded [34].

B. Impact of Regulators and Capacitors on Bus Voltage

Voltage regulators are used to maintain the bus voltages within a specific range by changing their tap positions to vary the turns ratio, thereby increasing or decreasing the downstream bus voltages. To reduce oscillations, deadbands and delays are incorporated in the controller logic for these devices.

Let V_r denote the input control voltage to the regulator. Usually, it is the voltage across the secondary winding of the regulator unless line drop compensation is used. Furthermore, let V_{min} and V_{max} denote the lower limit and the upper limit of the regulator deadband respectively. If V_r falls below V_{min}, the regulator tap position increases, which steps up V_r. Similarly, if V_r goes above V_{max}, the regulator tap position decreases causing a drop in V_r. These corrective control actions continue until V_r falls back within the regulator deadband. However, in case the regulator tap is already at an extreme position, no action is taken.

To incorporate the impact of regulators on bus voltage, we create a new voltage plane each time a regulator changes its tap position. Fig. 2 shows the planes for control voltage V_r for two different tap positions. The horizontal plane formed by $V_{min} = 0.98$ defines the lower limit for the regulator deadband. Plane 1 corresponds to the current position of the regulator tap and as such control voltage is confined to this plane based on (1). The intersection of the two planes forms the line AD. As soon as V_r enters the region defined by the

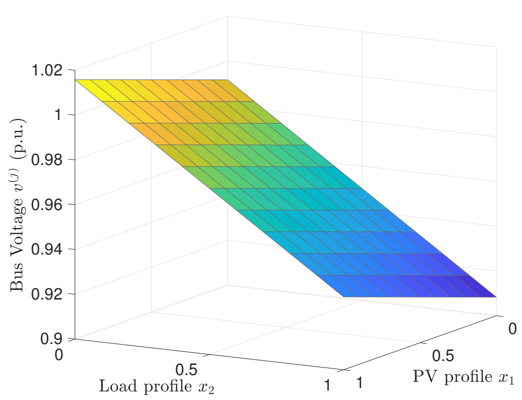

Fig. 1. Geometric interpretation of equation (1) for a single load and a single PV power injection profile.

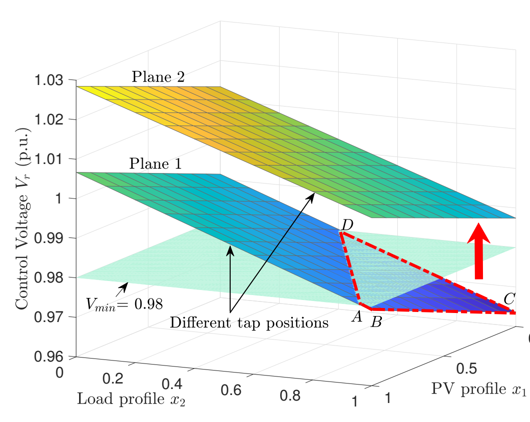

Fig. 2. Impact of regulator tap position on the control voltage plane. If V_r goes below V_{min}, a tap change is triggered causing V_r to shift from plane 1 to plane 2.

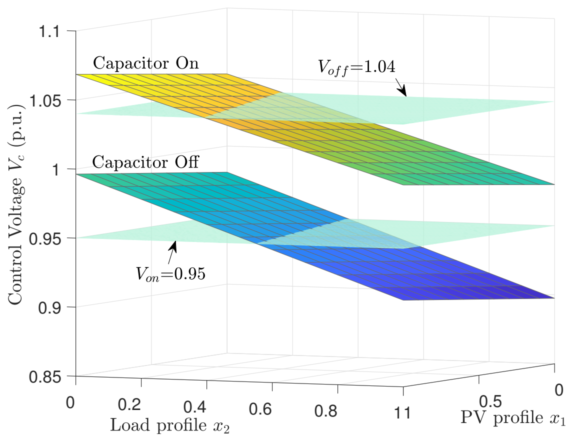

Fig. 3. Impact of capacitor state on the control voltage plane. If V_c goes below V_{on}, the capacitor turns on causing V_c to jump from one plane to the other and vice versa.

quadrilateral $ABCD$, a tap action is triggered. This is because every point in the region $ABCD$ corresponds to a value of V_r lower than V_{min}. The tap change causes V_r to jump from plane 1 to plane 2 and remain on it as long as it stays within the deadband. Therefore, by using multiple planes, we can predict the regulator tap positions and transitions.

Capacitors can regulate the reactive power in distribution circuits by switching on and off based on a control signal which can either be voltage, current, temperature or power factor. In our model, we consider capacitor banks which rely on a voltage signal for their operation. Unlike the regulators however, capacitors generally have only two states i.e. they are either connected (on) or disconnected (off). Let V_c denote the input control signal to the capacitor. If V_c falls below the minimum voltage threshold V_{on}, the capacitor switches to the on state. Conversely, if V_c goes beyond the maximum voltage threshold V_{off}, the capacitor switches off. Fig. 3 shows the planes for control voltage V_c corresponding to the two states of the capacitor.

C. Bus Voltage Model with Controllable Elements

For a distribution network with multiple controllable elements, we need to create a new plane for bus voltages each time a regulator tap action occurs or a capacitor changes its state. This is because each change creates a discrete jump in the bus voltages across the entire feeder. Based on the network topology, some buses will experience a small change while others will see a significant discontinuity in the voltage profile. Let s denote the system controller state which is the set of present states of all controllable elements in the circuit. For example, for a system with 2 regulators and 1 capacitor, $s = 1$ might correspond to regulator A at tap position 2, regulator B at tap position 6 and a capacitor that is in an off state. The voltage at bus j at time t is then given by,

$$v^{(j)}(t) = \mathcal{H}_s^{(j)} \mathbf{x}^\top(t) \tag{3}$$

where $\mathcal{H}_s^{(j)} = [\alpha_0^s, \beta_1^s, ..., \beta_p^s]$ is the vector of plane coefficients indexed by s and $\mathbf{x}(t) = [1, x_1(t), ..., x_p(t)]$ is the vector of power injections at time t. Equation (3) essentially states that each time the system controller state changes, a new set of plane coefficients characterize the voltage hyperplanes.

IV. Estimation using Multiple Linear Regression

The fidelity of the proposed linear sensitivity model (3) is greatly influenced by the estimation quality of the plane coefficients corresponding to a particular system controller state. In addition, the total possible system controller states can be considerably large depending upon the type and number of controllable elements in the feeder. For example, a feeder with 2 regulators (33 taps) and 2 capacitors, has a space of $33^2 \times 2^2$ possible system controller states. However, in practice this space of possible system controller states is extremely sparse [9]. To exploit this sparsity, the plane coefficients are estimated for only the unique system controller states that are observed during the time series simulation.

In order to accurately compute the plane coefficients and efficiently determine the controller states, we propose multiple linear regression based estimation methodology. The objective of this methodology is two-fold: computing the plane coefficients $\mathcal{H}_s^{(j)}$ and estimating control voltages to determine the controller states.

1) Computing Plane Coefficients: Using the framework of multiple linear regression, we can determine the plane coefficients for a given system controller state s. For n PV and k load profiles of power injections, we only need $n + k + 1$ observation to estimate $\mathcal{H}_s^{(j)}$. However, since bus voltages and power injections are not exactly linearly correlated, we use $n + 2k + 1$ observations to minimize the linearization error. To accurately capture the sensitivity of the bus voltages to each of the load/PV profiles, we propose the design matrix $\boldsymbol{X}$ with the following structure,

$$\boldsymbol{X} = \begin{bmatrix} 1 & x_1(t) & \dots & x_k(t) & 0 & 0 & \dots & 0 \\ 1 & \vdots & \dots & \vdots & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & x_1(t) & \dots & x_k(t) & 0 & 0 & \dots & 1 \\ \vdots & x_1(t)-\delta & & \vdots & 1 & 1 & \dots & 1 \\ \vdots & x_1(t)+\delta & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & & x_k(t)-\delta & 1 & 1 & \dots & 1 \\ 1 & x_1(t) & \dots & x_k(t)+\delta & 1 & 1 & \dots & 1 \end{bmatrix}$$

$$\underbrace{\qquad\qquad\qquad}_{k \text{ Load profiles}} \qquad \underbrace{\qquad\qquad}_{n \text{ PV profiles}}$$

where $\delta \in [0.05, 0.1]$ and $x_k(t)$ is the value of k^{th} load profile at time t. Following are some important observations regarding $\boldsymbol{X}$,

1) $\boldsymbol{X}$ has a full rank, $n + k + 1 \leq n + 2k + 1$
2) The number of observations required to estimate plane coefficients increase linearly with the number of profiles
3) PV profiles have greater temporal variability than load profiles, therefore the sensitivity of bus voltage to PV

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power injection is measured over the complete range i.e. from 0 to 1.

4) Sensitivity of bus voltage to load profiles is measured at the present value of load power injection and its δ-neighborhood.
5) The value of PV power injection is kept constant when measuring the sensitivity of the bus voltage to the load profiles and vice versa.

It is worthwhile mentioning here that the proposed estimation technique can accommodate other types of time series profiles as well including power output of an energy storage system or the charging of electric vehicles etc. After forming the design matrix $\boldsymbol{X}$, the vector of observed bus voltages $\boldsymbol{Y}^{(j)} \in \mathbb{R}^{n+2k+1}$ is obtained by solving the nonlinear power flow equations corresponding to each observation. Using the ordinary least squares estimator, the plane coefficients corresponding to a given system controller state s are then given by,

$$\mathcal{H}_s^{(j)} = (\boldsymbol{X}^\top \boldsymbol{X})^{-1} \boldsymbol{X}^\top \boldsymbol{Y}^{(j)} \tag{4}$$

The matrix $(\boldsymbol{X}^\top \boldsymbol{X})^{-1} \boldsymbol{X}^\top$ is called the Moore–Penrose pseudoinverse. It will always exist since $\boldsymbol{X}$ has a full rank [35].

2) Estimating Controller States: Accurately estimating the controller states is vital because of the dependency of plane coefficients on the system controller state. As mentioned earlier, the regulator tap position and the capacitor state are determined by the control voltages seen by the respective devices. Once the plane coefficients are computed, the control voltages can be easily estimated by evaluating (3) for the specific buses, which are being monitored by the controllers. If the estimated voltages are within the deadband, no controller action occurs. Otherwise the regulator tap position is changed or the capacitor is switch on/off according to the type of controller, control logic, and delays. This causes the system to transition to a new state for which the plane coefficients are again computed.

V. Fast QSTS Simulation using Linear Sensitivity Model

In this section, we discuss how to perform fast QSTS simulation using the linear sensitivity model. Let T_f and $\mathcal{N}$ denote the time horizon of the QSTS simulation and the set of all the buses in the circuit respectively. Furthermore, let $\mathcal{C} \subset \mathcal{N}$ denote a subset of buses whose voltages are monitored by the control elements.

Fig. 4 shows a detailed flow chart of the proposed algorithm. Upon initialization at $t = 0$, the circuit is compiled once to obtain the present controller states. The vector $\mathbf{x}$ consists of all the load/PV profiles of power injections at time t. The block in red, which is used to compute the plane coefficients for all the buses, is the only part of the algorithm that requires solving nonlinear power flow equations. However, once the plane coefficients corresponding to a system controller state s are computed, they are stored in a look up table using matrix indexing. In this manner, the algorithm keeps track of the system controller states encountered through time and reuses the plane coefficients every time a similar state is encountered. After estimating the control voltages, the next step is to establish if they lie within the deadbands of the respective devices. If they do, the controller states remain unchanged and the simulation steps through time, otherwise the controller logic determines the appropriate states of the controllers for the next time interval. After the completion of time series simulation, the post-processing block computes the important QSTS metrics that will be discussed in the subsequent section.

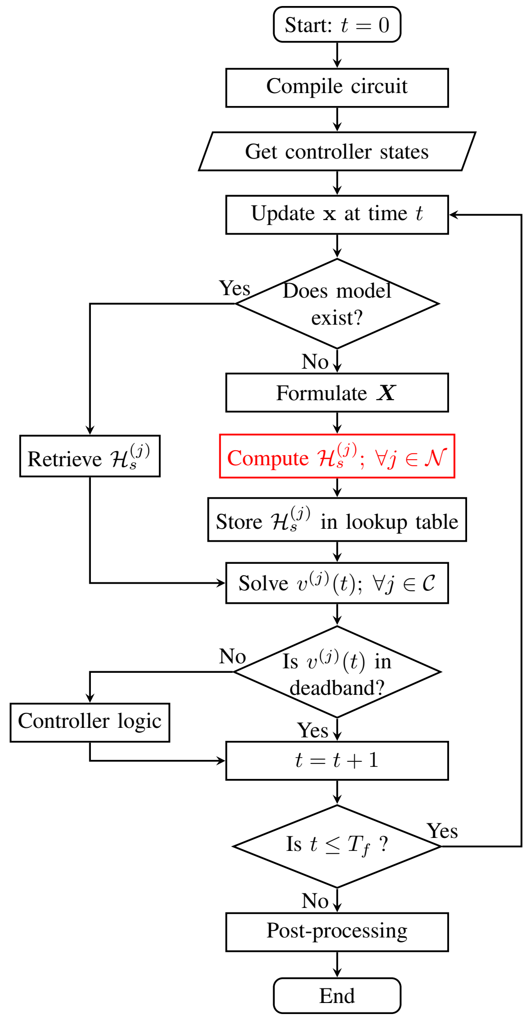

Fig. 4. Flow chart of the proposed algorithm for fast QSTS simulation.

The most important aspect of running a QSTS simulation is to accurately produce a set of output metrics that will quantify the impacts of PV on a distribution feeder. The proposed algorithm can be used to study voltage-related impacts which are listed in Table I, along with their acceptable absolute accuracy thresholds. These thresholds have been established in [7]. The motivation for these metrics comes from [3], where the authors present a detailed discussion on voltage issues arising in feeders with significant PV penetration. Metrics 1-2 can be easily computed since the proposed algorithm keeps track of controller states at each time instant. For metrics 3-5, the value of phase voltage at each bus is required, which can be obtained by evaluating (3), $\forall j \in \mathcal{N}$, at each time step in

TABLE I
VOLTAGE-RELATED PV IMPACT METRICS MEASURED BY QSTS

	QSTS Metrics	Error Threshold
1	Total regulator tap actions	10%
2	Total capacitor switching operations	20%
3	Highest/lowest voltage on the feeder	0.005 pu
4	Per phase highest/lowest voltage at each bus	0.005 pu
5	Duration of ANSI voltage violations	24 Hrs

the post-processing block. However, [7] suggests using larger time steps while estimating extreme feeder voltages and ANSI violations. We propose estimating phase voltages for all the buses only when the controller logic decides to update the system controller state. This is because extreme voltages are more likely to occur right before the controller state transitions. As for the duration of ANSI voltage violations, a value of time step anywhere between 5-15 minutes will yield an acceptable level of accuracy. In any case, depending upon the application specific requirements, the voltage at each bus can be obtained with the desired accuracy level by selecting the time step parameter in the post-processing block.

A. Analysis of Computation Time

The total time taken by the proposed algorithm is dependent on two factors. Firstly, it is a function of the number of input load/PV profiles. Recall that computing bus voltages requires evaluating (3), which is essentially a matrix multiplication operation. The dimension of the system is therefore dictated by the number of load/PV profiles rather than the total number of buses in the circuit. In contrast, solution of power flow equations is computationally much more time intensive because of the matrix inversion and multiple iterations.

Secondly, the total number of unique system states observed throughout the time horizon T_f also dictates the computation time of the algorithm. The proposed linear sensitivity model requires new plane coefficients every time the system state changes. Furthermore, for every new unique system state, equation (4) has to be evaluated which requires solving $n + 2k + 1$ nonlinear power flows. Consequently, the total number of power flows solved by the proposed algorithm, N_{PF}, is given by,

$$N_{PF} = (n + 2k + 1)N_s \tag{5}$$

where N_s is the total number of unique system controller states observed for the given time horizon. Moreover, equation (4) has to be evaluated a total of N_s times. This, however, is a trivial operation since it involves computing pseudoinverse of a matrix, the dimensions of which are dictated only by the number of power injection profiles. Compared to the brute-force approach, which solves power flows at each time step, the proposed algorithm drastically reduces the total number of power flows solved which in turn causes a significant reduction in the computation time.

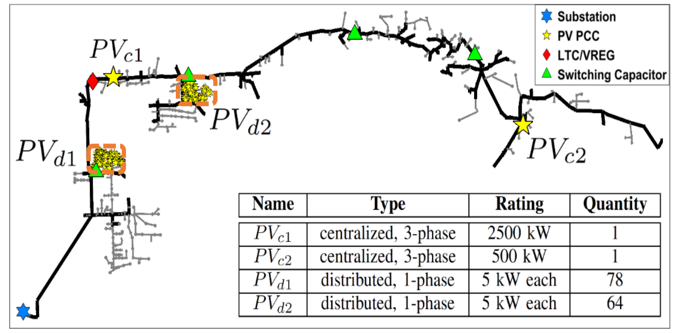

Fig. 5. Circuit diagram of the distribution feeder with various types of PV installations. Each type has a unique power injection profile assigned to it. In addition, there are a total of 1,111 single-phase and 317 three-phase loads attached to the feeder.

VI. SIMULATION CASE STUDY

In this section, we test the proposed fast QSTS simulation algorithm[1] on an actual distribution feeder [36], with 2969 buses (5469 nodes) as shown in Fig. 5. The length of the feeder is 21.37 km with a total of 9 controllable elements: a single three-phase substation load tap changing (LTC) transformer, 3 single-phase line voltage regulators, 3 three-phase and 2 single-phase switching capacitor banks. There are 1,111 single-phase customers and 317 three-phase customers on the feeder, each modeled individually connected on the low-voltage secondary networks. The feeder includes detailed models of the service transformers for the secondary system, including both wye and delta connected. All three-phase loads are designated as commercial while single-phase loads are assumed to be residential, accounting for 1.69 MW and 4.25 MW of the peak load respectively. In addition, other feeders connected to the substation transformer are also modeled as aggregate lumped loads with 12 MW of peak power. The mean phase-to-ground voltage imbalance in the feeder at rated load is 2.71%. Both load types are assigned a unique 1-second power injection profile generated from SCADA measurements of the feeder under consideration. Fig. 6 shows a boxplot for the year-long residential and commercial load profiles grouped by the hour.

A total of 144 PV systems are installed on the feeder, which are grouped into four categories based on their geographical locations, as shown in Fig. 5. Distributed PV systems are residential rooftop installations modeled on the low-voltage networks adjacent to the loads. The centralized PV systems are 3-phase utility-scale installations with their own inter-connection transformer. Each category of PV system is also assigned a unique 1-second power injection profile based on solar irradiance data. The raw irradiance measurements were obtained from DEMROES network at UC San Diego, which were then converted to year-long synthetic PV profiles [34]. In addition, all PV systems are injecting power at a constant power factor. We simulate two different scenarios to test the efficacy of the proposed algorithm.

[1]Fast QSTS simulation algorithm was implemented in MATLAB linked to OpenDSS through a COM interface. All simulations were performed on a Windows 10 machine with 32 GB of RAM and 3.50 GHz processor.

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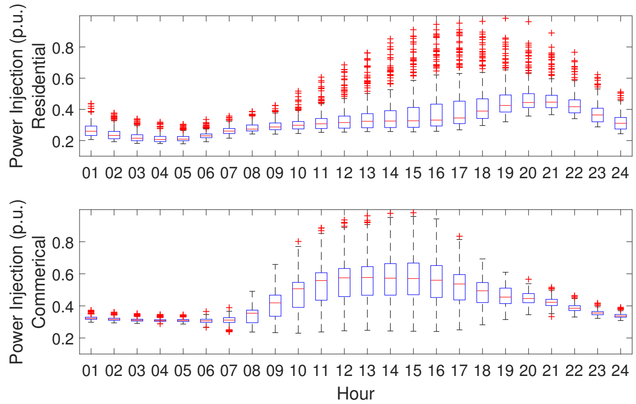


Fig. 6. Boxplot for the two load profiles. The bottom and top edges of the box indicate the 25th and 75th percentiles respectively, and the red line inside the box represents the median values.

Case A : Two Load Profiles with no PV system

In the first simulation case, the feeder has no PV installations. Moreover, since the total number of profiles in this case is only two, the behavior of bus voltages in a three dimensional space can be easily visualized. The baseline simulation results are obtained by running a year-long brute-force QSTS simulation with a residential and a commercial load profile. Fig. 7 shows the control voltages seen by the substation LTC as a function of the two load profiles. Each dot represents a power flow solution at a particular time instant in the time series simulation. The substation LTC is configured to regulate the control voltage to 124V with a deadband of 2V. The various colors represent power flow solutions corresponding to different tap positions. It is evident from the figure that the control voltages are confined to planes whose coefficients are dependent on the particular state of the controllable elements. These observations are coherent with our proposed linear sensitivity model. Since the substation LTC has a delay timer in the controller logic, a few power flow solutions are out of the deadband. Similar voltage planes were obtained for the IEEE-13 bus test feeder in [18].

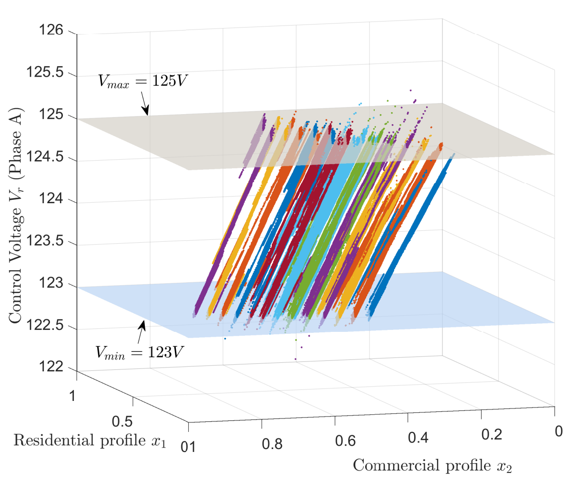


Fig. 7. Control voltages seen by the substation LTC in a 1-second year-long brute-force QSTS simulation (31.5 million points), with a different color for each tap position.

TABLE II
(CASE A) ACCURACY AND TIMING COMPARISON

QSTS Metric	Brute-force	Fast QSTS (error)
Regulator tap actions:		
Sub LTC (3-ϕ)	2742	+0.14%
VReg 1 (A-ϕ)	3125	-0.12%
VReg 2 (B-ϕ)	4177	-0.31%
VReg 3 (C-ϕ)	3872	-0.15%
Capacitor switches:		
Cap 1 (3-ϕ)	628	0%
Cap 2 (3-ϕ)	0	0
Cap 3 (3-ϕ)	44	0%
Cap 4 (A-ϕ)	464	-1.29%
Cap 5 (B-ϕ)	722	0%
Time Taken	29 hours	2.68 minutes

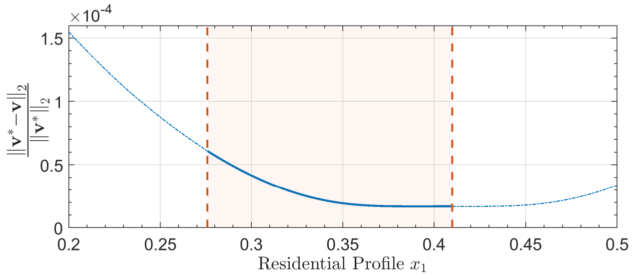
Table II presents an accuracy comparison of the QSTS metrics 1-2 computed using brute-force and the proposed fast QSTS algorithm using linear sensitivity model. Since the voltage planes are not exactly linear, the proposed algorithm produces a small error in estimating the controller states. Let $\mathbf{v}^*$ denote the voltage vector obtained by solving the actual nonlinear power flow equations. A plot of the relative error introduced due to the linear sensitivity model is shown in Fig. 8. In order to obtain the plane coefficients, the linearization of voltage-power manifold is done around the operating point $(x_1, x_2) = (0.4056, 0.3676)$, which is essentially the value of power injection profiles at $t = 1$. The shaded region represents the permissible values of power injections for which the control voltages of all the controllable elements remain within their respective deadbands before changing states. As shown, the error is well behaved within this region with a value of less than 0.0075%. Once the value of power injection profiles leave this region, a new set of plane coefficients will be computed as discussed in Section V.

The proposed fast QSTS algorithm computes all voltage-related impact metrics in less than 3 minutes as compared to the brute-force solution which takes almost 29 hours to complete (see Table II). The 99.85% reduction in computation time is primarily due to the fact that the proposed algorithm only solved 7,508 power flows for the year-long simulation as opposed to 31.5 million, solved by the brute-force approach.

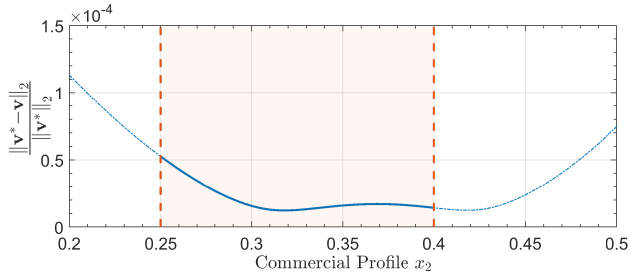
Case B : Two Load Profiles with significant PV

In this case, we run the QSTS simulation with all four categories of PV systems connected to the feeder, each following its own unique power injection profile. The PV penetration is approximately 50% of the peak load. Fig. 9 shows a comparison of the states of voltage regulators through time. The plot in blue corresponds to the actual controller states obtained through the brute-force solution whereas, the overlaid red plot shows the estimated states obtained by running the proposed fast QSTS algorithm. It can be seen that the fast QSTS algorithm is extremely accurate at predicting the actual states of the controllers through time.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSTE.2018.2833748, IEEE Transactions on Sustainable Energy



(a) Relative error for variation in residential profile



(b) Relative error for variation in commercial profile

Fig. 8. Error introduced due to the linear sensitivity model for a given system controller state. For (a), x_2 is fixed at 0.3676, while for (b) x_1 is fixed at 0.4056.

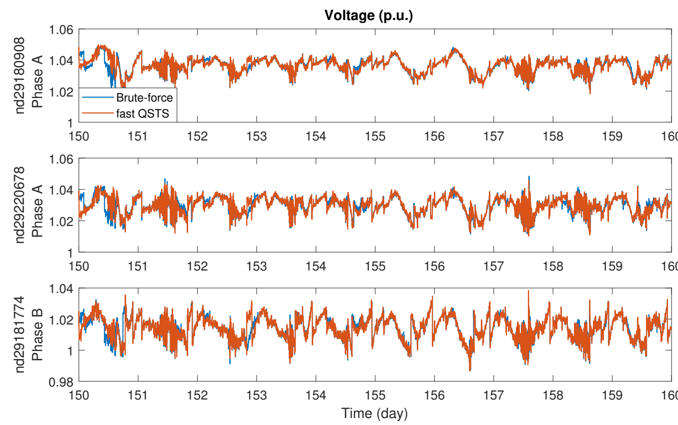


Fig. 10. Comparison of phase voltages of three different buses along the feeder. All the voltages are in p.u. on a 6.92 kV base.

Fig. 10 plots the phase voltages, at each time step, of three different buses along various locations in the feeder, the first one being close to PV_{d1}, the second one near PV_{d2} and the third one upstream of PV_{c2}. The estimated voltages using the fast QSTS algorithm are plotted in red whereas the brute-force results are shown in blue. It is important here to note that failure to predict accurate system controller state by the fast QSTS algorithm results in an error in the estimated phase voltages. This is because the plane coefficients are not updated corresponding to the actual state of the system at that time. This is evident in Fig. 10, where a missed capacitor state transition between day 150 and 151 causes an error in the actual and predicted phase voltages.

A comparison of the accuracy for various QSTS metrics is presented in Table III. It can be seen that installing PV in the feeder causes a significant increase in the number of controller actions. This is due to the highly variable PV power output. The fast QSTS algorithm produces an average absolute estimation error of 1.25% and 2.24% for the regulator and capacitor states respectively. In addition, the proposed algorithm is able to accurately predict the duration of ANSI violations as well as extreme voltages well under the specified accuracy thresholds. A 15 minute time step is chosen to obtain the duration of ANSI voltage violations. Table IV shows that the fast QSTS algorithm takes less than 10 minutes to compute all the voltage-related PV impact metrics which corresponds to a speed increase of more than 180 times. Because of the additional profiles and more unique controller states, the total number of power flows solved in this simulation case is greater than in case A, but it is still 1000 times less than solving a power flow for every second for a year-long QSTS simulation.

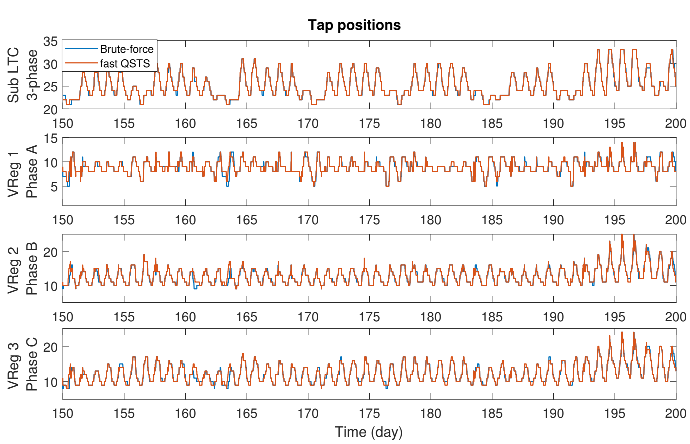


Fig. 9. Comparison of voltage regulator states through time. All regulators have a total of 33 tap positions.

TABLE III
(CASE B) QSTS VOLTAGE METRICS COMPARISON

QSTS Metric	Brute-force	Fast QSTS (error)
Regulator tap actions:		
Sub LTC (3-ϕ)	2746	-2.11%
VReg 1 (A-ϕ)	5515	+0.50%
VReg 2 (B-ϕ)	5055	+1.85%
VReg 3 (C-ϕ)	5274	+0.56%
Capacitor switches:		
Cap 1 (3-ϕ)	352	-3.97%
Cap 2 (3-ϕ)	54	0%
Cap 3 (3-ϕ)	30	0%
Cap 4 (A-ϕ)	544	-6.98%
Cap 5 (B-ϕ)	740	-0.27%
Feeder phase voltage:		
Highest	1.0631 p.u.	-0.0001 p.u.
Lowest	0.9010 p.u.	+0.0004 p.u.
Duration of ANSI violations:		
Over voltage	235.2 Hrs	-2.19 Hrs
Under voltage	1009.6 Hrs	-13.28 Hrs
Per phase voltage (each bus):	total of 5469 nodes	
Highest	0.0013 p.u. (mean error)	
Lowest	0.0007 p.u. (mean error)	

TABLE IV
(CASE B) TIMING COMPARISON

	Brute-force	Fast QSTS	% Reduction
Time Taken	29 hours	9.60 minutes	99.45%
Power flow solutions	31.5 million	32,364	99.90%

VII. CONCLUSION

A fast and novel quasi-static time series (QSTS) simulation algorithm is presented in this paper. It facilitates detailed analysis of voltage-related PV impacts on realistic, unbalanced 3-phase distribution feeders with discrete-step controllers such as voltage regulators and capacitor banks. Inspired from predictive modeling in machine learning, the proposed algorithm uses multiple linear regression to develop a linear sensitivity model for each unique system controller state experienced in the time series simulation. Consequently, instead of solving power flows at each time step, a solution of the linear sensitivity model yields the phase voltages at every bus across the entire feeder. The proposed fast QSTS algorithm is able to evaluate various voltage-related PV impacts with high accuracy in less than 10 minutes for a real 2969 bus (5469 node) distribution feeder with 9 voltage regulating devices and 6 high resolution load/PV profiles of power injections. This represents a computational time reduction of 99.45% as compared to the brute-force QSTS simulation.

The significant computational time reduction means that utilities can more efficiently utilize QSTS simulation to evaluate PV interconnection requests. Because of the scalability of this algorithm, various types of technologies and time series profiles can be simulated with QSTS, including load, PV, energy storage and electric vehicle charging etc. Implementation of this algorithm in commercial software can even further reduce the computation time and increase its applicability to various power system applications including PV hosting capacity analysis and distribution system planning.

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