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# ANALYSIS OF THE TAP CHANGER RELATED VOLTAGE COLLAPSE PHENOMENA FOR THE LARGE ELECTRIC POWER SYSTEM

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## Abstract

It is shown in this paper that nonsmooth events such as tap changing can be integrated into the framework of a recently presented general theory [15, 17] on the dynamics of the large power system modelled by smooth equations. The boundary to the region of feasible operations starting at a stable operating point, the feasibility boundary, is shown to be composed of five types of segments including two types connected with tap changers. These segments are defined as zero sets of explicitely known functions and thus are relatively computable (at roughly the load flow level). These new results offer a powerful new approach to system security when tap changers are present. No reference needs to be made to $P-V$ curves of the large power system, an obstacle in more conventional approaches.

## 1 Introduction

Voltage collapse caused by locally controlled tap changers is investigated in this paper. The methods proposed can be extended to include other local discrete controls such as locally controlled load dropping. Locally controlled automatic tap changers have been the subject of much interest and intensive studies [1, 8, 9, 10, 15, 19, 20]. Tap changers are used in two different ways in the power transmission system:

1. Within the transmission system itself in a way where there are loops of transmission lines closed through them. The purpose is to control the distribution of reactive power flow within the transmission system (or active power flow with out of phase taps) and hence its voltage and stability. This application, normally under secondary control, has no conspicuous special role in voltage collapse.

2. At subtransmission or distribution substations to keep the voltage on the secondary side within tolerances in face of voltage variations on the primary side. Under uncoordinated local control, combined with load characteristics and system conditions, this type can cause a puzzling cascading operation of rising taps and falling secondary voltage into collapse as well as other irregularities.

Analysis of the cascading voltage descent problem associated with the latter type of tap changers is the main objective of this paper. Algebraic conditions have been developed recently to locate the cascading voltage descent problem associated with the on-line automatic tap changers (or ULTC's) under certain assumptions [15, 19]. It will be shown that the voltage descent is typically due to a saddle-node type bifurcation associated with the discrete tap changer control. Algebraic conditions are also developed here to detect possible 'hunting' phenomena among several tap-changers.

The cascading descent problem mentioned above is a parameter space event in the sense that the inception of this phenomenon is marked by an operating parameter such as the load and voltage set point etc. crossing a 'feasibility limit'. The notion of the 'feasibility region' is proposed to include all the parameters which correspond to stable system operation and the boundary of this region, the feasibility boundary, then defines the feasibility limits for the parameters. For a differential-algebraic system described by smooth functions, the concept of feasibility region was introduced in [23, 14, 15, 17]. Feasibility is a generic concept and distinct definitions exist in other problem areas, such as load flow [3] or optimization. In this paper, we extend the notion of the feasibility region as defined in [15, 17] to include local control devices such as the automatic tap changers. Even though we consider only the automatic tap changers, other local controls such as locally controlled load dropping, can also be treated similarly. The main feature of such control mechanisms is that the control is purely local. For instance, the tap changer will increase (reduce) the tap setting, when its terminal voltage is below (above) the voltage set point. The setpoint itself is an input, manual or coming from a secondary control. Exploiting this decoupled nature of the control, we will develop a simple algebraic condition to test the stability of the control dynamics [15]. (In [19], the same stability condition was proposed independently). This will then allow us to directly identify the new segments of the feasibility

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boundary introduced by these local control devices. One type of the new boundary segments corresponds to the cascading voltage descent problem, which will be shown to be typically caused by a static saddle-node type bifurcation associated with the discrete tap changer control. We show that the entire feasibility boundary is composed of zero sets of explicit analytic functions and thus is accessible to computation. Hence, it fits directly into the security monitoring and operation planning process and can be very useful there. Questions of transient behavior involving tap changers seem to be less pressing, however, it has a sizeable literature.

# 2 Modeling Issues

Many problems in the dynamic analysis of the large power system like voltage stability analysis can be modeled (*within the quasi-stationary range* [15]) by a parameter dependent differential-algebraic system of the form

$$\dot{x} = f(x, y, p) \quad , \quad f : \mathbb{R}^{n+m+p} \to \mathbb{R}^n \tag{1}$$

$$0 = g(x, y, p) \quad , \quad g : \mathbb{R}^{n+m+p} \to \mathbb{R}^m \tag{2}$$

$$x \in X \subset \mathbb{R}^n, \;\; y \in Y \subset \mathbb{R}^m, \;\; p \in P \subset \mathbb{R}^p$$

In the state space $X \times Y$ dynamic state variables $x$ and instantaneous state variables $y$ are distinguished. Typical dynamic state variables are the time dependent values of generator voltages and rotor phases, instantaneous variables are bus voltages and other load flow variables. The parameter space $P$ is composed of system parameters (the system topography, i.e. what is energized, and equipment constants e.g. inductances), and operating parameters (such as loads, generation, voltage setpoints etc.).

Modeling and problem formulation aspects of tap changers are treated in detail next.

(1) *Physical features:* Traditionally, to operate a tap changer under load, it will be necessary first to bridge two transformer taps through appropriate inductors or resistors with center taps, and then interrupt the circulating current at the old tap. This involves considerable mechanical operation of contacts and switches and the use of small interruptors (often vacuum type). Operating these mechanisms takes a considerable and fixed amount of time which is independent of the deviation of the bus voltage from the set point. The point is that the system experiences one (or two) fixed magnitude step jumps of the transformation ratio per tap change (usually less than 1%) and a minimum time interval, also of fixed magnitude of many seconds between consecutive changes (depending on type, 40-100 sec., 3-8 seconds, or, in new thyristor devices, under 1 sec. This paper applies mostly to the former two types which represent the vast majority here.) It is clear that the response to small step increases, placed many seconds apart, will break down to two decoupled phenomena.

1. A transient response to the small step jump is normally quite harmless, unless the system is operating with inadequate security that is much too close to its feasibility or stability boundary. This transient decays during a small initial part of the minimum delay preceding the next tap change.
2. Thus the success of a tap change is expressed by the steady state voltage $E_L^s$ reached after the last tap change and before the next one. This voltage is independent of the transient which has died out by that time. So typically only the steady state value is significant.

The traditional tap changer operating control is a simple device which switches one step up or down when the deviation between the steady state load voltage $E_{Li}^s$ and the reference setting $E_{ri}$ reaches or exceeds the tolerance value, $\Delta E$ [9, 4]. Thus

$$n_{k+1} = \begin{cases} n_k + \Delta n & \text{if } E_{Li}^s - E_{ri} < -\Delta E \\ n_k & \text{if } |E_{Li}^s - E_{ri}| \leq \Delta E \\ n_k - \Delta n & \text{if } E_{Li}^s - E_{ri} > \Delta E \end{cases} \tag{3}$$

where $\Delta n$ is the tap step size (usually equal or slightly less than $\Delta E$) and $E_{Li}^s$ is the steady state voltage at the load $i$ and as such the solution of

$$0 = f(x, y, n, p), \;\; 0 = g(x, y, n, p). \tag{4}$$

In equation (4), for convenience, the vector $n$ of tap positions is displayed separately from other parameters. Its nature may vary depending on the problem. If we denote bus voltages by $z_1 = \{E_i^s\}$, the load voltages (secondaries of tap changers) by $z_2 = \{E_{Li}^s\}$ and the remaining (dynamic and instantaneous) state variables by $z_3$, and all the other parameters are grouped as $p$, then the system at steady state can be written comprehensively as

$$0 = h(z_1, z_2, z_3, n, p). \tag{5}$$

This model is analyzed in detail in section 3. Note that this analysis (See also subsection (2) below) applies to the traditional,i.e. slow tap changer.

There are a variety of questions connected with the tap changer that can be asked. From an operating, security monitoring and planning angle point of view, the most useful is the following one.

(2)*Effect of the tap changer on the feasibility boundary in the parameter space :* The actual postchange stationary voltage will depend both on the tap size and the system composition. As long as there is a rise in the secondary voltage above the set-point, no critical problem arises. However, because of load characteristics and other system features, the stationary voltage can actually go down in response to an upward tap change. When that happens, the simple automatic control device (3) set for local control (and even an inexperienced operator) will order another upward step initiating a chain reaction of descending and ultimately collapsing voltage. When a number of tap changers are involved, this scenario generalizes to the positive definiteness of the sensitivity matrix, i.e. the sensitivity of the load voltages to the tap changer positions. When this matrix is positive definite, for any load bus then, if the tap ratio is raised, the load bus voltage also increases, which is the desired effect. But, if the sensitivity matrix is not positive definite, then there is at least one direction in which the raise of the tap ratio may lead to a cascaded voltage collapse. It is later shown that the tap changer control becomes unstable when the sensitivity matrix is not positive definite. In the parameter space the boundary of after switch tap positions where the cascading phenomenon commences is where the sensitivity matrix becomes indefinite. Thus the boundary can be pinpointed by postulating an arbitrarily small tap size. For a finite tap size the actual boundary for the initiating step would then be lower by one step size. Over a region of system conditions, this condition defines a section of the feasibility boundary in parameter space where the cascading phenomenon sets in, i.e. local stability of the tap changer control is lost beyond this boundary. This leaves the small step boundary as the core of the answer which can be readily adjusted to the actual step size (See the details of analysis in section 3).

(3) *Transient phenomena connected with the tap changer in the state space :* The traditional slow tap changer has no real dynamic characteristics beyond (3) and (5). In other words, when the voltage is off the reference by the tap size, a tap change is implemented regardless of anything else. So on this type of device there is no $\dot{n}$ that is tap rate term–in fact the tap change is always strictly one tap at a time regardless of the *size* of the deviation. So this is a discrete element. Continuous models have been proposed [9] to approximate the discrete model by assuming that the tap size is small and that the tap changer has an option of not switching when the voltage is within the neutral band $|E_L^s - E_r| \leq \Delta E$. It is tempting to introduce a continuous approximation [1] of the form

$$\dot{E}_L^s = \frac{1}{T}(E_L^s - E_r) \tag{6}$$

or

$$\dot{n} = \frac{1}{T}(E_L^s - E_r). \tag{7}$$

No convincing justification of this approximation seems to exist for the traditional slow tap changers. In fact, originally it was simply stated as an unsupported assumption [1] and later users simply quote this reference. Since the normal dynamic elements (generator, rotor angle, excitation control, load dynamics etc.) are much too fast to couple into the tap changing cycle, the practical meaning of this assumption is dubious unless a much slower $f, g$ system is substituted, like boiler dynamics or some AGC action. On the other hand, using very fast thyristor type tap changers, it would be quite easy to implement an actual controller with the control law of (6) or (7) in the usual transient stability range. In this latter case, if such a control dynamics exists, then the system itself would need to be modeled in its full dynamic form, that is $\dot{x} = f(x, y, p), 0 = g(x, y, p)$ where now $\dot{x}$ includes $\dot{E}_{Li}^s$ or $\dot{n}$ and $f$ includes equation (6) or (7). Technology is still evolving for such fast switching of tap changers and this issue will not be discussed in this paper. These phenomena however are directly covered by the general dynamic theory presented in [15, 17].

The new introduction in subsection (2) above of studying the effect of the tap changer on the feasibility boundary seems to be providing the most pertinent information connected with the special type of collapse (the cascading one) which is not quite dynamic in the ordinary sense and which is an important practical problem in operation.

# 3 Mathematical Analysis

## 3.1 Feasibility boundary without tap changers

As a first step, it will be assumed that automatic tap changer controls are not present, i.e the transformer tap settings are taken as parameters. Let us define the equilibrium points or the steady state solutions as $EQ$,

$$\begin{aligned} EQ &= \{(x, y, p) \in X \times Y \times P : f(x, y, p) = 0, \\ &\qquad g(x, y, p) = 0\} \\ OP &= \{(x, y, p) \in EQ : \ D_y g \ \text{ is nonsingular and} \\ &\qquad J = D_x f - D_y f (D_y g)^{-1} D_x g \\ &\qquad \text{has eigenvalues with negative real part}\}; \end{aligned}$$

It can be seen that $OP$ is the subset of stable equilibria, hence consists of possible candidates for system operating points.

**Definition 1** *Given a stable equilibrium $z_s^0 = (z_0, y_0)$ (connected to a load flow solution) for parameter value $p_0$, the connected component $F$ of $OP$ which contains $(z_0, y_0, p_0)$ is called the* feasibility region *of $z_s^0$. Its boundary (relative to $EQ$) is the* feasibility boundary.

Thus the feasibility region is defined as a subset of $X \times Y \times P$, which consists of all possible operating conditions which can be reached from $(z_0, y_0, p_0)$ by continuous variations of the parameters while maintaining stability. The boundary of the feasibility region is composed of zero sets and thus is accessible to numerical computation.

## 3.2 Feasibility boundary without tap changers

**Theorem 1** *[15] For a system defined in equations (1) and (2), the feasibility boundary of a feasibility region $F$ consists of three zero sets*

$$\partial F = (\partial F \cap C_S) \cup (\partial F \cap C_Z) \cup (\partial F \cap C_H), \tag{8}$$

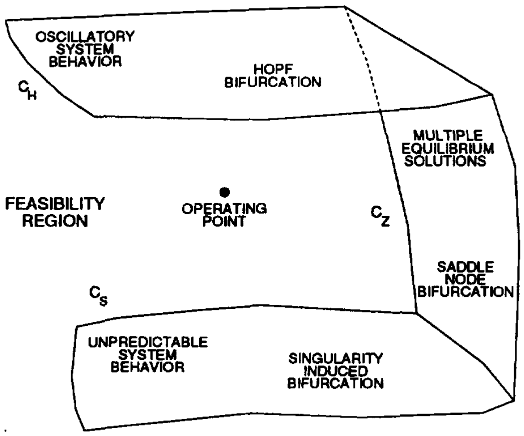


Figure 1: Conceptual sketch of the feasibiity boundary

*where*

$$
\begin{aligned}
C_S &= \{(x,y,p) \in EQ : \det D_y g = 0\}, \\
C_Z &= \{(x,y,p) \in EQ : \det D_y g \neq 0, \\
&\qquad \det \begin{pmatrix} D_x f & D_y f \\ D_x g & D_y g \end{pmatrix} = 0\} \\
C_H &= \{(x,y,p) \in EQ : \det D_y g \neq 0, \\
&\qquad \det \begin{pmatrix} D_x f & D_y f \\ D_x g & D_y g \end{pmatrix} \neq 0, \\
&\qquad \det(H_{n-1}(J)) = 0\}
\end{aligned}
$$

*with*

$$J = D_x f - D_y f (D_y g)^{-1} D_x g \tag{9}$$

*and $H_{n-1}(J)$ is the corresponding Hurwitz matrix of order $n-1$.*

Thus the feasibility boundary can be computed as zero sets and generically most of the boundary points consist of either a saddle-node bifurcation ($C_S$, zero eigenvalue), a Hopf bifurcation ($C_H$, pure imaginary eigenvalue) or a singularity induced bifurcation ($C_Z$, eigenvalue at infinity) [15, 17]. For a conceptual sketch see Figure 1.

### 3.3 Feasibility boundary with tap changers

As pointed out in Section 2, the pertinent model for studying the difficulties resulting from locally controlled tap changers is the one describing the steady state reached after the tap change. This model (5) will now be analysed for the restriction of the feasibility region caused by these types of tap changers. Let $d$ denote the number of buses with tap changers. If bus $i$ is equipped with tap changer, then $z_{2i} = z_{1i} n_i$, so that $d = dim(z_1) = dim(z_2) = dim(n)$. Substituting $z_{1i} = z_{2i}/n_i$ in (5) gives

$$0 = h_1(z_2, z_3, n, p) \tag{10}$$

where the number of equations is equal to the sum of the dimensions of $z_2$ and $z_3$. Suppose $n$ and $p$ are within the feasibility region and away from the feasibility boundary as introduced in this section. Then, since $(D_{z_2}h_1, D_{z_3}h_1)$ is nonsingular, locally near any such point the solutions to equation (10) can be represented as

$$z_2 = z_2(n,p), \quad z_3 = z_3(n,p). \tag{11}$$

The tap-changer continues switching until $z_{2i} = E_{ri}$ for all load buses. Hence, for the system with automatic tap changer control, the solution of (10) with $z_{2i} = E_{ri}$ defines the equilibrium state. The stability of the tap-changer control is determined by the Jacobian

$$J_d := -D_n z_2 = (I_d, 0)\left(D_{z_2}h_1, D_{z_3}h_1\right)^{-1} D_n h_1. \tag{12}$$

The equilibrium is stable if $-D_n z_2$ is stable (negative definite) and stability is lost when it has eigenvalues on the imaginary axis. This can be demonstrated by approximating the discrete dynamics of the tap-changer control with a continuous model as follows: The tap-changer changes the tap size by a small step if the voltage differs from the set point voltage by more than its allowed tolerance value. Usually, the tap-step size and the tolerance values are small, about 1% in p.u. Hence, we can approximate the discrete model by a continuous model of the form

$$\dot{n}_i = f_i(z_{2i}) = \begin{cases} c & \text{if } z_{2i} < E_{ri} \\ -c & \text{if } z_{2i} > E_{ri} \end{cases} \tag{13}$$

where $c$ is a suitable positive constant representing the tap size as justified by any safety margins included in the actual switching law [9]). The system described by equation (13) is not smooth at $z_{2i} = E_{ri}$, but the 'slope' $D_{z_{2i}} f_i$ can be taken arbitrarily large negative. To analyse the local stability, choose a suitable smooth function $\tilde{f}_i$ which approximates $f_i$ to the desired accuracy and, in addition is such that $D_{z_2}\tilde{f} = -kI_d$, where $k$ is a positive constant. The Jacobian of the system $\dot{n}_i = \tilde{f}_i(z_{2i})$ is then given by $kJ_d$. Hence, the approximated system is stable if all the eigenvalues of $J_d$ are in $\mathbb{C}^-$, and is unstable if there exists at least one eigenvalue in $\mathbb{C}^+$. Considering the limiting case for $f_i(z_{2i})$, as the slope goes to $-\infty$, the result follows. Hence, the segment(s) of the feasibility boundary induced by the tap changer are pinpointed (at least after taking the closure, if necessary) by the parameters where the sensitivity matrix $(-J_d)$ has eigenvalues on the imaginary axis.

The new boundary is then where $J_d$ loses stability, i.e. where the eigenvalues of $J_d$ cross the imaginary axis (as we stay away from $\partial F$, eigenvalues do not blow up), i.e. for either det $(J_d) = 0$ (zero eigenvalue) or $\det(H_{n-1}(J_d)) = 0$ (purely imaginary eigenvalues) [24]. The Jacobian, $J_d$ can be simplified as

$$J_d(z_2, z_3, n, p) = J_d(E_{ri}, z_3, \left(\frac{E_r}{z_1}\right)_i, p) = \tilde{J}_d(z_1, z_3, p).$$

Essentially, the tap-changer tries to track the solutions of

$$0 = h(z_1, z_2, z_3, n, p) \quad \text{where } z_{2i} = E_{ri}, \ n_i = \frac{E_{ri}}{z_{1i}}.$$

Rewriting these equations, the stationary solutions with the tap-changer control are the solutions of

$$0 = h_2(z_1, z_3, p) \tag{14}$$

and the new feasibility boundaries are defined by the conditions

$$0 = det(\tilde{J}_d) \text{ and } 0 = \det(H_{n-1}(\tilde{J}_d)).$$

This discussion can now be summarized as follows: The set of steady-state solutions with tap-changer control is defined by

$$\begin{aligned} EQ_{TC} &= \{(z_1, z_2, z_3, n, p) : h(z_1, z_2, z_3, n, p) = 0, \\ &\quad z_{2i} = E_{ri},\ n_i = \frac{E_{ri}}{z_{1i}}\} \end{aligned}$$

The set of stable solutions $OP_{TC}$ within set $EQ_{TC}$ is defined by those points where $J_d$ is negative definite

**Definition 2** *The connected component of the stable equilibria $OP_{TC}$ which contains a specific solution $(z_s, n_s, p_0)$ is defined as the* feasibility region with tap-changer control *and is denoted by $F_{TC}$.*

**Theorem 2 (Feasibility Boundary Theorem with Tap Changer)** *For a system containing locally controlled automatic tap changers, the feasibility boundary $\partial F_{TC}$ of a stable equilibrium point $(z_s, n_s, p_0)$ consists of five zero sets*

$$\partial F_{TC} = \partial F_{TC} \cap \{C_S \cup C_Z \cup C_H \cup C_{Z,TC} \cup C_{H,TC}\}.$$

*The sets $C_S$, $C_Z$ and $C_H$ are as defined in Theorem 1; $C_{Z,TC}$ and $C_{H,TC}$ are defined as*

$$\begin{aligned} C_{Z,TC} &= \{(z_1, z_3, p) : h_2(z_1, z_3, p) = 0, \\ &\quad \det(\tilde{J}_d)(z_1, z_3, p) = 0, \\ &\quad \det(D_{z_2}h_1(z_2, z_3, p), D_{z_3}h_1(z_2, z_3, p)) \neq 0\} \end{aligned}$$

*and*

$$\begin{aligned} C_{H,TC} &= \{(z_1, z_3, p) : h_2(z_1, z_3, p) = 0, \\ &\quad \det(\tilde{J}_d)(z_1, z_3, p) \neq 0, \\ &\quad \det(D_{z_2}h_1(z_2, z_3, p), D_{z_3}h_1(z_2, z_3, p)) \neq 0, \\ &\quad \det H_{n-1}(\tilde{J}_d)(z_1, z_3, p) = 0, \} \end{aligned}$$

*where $H_{n-1}(\tilde{J}_d)$ is the Hurwitz determinant of the Jacobian $\tilde{J}_d$ and is defined exactly as in the definition of $C_H$.*

# 4 Illustrative Examples

The results of this paper are applicable to any size system, however in this paper, we illustrate the results on a triangular three bus system shown in Figure 2, consisting of two generators and a load bus. For convenience, the figure shows both the generator and the ULTC together

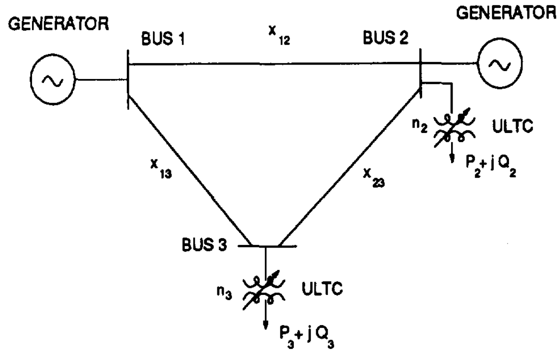


Figure 2: Three bus power system

at Bus 2, though only one of the two is assumed to be active. For both generators, the generator voltage dynamics is represented by the single axis model ($E_i'$). The excitation ($E_{fd_i}$) control which regulates the respective bus voltage $E_i$ is modeled by a first order differential equation as in [14, 18].

$$T'_{d_{0i}}\dot{E}_i' = -\frac{x_{d_i}}{x'_{d_i}}E_i' + \frac{x_{d_i} - x'_{d_i}}{x'_{d_i}}E_i\cos(\theta_i - \delta_i) + E_{fd_i} \tag{15}$$

$$T_i\dot{E}_{fd_i} = -K_i(E_i - E_{ref_i}) - (E_{fd_i} - E^0_{fd_i}) \tag{16}$$

for $i = 1, 2$. The dynamics of the generator angles ($\theta_i$) are represented by the swing equations,

$$J_i\ddot{\theta}_i + b_i\dot{\theta}_i = P_{T_i} - \frac{1}{x'_{d_i}}E_i'E_i\sin(\theta_i - \delta_i) \tag{17}$$

for $i = 1, 2$. The loads $P_2$, $Q_2$, $P_3$ and $Q_3$ are asssumed to be static consisting of the three basic components

$$P_i = P_i^0 + M_i(E_in_i) + G_i\,(E_in_i)^2 \tag{18}$$

$$Q_i = Q_i^0 + H_i(E_in_i) + B_i\,(E_in_i)^2 \tag{19}$$

for $i = 2, 3$. Here $n_i$ stands for the tap ratio of the ULTC's and hence $E_in_i$ (transformer secondary voltage) corresponds to the load voltage. The reference voltage for the tap changer controls are taken to be 1 p.u. and the transmission lines are assumed to be lossless. Thus equations (15)–(17) together define the dynamic equations $f$ in (1) and the power balance equations which can be easily written for buses 1, 2 and 3 define the algebraic constraints $g$ in (2). As explained in Sections 3.2 and 3.3, we can now construct the feasibility regions for our system in the parameter space using Theorems 1 and 2. For practical parameter values, two cross sections of the feasibility boundary for the normal operating point are shown in Figures 3 and 4, for variation of the parameters $K_1$, the excitation control gain for Generator 1 and $P_3$, the real power load at Bus 3.

For the simulation shown in Figure 3, the generator at Bus 2 is not considered, i.e. the system consists of the

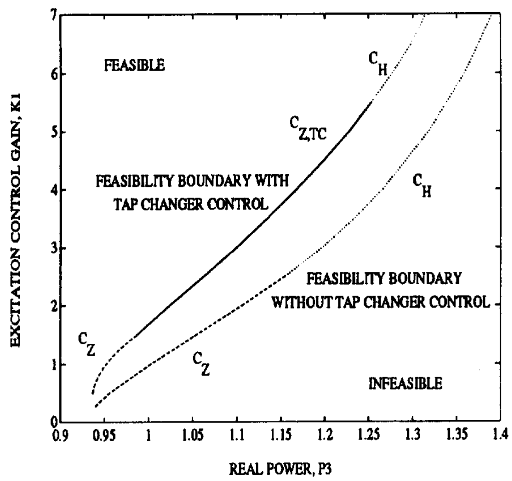


Figure 3: Feasibility boundaries with and without tap changer control with one generator and two load buses. The parameter values are $x_{12} = 0.2$, $x_{13} = 0.3$, $x_{23} = 0.5$, $x_{d_1} = 1.0$, $x'_{d_1} = 0.2$, $b_1 = 1.5$, $J_1 = 5$, $T_{d_{0_1}} = 8$, $T_1 = 1.0$, $E_{fd_{0_1}} = 1.9$, $E_{ref_1} = 1.1$, $Q_2 = 0.5P_2$, $P_2^0 = 0.1$, $M_2 = 0.05$, $G_2 = 0.05$, $Q_3 = 0.4P_3$ where $P_3$ consists of 80% constant power, 10% current and 10% impedance components.

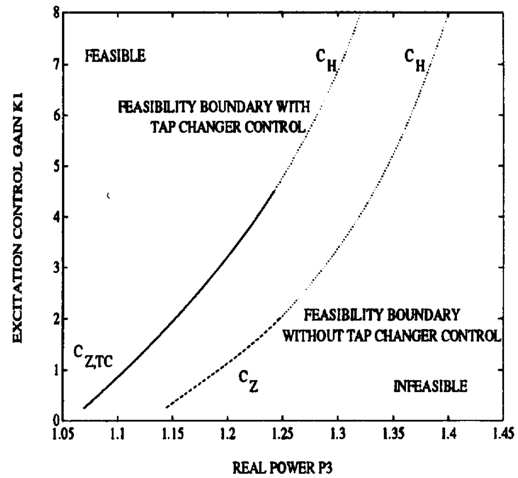


Figure 4: Feasibility boundaries with and without tap changer control with two generators and one load bus. The parameter values are $n_2 = 1$, $x_{d_2} = 1.0$, $x'_{d_2} = 0.25$, $b_2 = 1$, $J_2 = 3$, $T_{d_{0_2}} = 6$, $T_2 = 0.8$, $K_2 = 4$, $E_{fd_{0_1}} = 1.8$, $E_{ref_2} = 1.0$, $E_{ref_1} = 1.05$, $Q_2 = 0.4P_2$, $P_2^0 = 0.1$, $M_2 = 0.03$, $G_2 = 0.03$, $Q_3 = 0.5P_3$ where $P_3$ consists of 80% constant power, 10% current and 10% impedance components. Other parameter values are as shown in Figure 3.

generator at bus 1 and two load buses 2 and 3. When the tap changer controls are not present (i.e. with $n_2 = n_3 = 1$), the feasibility boundary consists of two zero sets, namely, $C_Z$ (the saddle node bifurcation) at low control gain values of $K_1$, and $C_H$ (the Hopf bifurcation) at higher gain values. When the tap changer controls are active, the load voltages are maintained at the reference value (1 p.u.), but it can be seen that the feasibility region is now bounded by three zero sets, $C_Z$ and $C_H$ along with $C_{Z,TC}$ (at medium control gains) which corresponds to the tap changer related saddle node bifurcation.

Both generators are considered for the simulation in Figure 4, but the tap ratio setting is $n_2 = 1$ for the ULTC at bus 2. When the tap changer control at Bus 3 is inactive (i.e. with $n_3 = 1$), the feasibility boundary again consists of the two zero sets $C_Z$ and $C_H$ as in Figure 3. When the tap changer control is present, the feasibility boundary now only consists of two zero sets $C_H$ and $C_{Z,TC}$ which is connected with the cascading descent instability of the tap changer control.

For this three bus system, for the parameter values as shown, the feasibility boundary for the operating point consists of the three zero sets $C_Z$, $C_H$ and $C_{Z,TC}$. However for the large system, by Theorem 2, the feasibility boundary may also consist of two other zero sets $C_S$ and $C_{H,TC}$. Since the feasibility boundary can be solved as the (five) zero sets of analytic functions, *Theorem 2 provides a practical method for computing rigorous security criteria for the large power system*, when discrete control devices such as tap changers are included in the analysis.

# 5 Discussions

The power system with locally controlled tap changers, is a hybrid dynamic system with continuous dynamics defined in equations (1) and (2), along with the slower discrete dynamics defined in equations (3). We will next consider the physical implications of the five different segments in the feasibility boundary.

After a tap change (more generally after any parameter change), if the system finds itself on the other (unstable) side of the three boundary segments, $C_2$, $C_S$ and $C_H$ then the transient will diverge away from the operating point, since the equilibrium point for the dynamics (1) and (2) is unstable there. The exact nature of the transient may depend on the specific type of boundary encountered. For points near the saddle node bifurcations in the set $C_Z$, by the Center manifold theorem and by the saddle node bifurcation theorem, the transient will slide along the center manifold [12]. For points near the Hopf bifurcations in the set $C_H$, the transient will be oscillatory on the center manifold by the Hopf bifurcation theorem. Also generally near the feasibility boundary segment $C_H$, all the other $(n-2)$ eigenvalues are stable, hence it follows that locally the center manifold is attracting [5]. Hence the behavior of the transients near the boundary segment $C_H$ are charaterized by the dynamics on the center manifold. Now applying the Hopf bifurcation theorem, the transient may either degenerate into a stable periodic orbit in the case of a supercritical Hopf bifurcation, or may be unstable (oscillatory with increasing amplitude) in the case of the subcritical Hopf bifurcation. For the singularity induced bifurcation points in the set $C_S$, the system behavior becomes unpredictable [15]. $C_S$ plays no role in the feasibility boundaries in Figures 3 and 4, although it can be shown to be active for the feasibility boundary of the operating point [16].

Next let us consider the feasibility boundary segment $C_{Z,TC}$. The system equilibria with the tap changer control correspond to the zero set of the functions $h_2(z_2, z_3, p)$ in the equation (14) and the Jacobian of $h_2$ is $\tilde{J}_d = (D_{z_2}h_2,\ D_{z_3 h_2})$. Since $\det(\tilde{J}_d) = 0$ in the set $C_{Z,TC}$, the Jacobian $\tilde{J}_d$ has zero eigenvalues there. By the genericity of the saddle node bifurcation theorem, then most of the points in the set $C_{Z,TC}$ (this excludes the points of transition in Figures 3 and 4) correspond to the ocurrence of static saddle node bifurcations. For applying the static version of the saddle node bifurcation theorem, let us introduce the necessary transversality conditions first.

- **(SN1TC)** The Jacobian $\tilde{J}_d$ has a simple zero eigenvalue with right eigenvector $v$ and left eigenvector $w$ and there is no other eigenvalue on the imaginary axis.
- **(SN2TC)** $w(D_p h_2) \neq 0$
- **(SN3TC)** $w(D^2_{\tilde{x}} h_2(v, v)) \neq 0$ where $\tilde{x}$ corresponds to the $x$-coordinates $\tilde{x} = (x_2, x_3)$. □

Define the set $B_{SN,TC}$ as the points in the set $C_{Z,TC}$ where the transversality conditions (SN1TC), (SN2TC) and (SN3TC) are satisfied. Then by Sotomayor's saddle node bifurcation theorem [11], we conclude that a Saddle node bifurcation occurs for points in the set $B_{SN,TC}$ and moreover generically

$$\overline{B_{SN,TC}} = C_{Z,TC}. \tag{20}$$

Therefore, near the bifurcation points in the set $B_{SN,TC}$, two solutions of the equation (14) meet and disappear. In other words, for most of the boundary points in the set $C_{Z,TC}$, the stable equilibrium of the tap changer control meets an unstable equilibrium and disappears at the boundary.

Suppose the system parameter crosses the feasibility boundary segment $B_{SN,TC}$. Then, since the tap changer control has no solution on the other side of the boundary (such as to the right of the curve $C_{Z,TC}$ in Figure 3), the tap changer control dynamics in equations (3) will diverge away from the previous operating point. The tap changer control will continuously order more tap changes, chasing a new equilibrium for the control dynamics in (3). This sequence may either take the system across the feasibility boundary (so that the continuous dynamics in (1) and (2) itself becomes unstable, and the transient diverges as explained above) or the taps may run out (then the voltage is nonviable) (see [21]). In any case, stable system operation at the equilibrium stops before the combined feasibility boundary (Figures 3 and 4).

Since the Jacobian $\tilde{J}_d$ has purely imaginary eigenvalues in the set $C_{H,TC}$, from Hopf bifurcation theorem, we conjecture that the feasibility boundary points in the set $C_{H,TC}$ correspond to the emergence of stable limit cycles for the tap changers (possible hunting phenomenon) or the annihilation of the region of attraction for the equilibrium. The exact nature of the instability mechanism needs more investigation.

In summary, the feasibility boundary with tap changer control consists of five zero sets, hence it is relatively accessible to computation. It is clear that the system operation at the stable equilibrium point becomes unstable at the feasibility boundary. Therefore *the distance of the operating point to the feasibility boundary serves as a rigorous, computable and efficient, and so, a powerful tool for secure system operation.*

# 6 Conclusion

A principal objective of system operation is to maintain security. For system security, it is essential that the operating point has adequate safety margins of feasibility to survive normal operating activities and changes and also a specified set of potential disturbances such as the "first contingencies." In the parameter space the *feasibility boundary* is identified as being composed of five zero sets defined by algebraic equations and hence relatively easily computable. This then provides information not just on whether the (existing or proposed) operating point is feasible (this can be checked by direct computation) but also on its distance from the boundary that is its margin of security. Information is further gained on the security of a proposed operating change in the parameter space, specifically its security against any collapse event during the proposed change. This includes the consideration of the troublesome automatic tapchanger problem which can be directly computed as one (or two) additional types of (restrictive) segments to the feasibility boundary.

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