Three-Phase Power Flow Calculations Using the Current Injection Method

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***Abstract*—This paper presents a new sparse formulation for the solution of unbalanced three-phase power systems using the Newton–Raphson method. The three-phase current injection equations are written in rectangular coordinates resulting in an order $6n$ system of equations. The Jacobian matrix is composed of 6 $\times$ 6 block matrices and retains the same structure as the nodal admittance matrix. Practical distribution systems were used to test the method and to compare its robustness with that of the backward/forward sweep method.**

***Index Terms*—Unbalanced Power Flow, Current Injection Power Flow, Newton Raphson Power Flow, Distribution Power Flow.**

I. Introduction

THE classical Newton [1] and fast decoupled load flow [2], in polar coordinates, for positive sequence networks have been extensively applied in power flow analysis since the early seventies. The majority of the developments in this area has been concerned with large scale interconnected systems and balanced operation has been assumed in most cases. A notable exception has been the class of studies on the flow of harmonics and several three-phase power flow algorithms have been proposed to deal with this problem [3]–[6].

Electric power distribution systems are characterized by low R/X rations and unbalanced operation. These characteristics impose serious challenges for the development of efficient computational power flow techniques. Two basic approaches have been used to deal with this problem: (*i*) Newton and Newton like methods [7]–[9] and (*ii*) load flow for radial networks [10]–[13].

In [7] the network radial structure is explored to express the Jacobian matrix as an product of UDU^t, where U is a constant triangular superior matrix and D is a diagonal natrix, the elements of which are updated at every iteration.

In [8] the power flow equations are expressed as a function of new variables that replace the V_i^2, $V_iV_j \sin\theta_{ij}$ and $V_iV_j \cos\theta_{ij}$ terms. The resulting system of equations has order $3n$ and good convergence properties were attained when the method was applied to balanced networks.

A three-phase power flow formulation is described in [9] where the Jacobian matrix is presented in complex form, but some simplifications are introduced by neglecting the component of the mismatch arising from voltage changes.

A compensation based technique is proposed in [10], for weakly meshed networks. In a first step, the radial part of the network is solved using the well known forward/backward sweep technique; in a second step, the meshes are introduced using nodal current injections compensation. In [11] an improved version of this method has been presented, in which the branch power flows are used instead of the branch currents.

In [12] a Z_{bus} based approach is presented, in which the voltage of each bus is considered to arise from two different contributions: the specified voltage sources (PV buses) and the equivalent current (PQ buses). In [13] further extensions of the methods of [10] and [11] are presented with emphasis on the modeling of unbalanced loads.

In [14] a new power flow formulation based on current injections (CIM), is described. The current injection equations are written in rectangular coordinates and the order $2n$ bus admittance matrix is composed of 2×2 blocks. A new dependent variable (ΔQ) is introduced for each PV bus, together with an additional equation imposing the condition of zero bus voltage deviation. Except for PV buses, the Jacobian matrix has the elements of the off-diagonal blocks equal to those of the nodal admittance matrix. The elements of the diagonal blocks are updated at every iteration, according with the load model being considered for that bus. This method was tested on large scale systems and has achieved an average 30% speedup, when test benched against a state of the art production grade Newton Raphson power flow.

In the present paper, the CIM technique presented in [14] is extended for unbalanced three-phase networks.

II. Three-Phase Current Injection Power Flow

A. Basic Equations

The three-phase current mismatches for a given bus k are:

$$\Delta I_k^2 = \frac{(P_k^{sp})^s - j(Q_k^{sp})^s}{(E_k^s)^*} - \sum_{i\in\Omega_k}\sum_{t\in\alpha_p} Y_{ki}^{st}E_i^t \qquad (1)$$

where:

$s, t \in \alpha_p$
$\alpha_p = \{a, b, c\}$
$k = \{1, \cdots, n\}$, n is the total number of buses.
Ω_k — set of buses directly connected to bus k.

$$E_k = V_{r_k} + jV_{m_k} \qquad (2)$$

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$$(P_k^{sp})^s = P_{g_k}^s - P_{l_k}^s \tag{3}$$
$$(Q_k^{sp})^s = Q_{g_k}^s - Q_{l_k}^s \tag{4}$$

$(P_k^{sp})^s$, $(Q_k^{sp})^s$ — specified active and reactive powers at bus k for a given phase s.

$P_{g_k}^s$, $Q_{g_k}^s$ — active and reactive powers of generators for a given phase s.

$P_{l_k}^s$, $Q_{l_k}^s$ — active and reactive powers of loads for a given phase s.

$Y_{ki}^{st} = G_{ki}^{st} + jB_{ki}^{st}$ — is the nodal admittance bus matrix element.

The effect of voltage level on the system loads is represented by second order polynomial equations, as follows:

$$P_{l_k}^s = P_{0_k}^s + P_{1_k}^s V_k + P_{2_k}^s V_k^2 \tag{5}$$

$$Q_{l_k}^s = Q_{0_k}^s + Q_{1_k}^s V_k + Q_{2_k}^s V_k^2 \tag{6}$$

where:

$P_{0_k}^s$, $Q_{0_k}^s$ — constant power components of phase s load at bus k.

$P_{1_k}^s$, $Q_{1_k}^s$ — constant current components of phase s load at bus k.

$P_{2_k}^s$, $Q_{2_k}^s$ — constant impedance components of phase s load at bus k.

$V_k = |E_k|$ — absolute value of bus k voltage.

Equation (1) can be expressed in terms of its real and imaginary parts as follows:

$$\Delta I_{r_k}^s = \frac{(P_k^{sp})^s V_{r_k}^s + (Q_k^{sp})^s V_{m_k}^s}{(V_{r_k}^s)^2 + (V_{m_k}^s)^2} - \sum_{i=1}^{n} \sum_{t \in \alpha_p} \left(G_{ki}^{st} V_{r_i}^t - B_{ki}^{st} V_{m_i}^t \right) \tag{7}$$

$$\Delta I_{m_k}^s = \frac{(P_k^{sp})^s V_{m_k}^s - (Q_k^{sp})^s V_{r_k}^s}{(V_{r_k}^s)^2 + (V_{m_k}^s)^2} - \sum_{i=1}^{n} \sum_{t \in \alpha_p} \left(G_{ki}^{st} V_{m_i}^t - B_{ki}^{st} V_{r_i}^t \right) \tag{8}$$

Equations (7) and (8) are written in terms of the specified and calculated values, leading to:

$$\Delta I_{r_k}^s = \left(I_{r_k}^{sp} \right)^s - \left(I_{r_k}^{calc} \right)^s \tag{9}$$
$$\Delta I_{m_k}^s = \left(I_{m_k}^{sp} \right)^s - \left(I_{m_k}^{calc} \right)^s \tag{10}$$

Applying Newton's method to (7) and (8), the following set of linear equations is obtained:

$$\begin{bmatrix} \Delta I_{m_1}^{abc} \\ \Delta I_{r_1}^{abc} \\ \hline \Delta I_{m_2}^{abc} \\ \Delta I_{r_2}^{abc} \\ \hline \vdots \\ \hline \Delta I_{m_n}^{abc} \\ \Delta I_{r_n}^{abc} \end{bmatrix} = \left[\begin{array}{c|c|c|c} (\mathbf{Y_{11}^{\bullet}})^{\mathbf{abc}} & \mathbf{Y_{12}^{abc}} & \cdots & \mathbf{Y_{1n}^{abc}} \\ \hline \mathbf{Y_{21}^{abc}} & (\mathbf{Y_{22}^{\bullet}})^{\mathbf{abc}} & \cdots & \mathbf{Y_{2n}^{abc}} \\ \hline \vdots & \vdots & \vdots & \vdots \\ \hline \mathbf{Y_{n1}^{abc}} & \mathbf{Y_{n2}^{abc}} & \cdots & (\mathbf{Y_{nn}^{\bullet}})^{\mathbf{abc}} \end{array} \right] \cdot \begin{bmatrix} \Delta V_{r_1}^{abc} \\ \Delta V_{m_1}^{abc} \\ \hline \Delta V_{r_2}^{abc} \\ \Delta V_{m_2}^{abc} \\ \hline \vdots \\ \hline \Delta V_{r_n}^{abc} \\ \Delta V_{m_n}^{abc} \end{bmatrix} \tag{11}$$

The off-diagonal elements are identical to the corresponding elements of the bus admittance matrix. Since three-phase rectangular coordinates are used, each element will be a 6×6 block having the following structure:

$$\mathbf{Y_{im}^{abc}} = \begin{bmatrix} B_{im}^{abc} & G_{im}^{abc} \\ G_{im}^{abc} & -B_{im}^{abc} \end{bmatrix}; \qquad i, m = 1, \cdots, n \tag{12}$$

The diagonal elements are given by:

$$(\mathbf{Y_{kk}^{\bullet}})^{\mathbf{abc}} = \begin{bmatrix} (B'_{kk})^{abc} & (G'_{kk})^{abc} \\ (G''_{kk})^{abc} & (B''_{kk})^{abc} \end{bmatrix} \tag{13}$$

where

$$(B'_{kk})^{abc} = B_{kk}^{abc} - \begin{bmatrix} a_k^a & & \\ & a_k^b & \\ & & a_k^c \end{bmatrix} \tag{14}$$

$$(G'_{kk})^{abc} = G_{kk}^{abc} - \begin{bmatrix} b_k^a & & \\ & b_k^b & \\ & & b_k^c \end{bmatrix} \tag{15}$$

$$(G''_{kk})^{abc} = G_{kk}^{abc} - \begin{bmatrix} c_k^a & & \\ & c_k^b & \\ & & c_k^c \end{bmatrix} \tag{16}$$

$$(B''_{kk})^{abc} = -B_{kk}^{abc} - \begin{bmatrix} d_k^a & & \\ & d_k^b & \\ & & d_k^c \end{bmatrix} \tag{17}$$

The elements, a_k^s, b_k^s, c_k^s and d_k^s ($s \in \alpha_p$) are dependent on the load model adopted for each phase at a given bus k, and are discussed in the Appendix. It will be seen from the expressions in the Appendix, that only a few elements of the diagonal (6×6) blocks given by (13) will need to be recalculated at every iteration.

The current mismatch of (11) for a given bus k and phase s can be expressed as:

$$\Delta I_{rk}^s = \frac{V_{rk}^s \Delta P_k^s + V_{mk}^s \Delta Q_k^s}{(V_{rk}^s)^2 + (V_{mk}^s)^2} \tag{18}$$

$$\Delta I_{mk}^s = \frac{V_{mk}^s \Delta P_k^s + V_{rk}^s \Delta Q_k^s}{(V_{rk}^s)^2 + (V_{mk}^s)^2} \tag{19}$$

The active and reactive power mismatches ΔP_k^s and ΔQ_k^s are given by:

$$\Delta P_k^s = (P_k^{sp})^s - \left(P_k^{calc}\right)^s \tag{20}$$

$$\Delta Q_k^s = (Q_k^{sp})^s - \left(Q_k^{calc}\right)^s \tag{21}$$

where:

$$\left(P_k^{calc}\right)^s = V_{rk}^s \left(I_{rk}^{calc}\right)^s + V_{mk}^s \left(I_{mk}^{calc}\right)^s \tag{22}$$

$$\left(Q_k^{calc}\right)^s = V_{mk}^s \left(I_{rk}^{calc}\right)^s - V_{rk}^s \left(I_{mk}^{calc}\right)^s \tag{23}$$

Sparse matrix techniques, using the well-known Tinney-2 ordering scheme [1], are used to solve the linear system of equations (11) for the voltage increments. The voltages are updated at each iteration, as follows:

$$\left(V_{rm_k}^{abc}\right)^{h+1} = \left(V_{rm_k}^{abc}\right)^h + \left(\Delta V_{rm_k}^{abc}\right)^h \tag{24}$$

where:

$$\left(V_{rm_k}^{abc}\right) = \left[V_{r_k}^a \quad V_{r_k}^b \quad V_{r_k}^c \quad V_{m_k}^a \quad V_{m_k}^b \quad V_{m_k}^c\right]^t \tag{25}$$

B. Representation of PV Buses

The representation of PV buses is becoming increasingly important on distribution networks, mostly due to the introduction of cogeneration plants in such systems.

The methodology adopted in [14] has been extended for three-phase networks. Thus, assuming that a PV bus k is directly connected to buses i and l, equation (11) can be expressed as shown in (26), at the bottom of the page, where:

$$(Y_{kk}^{\bullet\bullet})^{abc} = \begin{bmatrix} M & O \\ N & P \end{bmatrix} \tag{27}$$

The elements of the order 3 sub-matrices M and N are defined as:

$$m_{kk}^{st} = G_{kk}'^{st} - B_{kk}'^{st} \frac{V_{m_k}^t}{V_{r_k}^t} \tag{28}$$

$$n_{kk}^{st} = G_{kk}''^{st} - B_{kk}''^{st} \frac{V_{m_k}^t}{V_{r_k}^t} \tag{29}$$

where s, $t \in \alpha_p$ and G', G'', B' and B'' are given by (14) to (17).

O and P are diagonal sub-matrices given by [14]:

$$O = \begin{bmatrix} \dfrac{V_{r_k}^a}{(V_k^a)^2} & 0 & 0 \\ 0 & \dfrac{V_{r_k}^b}{(V_k^b)^2} & 0 \\ 0 & 0 & \dfrac{V_{r_k}^c}{(V_k^c)^2} \end{bmatrix} \tag{30}$$

$$\begin{bmatrix} \Delta I_{m_1}^{abc} \\ \Delta I_{r_1}^{abc} \\ \hline \vdots \\ \hline \Delta I_{m_i}^{abc} \\ \Delta I_{r_i}^{abc} \\ \hline \vdots \\ \hline (\boldsymbol{\Delta}\mathbf{I}_{\mathbf{m_k}}^{\bullet})^{\mathbf{abc}} \\ (\boldsymbol{\Delta}\mathbf{I}_{\mathbf{r_k}}^{\bullet})^{\mathbf{abc}} \\ \hline \vdots \\ \hline \Delta I_{m_l}^{abc} \\ \Delta I_{r_l}^{abc} \\ \hline \vdots \end{bmatrix} = \left[\begin{array}{c|c|c|c|c|c|c|c} (Y_{11}^{\bullet})^{abc} & \cdots & Y_{1i}^{abc} & \cdots & (\mathbf{Y}_{\mathbf{1k}}^{\bullet\bullet})^{\mathbf{abc}} & \cdots & Y_{1l}^{abc} & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hline Y_{i1}^{abc} & \cdots & (Y_{ii}^{\bullet})^{abc} & \cdots & (\mathbf{Y}_{\mathbf{ik}}^{\bullet\bullet})^{\mathbf{abc}} & \cdots & Y_{il}^{abc} & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hline Y_{k1}^{abc} & \cdots & Y_{ki}^{abc} & \cdots & (\mathbf{Y}_{\mathbf{kk}}^{\bullet\bullet})^{\mathbf{abc}} & \cdots & Y_{kl}^{abc} & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hline Y_{l1}^{abc} & \cdots & Y_{li}^{abc} & \cdots & (\mathbf{Y}_{\mathbf{lk}}^{\bullet\bullet})^{\mathbf{abc}} & \cdots & (Y_{ll}^{\bullet})^{abc} & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array}\right] \begin{bmatrix} \Delta V_{r_1}^{abc} \\ \Delta V_{m_1}^{abc} \\ \hline \vdots \\ \hline \Delta V_{r_i}^{abc} \\ \Delta V_{m_i}^{abc} \\ \hline \vdots \\ \hline \boldsymbol{\Delta}\mathbf{V}_{\mathbf{m_k}}^{\mathbf{abc}} \\ \boldsymbol{\Delta}\mathbf{Q}_{\mathbf{k}}^{\mathbf{abc}} \\ \hline \vdots \\ \hline \Delta V_{r_l}^{abc} \\ \Delta V_{m_l}^{abc} \\ \hline \vdots \end{bmatrix} \tag{26}$$

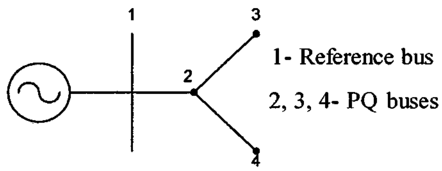

Fig. 1. Radial system.

$$P = \begin{bmatrix} -\frac{V_{m_k}^a}{(V_k^a)^2} & 0 & 0 \\ 0 & -\frac{V_{m_k}^b}{(V_k^b)^2} & 0 \\ 0 & 0 & -\frac{V_{m_k}^c}{(V_k^c)^2} \end{bmatrix} \tag{31}$$

The off-diagonal elements in (26) are given by:

$$(Y_{lk}^{\bullet\bullet})^{abc} = \begin{bmatrix} Q & U \\ R & W \end{bmatrix} \tag{32}$$

The elements of the (3×3) sub-matrices U and W are all zero and the elements of the sub-matrices Q and R are defined as:

$$q_{lk}^{st} = G_{lk}^{st} - B_{lk}^{st} \frac{V_{m_k}^t}{V_{r_k}^t} \tag{33}$$

$$r_{lk}^{st} = G_{lk}^{st} - B_{lk}^{st} \frac{V_{m_k}^t}{V_{r_k}^t} \tag{34}$$

It is seen from the above equations that both the diagonal and off-diagonal elements in column k of (26) are functions of the real and imaginary components of the voltage. Thus the Jacobian matrix would have to be updated at every iteration.

The three-phase current mismatches at bus k are given by:

$$(\Delta I_{m_k}^{\bullet})^{abc} = \left[\frac{V_{m_k}^a \Delta P_k^a}{(V_k^a)^2} \quad \frac{V_{m_k}^b \Delta P_k^b}{(V_k^b)^2} \quad \frac{V_{m_k}^c \Delta P_k^c}{(V_k^c)^2} \right]^t \tag{35}$$

$$(\Delta I_{r_k}^{\bullet})^{abc} = \left[\frac{V_{r_k}^a \Delta P_k^a}{(V_k^a)^2} \quad \frac{V_{r_k}^b \Delta P_k^b}{(V_k^b)^2} \quad \frac{V_{r_k}^c \Delta P_k^c}{(V_k^c)^2} \right]^t \tag{36}$$

C. Jacobian Matrix Structure

A powerful feature of the proposed TCIM method stems from the fact that the Jacobian matrix contains partitions that are identical to the bus admittance matrix. This will be illustrated through some simple examples.

1) Sample System Having only PQ Buses: Fig. 1 shows a sample system having a slack bus and three PQ buses. The corresponding Jacobian matrix structure using the TCIM method is shown in Fig. 2. It is seen that only 1/3 of the elements belonging to the diagonal blocks need to be updated. The off-diagonal (6×6) blocks are given by (12) and are thus constant.

2) Sample System Having a PV Bus: The system of Fig. 1 has been expanded to include a cogeneration plant at bus 5, which is represented by a PV bus, as shown in Fig. 3. The corresponding Jacobian matrix is shown in Fig. 4, where it is seen that, in addition to the block diagonal elements corresponding to PQ buses, the presence of a PV bus requires that half of the nonzero elements of column 5 should be recalculated at every iteration.

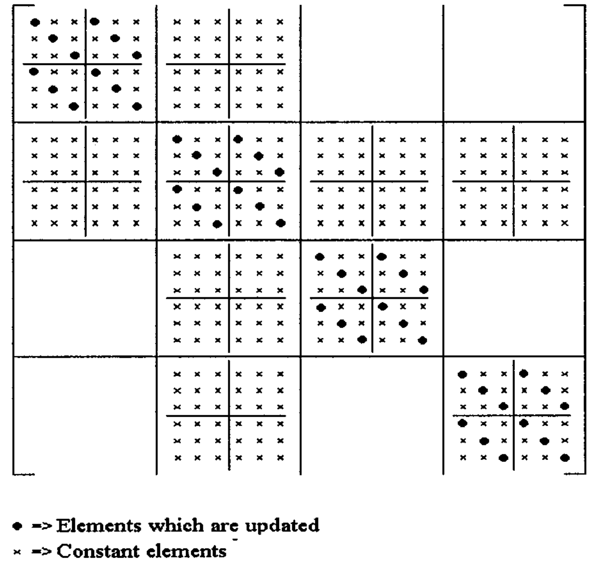

Fig. 2. Jacobian matrix structure for the system of Fig. 1.

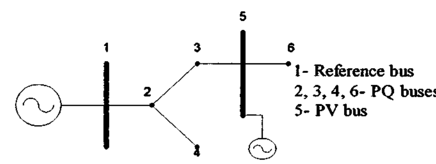

Fig. 3. Radial system with a PV bus.

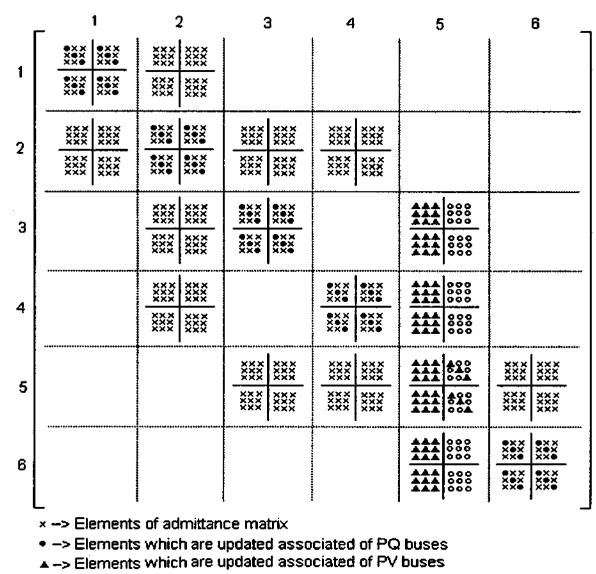

Fig. 4. Jacobian matrix structure for the system of Fig. 3.

III. Proposed Algorithm

Step 1: Initialize the iteration count and set voltages to initial value.

Step 2: Compute three-phase active and reactive power injections, equations (22) and (23);

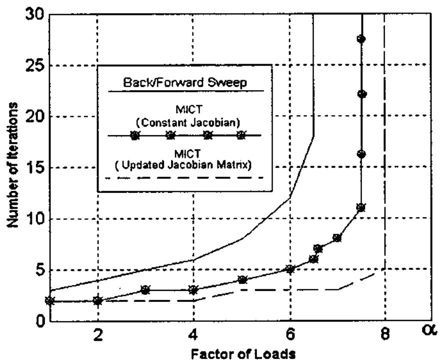

Fig. 5. Comparative performance for constant power load model.

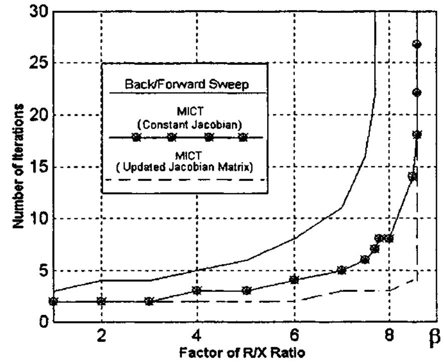

Fig. 6. Comparative performance for increased R/X ratio of conductors.

Step 3: Compute power mismatches. equations (20) and (21);
Step 4: Test for convergence: if $\max\{|(\Delta P_k^s)^{(h)}|\} \leq \varepsilon_p$ and $\max\{|(\Delta Q_k^s)^{(h)}|\} \leq \varepsilon_q$ then print results; else go to step 5.
Step 5: Compute the Jacobian matrix;
Step 6: Solve equation (26) for the voltage increments.
Step 7: update the voltages, equation (24), and go to step 2.

IV. Application to Practical Systems

The method has been tested on practical distribution systems operated by CEMIG-Companhia Energética de Minas Gerais, Brazil.

In order to verify the robustness of the proposed TCIM technique, balanced and unbalanced network operating conditions were tested. Constant impedance and constant power load models were used. The initial values of thc load and of the R/X ratios of the feeders were gradually increased up to the point were convergence was no longer attained.

In all cases the backward/forward sweep method was also applied for comparison purposes.

A. Balanced Operation

A 23 bus, 13.8 kV distribution system having a total load of 4.446 MVA, was chosen to test the method.

1) Constant Power Load: Fig. 5 shows the number of iterations required to find the solution when the total load was varied by a factor α. The performance obtained when the Jacobian matrix was not updated during the iteration process is also shown in this figure.

Fig. 6 shows the performance of the methods when the initial R/X ratio of the conductors were increased by factor β. It is seen that the TCIM is numerically very robust and requires less iterations then the backward/forward sweep.

2) Constant Impedance lLad: The results are shown in Figs. 7 and 8. Essentially the same conclusions as above are applicable in this case. Comparing Figs. 5 and 7, it will be seen that the backward/forward sweep method has the same convergence characteristics for either type of load model, whereas the TCIM method converges in less iterations for constant power loads.

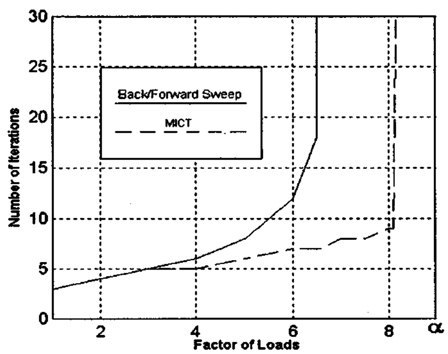

Fig. 7. Comparative performance for constant power load model.

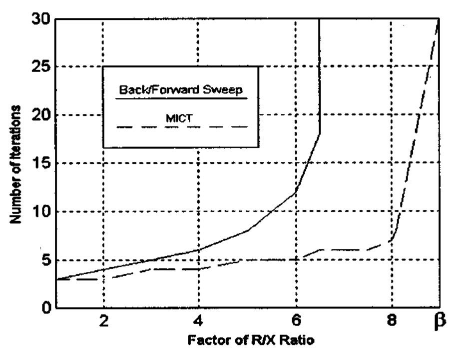

Fig. 8. Comparative performance for increased R/X ratio of conductors.

B. Unbalanced Systems

A 57 bus, 13.8 kV, 3.217 MVA distribution system composed of three-phase, two-phase and single-phase lines, was selected in this case.

From the results shown in Figs. 6–9 it is seen that the robustness of TCIM still holds.

1) Constant Power Load:

2) Constant Impedance Load:

V. Conclusions

The three-phase current injection method-TCIM, proposed in this paper, has been implemented and tested on practical three-phase balanced and unbalanced distribution systems. The polynomial load model has been incorporated in the load-flow for-

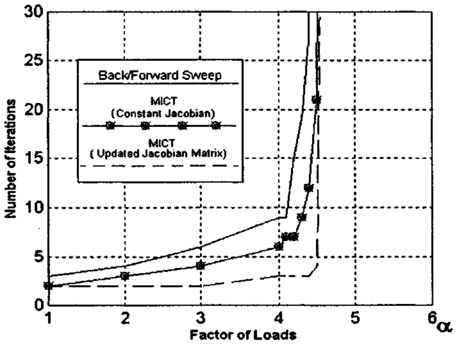

Fig. 9. Comparative performance for constant power load model.

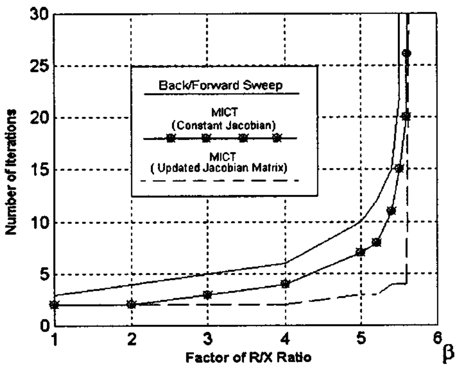

Fig. 10. Comparative performance for increased R/X radtio of conductors.

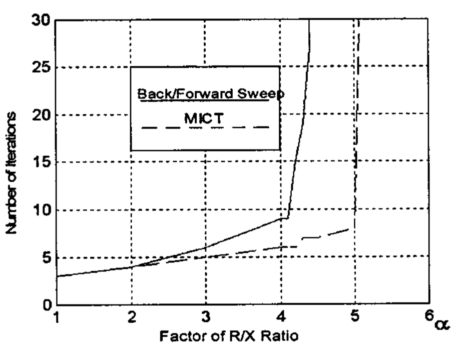

Fig. 11. Comparative performance for constant power load model.

mulation, and thus constant impedance, constant power and constant impedance loads can be easily represented.

The results have shown that the technique is very robust and converges in less iterations than the backward/forward sweep method specially for heavily loaded systems.

An interesting property of the TCIM formulation is that the structure of the Jacobian matrix is the same as the bus admittance matrix and thus retains its sparsity properties. Moreover the number of elements that has to be recalculated during the iteration process is very small. For strictly radial distribution systems, with no cogeneration plants, the Jacobian matrix will be constant.

The representation of voltage regulators and FACTS devices is currently being implemented and will be reported in the near future.

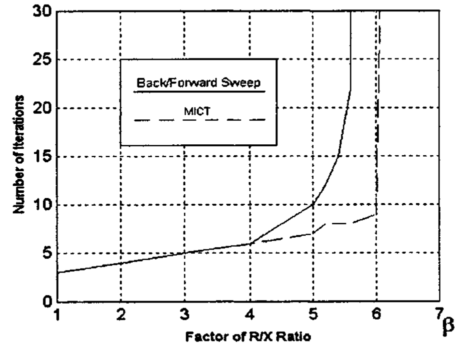

Fig. 12. Comparative performance for increased R/X ratio of conductors.

APPENDIX A

The elements a_k^s, b_k^s, c_k^s and d_k^s shown in equations (14), (15), (16) and (17) are dependent on the load model. These elements are used to update the Jacobian matrix and are given by:

$$a_k^s = \frac{Q_{0_k}^s\left[\left(V_{r_k}^s\right)^2 - \left(V_{m_k}^s\right)^2\right] - 2V_{r_k}^s V_{m_k}^s P_{0_k}^s}{(V_k^s)^4} + \frac{V_{r_k}^s V_{m_k}^s P_{1_k}^s + Q_{1_k}^s\left(V_{m_k}^s\right)^2}{(V_k^s)^3} + Q_{2_k}^s \tag{37}$$

$$b_k^s = \frac{P_{0_k}^s\left[\left(V_{r_k}^s\right)^2 - \left(V_{m_k}^s\right)^2\right] + 2V_{r_k}^s V_{m_k}^s Q_{0_k}^s}{(V_k^s)^4} - \frac{\left(V_{r_k}^s V_{m_k}^s Q_{1_k}^s + P_{1_k}^s\left(V_{r_k}^s\right)^2\right.}{(V_k^s)^3} - P_{2_k}^s \tag{38}$$

$$c_k^s = \frac{P_{0_k}^s\left[\left(V_{m_k}^s\right)^2 - \left(V_{r_k}^s\right)^2\right] - 2V_{r_k}^s V_{m_k}^s Q_{0_k}^s}{(V_k^s)^4} + \frac{\left(V_{r_k}^s V_{m_k}^s Q_{1_k}^s - P_{1_k}^s\left(V_{m_k}^s\right)^2\right.}{(V_k^s)^3} - P_{2_k}^s \tag{39}$$

$$d_k^s = \frac{Q_{0_k}^s\left[\left(V_{r_k}^s\right)^2 - \left(V_{m_k}^s\right)^2\right] - 2V_{r_k}^s V_{m_k}^s \left(P_{0_k}^s\right.}{(V_k^s)^4} + \frac{\left(V_{r_k}^s V_{m_k}^s P_{1_k}^s - Q_{1_k}^s\left(V_{r_k}^s\right)^2\right.}{(V_k^s)^3} - Q_{2_k}^s \tag{40}$$

where $s \in \alpha_p$ and $P_{0_k}^s$, $P_{1_k}^s$, $P_{2_k}^s$, $Q_{0_k}^s$ $Q_{1_k}^s$ and $Q_{2_k}^s$ are defined in equations (5) and (6).

A1—Constant Power Load

If constant power load representation is adopted, the expressions to update the Jacobian matrix are very simple, as follows:

$$a_k^s = \frac{(Q_k')^s\left[\left(V_{r_k}^s\right)^2 - \left(V_{m_k}^s\right)^2\right] - 2V_{r_k}^s V_{m_k}^s (P_k')^s}{(V_k^s)^4} \tag{41}$$

$$b_k^s = \frac{(P_k')^s\left[\left(V_{r_k}^s\right)^2 - \left(V_{m_k}^s\right)^2\right] + 2V_{r_k}^s V_{m_k}^s (Q_k')^s}{(V_k^s)^4} \tag{42}$$

$$c_k^s = -\, b_k^s \tag{43}$$

$$d_k^s = a_k^s \tag{44}$$

A2—Constant Impedance Load

If constant impedance load representation is considered, the Jacobian matrix will remain constant during the iteration process. In this case the elements a_k^s, b_k^s, c_k^s e d_k^s are given by:

$$a_k^s = -\, d_k^s = Q_{2k}^s \tag{45}$$

$$b_k^s = c_k^s = -P_{2k}^s \tag{46}$$

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