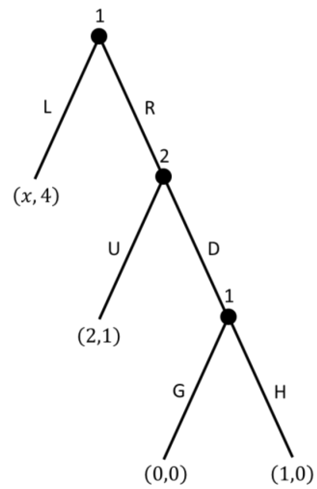
Homework 2

Due: 2018 Apr 19
Please the submit the homework before the class starts in the classroom

Problem 1 (Extensive form game with perfect information)

Consider the following extensive form game:



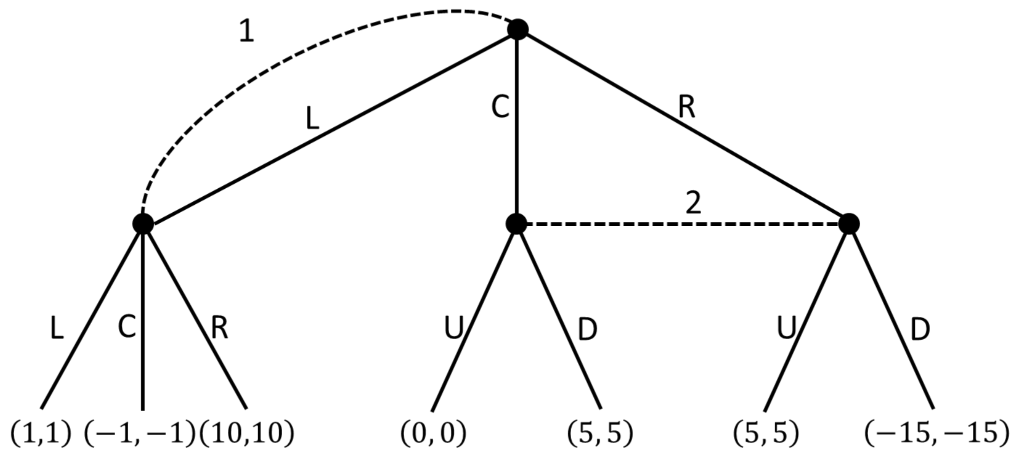
(1) when $x = 1$, find the all pure strategy Nash equilibria and subgame perfect Nash equilibria

(2) Find the range of x for which (R, U) is the unique subgame perfect NE outcome

(3) Find the range of x for which L is a Nash equilibrium outcome

Problem 2 (Imperfect information games, Pure vs. Mixed vs. Behavioral)

Consider the two-player game of imperfect information given in Figure 1. It is a common-value game, so the value at a leaf defines the payoff of both players. You must justify your answers.



(1) List all pure strategy Nash equilibria ("none exists" is a possible answer).

(2) Now we include mixed (but not behavioral) strategies. List all of the Nash equilibria (excluding any that you already found in part (1)). As there could be an infinite number of mixed Nash equilibria, you should use variables and give ranges over which strategy profiles constitute Nash equilibria.

(3) What is the highest expected payoff obtainable if the players use behavioral strategies?

Problem 3 (An Infinitely repeated game: Tit-for-Tat strategy)

Consider the following infinitely repeated version of the following game:

	C	D
C	4, 4	0, 6
D	6, 0	1, 1

The payoff of player i to any infinite sequence of payoffs $\{u_i^t\}$ is given by the normalized discounted sum of payoffs:

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}u_i^t$$

where $0 < \delta < 1$

(1) For what values of δ, if any, does it constitutes a subgame perfect equilibrium when both player choose this strategy?

- Choose C in period 1.
- Choose C after any history in which the previous periods' outcome was (C, C)
- Choose D after nay other history

(2) For what values of δ, if any, does it constitutes a subgame perfect equilibrium when both player choose this strategy?

- Choose C in period 1.
- Do whatever our opponent did in the previous period.

Which is called Tit-for-Tat strategy.

Problem 4 (Extensive form games)

Consider the following Rock (R), Paper (P), Scissors (S) game. Suppose that, with probability a, player 1 faces a rational opponent (who believes there to be common knowledge of rationality) and, with probability $1 - a$, she faces an opponent who will play P for sure. That is, before the game, Nature selects player 2's type and player 1 does not see Nature's choice.

(1) Draw the extensive form of this Bayesian game:

(2) Denote strategies for player 2 as $(P, P), (P, R), (P, S)$. Complete the following normal form representation of the game: (Since this is a zero-sum game, simply write player 1's expected payoff in that situation in each cell.)

(3) Find all equilibria of this game for $a = 1/3$.

Problem 5 (Folk Theorem)

Consider the following the following game:

	L	R
U	0, 6	1, 2
D	2, 1	0, 0

(1) What is the maximin payoff for each player and what strategy does the other player have to play to force that payoff (i.e. the other player's minimax strategy)?

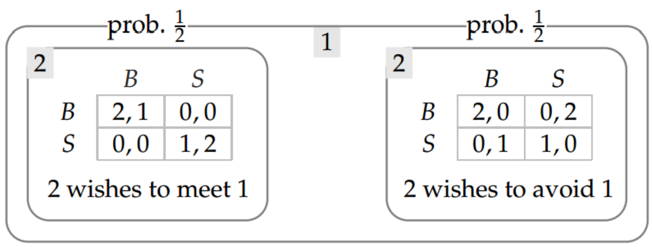
(2) Which of the following payoffs are feasible and enforceable for this game? (check if a strategy satisfies the two conditions)

	Yes	No
1. $(0, 0)$		
2. $(0, 6)$		
3. $(1, 2)$		
4. $(1, 3.5)$		
5. $(1, 0.5)$		
6. $(2, 2)$		

(3) Consider the infinitely repeated version of this game with a limit average reward, describe a Nash equilibrium of the repeated game that achieves a payoff of (1,3).

Problem 6 (Battle of Sex game with incomplete information)

Consider a variant of the situation modeled by Battle of Sex in which player 1 is unsure whether player 2 prefers to go out with her or prefers to avoid her, whereas player 2 knows player 1's preferences. Specifically, suppose player 1 thinks that with probability 1/2 player 2 wants to got out with her, and with probability 1/2 player 2 wants to avoid her. That is, player 1 thinks that with probability 1/2 she is playing the game on the left of the figure below and with probability 1/2 she is playing the game on the right. Find Bayesian Nash equilibria



Problem 7 (Cournot's duopoly game with incomplete information)

Two firms compete in selling a good; one firm does not know the other firm's cost function. How does the imperfect information affect the firms' behavior? Assume that both firms can produce the good at constant unit cost. Assume also that they both know that firm 1's unit cost is c, but only firm 2 knows its own unit cost; firm 1 believes that firm 2's cost is c_L with probability θ and c_H with probability $1 - \theta$, where $0 < \theta < 1$ and $c_L < c_H$.

The firms' payoffs are their profits; if the actions chosen are $(q1, q2)$ and the state is I (either L or H) then firm 1's profit is $q_1(P(q_1 + q_2) - c)$ and firm 2's profit is $q_2(P(q_1 + q_2) - c_I)$, where

$$P(q1 + q2) = \begin{cases} a - (q_1 + q_2) & if\ a \geq (q_1 + q_2) \\ 0 & \text{otherwise} \end{cases}$$

is the market price when the firms' outputs are q_1 and q_2.

(1). For values of c_H and c_L close enough that there is a Nash equilibrium in which all outputs are positive, find this equilibrium.

(2) Compare this equilibrium with the Nash equilibrium of the game in which firm 1 knows that firm 2's unit cost is c_L, and with the Nash equilibrium of the game in which firm 1 knows that firm 2's unit cost is c_H.

Problem 8 (Signaling game)

A signaling game is a two-player game in which Nature selects a game to be played according to a commonly known distribution (as in a Bayesian game), player 1 is informed of Nature's choice, then player 2 chooses an action knowing player's action but not Nature's choice. Consider the signaling game where Nature chooses between the following two games with equal probability.

	L	R
U	5, -5	1, -1
D	4, -4	0, 0

	L	R
U	0, 0	2, -2
D	2, -2	6, -6

(1) What is player 1's optimal play in each subgame? If player 1 uses this strategy in the signaling game, what will player 1's expected payoff be if player 2 best responds?

(2) Draw an extensive-form game (with Nature) of this signaling game (in a form of an extensive-form game with Nature). Make sure to include the distribution that Nature follows.

(3) Find a Nash Equilibrium of the induced extensive-form game. Does it differ from the strategy specified in part (1)? Why?