# MODELING AND ANALYSIS OF UNDER-LOAD TAP-CHANGING TRANSFORMER CONTROL SYSTEM APPLICATIONS

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This paper considers various applications of under-load tap-changing (ULTC) transformers directed by manual command or automatic voltage regulators, as used for voltage and reactive flow control in transmission and distribution networks. In view that the operation of tap changers is rather slow and that consequently, the existing ULTC transformers are at best only steady-state voltage/VAR correction devices, steady-state relations for different ULTC transformer applications are derived. They are aimed for practical computational use in various ULTC transformer control schemes.

## 1. INTRODUCTION

Under-load tap-changing (ULTC) transformers are presently in wide use in transmission and distribution networks, whenever the control of voltages and/or reactive power flows is required. This control is of discontinuous step-by-step and slow acting type, performed either manually or by automatic voltage regulators, which initiate the operation of the motor-drive unit associated to the tap changer. Such an automatic voltage-reactive flow control system is formed of inherently simple components, but the overall control system is a complex one due to the presence of nonlinearities, time-delays and disturbances [1].

There are several important applications of manually or automatically regulated ULTC transformers [1,2]. Among them, the most frequent are for

- Transmission/Distribution network voltage control.
- Reactive flow control between two high-voltage networks. connected through an ULTC transformer.
- Generator voltage/Reactive generation control.

The principal aim of this paper is to give a comprehensive consideration of practical engineering utilization of manually or automatically controlled ULTC transformers in all of these main areas of applications.

The paper presents appropriate models and analytical expressions for such applications, with main emphasis to steady-state considerations of voltage and reactive flow control.

## 2. ULTC TRANSMISSION/DISTRIBUTION TRANSFORMERS

ULTC transformers presently represent the main means for the voltage control in transmission and distribution networks.

Fig.1. gives the schematic representation and phasor diagrams of an ULTC transformer supplying constant impedance load for two different tap ratios $a_1$ and $a_2 (a_2 < a_1)$, and constant primary line-to-line supply voltage $\underline{V}_1$.

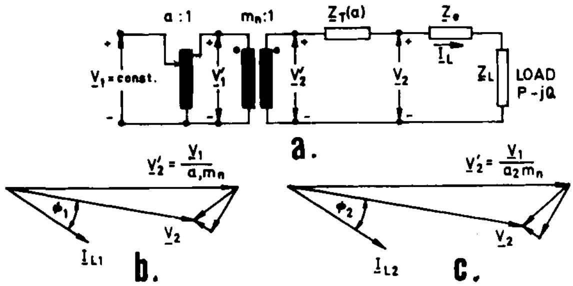


Fig. 1. Schematic representation (a) and phasor diagrams (b and c) of a step-down ULTC transformer, supplying constant impedance load, for two different tap-ratios $a_1$ and $a_2$ $(a_2 < a_1)$

The change of the complex line-to-line secondary voltage $\underline{V}_2$, due to the tap-ratio change from $a_1$ to $a_2$, according to the impedance diagram at Fig. 1a, with all quantities expressed in their physical values, is

$$\Delta\underline{V}_2 = \underline{V}_2(a_2) - \underline{V}_2(a_1) = \left[\frac{\underline{V}_1}{a_2 m_n} - \sqrt{3}\underline{Z}_T(a_2)\underline{I}_{L2}\right] - \left[\frac{\underline{V}_1}{a_1 m_n} - \sqrt{3}\underline{Z}_T(a_1)\underline{I}_{L1}\right] =$$
$$= \frac{\underline{V}_1}{m_n}\left(\frac{1}{a_2} - \frac{1}{a_1}\right) - \sqrt{3}\left[\underline{Z}_T(a_2)I_{L2} - \underline{Z}_T(a_1)\underline{I}_{L1}\right], \qquad (1)$$

where $Z_T(a)$ is the transformer impedance expressed in Ohms referred to the rated secondary voltage,

$m_n = \frac{V_{1r}}{V_{2r}} = \frac{N_1}{N_2}$ is rated transformer turn-ratio,

$V_{1r}$, $V_{2r}$ are primary and secondary rated voltages, that correspond to $N_1$ and $N_2$ nominal turns on primary and secondary windings, respectively, while

$a = \frac{V_1}{V_{1r}}$ is the transformer tap-ratio (tap winding is on the primary, high-voltage side),

$m = m_n a$ is actual transformer turn-ratio.

Assuming that the variation of load current $I_L$ is small, when the tap-ratio a changes from $a_1$ to $a_2$ ($\underline{I}_{L2} \approx \underline{I}_{L1} = \underline{I}_L = (P_L - jQ_L)/\sqrt{3}\underline{V}_2^*$) and by neglecting the winding resistance ($\underline{Z}_T(a) \approx jX_T(a)$), yields to the simplification of (1), so that

$$\Delta\underline{V}_2 = \frac{\underline{V}_1}{m_n}\left(\frac{1}{a_2} - \frac{1}{a_1}\right) - j\left[X_T(a_2) - X_T(a_1)\right]\frac{P_L - jQ_L}{\underline{V}_2^*}. \qquad (2)$$

Then, the magnitude of voltage change $\Delta\underline{V}_2$ assuming $\underline{V}_1 = V_1\underline{/0^0}$ is

$$|\Delta\underline{V}_2| = [(\Delta V_2)^2 + (\delta V_2)^2]^{1/2}, \tag{3}$$

where

$$\Delta V_2 = \mathrm{Re}(\Delta\underline{V}_2) \approx \frac{V_1}{m_n}\left(\frac{1}{a_2} - \frac{1}{a_1}\right) + m_n \frac{a_2X_T(a_2) - a_1X_T(a_1)}{V_1} Q_L$$

$$\delta V_2 = I_m(\Delta\underline{V}_2) \approx -m_n \frac{a_2X_T(a_2)-a_1X_T(a_1)}{V_1} P_L . \tag{4}$$

Relations (1)-(4) are also valid, with all parameters and variables expressed in per units (pu). Then, for $m_n$ = 1,00 pu and for practical pu values of $V_1$, $P_L$, $Q_L$ and $X_T$, $|\Delta V_2| >> |\delta V_2|$, so that

$$|\Delta\underline{V}_2| \approx \Delta V_2. \tag{5}$$

In practice, the supply voltage $V_1$ also varies; then the phasor diagram at Fig. 1. must be modified accordingly.

## 3. RADIAL TRANSMISSION/DISTRIBUTION SYSTEM WITH TWO CASCADED ULTC TRANSFORMERS

Very often tap-changing transformers are provided in both, the transmission and distribution voltages, connected in series with the feeder line, as shown in Fig. 2. Let $a_s$ and $a_r$ be tap ratios of ULTC transformers on the sending and receiving ends, respectively. Then, it is interesting to determine the tap ratios $a_s$ and $a_r$ which will retain some specified relationship between steady-state magnitudes of voltages $\underline{V}_1$ and $\underline{V}_2$. With all quantities expressed in physical units (Volts, Amperes, Ohms, etc.) in view of the impedance diagram from Fig. 2b, the following relations are valid:

$$\underline{V}_s' = \frac{\underline{V}_1}{a_s m_{n1}} ; \qquad \underline{V}_r' = a_r m_{n2} \underline{V}_2 . \tag{6}$$

Also, the system impedance, transferred to line voltage is

$$\underline{Z} = R + jX = Z_{T1}(a_s) + \underline{Z}_e + a_r^2 \underline{Z}_{T2}(a_r). \tag{7}$$

The line-to-line voltage drop in the whole system is

$$\Delta\underline{V} = \underline{V}_s' - \underline{V}_r' = \frac{\underline{V}_1}{a_s m_{n1}} - a_r m_{n2}\underline{V}_2 = (R + jX) \frac{P-jQ}{a_r m_{n2} \underline{V}_2^*} \tag{8}$$

Assuming $\underline{V}_2 = V_2\underline{/0^0}$ and by neglecting the quadrature voltage drop, expression (8) becomes

$$|\Delta\underline{V}| \approx \Delta V = \frac{V_1}{a_s m_{n1}} - a_r m_{n2} V_2 = \frac{RP+XQ}{a_r m_{n2} V_2} . \tag{9}$$

After the simple rearrengment, Equation (9) gets the form

$$(m_{n2}V_2)^2 - \frac{V_1}{a_s a_r m_{n1}} m_{n2}V_2 + \frac{RP + XQ}{a_r^2} = 0, \tag{10}$$

giving as the solution for $V_2$

$$m_{n2}V_2 = \frac{1}{2a_s a_r} \left\{ \frac{V_1}{m_{n1}} \pm \left[\left(\frac{V_1}{m_{n1}}\right)^2 - 4(RP+XQ)\right]^{1/2} \right\}. \tag{11}$$

Equation (10) connects $V_1$,$V_2$,$a_s$ and $a_r$, for a known load (P,Q) and an approximate value of system impedance ($\underline{Z}$=R+jX). Its solution (11) requires the additional specifications in view of variables involved.

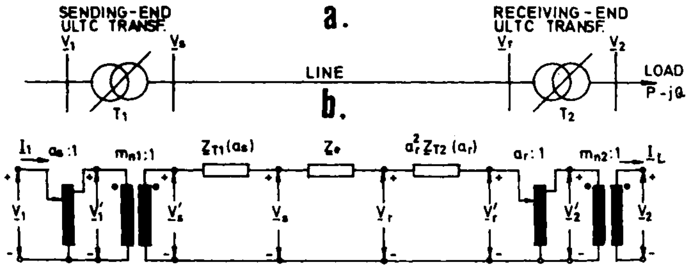


Fig. 2. Radial transmission/distribution system with two cascaded ULTC transformers: a. One-line diagram; b. Equivalent impedance diagram

## 4. COMBINED USE OF THREE-WINDING ULTC TRANSFORMERS FOR TAP-RATIO AND REACTIVE POWER INJECTION CONTROL IN TRANSMISSION NETWORKS

The common practice for reactive flow control in transmission networks is to use the tertiary of three-winding transmission transformers for the reactive power injection via synchronous condensers, or capacitor/ /reactor banks, as shown in Fig. 3a. Then, for a given load condition, it is necessary to find the transformer tap ratio for some specified reactive generation/consumption on the tertiary bus. By representing the three-winding transformer with its equivalent star (or Y) connection and by neglecting the winding resistances and transformer shunt losses, the impedance diagram of the system under consideration is shown in Fig. 3b.

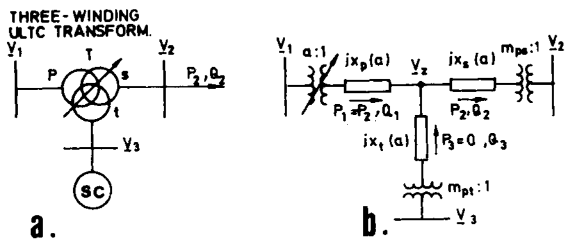


Fig. 3. Three-winding transmission ULTC transformer, with synchronous condenser connected to its tertiary: a. One-line diagram; b. Equivalent impedance diagram

For a given secondary load ($P_2$,$Q_2$), assuming $P_3 \approx 0$, the complex line-to--line voltage drop between buses $\underline{V}_1$ and $\underline{V}_2$ is

$$\Delta\underline{V} = \frac{\underline{V}_1}{a} - m_{ps}\underline{V}_2 \approx \frac{X_p(a)Q_1}{\underline{V}_Z^*} + \frac{X_s(a)Q_2}{m_{ps}\underline{V}_2^*} + j\left(\frac{X_p(a)}{\underline{V}_Z^*} + \frac{X_s(a)}{m_{ps}\underline{V}_2^*}\right)P_2 =$$

$$= \frac{X_p(a)(Q_2-Q_3)}{\underline{V}_Z^*} + \frac{X_s(a)Q_2}{m_{ps}\underline{V}_2^*} + j\left(\frac{X_p(a)}{\underline{V}_Z^*} + \frac{X_s(a)}{m_{ps}\underline{V}_2^*}\right)P_2, \quad (12)$$

where reactances $X_p(a)$ and $X_s(a)$ are expressed in Ohms transferred to rated primary voltage. The reactance $X_s(a)$ is small, comparing to $X_p(a)$

(it is usually close to zero and even may be negative). Then, $\underline{V}_Z \approx m_{ps}\underline{V}_2$ and assuming $\underline{V}_2 = V_2\angle 0^0$; $\underline{V}_1 = V_{1d} + jV_{1q}$, the separation of real and imaginary parts in (13) yields

$$\Delta V = \frac{V_{1d}}{a} - m_{ps}V_2 = \frac{X_p(a)(Q_2-Q_3)+X_s(a)Q_2}{m_{ps}V_2} = \frac{X_{ps}(a)Q_2-X_p(a)Q_3}{m_{ps}V_2}$$

$$\delta V = \frac{V_{1q}}{a} = \frac{[X_p(a)+X_s(a)]P_2}{m_{ps}V_2} = \frac{X_{ps}(a)P_2}{m_{ps}V_2}, \tag{13}$$

where $X_p(a) + X_s(a) = X_{ps}(a)$.

Then

$$V_1^2 = V_{1d}^2 + V_{1q}^2 = a^2[m_{ps}V_2 + \frac{X_{ps}(a)Q_2-X_p(a)Q_3}{m_{ps}V_2}]^2 + a^2[\frac{X_{ps}(a)P_2}{m_{ps}V_2}]^2$$

yields to

$$a^2\{(m_{ps}V_2)^2+[X_{ps}(a)Q_2-X_p(a)Q_3]^2+[X_{ps}(a)P_2]^2\}-V_1^2(m_{ps}V_2)^2=0. \tag{14}$$

Equation (14) gives the relationsnip among $V_1$,$V_2$,a and $Q_3$, assuming that approximate values of reactances $X_p(a)$, $X_s(a)$ and secondary load $P_2$,$Q_2$ are known.

## 5. REACTIVE FLOW CONTROL BETWEEN TWO HIGH-VOLTAGE NETWORKS CONNECTED THROUGH AN ULTC TRANSFORMER

Transmission networks of different voltage levels are usually interconnected through ULTC transformers (Fig.4a). If such networks are inherently the infinite-bus networks, the ULTC transformers are used as means for the reactive flow control between connected networks. By using the equivalent impedance diagram for such a system, shown in Fig.4b, assuming the tap winding is placed on the transformer primary winding, and by neglecting transformer resistances and shunt losses, the following equation holds:

$$\Delta\underline{V} = \frac{\underline{V}_1}{a} - m_n\underline{V}_2 \approx jX_T(a)\frac{P-jQ}{m_n\underline{V}_2^*}, \tag{15}$$

where the reactance $X_T(a)$ is expressed in Ohms transferred to rated primary voltage. Assuming $\underline{V}_2 = V_2\angle 0^0$; $\underline{V}_1 = V_{1d} + jV_{1q}$, further derivation of (15) yields

$$\Delta V = \frac{V_{1d}}{a} - m_nV_2 = \frac{X_T(a)Q}{m_nV_2}\ ;\ \delta V = \frac{V_{1q}}{a} = \frac{X_T(a)P}{m_nV_2}\ ; \tag{16}$$

Then

$$V_1^2 = V_{1d}^2 + V_{1q}^2 = [\frac{(m_nV_2)^2 + X_T(a)Q}{m_nV_2}a]^2 + [\frac{X_T(a)P}{m_nV_2}a]^2,$$

yields

$$a^2\{[(m_nV_2)^2+X_T(a)Q]^2 + [X_T(a)P]^2\} - V_1^2(m_nV_2)^2 = 0. \tag{17}$$

Equation (17) gives the relationship among $V_1$,$V_2$,a, and transformer loads P,Q, for $X_T(a)$ approximately known.

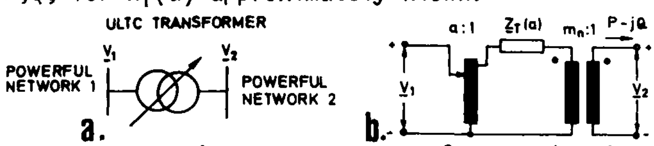


Fig.4. Interconnection of two powerful networks through an ULTC transformer: a. One-line diagram; b. Equivalent impedance diagram

## 6. GENERATOR STEP-UP ULTC TRANSFORMERS USED FOR GENERATOR VOLTAGE AND REACTIVE GENERATION CONTROL

Generator step-up transformers are often built as ULTC units. Such an arrangement is needed for full reactive utilization of generators under various operating conditions. To find the tap ratio of a generator step-up ULTC transformer, consider the scheme given in Fig.5, which represents a generator connected to a powerful network. Assuming the tap winding is located on the high voltage side of the transformer, the complex line-to-line voltage drop in the transformer is given by the relation

$$\Delta \underline{V} = \underline{V}_G m_n - \frac{\underline{V}_N}{a} = \underline{Z}_T(a)\,\frac{P-jQ}{\underline{V}_N^*/a} \approx jX_T(a)\,\frac{P-jQ}{\underline{V}_N^*}\,a, \tag{18}$$

where the reactance $X_T$ is given in Ohms, transferred to rated voltage on the network side.

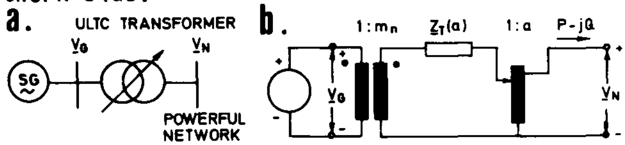


Fig.5. Generator connected to a powerful network through an ULTC transformer: a. One-line diagram; b. Equivalent impedance diagram

Assuming $\underline{V}_N = V_N\angle 0^0$; $\underline{V}_G = V_{Gd} + jV_{Gq}$, Equation (18) gives

$$\Delta V = m_n V_{Gd} - \frac{V_N}{a} = \frac{X_T(a)Q}{V_N}\,a; \quad \delta V = m_n V_{Gq} = \frac{X_T(a)P}{V_N}\,a. \tag{19}$$

Then

$$(m_n V_G)^2 = (m_n V_{Gd})^2 + (m_n V_{Gq})^2 = \left[\frac{V_N}{a} + \frac{X_T(a)Q}{V_N}\,a\right]^2 + \left[\frac{X_T(a)P}{V_N}\right]^2 a^2. \tag{20}$$

After simple rearrangement, the equation (20) gives the mutual relationship among $V_G$,P,Q and a, for a known $V_N$ and approximate value of $X_T(a)$, as

$$[V_N^2 + X_T(a)Qa^2]^2 + [X_T(a)Pa^2]^2 - [(aV_N)(m_n V_G)]^2 = 0. \tag{21}$$

## 7. CONCLUSION

Various problems of ULTC transformer applications for voltage/reactive power flow control have been considered. Useful steady-state relations for each of considered applications were developed, by taking into account engineering requirements and physical nature of control problems. Considerations were limited to the stationary ULTC transformer control behavior, due to the slow-speed character of the control process. Relations presented here will provide for better understanding and adequate utilization of the voltage-reactive control capabilities of ULTC transformers in transmission and distribution networks.

## 8. REFERENCES


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