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MODELING AND ANALYSIS OF UNDER-LOAD TAP-CHANGING TRANSFORMER CONTROL SYSTEMS

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Abstract - This paper considers the problem of modeling and analysis of under-load tap-changing (ULTC) transformer control systems. A nonlinear system model is derived, suitable for analysis of voltage and reactive power flow control applications of ULTC transformers in the consideration of mid-term and long-term dynamics and steady-state behavior of power systems. The model is verified with the example of a distribution ULTC transformer, used for the voltage control. As an illustration of the feasibility of the model in various voltage and reactive flow control applications, some digital simulation results of such a distribution voltage control are also presented.

INTRODUCTION

Modern power systems encompassing several generating plants and an extremely high number of consumers employ a variety of transmission and distribution voltages. Presently, they cover large geographic territories and have a number of interconnection and distribution points through which the electric energy is transmitted or delivered. At any point, where a change of voltage level is required, transformers must be applied. Such transformers are usually built with fixed turn-ratios, or sometimes they are equipped with tap terminals for turn-ratio control. Tap changers are needed for such a control, which results in voltage change on the regulated bus and in change of reactive power flow through the transformer. The operation of tap changers may be twofold: in deenergized state, or under load. The first option requires that the transformer to be tapped is tripped-off, while the second one represents a desirable operation practice, since all tap-changer manipulations in a normal transformer operation are performed, without interruption of service. These manipulations require that some additional accessories be associated with the transformer tap changer. Together, the various components used for under-load tap changing (ULTC) of a transformer make a transformer turn-ratio or transformer voltage-reactive (or VAR) flow control system. The brain of this system is an automatic voltage regulator. Its role is to detect voltage variations compared to the reference voltage value at the controlled bus (or variations of the reactive flow through the transformer) and to initiate the operation of the motor-drive unit associated with the tap changer, by a control relay. In addition to a voltage regulator and motor-drive unit, this control system encompasses the ULTC transformer, tap-changer mechanism and auxiliary equipment (relays, switches, current and potential measuring transformers, compensators, etc.). Inherently, this is an electromechanical, slow-acting, discontinuous, step-by-step control system. Although all of the components of this system are simple devices, the overall system is a complex one, due to the presence of nonlinearities, time-delays and external disturbances.

A review of current literature [1] - [10] dealing with this subject shows that the modeling and dynamic analysis are not considered. Attention was paid to ULTC transformer construction [1], [2], tap-changer arrangements [1] - [4], description of voltage regulators [5] - [7], design philosophy in the application of ULTC transformers [2], [4], [7] and ULTC transformer representation in steady-state system studies [3], [8] - [10]. The principal aim of this paper is to fill this gap in present-day technical literature, by developing an appropriate dynamic model of an ULTC transformer control system, suitable for various studies of mid-term and long-term dynamics as well as the steady-state voltage and reactive power problems. The proposed nonlinear model is based on the results exhibited in Report [11]. This model is derived by considering the physical nature and mutual links among various elements that form the control system. Then, the verification of the model is performed by digital simulation of a distribution voltage control system.

The paper presents the derivation of a nonlinear ULTC transformer control system model, the simulation results obtained on a sample system example, and discussion of applications of automatically controlled ULTC transformers in current power system practice.

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SYSTEM DESCRIPTION

An ULTC transformer automatic control, general system block diagram is shown in Fig. 1. It consists of the following physically distinct parts:

- -ULTC transformer with tap-changing mechanism,
- -Motor-drive unit,
- -Interlocking protective devices,
- -Voltage regulator (includes measuring element, time-delay element and line-drop compensator),
- -Measuring current and potential transformers,
- -Additional auxiliary equipment.

DERIVATION OF SYSTEM MODEL

In view of general system block diagram from Fig. 1 and physical features and roles of particular elements in the overall system performance, the modeling of an ULTC transformer control system can be performed by the development of mathematical models of various elements forming this system and by suitable connection of these elements.

A structural block-diagram form of an ULTC transformer control system model is shown in Fig. 2, while the short review of mathematical models of various system components is given below.

Voltage Regulator Measuring and Time-Delay Elements

The measuring element of a voltage regulator is an adjustable dead-band relay element with hysteresis,

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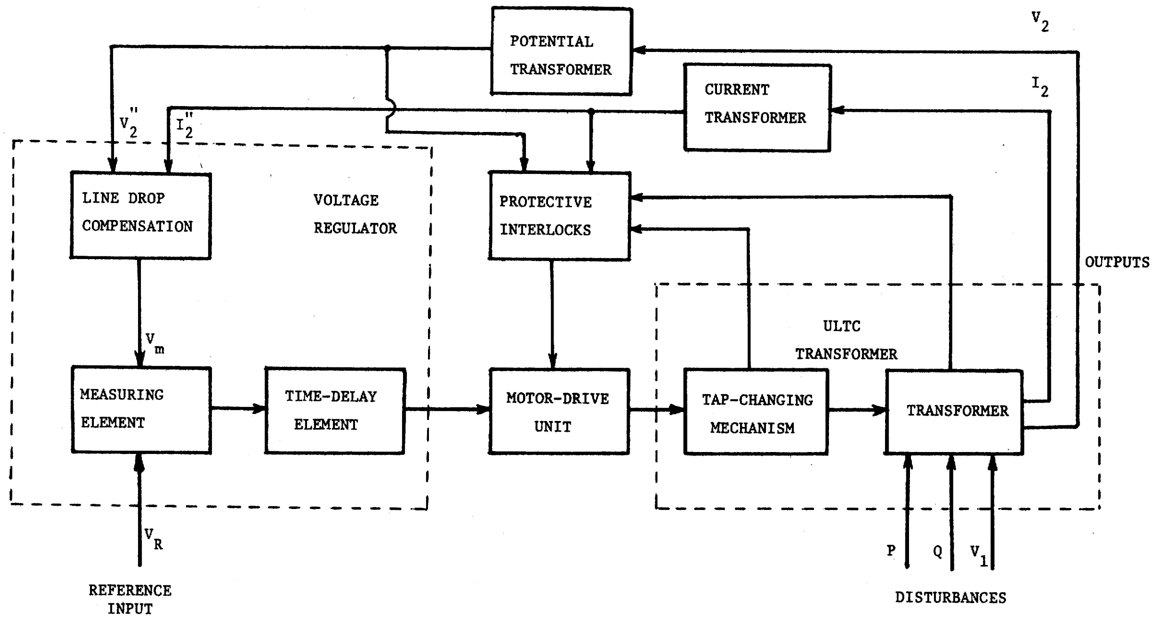

Figure 1. General structural block-diagram of an ULTC transformer control system.

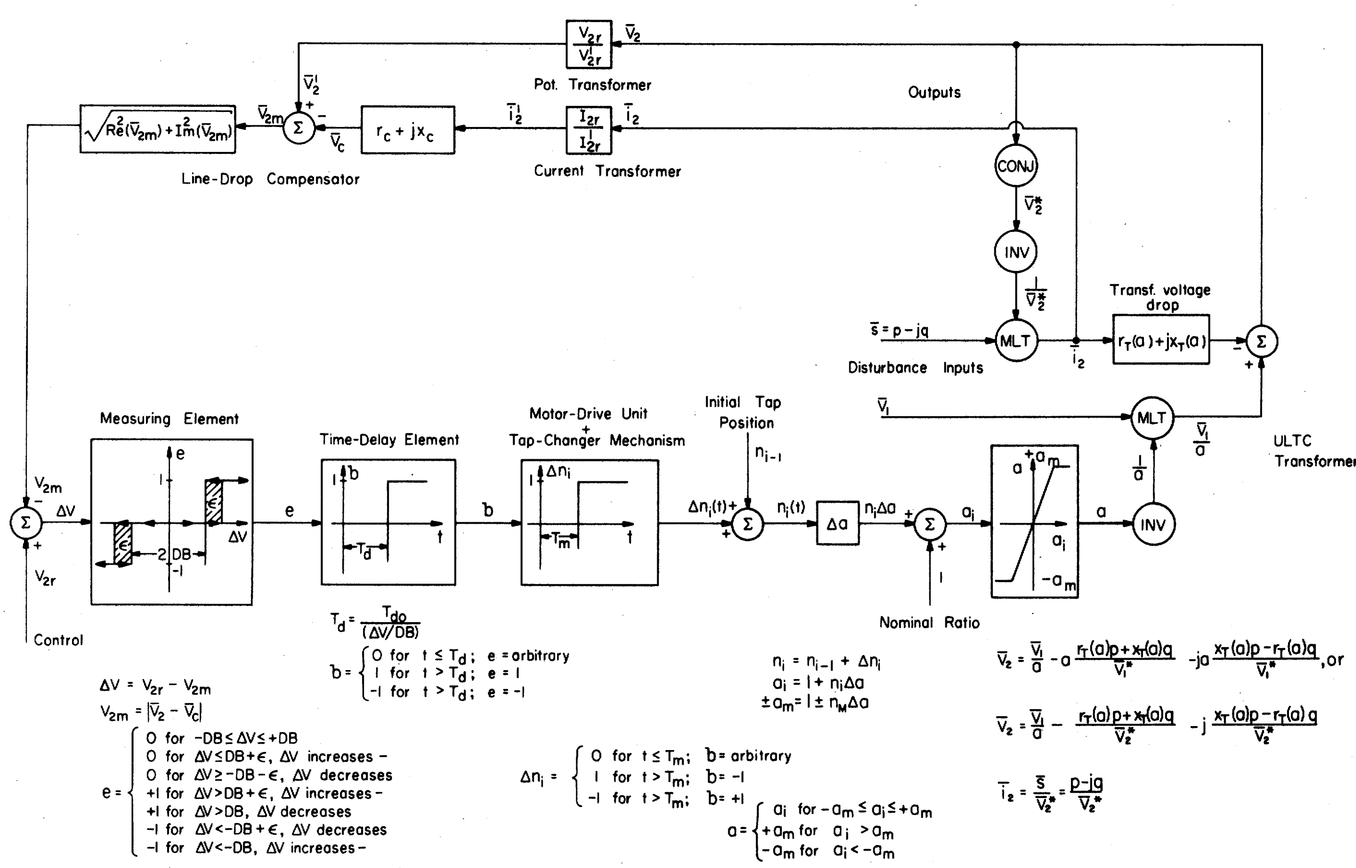

Figure 2. Structural block diagram of an ULTC transformer overall control system.

which can be described by the following expressions:
Voltage error (regulator input) is

$$\Delta v = v_{2r} - v_{2m} , \tag{1}$$

where

v_{2r} is the reference voltage, in pu,
v_{2m} is the measuring voltage, after line-drop compensator voltage is added, in pu.

Output from measuring element is

$$e = \begin{cases} 0 & \text{for } -DB \leq \Delta v \leq +DB \\ 0 & \text{for } \Delta v \leq DB + \varepsilon \quad , \ \Delta v \text{ increases} \\ 0 & \text{for } \Delta v \geq -(DB + \varepsilon) \ , \ \Delta v \text{ decreases} \\ 1 & \text{for } \Delta v > DB + \varepsilon \quad , \ \Delta v \text{ increases} \\ 1 & \text{for } \Delta v > DB \quad , \ \Delta v \text{ decreases} \\ -1 & \text{for } \Delta v < -(DB + \varepsilon) \ , \ \Delta v \text{ decreases} \\ -1 & \text{for } \Delta v < -DB \quad , \ \Delta v \text{ increases} \end{cases} , \tag{2}$$

where

DB is the adjustable dead band of the regulator, in pu,

ε is a hysteresis band of the regulator relay characteristic, giving various operating points depending upon the direction of change of voltage error at the entry of the measuring element, expressed in pu.

An adjustable time-delay relay is applied in order to reduce the effect of short-time voltage variations and to avoid unnecessary tap-position changes. The output of the time-delay element is

$$b = \begin{cases} 0 & \text{for} \quad t \leq T_d, \ e \text{ arbitrary} \\ 1 & \text{for} \quad t > T_d, \ e = 1 \ , \\ -1 & \text{for} \quad t > T_d, \ e = -1 \end{cases} \tag{3}$$

where T_d is the adjustable time delay of regulator operation in seconds. It is constant ($T_d = T_{do}$) for regulators with independent time delay, or a function of initial time-delay setting T_{do} voltage error Δv and dead-band DB, for regulators with variable time-delay characteristics.

An inverse-time delay characteristic is usually assumed, such that

$$T_d = \frac{T_{do}}{|\Delta v/DB|} \ . \tag{4}$$

Motor Drive-Unit and Tap-Changer Mechanism

This part of the system is inherently an integral element. Since the construction of a tap changing device and motor drive unit is made as a step-by-step "go-no go" mechanism, their operation could be represented as a constant time-delay relay element, with a counter. It can be described for any tap-changer operation i, and the tap position $n_i \ \varepsilon \ [0, \pm n_M]$, with the following expressions:

Incremental change of tap-position in i-th operation

$$\Delta n_i = \begin{cases} 0 & \text{for} \quad t \leq T_M, \ b \text{ arbitrary} \\ 1 & \text{for} \quad t > T_M, \ b = -1 \ , \\ -1 & \text{for} \quad t > T_M, \ b = +1 \end{cases} \tag{5}$$

Tap-changer position after i-th operation completed

$$n_i = n_{i-1} + \Delta n_i \ . \tag{6}$$

The transformer tap-ratio is then expressed as

$$a_i = 1 + n_i \Delta a = 1 + (n_{i-1} + \Delta n_i)\ \Delta a \ ;$$

$$a = \begin{cases} a_i & \text{for } -a_m \leq a_i \leq +a_m \\ +a_m & \text{for } a_i > +a_m \quad , \\ -a_m & \text{for } a_i < -a_m \end{cases} \tag{7}$$

where

$\pm a_m = 1 \pm n_M \Delta a$ is the limit value of the transformer tap-ratio a.

Under-Load Tap-Changing Transformer

In view of the fact that the operation of tap changers is rather slow, the existing ULTC transformers are at best only steady-state voltage correction devices. Thus a steady-state transformer model is used. It is derived in Appendix 1; the final expression describing a two-winding transformer behavior is

$$\overline{v}_2 = \frac{1}{a}\overline{v}_1 - a\,\frac{r_T(a)p + x_T(a)q}{\overline{v}^*_1} - ja\,\frac{x_T(a)p - r_T(a)q}{\overline{v}^*_1} , \tag{8}$$

or

$$\overline{v}_1 = a\overline{v}_2 + a\,\frac{r_T(a)p + x_T(a)q}{\overline{v}^*_2} + ja\,\frac{x_T(a)p - r_T(a)q}{\overline{v}^*_2} , \tag{9}$$

where

$\overline{v}_1$, $\overline{v}_2$ are phasors of primary and secondary transformer voltages, respectively,
$\overline{s} = p - jq$ is the transformer complex load,
$\overline{z}_T(a) = r_T(a) + jx_T(a)$ is the transformer leakage impedance (function of tap-ratio a).
* denotes the conjugate of the indicated phasor variable.

All quantities in (8) and (9) are expressed in pu, on transformer rated voltages and apparent power bases (see Appendix 1 for details).

Equations (8) and (9) exhibit a nonlinear relationship among $\overline{v}_1$, $\overline{v}_2$, a, p and q, giving the ULTC transformer block-diagram representation shown in Fig. 2.

Measuring Transformers

Both potential (PT) and current (CT) transformers are modeled as ideal, lossless transformers, with their turn ratios:

Potential transformer

$$m_{PT} = \frac{V'_{2r}}{V''_{2r}} \tag{10}$$

Current transformer

$$m_{cT} = \frac{I'_{2r}}{I''_{2r}} \,, \qquad (11)$$

where V'_{2r} and V''_{2r} are PT primary- and secondary-rated voltages; I'_{2r} and I''_{2r} are CT primary- and secondary-rated currents.

For $V'_{2r} = V_{2r}$ and $I'_{2r} = I_{2r}$, per unit values of m_{PT} and m_{CT} are equal to 1.00. If any discrepancies between rated bases V_{2r}, V'_{2r} and I_{2r}, I'_{2r} exist, then the relations giving the invariant primary values for voltage in volts and current in amperes of measuring transformers irrespectively of the selected bases give

$$\overline{v}'_2 V'_{2r} = \overline{v}_2 V_{2r}, \text{ or } \overline{v}'_2 = \overline{v}_2 \frac{V_{2r}}{V'_{2r}} \;; \qquad (12)$$

$$\overline{i}'_2 I'_{2r} = \overline{i}_2 I_{2r}, \text{ or } \overline{i}'_2 = \overline{i}_2 \frac{I_{2r}}{I'_{2r}} \;. \qquad (13)$$

Then, the pu values of PT and CT turn ratios are

$$\text{pu } m_{PT} = \frac{v'_{2r}}{v''_{2r}} = \frac{V_{2r}}{V'_{2r}} \qquad (14)$$

$$\text{pu } m_{CT} = \frac{i'_{2r}}{i''_{2r}} = \frac{I_{2r}}{I'_{2r}} \qquad (15)$$

Line-Drop Compensator (LDC)

A line-drop compensator (LDC) is used to correct the reference voltage v_{2r}, with the variation of transformer load. It simulates the voltage drop across an outgoing line connected to the ULTC transformer bus, by introducing the voltage drop on compensator impedance $\overline{Z}_c = R_c + jX_c$, which should map the line impedance voltage drop. This drop is calculated depending on the type of compensator and voltage regulator connection to the measuring transformers [3], [5]. In principle, the LDC compensates some definite portion k of line-voltage drop, so that the compensator voltage $\overline{V}_c$ in volts is determined from the expression

$$m_{PT}\overline{V}_c = (R_c + jX_c)\,\overline{I}''_2\, m_{PT} = k(R_L + jX_L)\,\overline{I}_2 \;, \qquad (16)$$

where $\overline{Z}_L = R_L + jX_L$ is the line impedance in ohms.

Compensator voltage drop $m_{PT}\overline{V}_c$ should be subtracted from the regulated transformer voltage $\overline{V}_2$, giving for the measuring voltage $\overline{V}_{2m}$ in volts the expression

$$\overline{V}_{2m} = \overline{V}''_2 - \overline{V}_c \,, \qquad (17)$$

or, in pu,

$$\overline{v}_{2m} = \overline{v}'_2 - \overline{v}_c = \overline{v}'_2 - (r_c + jx_c)\,\overline{i}'_2 = \overline{v}'_2 - k(r_L + jx_L)\overline{i}_2 \qquad (18)$$

$$v_{2m} = |\overline{v}_{2m}| = [\mathrm{Re}^2(\overline{v}_{2m}) + \mathrm{Im}^2(\overline{v}_{2m})]^{1/2}, \qquad (19)$$

where

$\overline{z}_c = r_c + jx_c$ is the LDC pu impedance and
$z_L = r_L + jx_L$ is the line pu impedance, calculated on V_{2r} and I_{2r} bases.

SYSTEM SIMULATION

To illustrate the behavior of an ULTC transformer control and to verify the derived system model, an example of a distribution ULTC transformer, 63 MVA, 110/36,75 kV rating is considered. Basic engineering data for the system are listed in Appendix 2.

The system simulation is made on a CYBER 175 digital computer system of the University of Illinois. The FORTRAN IV computer program for this simulation is written, which follows the system block diagram in Fig. 2. Several system simulations were performed by changing input variables (control reference v_{2r}, initial condition n_o and disturbances, $\overline{v}_1$, p and q) as well as control bandwidth DB retaining the system parameters r_T, x_T ε, r_c, x_c, m_{PT}, m_{CT}, T_{do}, T_m and Δa at fixed, nominal values. Simulation results exhibit good agreement with the performance of present-day ULTC transformer voltage regulation systems used for slow-acting distribution voltage control [2], [5], [6].

Figure 3 illustrates the variation of tap position n, when the transformer is loaded 42 MW and 31.5 MVAR for different values of primary voltage v_1 and various initial tap positions n_o, when the reference value v_{2r} of the regulated secondary voltage v_2 is taken to be 1.00 pu. Two different values of Dead-Band DB = 1% and 2% were considered, for all other system parameters fixed on values given in Appendix 2.

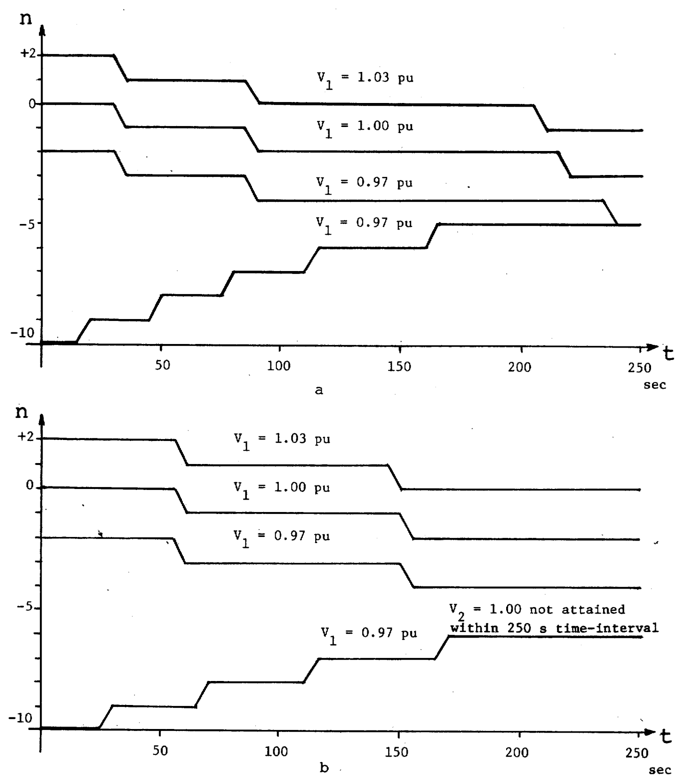

Figure 3. Variation of tap-position for different values of primary voltage V_1 and various initial tap-positions n_o, for V_{2r} = 1.00 pu.

a. Dead-Band = 1%
b. Dead-Band = 2%

Higher values of Dead-Band obviously make the control becomes slower, while the initial tap-position does not influence the final one.

APPLICATIONS

There are several important applications of automatically controlled ULTC transformers in present-day power systems. Among them, the most frequent are for

- Transmission/distribution network voltage control,
- Reactive flow control between two high-voltage networks connected through an ULTC transformer,
- Generator voltage/reactive generation control.

Page limitations prevent a detailed consideration of these applications, which are extensively studied in Reference [11].

CONCLUSION

The problem of mathematical modeling of an ULTC transformer control system was considered. Dynamic models of elements entering the ULTC transformer control system were developed as well as an overall control system. This derivation took into account the physical nature of different system elements and slow-speed character of such a control, neglecting the fast electromagnetic transient phenomena introduced by transformer turn-ratio variations. The obtained model is nonlinear and a discontinuous model, and encompasses some inherent time delays. It is suitable for the analysis of mid- and long-term power system dynamics as well as steady-state system performance. Digital system simulation, illustrated by an example of a distribution ULTC transformer, justified the theoretical derivations for a typical set of real engineering system data.

It is expected that the ULTC transformer model and analyses presented in this paper will provide for better understanding and more accurate representation of the voltage-reactive flow control systems than the purely steady-state models, proposed so far, because it simulates the real physical phenomena of interest for a slow-acting ULTC transformer control system.

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APPENDIX 1

A two-winding ULTC will be considered. As shown in Reference [11], the mathematical model of a three-winding ULTC transformer can be easily derived, by suitable modifications of a two-winding ULTC transformer model.

Assuming the symmetrical tapping range is always provided on high-voltage transformer windings, the transformer tap ratio a in any tap position $n \in [0, \pm n_M]$ is

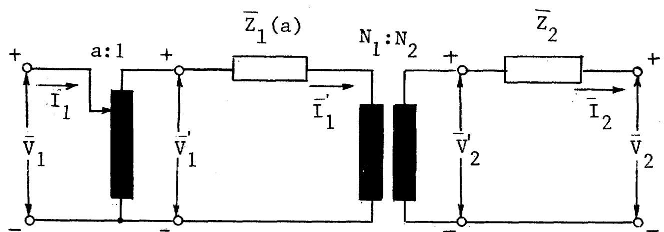

Figure 4. Simplified equivalent circuit of an ULTC transformer (tap-winding placed on the primary winding of the transformer).

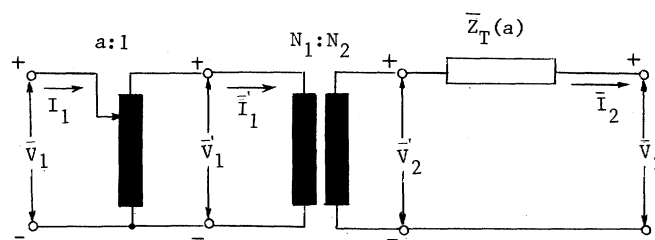

Figure 5. Reduced equivalent from Figure 4, after one side impedance referral.

$$a_n = \frac{1 \pm n\Delta a}{1} \; ; \quad n = 1, 2, \ldots, n_M \quad , \tag{A1-1}$$

where Δa represents the tap voltage in pu.

By neglecting the shunt losses, the equivalent circuit of an ULTC transformer, modelled in the form of two separate entities - a fixed nominal-tap transformer, cascaded with a lossless autotransformer, representing the tap-changing portion of the high voltage winding - is shown in Fig. 4.

Symbols used in Fig. 4 designated as follows (all per phase quantities):

$N_1 : N_2$ is the nominal turn-ratio of the ULTC transformer,
a:1 is the nominal tap ratio of the autotransformer,
$R_1(a)$ and R_2 are resistances; $X_1(a)$ and X_2 are leakage reactances of the transformer primary and secondary windings, respectively,
$\overline{I}_1$, $\overline{I}_1'$ and $\overline{I}_2$ are phasor currents,
$\overline{V}_1$, $\overline{V}_1'$, $\overline{V}_2'$ and $\overline{V}_2$ are phasor voltages.

Note that the autotransformer winding resistances and leakage reactances are attributed to the primary winding while all core losses and parallel magnetizing reactances are assumed negligible (thus: $R_1 = R_1(a)$; $X_1 = X_1(a)$).

The equivalent circuit from Fig. 4 can be further simplified by transferring all impedances to the transformer side opposite that of the autotransformer and by introducing the equivalent transformer impedance $\overline{Z}_T(a)$, which is reduced to the voltage $\overline{V}_2$

$$\overline{Z}_T^{(2)}(a) = [R_2 + R_1(a)(\frac{N_2}{N_1})^2] + j[X_2 + X_1(a)(\frac{N_2}{N_1})^2] \; . \tag{A1-2}$$

Then the equivalent circuit (or the steady-state model) of an ULTC transformer is given as a cascade of two ideal transformers, as shown in Fig. 5.

Now for the premise that both transformers in Fig. 5 are ideal, with turn-ratios a:1 and $N_1 : N_2$, the following relationships between currents and voltages exist:

$$\overline{I}_1' = a\overline{I}_1$$

$$\overline{I}_2 = \frac{N_1}{N_2}\overline{I}_1' = \frac{N_1}{N_2} a\overline{I}_1$$

$$\overline{V}_1' = \frac{\overline{V}_1}{a}$$

$$\overline{V}_2' = \frac{N_2}{N_1}\overline{V}_1' = \frac{1}{a}\frac{N_2}{N_1}\overline{V}_1$$

$$\overline{V}_2 = \overline{V}_2' - \overline{Z}_T^{(2)}(a)\,\overline{I}_2 = \frac{1}{a}\frac{N_2}{N_1}\overline{V}_1 - \overline{Z}_T^{(2)}(a)\frac{N_1}{N_2}a\overline{I}_1 \; . \tag{A1-3}$$

Converting Equation (A1-3) into pu quantities, by dividing both its sides with V_{2B}, such that

$$I_{1B}V_{1B} = I_{2B}V_{2B} = S_B , \tag{A1-4}$$

where subscript "B" designates the arbitrary base quantity of the indicated variable (if possible, the rated values of currents, voltages and apparent capacity of the transformer), yields

$$\overline{v}_2 = \frac{\overline{V}_2}{\overline{V}_{2B}} = \frac{1}{a}\frac{N_2}{N_1}\frac{\overline{V}_1}{V_{2B}} \cdot \frac{V_{1B}}{V_{1B}} + \overline{Z}_T^{(2)}(a)\frac{\overline{I}_2}{V_{2B}} \cdot \frac{I_{2B}}{I_{2B}} \quad , \text{or}$$

$$\overline{v}_2 = \frac{1}{a}\frac{N_2}{N_1}\overline{v}_1\frac{V_{1B}}{V_{2B}} - \overline{Z}_T^{(2)}(a)\,\overline{i}_2\frac{I_{2B}}{V_{2B}} \quad . \tag{A1-5}$$

Since

$$\frac{V_{1B}}{V_{2B}} = \frac{I_{2B}}{I_{1B}} = \frac{N_1}{N_2} \quad ,$$

$$\frac{V_{2B}}{I_{2B}} = Z_{TB}^{(2)} \quad , \tag{A1-6}$$

the Expression (A1-5), with all quantities expressed in pu, becomes

$$\overline{v}_2 = \frac{\overline{v}_1}{a} - \overline{z}_T(a)\,a\overline{i}_1 = \frac{\overline{v}_1}{a} - \overline{z}_T(a)\,\overline{i}_2 \quad ; \tag{A1-7}$$

or

$$\overline{v}_1 = a\overline{v}_2 + \overline{z}_T(a)\,a\overline{i}_2 = a\overline{v}_2 - \overline{z}_T(a)\,a^2\overline{i}_1 \quad , \tag{A1-8}$$

where the per-unit quantities of the transformer are defined as

$$\overline{i}_1 = \frac{\overline{I}_1}{I_{1B}} \; ; \quad \overline{i}_2 = \frac{\overline{I}_2}{I_{2B}} \; ; \tag{A1-9}$$

$$\overline{v}_1 = \frac{\overline{V}_1}{V_{1B}} \; ; \quad \overline{v}_2 = \frac{\overline{V}_2}{V_{2B}} \; ; \tag{A1-10}$$

$$\overline{z}_T = r_T + jx_T = \frac{\overline{Z}_T^{(2)}}{Z_{TB}^{(2)}} = \overline{Z}_T^{(2)} \cdot \frac{I_{2B}}{V_{2B}} \cdot \frac{3V_{2B}}{3V_{2B}} = \overline{Z}_T^{(2)} \cdot \frac{3S_B}{3V_B^2}$$

$$= \frac{\overline{Z}_T^{(2)} \cdot S_{B3\phi}}{V_{2B(L-L)}^2} = \frac{\overline{Z}_T^{(2)} S_r}{V_{2r(L-L)}^2} = \frac{Z_T^{(1)} S_r}{V_{1r(L-L)}^2} \; . \tag{A1-11}$$

In (A1-11) is assumed that $V_{1B} = V_{1r}$; $V_{2B} = V_{2r}$; $S_B = S_r/3$, (i.e., the base quantities of the transformers are equal to corresponding rated or nominal quantities, so that S_r represents the three-phase apparent capacity and $V_{1r(L-L)}$ and $V_{2r(L-L)}$ the line-to-line nominal voltages of the primary and secondary windings, respectively). With pu quantities introduced by Equations (A1-7) - (A1-11), the ideal transformer with turn ratio $N_1 : N_2$ in Fig. 5 disappears, thus simplifying this steady-state ULTC transformer model.

Equations (A1-7) and (A1-8) could be modified to a more familiar form, by introducing the load rather than currents. By neglecting transformer losses (small, compared to both its capacity and load), the following relations hold:

$\overline{S}_1 \approx \overline{S}_2 = P - jQ$ is three-phase transformer load;

$\overline{I}_2 = \frac{P - jQ}{3\overline{V}_2^*}$ is secondary transformer current, (A1-12)

where * designates the conjugate-complex variable.

Then

$$\overline{v}_2 = \frac{1}{a}\overline{v}_1 - a\frac{r_T(a)p + x_T(a)q}{\overline{v}_1^*} - ja\frac{x_T(a)p - r_T(a)q}{\overline{v}_1^*}\ ; \quad \text{(A1-13)}$$

$$\overline{v}_1 = a\overline{v}_2 + a\frac{r_T(a)p + x_T(a)q}{\overline{v}_2^*} + ja\frac{x_T(a)p - r_T(a)q}{\overline{v}_2^*}\ , \quad \text{(A1-14)}$$

where $p = P/S_B$; $q = Q/S_B$ are pu active and reactive loads, respectively.

Expressions (A1-13) and (A1-14) represent a steady-state ULTC transformer model.

APPENDIX 2

SYSTEM DATA USED IN SIMULATION STUDIES

1. ULTC Transformer

   S_r = 63/63/21 MVA, three-phase, three-winding unit
   V_{1r} = 110 kV, Y-connected, HV supply
   V_{2r} = 36.75 kV, Y-connected, distribution feeding
   V_{3r} = 10.50 kV, Δ-connected, compensating winding, not used for load supply
   I_{1r} = 331 A ; I_{2r} = 990 A
   m_{r1} = 110/36.75 = 2.993; m_{r2} = 110/10.5 = 10.476
   Tap winding: On HV side
   Tap range: $k_M = \pm 10 \times 1.5\%$
   Transformer nominal resistance : $r_{12}^0 = 0.5\%$
   Transformer nominal leakage reactance: $x_{12}^0 = 10.5\%$
   (Variation of r_{12} and x_{12} with the change of tap position neglected)
   Load: P = 42 MW; Q = 31.5 MVAR.

2. Tap changer mechanism

   T_M = 10 s ; n_o - variable

   $a_M = 15\%$

3. Voltage regulator

   DB - variable ; reference voltage: v_{2r} = 1.00 pu
   $\varepsilon = 0$
   T_{do} = 100 s; inverse time-delay characteristic.

   $r_c = 0$

   $x_c = 7.5\%$; additive compensation

4. Potential transformer

   $$m_{PT} = 35000V/100V = 350; \quad \frac{V_{2r}}{V'_{2r}} = \frac{36.75}{35} = 1.05$$

5. Current transformer

   $$m_{cT} = 1000A/5A = 200: \quad \frac{I_{2r}}{I'_{2r}} = \frac{990}{1000} = 0.99$$

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