DISCRETE MODELS OF SLOW VOLTAGE DYNAMICS FOR UNDER LOAD TAP-CHANGING TRANSFORMER COORDINATION

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ABSTRACT

In this paper, an approach to modeling, analysis, and design of slow distributed voltage control schemes is proposed. In particular, a dynamical voltage model governed by the under load tap-changing transformers as control tools is studied. Rigorous conditions are derived to predict when the LTC based scheme may be poorly coordinated and not able to maintain voltages within the limits. The proposition is that nonconvergence of LTC control scheme is one of the causes of a systemwide voltage collapse.

1. INTRODUCTION

Studies of recent voltage collapse related blackouts throughout the world [1],[2] clearly indicate that changes in voltage magnitudes after a large contingency cannot be neglected. State of the art in security assessment [3] is such that the main results pertain to the active power network under the assumption that voltages do not change significantly. Depending on the topology and composition of the network, this may or may not be justified. It is true that because of the definite time-scale separation between the dynamic changes in frequency and those in voltage due to secondary voltage control in the transient stability analysis, voltages can affect stability regions only as slowly varying parameters. It is the analysis of the slower phenomenon of voltage changes due to changes in the slow voltage controls (under load tap changing (ULTC) transformers, capacitors) on the load side that we study in this work. In this analysis, voltage changes must be taken into account; moreover, the fact that in voltage collapse caused blackouts frequency did not change appreciably can be used to justify, in the first analysis, voltage - reactive power changes due to slow controllers under the commonly accepted decoupling assumptions between the active power - frequency and reactive power - voltage. Under this assumption, we formulate a model to study this slow dynamics of load voltage changes due to local voltage control and reveal potential difficulties of the presently utilized schemes. We show that this model belongs to the class of nonlinear discrete type dynamical models. General analytical tools for analysis of these models are not available.

The main contribution of this paper is in the understanding of coordination issues of the LTC based distributed control. Explicit network configurations are introduced to test when the LTC control scheme may not be fully reliable and proposals are presented, on a theoretical level, to improve the performance of this slow and discrete control layer in power networks.

The approach is qualitatively different from existing approaches where the voltage collapse is studied on an oversimplified single generator, single load model. We claim that it is the improper coordination of different (local) voltage control tools which may be the major cause of voltage collapse. Theoretical analysis leading to procedures to prevent voltage collapse in a given network are suggested based on partial centralized control, i.e., on partial exchange of information within the network. And finally, a small network example is discussed together with the simulation results which illustrate the main theoretical conclusions.

2. BACKGROUND

Under load tap changing transformers (LTC's) are control tools used on a large power system to maintain local voltages within desirable limits. They are activated when a load voltage of the controlled bus deviates outside the allowable limits. Let the nominal tap position of the LTC in line connecting nodes i and j, controlling bus i, be denoted by $a^0_{i,j}$, let the corresponding nominal voltage at node i be V^0_i, and let the allowable voltage deviation be ΔV_i. When the operating conditions in the system change, each LTC in the system will change its tap position, if necessary to maintain local voltages within the given limits. This change will take the form of discrete equal-valued changes in the tap position, separated by the duration of the LTC operating cycle. At each change, the resulting bus voltages will be assumed to have settled in the new operating state which is assumed to satisfy the decoupled reactive power - voltage relations [4]. Enumerating buses in the power system so that the first n buses are the load buses and the remaining k buses are generator buses (including the slack bus), and using the notation:

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$V \epsilon R^n$ - is the vector of voltages at the load buses

$X \epsilon R^m$ - is the vector of voltages at the ULTC controlled load buses

$a \epsilon R^m$ - is the vector of current tap positions

$E \epsilon R^k$ - is the vector of voltages at generating buses

The reactive power - voltage equations at some nominal operating point (V^0, E, a^0) obey

$$Q_i(V, E, a) \Big|_{\substack{V = V^0 \\ a = a^0}} = 0, \; i = 1,\ldots,n \qquad (1)$$

with the controlled voltages given by

$$X^0 = C \cdot V^0 \qquad (2)$$

where $C \epsilon R^{mxn}$ is a matrix of zeros and ones extracting the controlled load voltages from the vector of load voltages.

The slow dynamics associated with the changes in the tap positions of the LTC can, in view of present practice [5], be expressed by the following set of discrete equations:

$$a_{i,j}(k + 1) = a_{i,j}(k) + d_i \cdot f(X_i - X_i^{ref}) \qquad (3)$$

where the subscripts i and j indicate that the ULTC located in the line between nodes i and j monitors and controls voltage at node i, d_i denotes the step-size in the change of the tap position during one operating cycle of the LTC and $f(X_i - X_i^{ref})$ is the control function governing the operating of the LTC and is given by

$$f(X_i - X_i^{ref}) = \begin{cases} 1 \;, & X_i - X_i^{ref} > \Delta V_i \\ 0 \;, & |X_i - X_i^{ref}| < \Delta V_i \\ -1 \;, & X_i - X_i^{ref} < -\Delta V_i \end{cases} \qquad (4)$$

Thus, if

$$|X_i - X_i^{ref}| < \Delta V_i \qquad (5)$$

no LTC action results, i.e., tap position of the i^{th} transformer remains unchanged. If, for some i, $|X_i - X_i^{ref}| > \Delta V_i$, the tap ratio is changed according to (3). Subtracting the nominal value $a_{i,j}^0$ from both sides of (3) and denoting the change in the tap position from the nominal by $b_{i,j}$, and the deviation of voltage at controlled buses from the reference values by x_i, we have

$$b_{i,j}(k) = a_{i,j}(k) - a_{i,j}^0 \qquad (6)$$

$$x_i(k) = X_i(k) - X_i^{ref}$$

and (3) can be written in the form

$$b_{i,j}(k + 1) = b_{i,j}(k) + d_i f_i(x_i) \;, \; i = 1,\ldots,m \qquad (7)$$

or in vector form

$$b(k + 1) = b(k) + Df(x) \qquad (8)$$

where $b \epsilon R^m$ is a vector with components $b_{i,j}$, $x \epsilon R^m$ is a vector with components x_i, D is a diagonal matrix with components d_i and f(x) is a vector function of the vector variable x with components $f_i(x_i)$ given by (4).

We wish to study the convergence of the decentralized control law embodied in (7), subject to the operating constraints (1). The problem is difficult, because (1) is nonlinear. To proceed, we make the additional assumption that we may linearize equation (1) around a nominal operating condition, with the purpose of understanding the nature of the discrete dynamic process characterized by (7) in the vicinity of this nominal operating point. Of course, the larger the region in which linearization is sufficiently accurate, the larger the region in which the results of this analysis would be valid.

Since linearization of the Q-V decoupled equations after normalization by V_i^0, respectively for i=1,...,n, has been reported to hold with good accuracy for a larger vicinity around that nominal operating point [6] than the linearization of the original power flow equations, this normalization will be applied first to (1). The implied assumption is that linearization around (V^0, a^0) will be acceptable in a region $A = \{a : |a_{i,j} - a_{i,j}^0| < \Delta a_i, \; i = 1, \ldots, m\}$ encompassing the actual tap ratio limits and the region $V = \{V : |V_i - V_i^0| < \Delta V_i, \; l = 1,\ldots,m\}$ of magnitude greater than the allowable limits ΔV_i and sufficiently large to consider variations of load voltages under the influence of variations of tap ratios. We will assume that in addition $V^0 = V^{ref}$. Under this assumption, the linearized Q-V equations at the k^{th} step in the discrete evolution of the voltage and tap ratio changes take the form

$$\frac{\partial Q}{\partial V}\left(V(k) - V^0\right) + \frac{\partial Q}{\partial a}\left(a(k) - a^0\right) = 0 \qquad (9)$$

with the asumption that $\frac{\partial Q}{\partial V}$ and $\frac{\partial Q}{\partial a}$ can be considered constant matrices in the considered region of variation of V and a. It follows, under the assumption that $\frac{\partial Q}{\partial V}$ is nonsingular that

$$V(k) - V^0 = -\left(\frac{\partial Q}{\partial V}\right)^{-1} \frac{\partial Q}{\partial a}\left(a(k) - a^0\right) \qquad (10)$$

and using (2) that

$$x(k) = X(k) - X^0 = C\left(V(k) - V^0\right) = -C\left(\frac{\partial Q}{\partial A}\right)^{-1}\left(\frac{\partial Q}{\partial a}\right) b(k) \qquad (11)$$

Now premultiplying (8) by

$$A = C\left(\frac{\partial Q}{\partial V}\right)^{-1}\left(\frac{\partial Q}{\partial a}\right) \qquad (12)$$

we have

$$Ab(k + 1) = Ab(k) + ADf(x) \qquad (13)$$

and utilizing (11)

$$x(k + 1) = x(k) - ADf(x) \qquad (14)$$

Equation (14) characterizes, in the form of a discrete system with the specific relay-type nonlinearity with a dead zone, the essential slow dynamic variation of voltage at the controlled nodes in function of tap ratio changes.

3. DISCRETE-TIME ANALYSIS

We now study the dynamics of voltage variations in the vicinity of the nominal operating point. Our goal is to determine the basic conditions under which the controlled voltages will return to their nominal values, and conversely, conditions under which some of the controlled voltages will diverge from their nominal values when voltages at the controlled buses are perturbed from the nominal and the control strategy embodied in (3), and leading to the discrete dynamics (14), is used to adjust the tap ratios and control the voltages.

Because of (4), we will let X_a denote the target set, defined by

$$X_a = \{x : |x| < |\Delta V_i|, \ i = 1,\ldots,m\} \qquad (15)$$

such that if $x \in X_a$, then all LTC's will be inactive since all controlled voltages are within prescribed limits, and if $x \notin X_a$, then at least one LTC will operate because at least one controlled voltage is outside the prescribed limits. Clearly, given $x \notin X_a$ of interest is whether x will converge to X_a, and if not, what are the conditions on A and D such that, at least for some points $x \notin X_a$ under the action of (14), x will not converge to X_a.

Before proceeding, we formulate certain simplyfing assumptions under which convergence will be considered in this paper. Specifically, we will assume:

(a) All duty cycles are of the same order of magnitude;

(b) All step sizes are of the same magnitude (thus we may take D = I in (14));

(c) The change in tap ratio is small enough so that voltage deviations after tap change are always smaller than the width of the allowable voltage deviations.

We will discuss subsequently the influence of these assumptions on convergence. With these assumptions, we may easily establish the following results:

THEOREM 1: Suppose A is diagonally dominant with all diagonal elements positive. Then the sequence x(k), k = 1,2,..., generated by (14) converges to X_a.

PROOF: From equation (14) we have

$$x_i(k+1) = x_i(k) - \sum_{j=1}^{m} a_{ij} f_j(x_j(k)) =$$

$$= x_i(k) - a_{ii} f_i(x_i(k)) - \sum_{\substack{j=1 \\ j \neq i}}^{m} a_{ij} f_j(x_j(k)) \qquad (16)$$

Invoking the diagonal dominance condition

$$|a_{ii}| > \sum_{j \neq i}^{m} |a_{ij}| \qquad (17)$$

and the fact that $|f_j(x_j(k))| = 1$ we see that the summation term in (16) is of smaller magnitude than the term $a_{ii} f_i(x_i(k))$, and so the summation cannot change the sign of the variation of $(x_i(k+1) - x_i(k))$ as defined by the term $a_{ii} f_i(x_i(k))$. Assumption (c) guarantees that the summation term will not superimpose on changes due to the first term in such a way as to cause a voltage jump over the allowed region. To see this, observe that from (14)

$$x_i(k+1) - x_i(k) + a_{ii} = \sum_{j \neq i}^{m} a_{ij} f_j(x_j(k))$$

for $x_i(k) > \Delta V_i$. Therefore

$$|x_i(k+1) - x_i(k) + a_{ii}| =$$

$$\left|\sum_{j\neq i}^{m} a_{ij} f_j(x_j(k))\right| < \sum_{j\neq i}^{m} |a_{ij}|\, |f_j(V_j(k))| < \sum_{j\neq i}^{m} |a_{ij}| < a_{ii}$$

from which follows

$$-a_{ii} < x_i(k+1) - x_i(k) + a_{ii} < a_{ii}$$

or

$$x_i(k) - 2a_{ii} < x_i(k+1) < x_i(k) \qquad (18)$$

Similarly for $x_i(k) < -\Delta V_i$, we obtain by repeating the procedure

$$x_i(k) < x_i(k+1) < x_i(k) + 2a_{ii} \qquad (19)$$

Therefore,

$$|x_i(k+1)| < |x_i(k)| \qquad (20)$$

whenever

$$a_{ii} < \Delta V_i \ . \qquad (21)$$

Since this is true for all i, and since A will retain the diagonally dominant property if any row and corresponding column are deleted (as must be considered when for certain $j, f_j(x_j(k)) = 0$ because the jth voltage is within prescribed limits), we see that x(k) converges to X_a. QED

The next theorem shows convergence by utilizing a Lyapunov function approach to stability of discrete systems [7]. Let Q be a diagonal matrix with positive elements. Then the function

$$V(x) = f(x)^T Qx \qquad (22)$$

is a candidate function for (14). To see this, observe that

$$V(x(k)) = f(x(k))^T Qx(k) = \sum_{i=1}^{m} q_{ii} f_i(x_i(k)) x_i(k)$$

and so in view of (4), we find that $V(x(k)) \geq 0$ for all x(k) and $V(x(k)) = 0$ for $x(k) \in X_a$.

THEOREM 2: Suppose there exists a diagonal Q > 0 such that

$$A^T Q + QA = P > 0 \ . \qquad (23)$$

Then the sequence x(k), k = 1,2,... generated by (14) converges to X_a.

PROOF: The proof of this theorem is rather lengthy because of the many different cases that have to be considered. In two consecutive iterations, we may have the following main cases: (a) $f(x(k+1)) = f(x(k))$, i.e., there is no change in the components of $f(x)$ in two consecutive steps, (b) only one component of $f(x)$ changes value in two consecutive iterations, and (c) more than one component of $f(x)$ changes value in two iterations. We now prove that the difference $V(x(k+1)) - V(x(k))$ decreases in consecutive iterations for cases (a) and (b) above and indicate how the results is shown to hold for case (c).

Consider the difference $V(x(k+1)) - V(x(k))$. We have

$$\begin{aligned} V(x(k+1)) &= x(k+1)^T Q f(x(k+1)) \\ &= f(x(k+1))^T Q(x(k) - Af(x(k))) \\ &= f(x(k+1))^T Q x(k) - f(x(k+1))^T QAf(x(k)) \\ &= f(x(k))^T Q x(k) + [f(f(k+1)) \\ &\quad - f(x(k))]^T Q x(k) - f(x(k+1))^T QAf(x(k)) \end{aligned} \tag{24}$$

We now consider separately the main cases that occur.

(a) $f(x(k+1)) = f(x(k))$, i.e., no change in the values of the components of $f(x)$ occurs in two consecutive steps.

In this case we have from (24)

$$\begin{aligned} V(x(k+1)) - V(x(k)) &= -f(x(k))^T QAf(x(k)) \\ &= -\frac{1}{2} f(x(k))^T (A^T Q + QA) f(x(k)) \\ &= -\frac{1}{2} f(x(k))^T P f(x(k)) < 0 \end{aligned}$$

and so the value of $V(x(k+1))$ is smaller than $V(x(k))$.

(b) $f_i(x(k))=0$, $f_i(x(k+1))=1$, while $f_j(x(k+1))= f_j(x(k))$, $j \neq i$;

Observe first that $f_i(x_i(k)) = 0$ and $f_i(x_i(k+1)) = 1$ implies that $x_i(k+1)$ is negative (i.e., more negative than $-\Delta V_i$) so

$$e_i^T x(k+1) = x_i(k+1) < 0 \ , \tag{25}$$

where $e_i^T = [0\ 0\ \ldots\ 1\ \ldots\ 0]$. Second, observe that we may relate $f(x(k+1))$ to $f(x(k))$ as follows:

$$f(x(k+1)) = \begin{bmatrix} x \\ x \\ \bullet \\ \bullet \\ \bullet \\ 1 \\ x \\ x \end{bmatrix} = \begin{bmatrix} x \\ x \\ \bullet \\ \bullet \\ \bullet \\ 0 \\ x \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \bullet \\ \bullet \\ \bullet \\ 1 \\ 0 \\ 0 \end{bmatrix} = f(x(k)) + e_i \tag{26}$$

Thus,

$$f(x(k+1)) - f(x(k)) = e_i \tag{27}$$

and substituting (27) into (24) we find

$$\begin{aligned} V(x(k+1)) &= V(x(k)) - f(x(k))^T QAf(x(k)) + e_i^T Q x(k+1) \\ &= V(x(k)) - \frac{1}{2} f(x(k))^T (A^T Q + QA) f(x(k)) + \\ &\quad + e_i^T Q x(k+1) \end{aligned} \tag{28}$$

Now, by assumption (23), the second term is negative. The last term need not be negative for arbitrary positive definite Q. In fact, since the second term is finite while the last term can be made as large as desired by the choice of $x(k+1)$, it follows that for most Q there are points in the state space for which $V(x(k+1)) - V(x(k))$ can be made positive.

The only exceptions are the cases when $Q = I$, as well as the case when Q is a diagonal matrix, because then $e_i^T Q x(k+1) = q_{ii} x_i(k+1) < 0$. Thus, the difference $V(x(k+1)) - V(x(k))$ is negative in this case.

A similar development can be used to show that the same is true for all other possible changes of only one component of $f(x)$. For example, if $f_i(x(k)) = 0$, and $f_i(x(k+1)) = -1$, then instead of (26), we have

$$f_i(x(k+1)) = f(x(k)) - e_i \tag{29}$$

and

$$e_i^T x(k+1) > 0 \ . \tag{30}$$

In this case, instead of (28) we have

$$\begin{aligned} V(x(k+1)) &= V(x(k)) - \frac{1}{2} f(x(k))^T (A^T Q + QA) f(x(k)) \\ &\quad - e_i^T Q x(k+1) \ , \end{aligned} \tag{31}$$

and, so in view of (30) again for Q diagonal, $V(x(k+1)) - V(x(k))$ is negative. Other cases can be shown to lead to a decreasing sequence of $V(x(k))$ in the same manner.

(c) More than one component of $f(x)$ changes in consecutive iterations.

In this case we will in general have

$$f(x(k+1)) = f(x(k)) + \sum_{i \in S_1} e_i - \sum_{j \in S_2} e_j \tag{32}$$

and by definition of S_1 and S_2

$$\begin{aligned} &e_i^T x(k+1) < 0, \quad \text{for all } i \in S_1 \\ &e_i^T x(k+1) > 0, \quad \text{for all } j \in S_2 \ . \end{aligned} \tag{33}$$

Then, by a parallel development, we have in this case that

$$\begin{aligned} V(x(k+1)) &= V(x(k)) - \frac{1}{2} f(x(k))^T (A^T Q + QA) f(x(k)) + \\ &\quad + \sum_{i \in S_1} e_i^T Q x(k+1) - \sum_{i \in S_2} e_i^T Q x(k+1) \end{aligned} \tag{34}$$

and in view of (33), we have that $V(x(k))$ is again a decreasing sequence, and so that $x(k)$ converges to X_a.

Theorems 1 and 2 provide important, practically verifiable conditions for the assessment of convergence. It is also immediately clear how different step sizes will affect the results since all that has to be considered is the matrix AD instead of the matrix A.

4. CONTINUOUS MODEL ANALYSIS

Consideration of simple examples, however, indicates that the class of systems for which convergence results is broader than captured by Theorems 1 and 2. These experimental indications are in agreement with convergence results that can be obtained when the discrete model (14) is approximated for purposes of analysis by a continuous model. We therefore present a qualitative analysis of the problem by resorting to an approximation of our discrete dynamic process by a continuous process.

We recall that (14) arises under the assumption that operating cycles of all LTC's have the same (or almost the same) duration. While we do not necessarily associate the counter k in (14) with time-instants, the discrete equation (14) can be viewed as a discretization of a continuous dynamic system. Because of the particular form of relay type nonlinearity in (14), we utilize the following device to construct a plausible continuous generator equation for (14). To this end, introduce ε and write (14) as

$$\frac{x(k+1) - x(k)}{\varepsilon} = -ADf(x) \tag{35}$$

and now let $\varepsilon \to 0$. We will approximate the result of this process by defining

$$\frac{x(k+1) - x(k)}{\varepsilon} \approx \frac{dx}{d\lambda} \tag{36}$$

and write the result as

$$\varepsilon \frac{dx}{d\lambda} = -ADf(x) \ , \tag{37}$$

where ε is a small parameter. The model (37) represents a totally singular problem in the theory of singular perturbations, which can therefore be studied in the so-called fast time scale, by defining

$$\varepsilon \frac{dx}{d\lambda} = \frac{dx}{d(\lambda/\varepsilon)} = \frac{dx}{d\tau} \cong \dot{x} \tag{38}$$

whereupon (18) becomes

$$\dot{x} = -ADf(x) \tag{39}$$

We now use (39) to study the behavior of $x(\tau)$ given $x_0 \notin X_a$. To this end, we again assume that $D = I$, which in effect implies that the tap ratio changes steps are the same for all LTC's (or that D is absorbed in A). This reduces (39) to

$$\dot{x} = -Af(x) \ . \tag{40}$$

It is now easy to show that using (40) we can recover the results of Theorems 1 and 2. In addition, we also prove the following stronger results:

THEOREM 3: Suppose A is such that there exists a $Q > 0$ and diagonally dominant such that

$$A^TQ + QA = P > 0 \tag{41}$$

Then (14) converges to X_a.

PROOF: We first show that

$$V(x) = x(k)^TQf(x(k)) \tag{42}$$

is a candidate Lyapunov function for the discrete process (14). First we note that if $x \in X_a$, then $f(x) = 0$ and $V(x) = 0$. Second, for all $x \notin X_a$, we show that $V(x) > 0$. We have

$$V(x) = x(k)^TQ(x(k)) = \sum_{i=1}^{m}\sum_{j=1}^{m} x_i(k)q_{ij}f_j(x_j(k)) \tag{43}$$

and so because $|f_i(x_i(k))| = 1$ and

$$\left|\sum_{j\neq i}^{m} q_{ij}f_j(x_j(k))\right| \le \sum_{j\neq i}^{m} |q_{ij}||f_j(x_j(k))| \le$$

$$\le \sum_{j\neq i}^{m} |q_{ij}| < |q_{ii}| \tag{44}$$

The last summation over j cannot, therefore, alter the sign of the expression $q_{ii}f_i(x_i(k)) + \sum_{j\neq i}^{m} q_{ij}f_j(x_j(k))$ with respect to the sign characterizing the term $q_{ii}f_i(x_i(k))$, and so the sign of each term in the summation (43) over i is the same as that of $q_{ii}x_if_i(x_i(k)) > 0$. Therefore, $V(x(k)) > 0$, and when $x(k) \in X_a$, then $V(x(k)) = 0$.

Now, differentiating (42), we obtain

$$\dot{V} = f(x)^TQ\dot{x} = -f(x)^TQAf(x) = -\frac{1}{2}f(x)^T(A^TQ + QA)f(x) \tag{45}$$

and so if condition (41) is satisfied $V(x) < 0$, and x converges to X_a.

5. STABILITY ENHANCEMENT

Our final result again resorts to the continuous model. Suppose that A is not diagonally dominant, and $(A^T + A)$ is not positive definite or that conditions in Theorem 3 cannot be established for a given A. Then we have no proof that x will converge to X_a. Suppose, however, that instead of the decentralized control law, a centralized control law is used. More precisely, suppose

$$\dot{x} = -Af(u) \tag{46}$$

and

$$u = Qx \tag{47}$$

where Q is as yet unspecified. This of course means that certain linear combinations of all controlled voltages is used to define the signal controlling the LTC.

We now proceed to show that whenever $-A$ is stable we can select a positive definite matrix Q in (47) and guarantee stability. Since $Q > 0$, take the Lyapunov function to be

$$V(x) = f(Qx)^TQx \tag{48}$$

This is a positive function because $f(u)^Tu \geq 0$ as shown in proof of Theorem 3. Then

$$\dot{V} = f(Qx)^T Q\dot{x}$$
$$= f(Qx)QAf(Qx)$$
$$= -\frac{1}{2} f(Qx)^T(A^TQ + QA)f(Qx) \tag{49}$$

and taking Q such that

$$+ A^TQ + QA = + P > 0, \tag{50}$$

it follows that $\dot{V} < 0$, and x converges to X_a. But this means that if $-A$ is Hurwitz, and P is a positive definite matrix selected to guarantee a certain rate of decrease of $\dot{V}(x)$, then Q in (47) should be taken as the solution of the Lyapunov equation (50). Thus, the implications of this result are that if $-A$ is Hurwitz, even if the decentralized control law does not stabilize the system, a centralized control law can always be selected to guarantee convergence. If $-A$ is not Hurwitz, it is not clear whether there are even centralized control laws that will always guarantee convergence, and so this is an open problem for which further research is necessary. In addition, it is necessary to show that the same results essentially hold for the actual discrete dynamics, where the finite step length is the major impediment to proving analogous results as for the continuous model. In addition, it will be necessary to consider the effect of different duty cycles, which essentially reduce to different sampling rates of different variables, but it is not clear that these could be reduced to a continuous model for qualitative analysis via, for example, multiple time scales in singularly perturbed models, and so this issue demands more research at the modeling level.

6. AN EXAMPLE

Ths six bus Ward-Hale system [8] will be used to illustrate the implications of our theoretical results. As can be seen from the one-line diagram of this system, Figure 1, it contains two ULTC transformers, locally controlling voltages V_3 and V_5. Thus, this example is of sufficient complexity to discuss the main issues of ULTC coordination with decentralized control strategies.

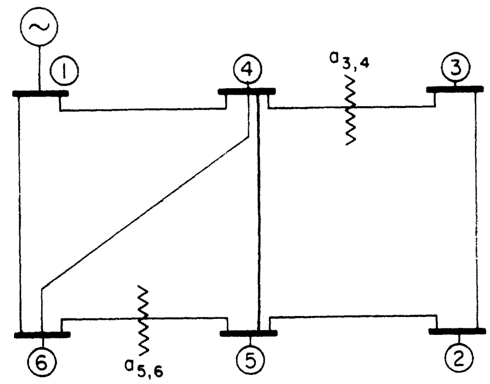

Figure 1. Ward-Hale Power System.

The standard Π section equivalent model for transmission lines is used [8]. For example, a line connecting nodes 4 and 3 has an equivalent circuit given in Figure 2.

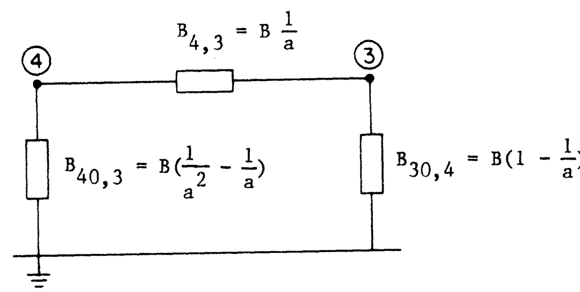

Figure 2. An Equivalent Line Model.

CASE #	INITIAL VOLTAGE POSITIONS	INITIAL TAP POSITIONS	REFERENCE VOLTAGES	REACTIVE POWER FOR DEMAND	CONVERGENCE	NUMBER OF ITERATIONS	FINAL STATE
I	V_3^0= .8307 , V_4^0=.9310 V_5^0=.8511 , V_6^0=.9058	t_{34}^0 =1.100 t_{56}^0 =1.025	V_3^{ref}=1.00 V_5^{ref}= .85	Q_3=.130 , Q_4=0.00 Q_5=.180 , Q_6=.050	NO	60	V_3=.9184 , V_4=.6557 V_5=.8407 , V_6=.7399
II	V_3^0=1.1005 , V_4^0=.9296 V_5^0=.9590 , V_6^0=.9190	t_{34}^0 =1.100 t_{56}^0 =1.025	V_3^{ref}=1.00 V_5^{ref}=1.00	Q_3=.130 , Q_4=0.00 Q_5= .180, Q_6=.050	YES	10	V_3=1.0074, V_4=.9228 V_5= .9904, V_6=.9067
III	V_3^0=1.1005 , V_4^0=.9296 V_5^0=.9590 , V_6^0=.9190	t_{34}^0 =1.100 t_{56}^0 =1.025	V_3^{ref}=1.00 V_5^{ref}=1.00	Q_3=-8.299,Q_4=0.00 Q_5=-1.80, Q_6=.050	YES	29	V_3=1.0090, V_4=1.0263 V_5=1.0088, V_6= .9318
IV	V_3^0=1.1005 , V_4^0=.9296 V_5^0=.9590 , V_6^0=.9190	t_{34}^0 =1.100 t_{56}^0 =1.025	V_3^{ref}=1.00 V_5^{ref}=1.00	Q_3=1.30 , Q_4=0.00 Q_5=1.80 ,Q_6=.050	YES	7	V_3=1.0088, V_4=.9098 V_5= .9980, V_6=.9145
V	V_3^0=1.1005 , V_4^0=.9296 V_5^0=.9590 , V_6^0=.9190	t_{34}^0 =1.100 t_{56}^0 =1.025	V_3^{ref}=1.00 V_5^{ref}=1.00	Q_3=1.30 , Q_4=0.00 Q_5=2.70 , Q_6=.050	NO	60	V_3=.9957 , V_4=.7519 V_5=.6929 , V_6=.5436

* All values are in per unit
** Capacitive load

TABLE 1

Its tap position dependent admittances are

$$B_{4,3} = \frac{B}{a} \tag{51}$$

$$B_{30,4} = B(1 - \frac{1}{a}) \tag{52}$$

$$B_{40,3} = B(\frac{1}{a^2} - \frac{1}{a}) \tag{53}$$

Parameter B represents the admittance of the line when $a = 1$.

The reactive power - voltage constraints (1) in this example need to be formulated for the load nodes $i = 3,4,5,6$ and take on the form [6],[9]

$$(\sum B_{ij} - B_i)V_i^2 - \sum_{\substack{j=1 \\ j\neq i}}^{n} B_i V_i V_j - \sum_{n+1}^{n+k} B_{ij} V_i V_j + Q_i = 0 \tag{54}$$

where B_{ij} and B_i are the susceptance of transmission line i_j and the shunt susceptance at bus i, respectively. After the normalization by V_i [6], the reactive current becomes

$$(\sum_{j\varepsilon Ci} B_{ij} - B_i)V_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} B_{ij} V_j - \sum_{n+1}^{n+k} B_{ij} V_j + \frac{Q_i}{V_i} = 0 \tag{55}$$

where C is the set of nodes directly connected to node i. The sensitivity matrix $(\frac{\partial Q}{\partial V})$ required for the control coordination studies here takes the form

$$\frac{\partial Q}{\partial V} = \begin{bmatrix} (\sum B_{3j} - B_3 - \frac{Q_3}{V_3^2}) & -B_{34} & -B_{35} & -B_{36} \\ -B_{34} & (\sum B_{4j} - B_4 - \frac{Q_4}{V_4^2}) & -B_{45} & -B_{46} \\ -B_{35} & -B_{45} & (\sum B_{5j} - B_5 - \frac{Q_5}{V_5^2}) & -B_{56} \\ -B_{36} & -B_{46} & -B_{56} & (\sum B_{6j} - B_6 - \frac{Q_6}{V_6^2}) \end{bmatrix} \tag{56}$$

Tap position dependent terms in the matrix $(\frac{\partial Q}{\partial V}) = m_{ij}, i,j = 1,\ldots,m$ are all along the main diagonal except for terms

$$B_{34} = B_{34}(a_{34}) \text{ and } B_{56} = B_{56}(a_{56}) .$$

With the numerical data given for normal operating conditions [8], this matrix reduces to

$$(\frac{\partial Q}{\partial V}) = \begin{bmatrix} (8.192 - \frac{Q_3}{V_3^2}) & -\frac{7.55}{a_{34}} & 0 & 0 \\ -\frac{7.55}{a_{34}} & (4.873 + \frac{7.55}{a_{34}^2} - \frac{Q_4}{V_4^2}) & 0 & -2.323 \\ 0 & 0 & (4.643 - \frac{Q_5}{V_5^2}) & -\frac{3.33}{a_{56}} \\ 0 & -2.323 & \frac{-3.33}{a_{56}} & (4.146 + \frac{3.33}{a_{56}^2} - \frac{Q_6}{V_6^2}) \end{bmatrix} \tag{57}$$

Similarly, the sensitivity matrix $(\frac{\partial Q}{\partial a})\Big|_0$ here takes the form

$$(\frac{\partial Q}{\partial a})\Big|_0 = \begin{bmatrix} \frac{\partial Q_3}{\partial a_{34}} & \frac{\partial Q_3}{\partial a_{56}} \\ \frac{\partial Q_4}{\partial a_{34}} & \frac{\partial Q_4}{\partial a_{56}} \\ \frac{\partial Q_5}{\partial a_{34}} & \frac{\partial Q_5}{\partial a_{56}} \\ \frac{\partial Q_6}{\partial a_{34}} & \frac{\partial Q_6}{\partial a_{56}} \end{bmatrix} = [n_{ij}]$$

with its elements n_{ij}, $i = 1,2$; $j = 1,2,3,4$ having values

$$n_{11} = \frac{\partial Q_3}{\partial B_{34}}\frac{\partial B_{34}}{\partial a_{34}} + \frac{\partial Q_3}{\partial B_{30,4}}\frac{\partial B_{30,4}}{\partial a_{34}} = \frac{7.55V_4}{a_{34}^2} \tag{58}$$

$$n_{12} = n_{22} = n_{31} = n_{41} = 0 \tag{59}$$

$$n_{21} = \frac{7.55}{a_{34}^2}(V_3 - \frac{2V_4}{a_{34}}) \tag{60}$$

$$n_{32} = \frac{3.33V_6}{a_{56}^2} \tag{61}$$

$$n_{42} = \frac{3.33}{a_{56}^2}(V_5 - \frac{2V_6}{a_{56}}) \tag{62}$$

For nominal operating conditions $a_{34}^0 = .909$ and $a_{56}^0 = .976$, with all the other nominal values as introduced in [8]. Under the decoupling assumption $\cos\theta_{ij} \approx 1$ for all $i,j = 1,\ldots,(n+k)$.

Matrix C defined in (2) for this system is

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{63}$$

Based on theoretical results presented in Sections 3 and 4, properties o f matrix $A = C(\frac{\partial Q}{\partial V})^{-1}(\frac{\partial Q}{\partial a})\Big|_0$ are critical for the proper coordination of ULTC controls.

Earlier developed results [9],[10] give conditions for matrix $(\frac{\partial Q}{\partial V})$ to be an M matrix (which is positive definite and thus $-(\frac{\partial Q}{\partial V})$ is stable). These conditions are strongly related to the conditions that decoupled Q-V network has a unique, physically meaningful solutions. From (57), it is obvious that matrix $(\frac{\partial Q}{\partial V})$ will lose, for example, diagonal dominance properties required by Theorem 1, if any loads are capacitive, but still may be an M matrix, as long as the capacitive loads are within certain bounds. Simulations have shown that this is the easiest way to destroy the desired properties of the A matrix. Table 1 shows results of simulations with different reactive power demand. With an increasing effort in industry to use more local capacitive control, this situation may be very realistic. It is important to notice that when the ULTC scheme does not converge (cases 4 and 5) the uncontrolled voltages V_4 and V_6 are far below acceptable voltage limits. The value of .50pu indicates that this poorly coordinated scheme might have led to the voltage collapse, and confirms our proposition that the coordination of the LTC controls has to be carefully designed, to prevent the voltage collapse phenomenon.

Results developed in Sections 4 and 5 give just sufficient conditions for the ULTC scheme to converge; if the conditions are not satisfied, simulations show that voltage collapse may occur. The more capacitive controls implemented, the worse convergence of the LTC scheme results. Also, Table 2 shows values of the matrix A for cases presented in Table 1. It is easy to see on this (2 x 2) matrix that conditions of Theorems 1 and/or 2 are violated in cases 4 and 5, and as a result, the ULTC scheme did not converge. It is important to note that the complexity of checking these conditions is significantly reduced since only the number of directly controlled loads determines the order of matrix A, in this case m = 2.

CASE NUMBER	MATRIX A' AT THE FIRST ITERATION	MATRIX A' AT THE LAST ITERATION
I	$\begin{bmatrix} .4152 & -.1143 \\ -.1617 & .4883 \end{bmatrix}$	$\begin{bmatrix} -.0074 & -.1337 \\ -.1606 & .3530 \end{bmatrix}$
II	$\begin{bmatrix} .87291 & -.0798 \\ .0024 & .5480 \end{bmatrix}$	$\begin{bmatrix} .8943 & -.0804 \\ .0129 & .5169 \end{bmatrix}$
III	$\begin{bmatrix} .2527 & -.0390 \\ -.0789 & .3162 \end{bmatrix}$	$\begin{bmatrix} .3576 & -.0380 \\ -.0675 & .2932 \end{bmatrix}$
IV	$\begin{bmatrix} 1.416 & .2550 \\ .3363 & 1.660 \end{bmatrix}$	$\begin{bmatrix} 1.483 & .1929 \\ .3697 & 1.460 \end{bmatrix}$
V	$\begin{bmatrix} .5885 & -3.832 \\ -2.124 & -10.487 \end{bmatrix}$	$\begin{bmatrix} 1.249 & -.2733 \\ -.0941 & -.0262 \end{bmatrix}$

TABLE 2. Simulations Results.

7. CONCLUSIONS

In this paper, conditions are derived for proper coordination of local voltage control tools, under load tap changing transformers, in particular. Instability due to LTC controls is shown to lead to the possibility of either hunting or voltage collapse. This paper is a drastic departure from typical studies related to voltage collapse where a simplified single generator, single load system is analyzed. The effects of decentralized ULTC control are studied here for the first time. Rigorous mathematical conditions are established which assure LTC operation which maintains voltages within the desired limits.

The established theoretical results give easy to check conditions when capacitive controls may destroy the LTC convergence scheme. With the conditions on the matrix $(\frac{\partial Q}{\partial V})$ to be an M matrix satisfied, further studies are still required, in order that the the product $A = C(\frac{\partial Q}{\partial V})^{-1}(\frac{\partial Q}{\partial V})$ preserves the necessary properties. These studies should lead to a variety of combinations for controlling load voltages properly. Further work is pursued in this direction.

Further research is necessary to show how the developed results are affected by different duty cycles of LTC's, which essentially reduce to different sampling rates of different variables.

And finally, simulations or larger realistic size power systems should be conducted where the effect of decentralization may be even worse.

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