Developments in the Newton Raphson Power Flow Formulation Based on Current Injections

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Abstract:
This paper describes a sparse Newton Raphson formulation for the solution of the power flow problem, comprising $2n$ current injection equations written in rectangular coordinates. The Jacobian matrix has the same structure as the $(2n \times 2n)$ nodal admittance matrix, in which each network branch is represented by a (2×2) block. Except for PV buses, the off-diagonal (2×2) blocks of the proposed Jacobian equations are equal to those of the nodal admittance matrix. The results presented show the proposed method leads to a substantially faster power flow solution, when compared to the conventional Newton Raphson formulation, expressed in terms of power mismatches and written in polar coordinates.

Keywords: Power Flow, Current Injection Mismatches, Newton Raphson

1 Introduction

The power flow problem involves determining voltages and line flows, in a large sparse electrical network, for a given load and generation schedule. Many important contributions have been made on this field along the years and only a few are referenced in this paper [1, 2, 3, 4, 5, 6] due to space limitations. An excellent review, though not so recent, can be found in [4]. Current developments include representation of recent technology devices [7, 8], voltage stability tools [9, 10, 11, 12] and more advanced solution techniques [13].

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A power flow formulation based on current injections and using state variables expressed in a mix of polar and rectangular coordinates is presented in [14]. Each PQ bus is represented by two equations comprising the real and imaginary components of the current injection mismatches expressed in terms of the voltage rectangular coordinates. A PV bus is represented by a single active power mismatch equation and the associated angle deviation.

A current injection algorithm based on the use of a constant nodal admittance matrix is described in [15]. This reference discusses, in a tutorial nature, that this algorithm can not be used for general power flow applications because a satisfactory method of modeling PV nodes has not yet been developed. The known methods for including a PV node nullify its speed of convergence. Reference [15] also comments that a breakthrough in the power flow solution would have an impact on applications such as power system dynamics, state estimation, contingency analysis, steady-state optimization, among others.

This paper proposes the use of the set of $2n$ current injection equations, written in rectangular coordinates, for both PQ and PV buses. A new dependent variable (ΔQ) is introduced for each PV bus together with an additional equation imposing the constraint of zero deviations in the bus voltage. Except for PV buses, the Jacobian matrix has the elements of the (2×2) off-diagonal blocks equal to those of the nodal admittance matrix expanded into real and imaginary coordinates. The elements of the (2×2) diagonal blocks need be updated at every iteration according to the bus load model being considered.

Though not representing a major breakthrough, this paper's concepts are expected to have an impact on some power system analysis applications. Power flow controls are not dealt with here, being the subject of another paper submitted by the authors to the same conference.

The notations adopted in the paper are the conventional ones whenever possible. The authors attempted to include in the text most expressions for the Jacobian elements and mismatch equations needed in the proposed method, so that others may duplicate or expand upon the results presented here.

2 Notation

n: number of system buses

h: iteration counter

$\Delta P_k + j\,\Delta Q_k$: complex power mismatch at bus k

$P_{G_{(k)}} + j\,Q_{G_{(k)}}$: generated complex power at bus k

$P_{L_{(k)}} + j\,Q_{L_{(k)}}$: complex power consumed by load at bus k

$P_k^{sp} + j\,Q_k^{sp}$: net scheduled complex power at bus k

$P_k^{calc} + j\,Q_k^{calc}$: calculated complex power at bus k

$\Delta I_{r_k} + j\,\Delta I_{m_k}$: complex current mismatch at bus k

$I_{r_k}^{sp} + j\,I_{m_k}^{sp}$: scheduled injected current at bus k

$I_{r_k}^{calc} + j\,I_{m_k}^{calc}$: calculated injected current at bus k

$V_{r_k} + j\,V_{m_k}$: complex voltage at bus k

θ_k, V_k: voltage angle and magnitude at bus k

$E_k^* = V_k e^{-j\theta_k}$: complex conjugate voltage phasor at bus k

$\Delta V_{r_k} + j\,\Delta V_{m_k}$: complex voltage mismatch at bus k

$G_{km} + j B_{km}$: $(k,m)th$ element of nodal admittance matrix

$\Delta\theta, \Delta V$: voltage angle, magnitude corrections

Matrices are shown in bold and vectors are shown in bold underlined

3 Current Injection Power Flow

3.1 Equations for PQ Buses

The complex current mismatch at a given PQ bus k is given by [14]:

$$\Delta I_k = \frac{P_k^{sp} - j\,Q_k^{sp}}{E_k^*} - \sum_{i=1}^{n} Y_{ki}\,E_i = 0 \tag{1}$$

where:

$$P_k^{sp} = P_{G_{(k)}} - P_{L_{(k)}} \tag{2}$$

$$Q_k^{sp} = Q_{G_{(k)}} - Q_{L_{(k)}} \tag{3}$$

The voltage dependence of the load powers is modeled in polynomial form:

$$P_{L_{(k)}} = P_{0_k}(a_p + b_p V_k + c_p V_k^2) \tag{4}$$

$$Q_{L_{(k)}} = Q_{0_k}(a_q + b_q V_k + c_q V_k^2) \tag{5}$$

where:

$$a_p + b_p + c_p = 1 \tag{6}$$

$$a_q + b_q + c_q = 1 \tag{7}$$

Equation (1) can be expanded into its real and imaginary components:

$$\Delta I_{r_k} = \frac{P_k^{sp}\,V_{r_k} + Q_k^{sp}\,V_{m_k}}{V_{r_k}^2 + V_{m_k}^2} - \sum_{i=1}^{n}(G_{ki}V_{r_i} - B_{ki}V_{m_i}) = 0 \tag{8}$$

$$\Delta I_{m_k} = \frac{P_k^{sp}\,V_{m_k} - Q_k^{sp}\,V_{r_k}}{V_{r_k}^2 + V_{m_k}^2} - \sum_{i=1}^{n}(G_{ki}V_{m_i} + B_{ki}V_{r_i}) = 0 \tag{9}$$

and written in compact form:

$$\Delta I_{r_k} = I_{r_k}^{sp} - I_{r_k}^{calc} \tag{10}$$

$$\Delta I_{m_k} = I_{m_k}^{sp} - I_{m_k}^{calc} \tag{11}$$

The Newton Raphson solution algorithm, applied to Equations (8) and (9), considering all system buses as being of the PQ type, is given by:

$$\begin{bmatrix} \Delta I_{m_1} \\ \Delta I_{r_1} \\ \hline \Delta I_{m_2} \\ \Delta I_{r_2} \\ \hline \vdots \\ \hline \Delta I_{m_n} \\ \Delta I_{r_n} \end{bmatrix} = \left[\begin{array}{c|c|c|c} \mathbf{Y}_{11}^* & \mathbf{Y}_{12}^* & \dots & \mathbf{Y}_{1n}^* \\ \hline \mathbf{Y}_{21}^* & \mathbf{Y}_{22}^* & \dots & \mathbf{Y}_{2n}^* \\ \hline \vdots & \vdots & \vdots & \vdots \\ \hline \mathbf{Y}_{n1}^* & \mathbf{Y}_{n2}^* & \dots & \mathbf{Y}_{nn}^* \end{array}\right] \begin{bmatrix} \Delta V_{r_1} \\ \Delta V_{m_1} \\ \hline \Delta V_{r_2} \\ \Delta V_{m_2} \\ \hline \vdots \\ \hline \Delta V_{r_n} \\ \Delta V_{m_n} \end{bmatrix} \tag{12}$$

which can be written in a compact form:

$$\underline{\mathbf{\Delta I}}_{mr} = \mathbf{Y}^*\,\underline{\mathbf{\Delta V}}_{rm} \tag{13}$$

where:

$$\mathbf{Y_{kk}^*} = \begin{bmatrix} B'_{kk} & G'_{kk} \\ G''_{kk} & B''_{kk} \end{bmatrix} \qquad \mathbf{Y_{km}^*} = \begin{bmatrix} B_{km} & G_{km} \\ G_{km} & -B_{km} \end{bmatrix}$$

$$B'_{kk} = B_{kk} - a_k \tag{14}$$

$$G'_{kk} = G_{kk} - b_k \tag{15}$$

$$G''_{kk} = G_{kk} - c_k \tag{16}$$

$$B''_{kk} = -B_{kk} - d_k \tag{17}$$

The terms a_k, b_k, c_k and d_k depend on the specified load and generation at bus k and also on the load model, being presented in the Appendix. Note that in (12), the imaginary components of the current mismatches are ordered first, so that matrix $\mathbf{Y}^*$ becomes diagonal dominant(a consequence of the fact that susceptances B_{km} are much larger than condutances G_{km}).

3.2 Calculation of Current Mismatches

The active and reactive power mismatches for bus k is given by:

$$\Delta P_k = P_k^{sp} - P_k^{calc} \tag{18}$$

$$\Delta Q_k = Q_k^{sp} - Q_k^{calc} \tag{19}$$

where:

$$P_k^{calc} = V_{r_k} I_{r_k}^{calc} + V_{m_k} I_{m_k}^{calc} \tag{20}$$

$$Q_k^{calc} = V_{m_k} I_{r_k}^{calc} - V_{r_k} I_{m_k}^{calc} \tag{21}$$

By simple manipulation of the above equations, the current mismatches in (10) and (11) can be expressed only in terms of power mismatches and voltages at bus k:

$$\Delta I_{r_k} = \frac{V_{r_k} \Delta P_k + V_{m_k} \Delta Q_k}{V_k^2} \tag{22}$$

$$\Delta I_{m_k} = \frac{V_{m_k} \Delta P_k - V_{r_k} \Delta Q_k}{V_k^2} \tag{23}$$

where:

$$V_k^2 = V_{r_k}^2 + V_{m_k}^2 \tag{24}$$

The calculation of real and imaginary current mismatches is straightforward for PQ buses, because the associated real and reactive power mismatches are known. The current mismatches given in Equations (22) and (23) are computed to form the vector of mismatches, shown in the left side of Equation (12). The formulation described up to this point looks pretty much standard. The major contribution lies in the representation of PV buses, as described in the next section.

3.3 Representation of PV Buses

For the sake of brevity, let us consider the equations for a single node k, solely connected to a slack bus. The current mismatch equations for this node, initially considered to be of the PQ type, are given by:

$$\begin{bmatrix} \frac{V_{m_k}}{V_k^2} \Delta P_k - \frac{V_{r_k}}{V_k^2} \Delta Q_k \\ \frac{V_{r_k}}{V_k^2} \Delta P_k + \frac{V_{m_k}}{V_k^2} \Delta Q_k \end{bmatrix} = \begin{bmatrix} B'_{kk} & G'_{kk} \\ G''_{kk} & B''_{kk} \end{bmatrix} \begin{bmatrix} \Delta V_{r_k} \\ \Delta V_{m_k} \end{bmatrix} \tag{25}$$

In the case of a PV bus, the reactive power mismatch ΔQ_k becomes a dependent variable. An augmented solution vector is then created, containing this dependent variable. An additional linearized equality constraint, imposing that $\Delta V_k = 0$ for a PV bus, is then used to eliminate the over-determination:

$$\Delta V_k = 0 = \frac{V_{r_k}}{V_k} \Delta V_{r_k} + \frac{V_{m_k}}{V_k} \Delta V_{m_k} \tag{26}$$

Thus, the augmented set of equations becomes:

$$\begin{bmatrix} \frac{V_{m_k}}{V_k^2} \Delta P_k \\ \frac{V_{r_k}}{V_k^2} \Delta P_k \\ 0 \end{bmatrix} = \begin{bmatrix} B'_{kk} & G'_{kk} & \frac{V_{r_k}}{V_k^2} \\ G''_{kk} & B''_{kk} & -\frac{V_{m_k}}{V_k^2} \\ \frac{V_{r_k}}{V_k} & \frac{V_{m_k}}{V_k} & 0 \end{bmatrix} \begin{bmatrix} \Delta V_{r_k} \\ \Delta V_{m_k} \\ \Delta Q_k \end{bmatrix} \tag{27}$$

Note the last row in matrix equation (27) establishes that:

$$\Delta V_{r_k} = -\frac{V_{m_k}}{V_{r_k}} \Delta V_{m_k} \tag{28}$$

which substituted into the first two rows of Equation (27) yields:

$$\begin{bmatrix} \frac{V_{m_k}}{V_k^2} \Delta P_k \\ \frac{V_{r_k}}{V_k^2} \Delta P_k \end{bmatrix} = \begin{bmatrix} (G'_{kk} - \frac{B'_{kk} V_{m_k}}{V_{r_k}}) & \frac{V_{r_k}}{V_k^2} \\ (B''_{kk} - \frac{G''_{kk} V_{m_k}}{V_{r_k}}) & -\frac{V_{m_k}}{V_k^2} \end{bmatrix} \begin{bmatrix} \Delta V_{m_k} \\ \Delta Q_k \end{bmatrix} \tag{29}$$

This implementation causes the power flow Jacobian matrix to maintain the $(2n \times 2n)$ nodal admittance matrix structure, irrespective of the number of PV buses in the system, with the new variable ΔQ_k replacing ΔV_{r_k} for each PV bus.

This Jacobian matrix has (2×2) blocks of three types, which are described below. The (2×2) diagonal block related to bus k, of the PV type, is given by:

$$Y_{kk}^{**} = \left[\begin{array}{c|c} G'_{kk} - \frac{B'_{kk} V_{m_k}}{V_{r_k}} & \frac{V_{r_k}}{V_k^2} \\ \hline B''_{kk} - \frac{G''_{kk} V_{m_k}}{V_{r_k}} & -\frac{V_{m_k}}{V_k^2} \end{array}\right] \tag{30}$$

The (2×2) off-diagonal blocks associated with a generic branch $k - l$ are given by:

$$Y_{lk}^{**} = \left[\begin{array}{c|c} G_{lk} - \frac{B_{lk} V_{m_k}}{V_{r_k}} & 0 \\ \hline -B_{lk} - \frac{G_{lk} V_{m_k}}{V_{r_k}} & 0 \end{array}\right] \tag{31}$$

$$Y_{kl}^{**} = Y_{kl}^{*} = \left[\begin{array}{c|c} B_{kl} & G_{kl} \\ \hline G_{kl} & -B_{kl} \end{array}\right] \tag{32}$$

The (2×2) diagonal block related to a PQ node (without any specified load), numbered i, is given by:

$$Y_{ii}^{**} = \left[\begin{array}{c|c} B_{ii} & G_{ii} \\ \hline G_{ii} & -B_{ii} \end{array} \right]$$

The current mismatches for bus k, of the PV type, are given by:

$$\Delta I_{m_k}^* = \frac{V_{m_k} \, \Delta P_k}{V_k^2} \tag{33}$$

$$\Delta I_{r_k}^* = \frac{V_{r_k} \, \Delta P_k}{V_k^2} \tag{34}$$

The proposed Newton Raphson algorithm is shown in Equation (35) in schematic form. Node k, of the PV type, is connected to the two PQ nodes i and l.

$$\left[\begin{array}{c} \vdots \\ \hline \Delta I_{m_i} \\ \Delta I_{r_i} \\ \hline \Delta I_{m_k}^* \\ \Delta I_{r_k}^* \\ \hline \Delta I_{m_l} \\ \Delta I_{r_l} \\ \hline \vdots \end{array} \right] = \left[\begin{array}{c|c|c|c|c} \vdots & \vdots & \vdots & \vdots & \vdots \\ \hline \cdots & \mathbf{Y}_{\mathrm{ii}}^* & \mathbf{Y}_{\mathrm{ik}}^{**} & \mathbf{Y}_{\mathrm{il}}^* & \cdots \\ \hline \cdots & \mathbf{Y}_{\mathrm{ki}}^* & \mathbf{Y}_{\mathrm{kk}}^{**} & \mathbf{Y}_{\mathrm{kl}}^* & \cdots \\ \hline \cdots & \mathbf{Y}_{\mathrm{li}}^* & \mathbf{Y}_{\mathrm{lk}}^{**} & \mathbf{Y}_{\mathrm{ll}}^* & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots \end{array} \right] \left[\begin{array}{c} \vdots \\ \hline \Delta V_{r_i} \\ \Delta V_{m_i} \\ \hline \Delta V_{m_k} \\ \Delta Q_k \\ \hline \Delta V_{r_l} \\ \Delta V_{m_l} \\ \hline \vdots \end{array} \right] \tag{35}$$

This equation can be written in compact form:

$$\underline{\mathbf{\Delta}}\mathbf{I}_{mr}^* = \mathbf{J}^* \, \underline{\mathbf{\Delta}}\mathbf{V}_{rm}^* \tag{36}$$

Nodes i, k and l need not be ordered adjacent, but appear as such in Equation (35) due to the space limitations.

Figure (1) shows the structure of the Jacobian matrix $\mathbf{J}^*$ for the IEEE 14-bus system, where buses 2, 3, 6 and 8 are of the PV type. Note that node 1 is the slack bus, and should be discarded to yield the power flow Jacobian matrix. Note also that node 7 is a PQ bus with no specified load.

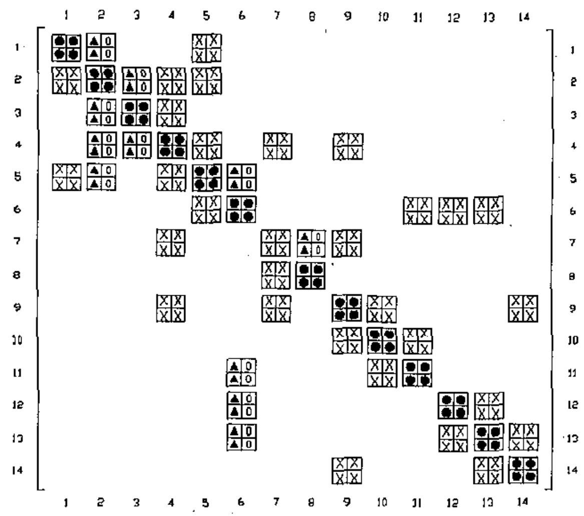
3.4 Bus Voltage Corrections and Convergence Characteristics

The bus voltage corrections in polar coordinates, at a generic iteration $(h+1)$ are given by:

$$V_k^{(h+1)} = V_k^{(h)} + \Delta V_k^{(h)} \tag{37}$$

$$\theta_k^{(h+1)} = \theta_k^{(h)} + \Delta\theta_k^{(h)} \tag{38}$$

where:



X = Nodal Admittance Matrix Elements
0 = Null Elements
● = Elements which are updated at every iteration
▲ = Elements associated with a PV node, also updated at every iteration

Figure 1: Jacobian Matrix Structure for the IEEE 14-bus System

$$\Delta V_k = \frac{V_{r_k}}{V_k} \Delta V_{r_k} + \frac{V_{m_k}}{V_k} \Delta V_{m_k} \tag{39}$$

$$\Delta\theta_k = \frac{V_{r_k}}{V_k^2} \Delta V_{m_k} - \frac{V_{m_k}}{V_k^2} \Delta V_{r_k} \tag{40}$$

Note that Equation (40) is the linearized form of:

$$\theta_k = \tan^{-1} \frac{V_{m_k}}{V_{r_k}} \tag{41}$$

It is worth noting that the convergence trajectory of the proposed method is identical to the Newton Raphson power mismatches formulation in polar coordinates.

3.5 Solution Algorithm

step 1: Assemble the nodal admittance matrix $\mathbf{Y}$
Initialize the iteration counter $h = 0$
Initialize voltages and angles V_k^h and θ_k^h; $k = 1, n$

step 2: Determine the current injections $\underline{\mathbf{I}} = \mathbf{Y}\underline{\mathbf{E}}$
Determine the active and reactive power mismatches (Equations (18), (19), (20),(21))

step 3: If $max(\underline{\mathbf{\Delta}}\mathbf{P}, \underline{\mathbf{\Delta}}\mathbf{Q}) \leq tolerance$:
then Go to step 5
else Determine the bus voltage corrections by using (35)
Update the bus voltages by using (37) and (38)

Increment the iteration counter $h = h + 1$

step 4: If $h \geq maximum\ number\ of\ iterations$:
then Go to step 5
else Go to step 2

step 5: Print the results

4 Implementation Aspects

The elements of the Jacobian matrix are assembled into (2×2) blocks, such that the network nodal admittance matrix structure is preserved. The (2×2) blocked system of equations is ordered by the Tinney-2 scheme [1] for computational efficiency.

An important feature of this formulation is that most of the (2×2) blocks of the Jacobian matrix $\mathbf{J}^*$ remain unchanged during the solution process. The diagonal (2×2) blocks, having specified generation or loads other than impedance loads, must be updated at every iteration.

5 Results

The proposed power flow formulation was validated through tests with the prototype computer program developed. Practical system models having 730, 1653 and 2111 buses, and described in Table (1), were utilized. The first two are models of the South-Southeast Brazilian System. The third is a planning model for the year 2001 of the North-South Brazilian Interconnection.

The maximum power mismatch for each iteration is shown in Table (2) which displays the convergence characteristics when using Equations (37) e (38). The convergence results in Table (2) are identical to those produced by a production grade program (program ANAREDE of CEPEL), utilizing the full Newton Raphson formulation and the power mismatch equations in polar coordinates. The convergence tolerance considered in the iterative process is 0.001 p.u. for both active and reactive power mismatches.

The CPU times required by the prototype program and the production grade program, when using a personal computer PENTIUM 100 - MHz., are shown in Table (3). The results show a consistent reduction in computational time (over 20 %) when using the prototype program.

Table 2: Convergence Characteristics for Voltage Corrections in Polar Form

Iter.	Maximum Mismatch (pu)		
	730-bus	1653-bus	2111-bus
0	-48.876521	82.506695	-116.202292
1	0.190831	3.514192	-1.398069
2	0.000496	0.047897	0.011986
3	–	-0.000873	-0.000022

Table 3: Total CPU Time

Systems	CPU Time(s)	
	Prod. Grade Program	Prototype Program
730-bus	0.350	0.219
1653-bus	0.850	0.680
2111-bus	1.230	0.990

The three test systems utilized have a percentage of PV buses varying from 7.3% to 14.1%. The CPU time impact when increasing the number of PV buses was verified by artificially increasing the number of PV buses on the test systems. The results are shown in Table (4), indicating there is a small reduction in CPU time, as the percentage of PV buses is increased. It should be noted that a PV bus model implies in setting a full column of the Jacobian matrix to zero (as shown in Figure (1) and Equation (31)), except for the (2×2) diagonal block (Equation (30)).

6 Conclusions

This paper proposes a novel approach for solving the power flow problem based on nodal current injections and a sparse $(2n \times 2n)$ Jacobian matrix which can be obtained faster than the conventional power flow Jacobian. The main advantage of this formulation lies in the calculation of the Jacobian matrix $\mathbf{J}^*$, because its off-diagonal elements (plus a few diagonal ones) are constant and equal to the terms of the nodal admittance matrix, except for the PV buses whose elements are obtained with a little additional effort. The method does not require at all the use of transcendental(sine, cosine) functions during the iterative process.

One of the contributions of this paper is the rehabilitation

Table 1: Test Systems

Test System	730-bus	1653-bus	2111-bus
Circuits	1146	2382	3236
PV Buses	103	121	177
Transformers	277	472	968
Load (MW)	28565	26703	44704

Table 4: CPU Time per Iteration, in Seconds, when Increasing the Number of PV Buses

Systems	Base Case	Percentage of PV Buses		
		20%	30%	40%
730-bus	0.109	0.094	0.097	0.094
1653-bus	0.227	0.206	0.205	0.206
2111-bus	0.330	0.301	0.301	0.301

of the current mismatch formulation, in full Newton form, for the practical solution of power flow problems.

The computer program developed in this work is still at a prototype stage, but consistently showed a 20% average speed-up when benchmarked with a state of the art, production grade Newton Raphson power flow. It is worth noting that the same code for sparse factorization and solution is used in both the prototype and production grade programs.

The authors have already developed a three phase power flow formulation based on this approach. Another application being considered is the exact algebraic representation of PV buses into a transient mid-term stability dynamic simulator. This is advantageous for mid-term voltage stability studies, where the generation remote from the study area, could be represented as PV buses, yielding large savings in computation with only a small loss in modeling accuracy.

Minor attempts have so far been made to speed up the solution by the use of partly constant approximate Jacobians during all or some iterations, but the results are not encouraging.

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APPENDIX

Terms Added to the Diagonal Elements of the Jacobian Matrix

The terms a_k, b_k, c_k and d_k shown in the Equations (14), (15), (16) and (17) are given by the following expressions:

$$a_k = \frac{Q_k'(V_{r_k}^2 - V_{m_k}^2) - 2\,V_{r_k}\,V_{m_k}\,P_k'}{V_k^4} + \frac{V_{r_k}\,V_{m_k}\,P_{0_k}b_p + Q_{0_k}b_q\,V_{m_k}^2}{V_k^3} + Q_{0_k}c_q \quad (42)$$

$$b_k = \frac{P_k'(V_{r_k}^2 - V_{m_k}^2) + 2\,V_{r_k}\,V_{m_k}\,Q_k'}{V_k^4} - \frac{V_{r_k}\,V_{m_k}\,Q_{0_k}b_q + P_{0_k}b_p\,V_{r_k}^2}{V_k^3} - P_{0_k}c_p \quad (43)$$

$$c_k = \frac{P_k'(V_{m_k}^2 - V_{r_k}^2) - 2\,V_{r_k}\,V_{m_k}\,Q_k'}{V_k^4} + \frac{V_{r_k}\,V_{m_k}\,Q_{0_k}b_q - P_{0_k}b_p\,V_{m_k}^2}{V_k^3} - P_{0_k}c_p \quad (44)$$

$$d_k = \frac{Q_k'(V_{r_k}^2 - V_{m_k}^2) - 2\,V_{r_k}\,V_{m_k}\,P_k'}{V_k^4} + \frac{V_{r_k}\,V_{m_k}\,P_{0_k}b_p - Q_{0_k}b_q\,V_{r_k}^2}{V_k^3} - Q_{0_k}c_q \qquad (45)$$

where:

$$P_k' = P_{G_{(k)}} - P_{0_k}a_p \qquad (46)$$

$$Q_k' = Q_{G_{(k)}} - Q_{0_k}a_q \qquad (47)$$

For the case where bus k has only a constant power load, the expressions are simplified to:

$$a_k = d_k = \frac{Q_k'(V_{r_k}^2 - V_{m_k}^2) - 2\,V_{r_k}\,V_{m_k}\,P_k'}{V_k^4} \qquad (48)$$

$$b_k = -c_k = \frac{P_k'(V_{r_k}^2 - V_{m_k}^2) + 2\,V_{r_k}\,V_{m_k}\,Q_k'}{V_k^4} \qquad (49)$$

These parameters are calculated, at every iteration of the power flow solution, and are used for updating the diagonal blocks of matrix $\mathbf{J}^*$.

When bus k has only a constant impedance load and no specified generation, these expressions are further simplified:

$$a_k = -d_k = Q_{0_k}c_q \qquad (50)$$

$$b_k = c_k = -P_{0_k}c_p \qquad (51)$$

Note that $Q_{0_k}c_q$ and $P_{0_k}c_p$ are pure admittance terms, which therefore remain constant during computation.

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