Optimal Reactive Power Flow with Exact Linearized Transformer Model in Distribution Power Networks

Bin Liu[1], Feng Liu[2], Shengwei Mei[3], Yue Chen[4]

1. Department of Electrical Engineering, Tsinghua University, Haidian District, Beijing 100084, P.R.China
E-mail: eeliubin@yeah.net

2. Department of Electrical Engineering, Tsinghua University, Haidian District, Beijing 100084, P.R.China
E-mail: lfeng@tsinghua.edu.cn

3. Department of Electrical Engineering, Tsinghua University, Haidian District, Beijing 100084, P.R.China
E-mail: meishengwei@tsinghua.edu.cn

4. Department of Electrical Engineering, Tsinghua University, Haidian District, Beijing 100084, P.R.China
E-mail: zjzx_cy@126.com

Abstract: Optimal reactive power flow (ORPF) in distribution power networks is to optimize the operation of reactive power devices to minimize the total operation cost or power loss. As the optimization model is constructed with AC power flow constraints and the strategies of compensating capacitor and transformer are discrete, ORPF problem in distribution power networks is essentially a Mixed Integer Nonconvex Nonlinear Programming (MINNLP) problem which is hard to solve mathematically. Based on the Branch Flow Model of distribution power networks, we proposed a method to exactly linearize the transformer model using piecewise linear technique. To efficiently solve the problem, the latest second-order cone (SOC) relaxation technique is adopted and the ORPF is relaxed to a Mixed Integer Second-order Cone Programming (MISOCP) problem. Further, the exactness of the SOC relaxation is discussed in this paper and the modified IEEE 33 and 69 bus system are employed to study the effectiveness of the proposed method.

Key Words: distribution power networks, optimal reactive power flow, piecewise linear, second-order cone

1 INTRODUCTION

Optimal reactive power flow (ORPF) in distribution power networks is to determine an operation point that can minimize the total operation cost or power loss, which essentially is an optimal power flow (OPF) problem. Different from transmission power networks, line resistance cannot be neglected in distribution power networks and the power injection balance is constructed based on AC power flow constraints which makes ORPF a nonconvex nonlinear problem. In ORPF, the decision variables include reactive power generation of SVG, transformer (TF) ratio and position of compensating capacitor (CP) of which the strategies are discrete. The discrete variables further make ORPF a Mixed Integer Nonconvex Nonlinear Programming (MINNLP) problem which belongs to a class of problem hard to solve.

Currently, the algorithms to solve ORPF problem can be mainly divided into three categories. The first category is relaxation-tuning method in which the discrete variables are relaxed to continuous variables and then modified back to discrete variables during the optimization process using different strategies, e.g. Gauss penalty function or Mixed Integer Quadratic Programming (MIQP) methods [2, 3]. The second category is based on nonlinear primal-dual algorithm [4~6]. The third category is modern optimization algorithms including genetic algorithm (GA), particle swarm optimization (PSO) etc. [7-9]. However, all the algorithms can only guarantee local optimality and the computation efficiency and robustness of these algorithms need further research.

Recently, solving AC-constrained OPF (ACOPF) based on convex relaxation is widely concerned as its ability to find global optima and polynomial time complexity [10-13], especially second-order cone (SOC) relaxation-based method to solve ACOPF in radial power networks. The method built on Branch Flow Model and relaxed quadratic constraints to SOC constraints which transformed ORPF to an efficiently solvable second-order cone programming (SOCP) problem. If the SOCP problem is equivalent to the original problem, solving the relaxed SOCP problem can obtain the global optima of the original problem. As the Mixed Integer Second Order Cone Programming (MISOCP) problem can also be solved by commercial solvers with high computation efficiency, this provides an alternative method to solve OPRF problem containing both continuous and discrete variables in distribution power networks. Ref [15, 16] studied three-phase ORPF problem in active distribution networks (ADN) and solved the problem based on MISOCP technique which showed the effectiveness of SOC relaxation technique. However, the transformer is not modeled and sufficient conditions to guarantee exactness of SOC relaxation provided need further validation.

Based on previous work, we studied ORPF with both continuous and discrete controllable devices. Specially, the transformer is incorporated and was exactly linearized using piecewise linear (PWL) technique based on the Branch Flow Model. The ORPF problem was then relaxed to a MISOCP problem applying the latest SOC relaxation technique. Moreover, the exactness of SOC relaxation was also discussed in our work with transformer incorporated.

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The paper is organized as follows: Part II introduces the Branch Flow Model and its relaxation format. The exact linearized transformer model is also presented in this part. Part III builds ORPF model containing both continuous and discrete decision variables and the convex relaxation method to solve this problem is discussed. The case study based on modified IEEE 33 and IEEE 69 bus systems are studied in Part IV and conclusions are given in Part V.

2 BRANCH FLOW MODEL

2.1 Branch Flow Model without Transformer

Branch Flow Model in distribution power networks was firstly proposed in [14, 17] of which the topology is given below.

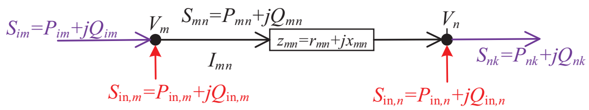

Fig.1 Branch Flow Model without transformer

In figure 1, the constraints of bus voltage, power flow can be formulated as follows.

$$v_m - v_n = 2(r_{mn}P_{mn} + x_{mn}Q_{mn}) - (r_{mn}^2 + x_{mn}^2)l_{mn} \tag{1-1}$$

$$l_{mn}v_m = P_{mn}^2 + Q_{mn}^2;\ l_{mn} \geq 0 \tag{1-2}$$

$$P_{\text{in},n} = \sum_{k:n\to k} P_{nk} - (P_{mn} - l_{mn}r_{mn}) \tag{1-3}$$

$$Q_{\text{in},n} = \sum_{k:n\to k} Q_{nk} - (Q_{mn} - l_{mn}x_{mn}) \tag{1-4}$$

where v_m, l_{mn} represent the square of voltage amplitude (SVA) of bus m and the square of current amplitude (SCA) of line $\#(mn)$ respectively. $r_{mn} + jx_{mn}$ is the reactance of line $\#(mn)$. $P_{mn} + jQ_{mn}$ is the apparent power (AP) of line $\#(mn)$ on the side of bus m. $P_{\text{in},n}, Q_{\text{in},n}$ are active and reactive power injections of bus n.

The quadratic constraint (1-2) is the main nonconvex nonlinear part when incorporated in ACOPF in radial power networks. The latest convex relaxation to solve ACOPF relaxes it to SOC constraint given below.

$$l_{mn}v_m \geq P_{mn}^2 + Q_{mn}^2;\ l_{mn} \geq 0 \tag{2}$$

Jabr proposed a conic power flow model using the SOC relaxation [18]. Steven Low et al further studied the OPF problem based on the relaxation and provided various sufficient conditions to guarantee the exactness of the relaxation [12, 13].

2.2 Branch Flow Model with Transformer

Incorporating transformer, the topology of Branch Flow Model can be depicted as follows.

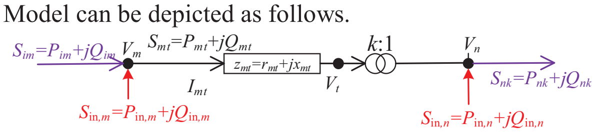

Fig.2 Branch Flow Model with transformer

To clearly describe the modeling method, we introduce a virtual bus t and the Branch Flow Model of line $\#(mt)$ can be formulated as follows.

$$v_m - v_t = 2(r_{mt}P_{mt} + x_{mt}Q_{mt}) - (r_{mt}^2 + x_{mt}^2)l_{mt} \tag{3-1}$$

$$l_{mt}v_m = P_{mt}^2 + Q_{mt}^2;\ l_{mt} \geq 0 \tag{3-2}$$

$$P_{tn} = P_{mt} - l_{mt}r_{mt} \tag{3-3}$$

$$Q_{tn} = Q_{mt} - l_{mt}x_{mt} \tag{3-4}$$

where $r_{mt} + jx_{mt}$ now represent the sum of reactance of line $\#(mn)$ and reactance of transformer on line $\#(mn)$.

The Branch Flow Model of line $\#(tn)$ is much more straightforward and is given below.

$$v_t = k_{mn}^2 v_n;\ l_{mt} = l_{tn} \tag{4-1}$$

$$l_{tn}v_t = P_{tn}^2 + Q_{tn}^2;\ l_{tn} \geq 0 \tag{4-2}$$

$$P_{\text{in},n} = \sum_{k:n\to k} P_{nk} - P_{tn} \tag{4-3}$$

$$Q_{\text{in},n} = \sum_{k:n\to k} Q_{nk} - Q_{tn} \tag{4-4}$$

where k_{mn} is the ratio of line $\#(mn)$.

Integrating (3-1)~(3-4), (4-1)~(4-4), Branch Flow Model with transformer can be formulated as follows.

$$v_m - z_{mn} = 2(r_{mn}P_{mn} + x_{mn}Q_{mn}) - (r_{mn}^2 + x_{mn}^2)l_{mn} \tag{5-1}$$

$$z_{mn} = k_{mn}^2 v_n \tag{5-2}$$

(1-2)~(1-4)

In (5-1) and (5-2), $r_{mn} + jx_{mn}$ represents the sum of reactance of line and the corresponding transformer whenever there is a transformer on line $\#(mn)$. However, we keep the symbol as $r_{mn} + jx_{mn}$ for simplicity.

The derived model is still nonconvex and nonlinear because of constraints $l_{mn}v_m = P_{mn}^2 + Q_{mn}^2$ and $z_{mn} = k_{mn}^2 v_n$. However, the former one can be relaxed to a SOC constraint by SOC relaxation technique. To the latter one, we will transform it to a Mixed Integer Linear (MIP) constraint using PWL technique.

The philosophy of PWL is to approximate the original nonlinear function with a series of linear functions by introducing auxiliary variables. The illustration of PWL of a univariate function is given in figure 3.

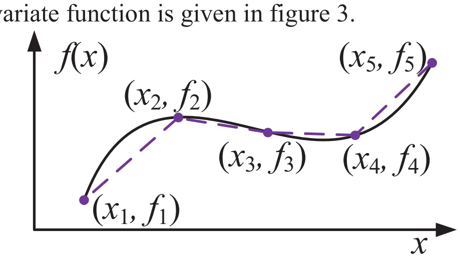

Fig.3 Piecewise linear of a univariate function

$$\boldsymbol{f(x) = \sum a_i f_i\,;\, x = \sum a_i x_i\,;\, a_1 \leq d_1;\, a_5 \leq d_4;}$$

$$\boldsymbol{a_i \leq d_{i-1} + d_i (2 \leq i \leq 4), a_i \geq 0, d_i \in \{0, 1\}\ \forall i}$$

$$\textstyle\sum_{i=1}^{5} a_i = 1; \sum_{i=1}^{4} d_i = 1$$

As showed in figure 3, for univariate function, PWL approximates the original curve by a series of segments. Obviously, the PWL is exact if the feasible region of the variable is a discrete set.

The PWL technique is also adaptable to nonlinear function with two or more variables. Specially, PWL for nonlinear function with two variables is to approximate the original surface by a series of polyhedrons and the exactness of PWL can be illustrated by the following theorem.

Theorem 1: Given the nonlinear function $z=f(x,y)$ with $x\in X, y\in Y$. If $f(x,y)$ has the form: $f(x,y)=x^m y$ and X is a discrete set, then $z=f(x,y)$ can be exactly linearized by PWL technique.

The proof of theorem 1 is omitted here for simplicity and the idea of the proof can be described as follows.

For each fixed $x_i\in X$, $z=x_i^m y$ is a linear function. Therefore, the original function $z=f(x,y)$ can be exactly linearized by a series of linear functions as follows.

$$z_i=x_i^m y, i=1,\cdots,n$$

Further, if Y is a box set, i.e. $Y=[y^{\min},y^{\max}]$, exactly linearizing $f(x,y)$ needs $2n$ auxiliary continuous variables and $n-1$ auxiliary binary variables where auxiliary binary variable is used to indicate which "part" x and y lie and the auxiliary continuous variable is used to exactly modeling the original function in this "part".

Obviously, formula (5-2) satisfies Theorem 1 as the feasible region of k_{mn} is discrete. Correspondingly, formula (5-2) can be exactly linearized as follows.

For $\#(mn)\in\mathcal{L}_{tf}$, the expression and accompanying constraints of k_{mn} can be formulated as follows.

$$\begin{gathered} k_{mn}=k_0+K_{mn}\Delta k_{mn} \\ K_{mn}^{\min}\le K_{mn}\le K_{mn}^{\max}; K_{mn}\in\mathbb{Z} \end{gathered} \tag{6}$$

where $\mathcal{L}_{tf}$ is the set of lines with transformer. $k_0,\Delta k_{mn}$ represent the initial and regulation step of transformer in line $\#(mn)$ respectively. $K_{mn},K_{mn}^{\min},K_{mn}^{\max}$ represent the possible, lower and upper limits of transformer tap in line $\#(mn)$ respectively. $\mathbb{Z}$ is the set of real integer variables.

Constraint (7) can be obtained integrating (5-2) and (6).

$$z_{mn}=v_n[\Delta k_{mn}^2 K_{mn}^2+2k_0\Delta k_{mn}K_{mn}+k_0^2] \tag{7}$$

Denoting feasible region of K_{mn} as $\{K_{mn,1},\cdots,K_{mn,\mathrm{A}}\}$, introducing 2A continuous variables and $\mathrm{A}-1$ binary variables can exactly linearize formula (7) resulting constraints (8-1)~(8-6).

$$v_n=\sum_{j=1}^{\mathrm{A}}x_{mn,1,j}v_n^{\min}+\sum_{j=1}^{\mathrm{A}}x_{mn,2,j}v_n^{\max} \tag{8-1}$$

$$K_{mn}=\sum_{j=1}^{\mathrm{A}}(x_{mn,1,j}+x_{mn,2,j})K_{mn,\mathrm{j}}\in\mathbb{Z} \tag{8-2}$$

$$\begin{gathered} x_{mn,1,j}\ge 0, x_{mn,2,j}\ge 0\ \ \forall j\in[1,\mathrm{A}] \\ d_{mn,j}\in\{0,1\}\ \ \forall j\in[1,\mathrm{A}-1] \end{gathered} \tag{8-3}$$

$$\sum_{j=1}^{\mathrm{A}}\left(x_{mn,1,j}+x_{mn,2,j}\right)=1;\sum_{j=1}^{\mathrm{A}-1}d_{mn,j}=1 \tag{8-4}$$

$$\begin{gathered} x_{mn,i,1}\le d_{mn,1}, x_{mn,i,\mathrm{A}}\le d_{mn,\mathrm{A}-1}\quad \forall i\in\{1,2\} \\ x_{mn,i,j}\le d_{mn,j-1}+d_{mn,j}\quad \forall i\in\{1,2\},\forall j \in[2,\mathrm{A}-1] \end{gathered} \tag{8-5}$$

$$\begin{gathered} z_{mn}=\Delta k_{mn}^2\sum_{j=1}^{\mathrm{A}}\left(x_{mn,1,j}v_n^{\min}+x_{mn,2,j}v_n^{\max}\right)K_{mn,j}^2 \\ +2k_0\Delta k_{mn}\sum_{j=1}^{\mathrm{A}}\left(x_{mn,1,j}v_n^{\min}+x_{mn,2,j}v_n^{\max}\right)K_{mn,j}+v_n k_0^2 \end{gathered} \tag{8-6}$$

where $v^{\max},v^{\min}$ are the upper and lower limits of SVA respectively.

Using PWL technique and SOC relaxation, the nonlinear nonconvex constraints in Branch Flow Model are eventually transformed to easy-handling SOC and MIL constraints.

3 ORPF MODEL IN DISTRIBUTION POWER NETWORKS

3.1 Optimal Reactive Power Flow Model

In distribution power networks, we denote the decision variables of compensating capacitor and reactive power of SVG as C_p,Q_r respectively. The active and reactive power injections of the root node (can be regarded as a virtual generator) is denoted by P_g and Q_g. We use x to represent all decision variables and the ORPF of distribution power networks based on Branch Flow Model can be formulated as follows.

$$\min_x \sum_{k\in\mathcal{N}}P_{g,k} \tag{9-1}$$

$$(1-1)\sim(1-4)\quad \forall(mn)\in\mathcal{L}\backslash\mathcal{L}_{tf} \tag{9-2}$$

$$(1-2)\sim(1-4),(5-1)\quad \forall(mn)\in\mathcal{L}_{tf} \tag{9-3}$$

$$P_{g,i}^{\min}\le P_{g,i}\le P_{g,i}^{\max}; Q_{g,i}^{\min}\le Q_{g,i}\le Q_{g,i}^{\max}\ \ \forall i\in\mathcal{N} \tag{9-4}$$

$$v_k^{\min}\le v_k\le v_k^{\max}\quad \forall n\in\mathcal{N} \tag{9-5}$$

$$P_{mn}^{\min}\le\ P_{mn}\ \le P_{mn}^{\max}\quad \forall(mn)\in\mathcal{L} \tag{9-6}$$

$$\begin{gathered} C_{p,n}=C_{p,n}^{\min}+H_n\Delta C_{p,n}; H_n^{\min}\le H_n\le H_n^{\max} \\ H_n\in\mathbb{Z}\quad \forall n\in\mathcal{N} \end{gathered} \tag{9-7}$$

$$Q_{r,n}^{\min}\le Q_{r,n}\le Q_{r,n}^{\max}\quad \forall n\in\mathcal{N} \tag{9-8}$$

$$(8-1)\sim(8-6)\quad \forall(mn)\in\mathcal{L}_{tf} \tag{9-9}$$

$$v_1=v_{rt} \tag{9-10}$$

where $\mathcal{N}$ is the bus set and v_{rt} is the fixed SVA value of the root bus.

$P_{\mathrm{in},n},Q_{\mathrm{in},n}$ in (1-3), (1-4) are expressed as formula (10).

$$\begin{gathered} P_{in,n}=P_{g,n}-P_{d,n} \\ Q_{in,n}=Q_{g,n}+C_{p,n}+Q_{r,n}-Q_{d,n} \end{gathered} \tag{10}$$

where P_d,Q_d represent active and reactive bus load.

ORPF is a MINNLP problem which is not easy to solve. However, applying the SOC relaxation technique, ORPF can be transformed to a MISOCP problem which can be solved using commercial solver Gurobi or Cplex with high computation efficiency. The relaxed model is denoted by rxORPF in this paper and the exactness of the relaxation will be discussed in the next section.

3.2 Discussion on the Exactness of SOC Relaxation

The sufficient condition to guarantee the exactness of SOC relaxation is important and crucial to apply the SOC relaxation to ORPF. Ref [19, 20] indicated that if the load over-satisfaction is allowed in ORPF, the SOC relaxation is exact. However, the obtained optima may be physically meaningless. Ref [21] showed that if all power in the lines flow from the root node toward to the end nodes, the SOC relaxation is exact if there's no upper limit of SVA. Ref [22] proved that the SOC relaxation is exact if parameter condition is satisfied and the upper limit of SVA are large enough or the upper limit of SVA is modified introducing additional constraints in ACOPF with strictly increasing objective function. Ref [15, 23] both validated the exactness of SOC relaxation after the ORPF is solved.

However, the sufficient conditions provided need further verification.

As discussed in [22], the sufficient condition given by [22] is tighter than that given by [21], we will check whether the condition is satisfied by given examples in case study according to the theorem given by [22]. We denote the parameter condition as C-22 and the details can be found in this literature which is omitted here for simplicity. Therefore, the SOC relaxation is exact if C-22 is satisfied and the upper limit of SVA is large enough or the upper limit of SVA is modified.

4 CASE STUDY

4.1 Case Illustration

The modified IEEE 33 and 69 bus system are adopted in this section to study the effectiveness of the proposed method. The topology of modified IEEE 33 bus system is given in figure 4 and its main parameters can be found in [26] with the voltage amplitude in the range [0.95, 1.05]. Parameters of virtual generator, new added compensating capacitors and SVG's are given in table 1 and table 2.

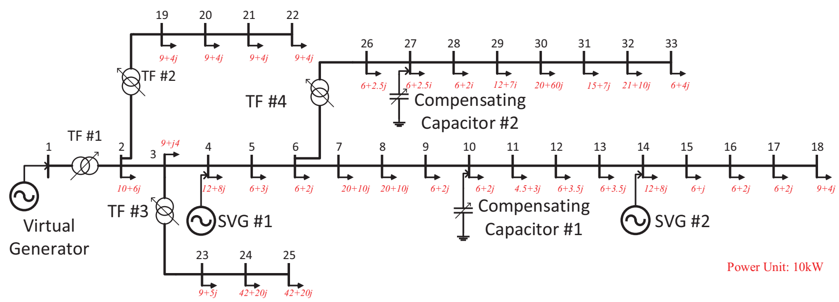

Fig.4 Topology of modified IEEE 33 bus system

Table 1 Parameters for modified IEEE 33 bus system

Devices/Parameters	Lower Limit	Upper Limit	Position Step Width
TF #1~4	0.95	1.05	0.005
Compensating Capacitor #1 (kVAR)	0	450	50
Compensating Capacitor #2 (kVAR)	0	300	50
SVG #1 (kVAR)	-10	500	Continuous
SVG #2 (kVAR)	0	400	Continuous
Virtual Generator (P:MW)	0	5	Continuous
Virtual Generator (Q:MVAR)	-1	3	Continuous

The parameters of IEEE 69 bus system can be found in [27] and the parameters for transformers, compensating capacitors, SVG's and virtual generator are the same with modified IEEE 33 bus system except that the active and reactive power generation ranges of virtual generator are modified to 0MW~10MW and -1MVAR~10MVAR respectively. The grid-connection information of discrete devices in modified IEEE 69 bus system is given in table 2.

Table 2 Grid-connection information of discrete devices in modified IEEE 69 bus system

Devices	Compensating Capacitor #1~2	SVG #1~2	TF #1~4
Grid-connected bus or line	10, 17	4, 14	1, 18, 22, 25

In this paper, the ORPF model will be coded with YALMIP [28] in MATLAB and solved by Gurobi 6.0 on IBM X230 Intel(R) Core i5, 2.60 GHz, 8GB RAM Personal Computer.

4.2 Computation Results

We firstly check C-22 for both IEEE 33 and 69 bus system and find that the condition is both satisfied.

A simple way to modify the upper limit of SVA given by [12] is dropping the upper limit of SVA. Thus, we solve rxORPF according the following procedure:

(S1) Solve rxORPF and check whether the SOC relaxation is exact;

(S2) If S1 fails, solve rxORPF without upper limit of SVA and check whether the SVA constraints are satisfied;

However, we solved the numerical examples and found that (S1) can always obtain a optima with exact SOC relaxation indicated by figure 5 and figure 6. Actually, for a wide range of IEEE benchmark or practical cases, (S1) can always find an optimal solution, i.e. the global optima of the original ORPF problem.

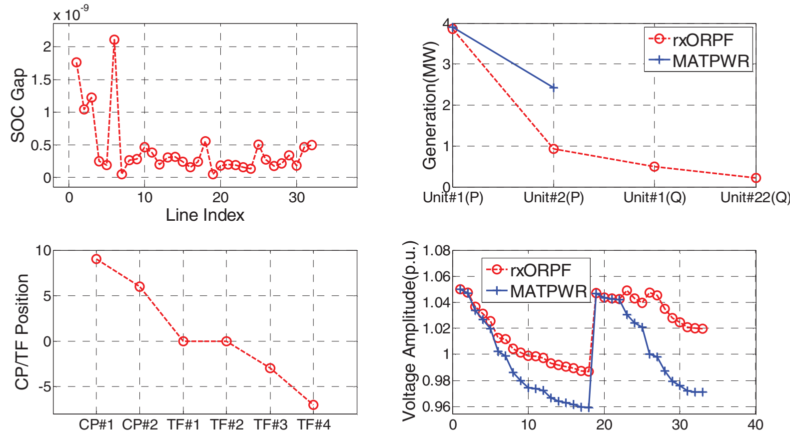

Fig.5 Computation results for modified IEEE 33 bus system

The computation results of modified IEEE 33 bus system is shown in figure 5. The computation result without reactive devices solve by MATPOWER is also presented.

In the modified IEEE 33 bus system case, the power loss is 23.88% less than the situation that contains no reactive power devices, i.e. transformers, SVG's and compensating capacitors.

Similar computation results can be obtained in modified IEEE 69 bus system given in figure 6. And the power loss is 23.88% less than the situation that contains no reactive power devices.

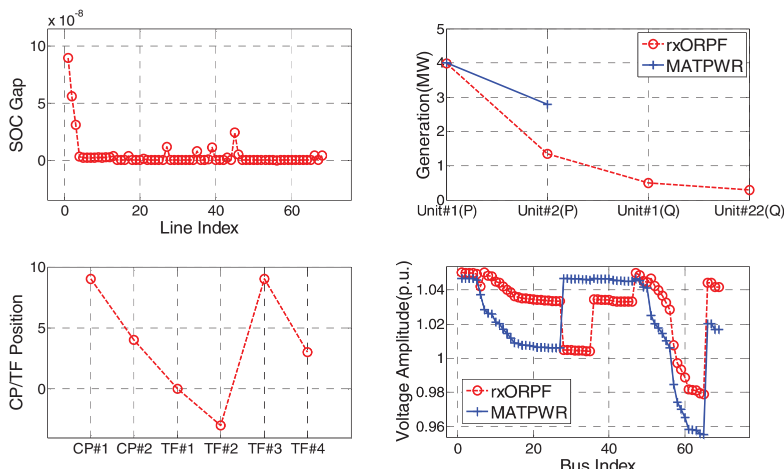

Fig.6 Computation results for modified IEEE 69 bus system

To further illustrate the computation efficiency of rxORPF, the ORPF model is also solved directly using MINLP solver BNB and Bonmin. The computation results are given in table 3.

Table 3 Comparison of various computation results to solve rxORPF and ORPF

System/parameters	Results	rxORPF(MISOCP)	ORPF(MINNLP)	
		Gurobi	BNB	Bonmin
IEEE 33	Time(s)	0.36	54.36	3.04
	Power Loss(kW)	143.6	143.6	143.6
IEEE 69	Time(s)	0.35	74.22	0.74
	Power Loss(kW)	189.9	189.9	failed

Table 4 shows that in most cases, the same objective can be obtained. However, the computation time of rxORPF is higher than ORPF despite of the solvers used to solve the MINNLP problem. Specially, the Bonmin failed to find a feasible solution in modified IEEE 69 bus system because the convergence of MINLP solver cannot be guaranteed when solving a MINNLP problem.

All the computation results showed the effectiveness of the correctness of the proposed exact linearized transformer model which can be easily incorporated into the ORPF model based on Branch Flow Model. The ORPF can still be solved with high computation efficiency after relaxed to a MISOCP problem under a widely applicable conditions.

5 CONLUSIONS

In this paper, we studied the optimal reactive power problem in distribution power networks. The piecewise linear technique is adopted to achieve exact linearization of transformer based on Branch Flow Model. Appling the latest second-order cone relaxation technique, the ORPF is relaxed to a MISOCP problem which can be solved by commercial solver Gurobi with high computation efficiency. The case study based on modified IEEE 33 and 69 system validated the effectiveness of the proposed method.

We also studied the condition to guarantee the exactness of SOC relaxation in this paper and found that the sufficient condition proposed in previous literature is applicable for cases with transformers. However, in practical application, the condition should be checked for each given numerical examples. How to incorporate uncertainties in distribution power networks, e.g. forecast error of load or distributed generators, need further research.

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