Theoretical Framework for Integrating Distributed Energy Resources into Distribution Systems

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***Abstract*—This paper focuses on developing a novel theoretical framework for effective coordination and control of a large number of distributed energy resources in distribution systems. The proposed framework provides a systematic view of the overall structure of the future distribution systems along with the underlying information flow, functional organization, and operational procedures. It is characterized by the features of being open, flexible and interoperable for managing active distribution systems. Under this framework, the energy consumption of various DERs is coordinated and controlled using market-based approaches in a hierarchical manner. The Volt/VAR control design is simultaneously considered in order to maintain stable nodal voltages within acceptable ranges. In addition, computational challenges associated with the proposed framework are also discussed with recommended practices.**

***Index Terms*—Distributed energy resources, distribution system, energy market, distributed market clearing, Volt/VAR control.**

I. Introduction

The increased penetration of renewable energy has imposed significant challenges to power system operation. Hence, the ISO requires flexibility and reserve to address the uncertainty and variability introduced by renewable energy. Therefore, new responsive distributed energy resources (DERs) such as controllable loads, distributed generators and inverter-interfaced energy resources will be needed to help manage the power grid in terms of both system efficiency and operational reliability. In order to harvest the benefits introduced by various DERs, it is necessary to develop appropriate coordination and control approaches to enable seamless integration of a large amount of DERs into the power distribution system. Such coordination and control will be one of critical functions in the next-generation distribution management system.

During the past few years, many studies have been dedicated to distributed approaches for coordinating distributed generators, such as two-level incremental cost consensus algorithm [1], distributed algorithm based on consensus and bisection method [2], and minimum-time consensus algorithm [3], just to name a few. Recently, the coordination between distributed generators and controllable loads has been reported in [4] and [5]. Although useful insights have been provided in these studies, the existing results cannot be directly extended and applied to practical applications. This is because that the controllable loads were simply modeled as a "generator" with negative generation, where the load characteristics and dynamics were totally ignored. Furthermore, they did not address the issue of designing real-time load control strategies to achieve the optimal power consumption. In [6], a hierarchical framework that relates short-term scheduling with real-time control was proposed for the first time for systematic DER integration. The underlying control strategy takes into account the detailed characteristics and dynamics of controllable loads, and addresses the issue of designing real-time control strategies. However, the power flow constraints of the distribution system were not considered therein.

In this paper, the framework proposed in [6] is extended to incorporate the power flow constraints of distribution systems to improve system fidelity. In order to be tractable and scalable with large-scale integration of DERs, the proposed framework decomposes the distribution system into three levels. In this way, different information will be processed at different levels to greatly reduces the burdens of computation and communication that would be required by centralized information processing. Under the proposed framework, the load serving entity (LSE) serves as the interface between DERs and the distribution system operator (DSO). It collects and aggregates information from DERs within its authority, and then disaggregates and dispatches signals received from the DSO back to DERs. From the view of the DSO, the distribution system becomes significantly reduced because it only needs to interact with the LSEs. Specifically, the proposed theoretical framework applies market-based approaches to realize the optimal energy management of various DERs. Two different market clearing strategies are discussed. By developing an energy market across the distribution system, the local objectives of individual DER owners can be respected, and their participation can be equitably rewarded as well. Besides the energy management, the Volt/VAR control design that utilize the reactive power control capability of PV inverters is also considered herein in order to maintain the real-time nodal voltage profiles within permissible ranges in the presence of system uncertainties.

II. System Modeling and Decomposition

Consider a power distribution system with N nodes, where node 0 represents the distribution substation connected to a transmission network. The DSO is located at the substation (node 0) and directly interacts with individual LSEs that are distributed throughout the distribution system. Individual LSEs communicate with various DERs under their authority to realize resource aggregation and control. This distribution system can be abstracted by a tree graph denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{0\} \cup \mathcal{N}$ with $\mathcal{N} = \{1, 2, \cdots, N-1\}$ denotes the set of nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denotes the set of distribution lines connecting different nodes. Individual LSEs can be abstracted by a tree graph denoted by $\mathcal{G}_m = (\mathcal{V}_m, \mathcal{E}_m)$, $m = 1, \ldots, M$, where $\mathcal{G}_m$ is connected, $\mathcal{V}_m \subset \mathcal{N}$ are disjoint sets from each other and $\mathcal{E}_m \subset \mathcal{V}_m \times \mathcal{V}_m$. Furthermore, let node $i_m \in \mathcal{V}_m$ denote the root of $\mathcal{G}_m$.

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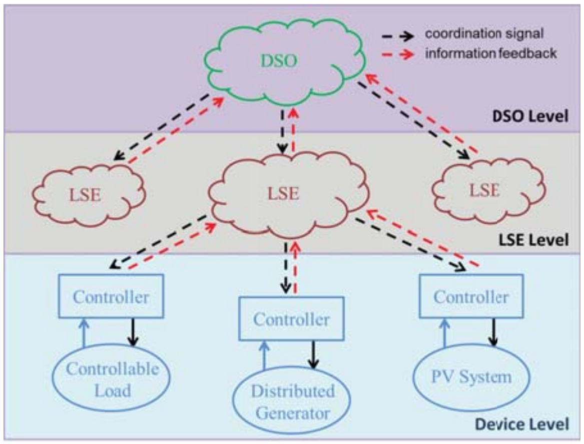

Fig. 1. Schematics of proposed theoretical framework.

For each node $i \in \mathcal{V}$, let $p_i \in \mathbb{R}$ and $q_i \in \mathbb{R}$ denote the *negative* of net real and reactive power injections, respectively, and let $v_i \in \mathbb{C}$ with $v_i = V_i \angle \theta_i$ denote the phasor for the line-to-ground voltage. For each distribution line $(i,j) \in \mathcal{E}$, let r_{ij} and x_{ij} denote the line resistance and reactance, respectively. Although it is more appropriate to consider the unbalanced three-phase distribution system, the subsequent framework development is based on the following balanced single-phase-equivalent power flow model for notational convenience and descriptive simplicity,

$$p_i = -V_i \sum_{j \in \mathcal{V}, j \neq i} V_j \left(g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij} \right) \tag{1a}$$

$$q_i = -V_i \sum_{j \in \mathcal{V}, j \neq i} V_j \left(g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij} \right), \tag{1b}$$

where $\theta_{ij} = \theta_i - \theta_j$, and $g_{ij} + \mathbf{i} b_{ij} = (r_{ij} + \mathbf{i} x_{ij})^{-1}$. However, the following results should be readily extended to account for the unbalanced three-phase power flow model. This can be done by adopting the multi-phase extension of (1) as shown in [7], [8]. For the system downstream of each node $i \in \mathcal{V}_m$, where various DERs such as controllable loads, distributed generators, and PV systems are connected, the associated power flow constraints is assumed to be neglected.

In this paper, a theoretical framework with hierarchical structure as shown in Fig. 1 is proposed to facilitate the large-scale integration of various DERs in distribution systems. Under the proposed framework, the DER coordination and control are realized in two stages. At the scheduling stage of slow time scale, the DSO sets up an energy market to realize optimal resource allocation by solving an optimization problem with information provided by individual LSEs. Then, the DSO sends the optimal dispatch signals to individual LSEs, which further disaggregate the signals and send them to DERs under their authority. At the operation stage of fast time scale, individual DERs carry out real-time control to follow the optimal dispatch signals. At the same time, various DERs that are capable of reactive power control will be systematically coordinated to maintain the nodal voltage profile within permissible ranges in the presence of operational uncertainties. In the following sections, the proposed coordination and control framework will be described in details at each level.

III. Device-level Control

When participating into the energy market, individual DERs are considered as self-interested agents with local objectives. They are automated to perform local control actions by interacting with other agents through the market aiming to maximize their local benefits. This local objective can be modeled as a surplus maximization problem. In this paper, individual DERs in the market are price taking agents, namely, the decisions made by individual agents will not significantly affect the market price. This is a standard assumption when the market involves a large number of players [9]. In the following subsections, the device-level control is introduced for three different types of DERs under node $i \in \mathcal{V}_m$, respectively.

A. Controllable Load

For the k-th controllable load connected to node i, $k = 1, \ldots, N_{mi}^{\mathrm{L}}$, the surplus maximization problem is given by

$$(\mathcal{P}_{\mathrm{DER,L}}) \quad \underset{p_{ik,\mathrm{L}}}{\text{Maximize}} \quad U_{ik,\mathrm{L}}\left(p_{ik,\mathrm{L}}\right) - \lambda_i p_{ik,\mathrm{L}} \tag{2a}$$

$$\text{subject to} \quad (p_{ik,\mathrm{L}}, q_{ik,\mathrm{L}}) \in \Omega_{ik,\mathrm{L}} \tag{2b}$$

where $U_{ik,\mathrm{L}}(\cdot)$ is the utility function characterizing the satisfaction of energy consumption, λ_i denotes the energy price at node i, and $\Omega_{ik,\mathrm{L}}$ denotes the feasible set of real and reactive power[1], $p_{ik,\mathrm{L}}$ and $q_{ik,\mathrm{L}}$, of the controllable load. For controllable loads, the feasible set $\Omega_{ik,\mathrm{L}}$ is usually in the following form,

$$\Omega_{ik,\mathrm{L}} = \left\{ (p_{ik,\mathrm{L}}, q_{ik,\mathrm{L}}) \;\middle|\; \begin{array}{c} p_{ik,\mathrm{L}}^{\min} \leq p_{ik,\mathrm{L}} \leq p_{ik,\mathrm{L}}^{\max} \\ q_{ik,\mathrm{L}} = \alpha_{ik,\mathrm{L}} p_{ik,\mathrm{L}} \end{array} \right\},$$

where $\alpha_{ik,\mathrm{L}}$ is a constant. For example, $\alpha_{ik,\mathrm{L}}$ is nonzero for residential air conditioners but zero for electric resistance water heaters. When λ_i is received from the m-th LSE, where $i \in \mathcal{V}_m$, the desired power consumption can be determined by solving $(\mathcal{P}_{\mathrm{DER,L}})$ defined in (2) as

$$p_{ik,\mathrm{L}}^{*}(\lambda_i) = \underset{p_{ik,\mathrm{L}}}{\operatorname{argmax}} \; U_{ik,\mathrm{L}}\left(p_{ik,\mathrm{L}}\right) - \lambda_i p_{ik,\mathrm{L}}.$$

Then the local controller has to maintain the power consumption, $p_{ik,\mathrm{L}}$, to follow $p_{ik,\mathrm{L}}^{*}$. However, there are two critical challenges associated with local controller design for controllable loads. The first challenge is that the energy consumption of controllable loads with discrete instantaneous power cannot not be directly controlled and is often affected by other inputs. It is important to determine locally acceptable control inputs so that controllable loads can follow the optimal dispatch signals obtained at the scheduling stage. The second challenges is that it is not straightforward to determine the required utility functions of controllable loads for market participation. To accurately characterize utility functions, it is necessary to quantitatively relate the marginal utility of individual customers for power consumption to the local control input based on the preferences of individual customers. That is, local controller design has to be linked with individual economic preferences so that control decisions can be driven by the economic signal. In [10], a practical method is presented for air conditioners to extract such a relationship. Although it has been specifically presented for air conditioners, the underlying control philosophy can be easily extended and applied to other types of controllable loads.

[1] For a given period, "power" and "energy" are interchangeable in this paper.

B. Distributed Generator

For the k-th distributed generator connected to node i, $k = 1, \ldots, N_{mi}^{\mathrm{G}}$, the surplus maximization problem is given by

$$(\mathcal{P}_{\mathrm{DER,G}}) \quad \underset{p_{ik,\mathrm{G}}}{\text{Maximize}} \quad \lambda_i p_{ik,\mathrm{G}} - C_{ik,\mathrm{G}}\left(p_{ik,\mathrm{G}}\right) \tag{3a}$$
$$\text{subject to} \quad (p_{ik,\mathrm{G}}, q_{ik,\mathrm{G}}) \in \Omega_{ik,\mathrm{G}} \tag{3b}$$

where $C_{ik,\mathrm{G}}(\cdot)$ is the cost function characterizing the cost of energy production, λ_i denotes the energy price at node i, and $\Omega_{ik,\mathrm{G}}$ denotes the feasible set of real and reactive power, $p_{ik,\mathrm{G}}$ and $q_{ik,\mathrm{G}}$, of the distributed generator. When λ_i is received from the m-th LSE, where $i \in \mathcal{V}_m$, the desired power generation can be determined by solving $(\mathcal{P}_{\mathrm{DER,G}})$ defined in (3) as

$$p_{ik,\mathrm{G}}^{*}(\lambda_i) = \underset{p_{ik,\mathrm{G}}}{\operatorname{argmax}} \ \lambda_i p_{ik,\mathrm{G}} - C_{ik,\mathrm{G}}\left(p_{ik,\mathrm{G}}\right).$$

It is straightforward for distributed generators to follow the desired power generation $p_{ik,\mathrm{G}}^{*}$ because their generation level can be continuously adjusted by well established controllers. Furthermore, it is also simple to determine the cost functions of distributed generators based on easily identified factors such as generator operational cost, fuel efficiency, and fuel cost.

C. PV System

For the k-th PV system connected to node i, $k = 1, \ldots, N_{mi}^{\mathrm{I}}$, the surplus maximization problem is given by

$$(\mathcal{P}_{\mathrm{DER,I}}) \quad \underset{p_{ik,\mathrm{I}}}{\text{Maximize}} \quad \lambda_i p_{ik,\mathrm{I}} \tag{4a}$$
$$\text{subject to} \quad (p_{ik,\mathrm{I}}, q_{ik,\mathrm{I}}) \in \Omega_{ik,\mathrm{I}} \tag{4b}$$

where λ_i denotes the energy price at node i, and $\Omega_{ik,\mathrm{I}}$ denotes the feasible set of real and reactive power, $p_{ik,\mathrm{I}}$ and $q_{ik,\mathrm{I}}$, of the PV system. For PV systems, the feasible set $\Omega_{ik,\mathrm{I}}$ is usually of the following form,

$$\Omega_{ik,\mathrm{I}} = \left\{ (p_{ik,\mathrm{I}}, q_{ik,\mathrm{I}}) \,\middle|\, \begin{array}{c} 0 \le p_{ik,\mathrm{I}} \le p_{ik,\mathrm{I}}^{\max} \\ p_{ik,\mathrm{I}}^2 + q_{ik,\mathrm{I}}^2 \le s_{ik,\mathrm{I}}^2 \end{array} \right\},$$

where $p_{ik,\mathrm{I}}^{\max}$ is the forecasted maximal power, and $s_{ik,\mathrm{I}}$ is the capacity of the PV inverter. It is clear that the solution to the problem $(\mathcal{P}_{\mathrm{DER,I}})$ is $p_{ik,\mathrm{I}} = p_{ik,\mathrm{I}}^{\max}$. It implies that PV systems would like to maximize solar power extraction under all conditions, which can be achieved by applying the technique of maximum power point tracking (MPPT).

IV. LSE-level Aggregation

At the scheduling stage, individual DERs send energy bids along with constraints to the LSE in order to participate into the energy market. Then the LSE aggregates individual information together to obtain the LSE-level energy bid and constraint, which will be finally sent to the DSO for market clearing. Let p_{i_m} and q_{i_m} denote the real and reactive power flow from node i_m into the m-th LSE, respectively. That is,

$$p_{i_m} + \mathbf{i} q_{i_m} = \sum_{h \in \mathcal{V}_{i_m}} v_{i_m} \left(\frac{v_{i_m} - v_h}{r_{i_m h} + \mathbf{i} x_{i_m h}} \right)^{*},$$

where $\mathcal{V}_{i_m} = \{h : (i_m, h) \in \mathcal{E}_m\}$ and $(\cdot)^*$ denotes the complex conjugate. The information aggregation at the LSE level is achieved by solving the following optimization problem,

$$(\mathcal{P}_{\mathrm{LSE}}) \quad \underset{p_m}{\text{Maximize}} \quad \sum_{i \in \mathcal{V}_m} U_i\left(p_i\right) - \lambda_{i_m} p_{i_m} \tag{5a}$$
$$\text{subject to} \quad \text{(1a)–(1b)}, \ \forall i, j \in \mathcal{V}_m, \tag{5b}$$
$$v_i^{\min} \le v_i \le v_i^{\max} \tag{5c}$$
$$l_{ij}^{\min} \le |l_{ij}(v_i, v_j)| \le l_{ij}^{\max} \tag{5d}$$
$$(p_i, q_i) \in \Omega_i, \tag{5e}$$

where $l_{ij}(v_i, v_j) \in \mathbb{C}$ denotes the line flow from node i to node j. In the above optimization problem $(\mathcal{P}_{\mathrm{LSE}})$, $U_i(\cdot)$ denotes the aggregated utility function derived from energy bids received from individual DERs under node i, λ_{i_m} is the energy price received from the DSO, and p_i and q_i are calculated as

$$p_i = -\sum_{k=1}^{N_{mi}^{\mathrm{G}}} p_{ik,\mathrm{G}} - \sum_{k=1}^{N_{mi}^{\mathrm{I}}} p_{ik,\mathrm{I}}^{\max} + \sum_{k=1}^{N_{mi}^{\mathrm{L}}} p_{ik,\mathrm{L}} + p_{i,\mathrm{L}}^{\mathrm{uc}} \tag{6}$$
$$q_i = -\sum_{k=1}^{N_{mi}^{\mathrm{G}}} q_{ik,\mathrm{G}} - \sum_{k=1}^{N_{mi}^{\mathrm{I}}} q_{ik,\mathrm{I}} + \sum_{k=1}^{N_{mi}^{\mathrm{L}}} q_{ik,\mathrm{L}} + q_{i,\mathrm{L}}^{\mathrm{uc}}, \tag{7}$$

where $p_{i,\mathrm{L}}^{\mathrm{uc}}$ and $q_{i,\mathrm{L}}^{\mathrm{uc}}$ are the forecasted uncontrollable real and reactive power, respectively, and Ω_i is the aggregated feasible set for p_i and q_i defined as $\Omega_i = \Omega_{i,\mathrm{G}} \uplus \Omega_{i,\mathrm{I}} \uplus \Omega_{i,\mathrm{L}}$ with

$$\Omega_{i,\mathrm{b}} = \biguplus_{k=1}^{N_{mi}^{\mathrm{b}}} \Omega_{ik,\mathrm{b}}, \quad b \in \{\mathrm{G}, \mathrm{I}, \mathrm{L}\},$$

where $\uplus$ denotes the Minkowski sum. By solving $(\mathcal{P}_{\mathrm{LSE}})$ for each given price λ_{i_m} received from the DSO, the relationship between p_{i_m} and λ_{i_m} can be determined, which leads to the utility function $U_{i_m}(p_{i_m})$ of the m-th LSE.

V. DSO-level Coordination

At the scheduling stage, the DSO sets up an energy market in order to realize optimal resource allocation among various DERs in the distribution system. To proceed, the distribution system $\mathcal{G}_m$ under the authority of the m-th LSE is replaced by node i_m in the distribution system. Then the set of nodes for the reduced distribution system is defined as

$$\mathcal{V}_d = \left(\mathcal{V} \setminus \left(\bigcup_{m=1}^{M} \mathcal{V}_m \right) \right) \cup \{i_1, i_2, \ldots, i_M\}.$$

Recall that p_{i_m} and q_{i_m} are the negative of net real and reactive power injections, respectively, at node i_m. Let p_0 and q_0 denote the real and reactive power flow from node 0 into the DSO, respectively. That is,

$$p_0 + \mathbf{i} q_0 = \sum_{h \in \mathcal{V}_0} v_0 \left(\frac{v_0 - v_h}{r_{0h} + \mathbf{i} x_{0h}} \right)^{*},$$

where $\mathcal{V}_0 = \{h : (0, h) \in \mathcal{E}\}$. Then the market clearing is equivalent to solving the following optimization problem,

$$(\mathcal{P}_{\mathrm{DSO}}) \quad \underset{p_m}{\text{Maximize}} \quad \sum_{i_m \in \mathcal{V}_d} U_{i_m}\left(p_{i_m}\right) - \lambda_0 p_0 \tag{8a}$$
$$\text{subject to} \quad \text{(1a)–(1b)}, \ \forall i, j \in \mathcal{V}_d, \tag{8b}$$
$$v_i^{\min} \le v_i \le v_i^{\max} \tag{8c}$$
$$l_{ij}^{\min} \le |l_{ij}(v_i, v_j)| \le l_{ij}^{\max} \tag{8d}$$
$$(p_{i_m}, q_{i_m}) \in \Omega_{i_m}, \tag{8e}$$

where λ_0 is the locational marginal price (LMP) received from the wholesale market at the ISO level.

The optimization problem $(\mathcal{P}_{\text{DSO}})$ can actually be solved using two different methods. That is, there exist two distinct ways for market clearing at the DSO level.

A. Centralized Market Clearing

When there is no privacy issue between individual LSEs and the DSO, the market clearing can be achieved in a centralized way. In this case, individual LSEs are willing to share local information with the DSO. They will directly submit to the DSO the aggregated utility function $U_{i_m}(\cdot)$ and feasible set Ω_{i_m} that by solving the optimization problem $(\mathcal{P}_{\text{LSE}})$ based on the information received from DERs. Then the DSO will solve the optimization problem $(\mathcal{P}_{\text{DSO}})$ by itself and determine the nodal prices λ_{i_m}. After that, the DSO will send these prices back to individual LSEs so that they can solve the optimization problem $(\mathcal{P}_{\text{LSE}})$ to obtain disaggregated dispatch signals for various DERs under their authority. Examples of centralized market clearing can be found in [10], [11].

B. Distributed Market Clearing

When there is privacy issue between individual LSEs and the DSO, the market clearing has to be achieved in a distributed way. In this case, individual LSEs would like to preserve local privacy and, thus, do not want to share the aggregated utility function $U_{i_m}(\cdot)$ and feasible set Ω_{i_m} with the DSO. Hence, the DSO does not have all the required information to solve the optimization problem $(\mathcal{P}_{\text{DSO}})$ by itself. Instead, the DSO will initialize the nodal prices λ_{i_m} and sends them to individual LSEs. After solving $(\mathcal{P}_{\text{LSE}})$ based on the received λ_{i_m}, individual LSEs will send the net real power injection p_{i_m} and the corresponding feasible set of q_{i_m} back to the DSO. Based on these feedback information, the DSO will update the nodal prices λ_{i_m} in order to reduce the mismatch, and send the new prices to individual LSEs again. This process will be repeated until all the nodal prices λ_{i_m} converge. Then individual LSEs can finally solve the optimization problem $(\mathcal{P}_{\text{LSE}})$ to obtain disaggregated dispatch signals for various DERs under their authority. Examples of distributed market clearing can be found in [3], [12].

C. Computational Challenges

When applying the proposed theoretical framework to the practical distribution systems, there is a common computational challenge to solving both $(\mathcal{P}_{\text{LSE}})$ and $(\mathcal{P}_{\text{DSO}})$. This challenge is imposed by the non-convex feasible regions associated with nonlinear power flow equations defined in (1). In order to overcome this challenge, different representations of power flow equations have to be considered. In fact, the power flow of a distribution network can be described by the following angle-relaxed branch flow model,

$$p_{ij} = \sum_{h:(j,h)\in\mathcal{E}} p_{jh} + p_j + r_{ij}L_{ij} \tag{9a}$$

$$q_{ij} = \sum_{h:(j,h)\in\mathcal{E}} q_{jh} + q_j + x_{ij}L_{ij} \tag{9b}$$

$$V_i^2 - V_j^2 = 2\,(r_{ij}p_{ij} + x_{ij}q_{ij}) - (r_{ij}^2 + x_{ij}^2)L_{ij} \tag{9c}$$

$$L_{ij} = \frac{p_{ij}^2 + q_{ij}^2}{V_i^2}, \tag{9d}$$

where p_{ij} and q_{ij} are the active and reactive power flows along the distribution line $(i,j)\in\mathcal{E}$, p_j and q_j are defined in (6) and (7), respectively, and $L_{ij} = |l_{ij}|^2$ is the squared current magnitude. It has been shown in [13] that the above relaxation (dropping the angle information in voltage) is exact for radial distribution systems with tree topology. It is worth mentioning that, even though the above angle-relaxed branch flow model is still nonlinear (and non-convex) in the decision variables, all optimization problems $(\mathcal{P}_{\text{LSE}})$ and $(\mathcal{P}_{\text{DSO}})$ can be solved with this model. Particularly, it has been shown in [14] that the convex relaxation replacing (9d) by

$$L_{ij} \geq \frac{p_{ij}^2 + q_{ij}^2}{V_i^2}$$

is exact for optimal power flow problems.

VI. Real-time Voltage Control

The optimal solution to the above optimization problems $(\mathcal{P}_{\text{DSO}})$, $(\mathcal{P}_{\text{LSE}})$ and $(\mathcal{P}_{\text{DER}})$ associated with the energy market defines the scheduled operating point of the distribution system for each scheduling period. However, the actual operating point may deviate from the scheduled one due to high volatility of renewable energy, inaccurate prediction of uncontrollable load consumptions, or unexpected contingency events. In particular, the nodal voltages will inevitably deviate from the scheduled operating point, and even become unstable in the worst case. Hence, it is necessary to deploy appropriate (centralized or distributed) Volt/VAR control strategies to maintain the stable and reliable system operation.

The fundamental objective of the local Volt/VAR control problem is to stabilize the nodal voltages of the distribution system, which has been studied extensively in the literature. In practice, reactive power injection into the distribution system plays a major role in helping regulate the voltage profile. Traditionally, the on-load tap changers, capacitor banks and shunt capacitors pre-installed in the distribution system are controlled to provide reactive power support. However, these traditional devices can only be operated in several discrete modes and are relatively inaccurate. Very recently, with the increasing penetration of renewable energy resources (e.g. wind and solar), the idea of using inverters associated with these DERs to provide reactive power support has been extensively investigated (see, for example, [15]–[17]).

In order to facilitate the development of computationally tractable Volt/VAR control strategies, a linearized approximation of the nonlinear power flow equations (1) is often utilized in the literature. Note that the angle-relaxed branch flow model (9) can be linearized by assuming that the loss over each line $(i,j)\in\mathcal{E}$ is negligible, i.e. $r_{ij}L_{ij}\approx 0$ and $x_{ij}L_{ij}\approx 0$ and the voltage profiles across the distribution system is relatively flat, i.e. $V_i \approx 1$ in per unit. Hence, the resulted linearized power flow equations are given by

$$p_{ij} = \sum_{h:(j,h)\in\mathcal{E}} p_{jh} + p_j \tag{10a}$$

$$q_{ij} = \sum_{h:(j,h)\in\mathcal{E}} q_{jh} + q_j \tag{10b}$$

$$V_i - V_j = r_{ij}p_{ij} + x_{ij}q_{ij}, \tag{10c}$$

which can be written in the following compact form,

$$\boldsymbol{V} = \boldsymbol{R}\boldsymbol{p} + \boldsymbol{X}\boldsymbol{q} + \boldsymbol{V}_0,$$

where $\boldsymbol{R}$ and $\boldsymbol{X}$ are constant matrices depending only on the configuration of the distribution system, $\boldsymbol{V}$, $\boldsymbol{p}$ and $\boldsymbol{q}$ are the stacked vectors of voltage magnitudes, active power and reactive power, respectively, and $\boldsymbol{V}_0$ is the vector with all the elements equal to the voltage magnitude of node 0. This simplification is primarily introduced for the Volt/VAR control design and has been widely adopted [15]–[17]. The readers are referred to [15], [16] for detailed derivations of the matrices R and X and more discussions. Let $\boldsymbol{q}_I$ denote the vector of controllable reactive power at individual nodes. The relationship between $\boldsymbol{q}_I$ and the voltage profile $\boldsymbol{V}$ of the distribution system can be modeled as

$$\boldsymbol{V} = \boldsymbol{X}\boldsymbol{q}_I + \tilde{\boldsymbol{V}}, \tag{11}$$

where $\tilde{\boldsymbol{V}}$ denotes the voltage profiles without any reactive power control in the system.

The Volt/VAR control problem seeks to determine a control policy $\boldsymbol{u}$ for controllable reactive power so that the following dynamical system has a stable equilibrium,

$$\begin{cases} \boldsymbol{V}(k+1) = \boldsymbol{X}\boldsymbol{q}_I(k+1) + \tilde{\boldsymbol{V}} \\ \boldsymbol{q}_I(k+1) = \boldsymbol{u}(\boldsymbol{q}_I(k), \boldsymbol{V}(k); \boldsymbol{V}^*), \end{cases}$$

where k denotes the discrete time, and $\boldsymbol{V}^*$ is the vector of optimal nodal voltages specified by the scheduled operating point. Currently, most existing works focus on fully decentralized (local) control policies, i.e.

$$q_{i,I}(k+1) = u_i(q_{i,I}(k), V_i(k); V_i^*).$$

One of the extensively studied control policy is the so-called droop-like controller which is given by

$$q_{i,I}(k+1) = [-K_i(V_i(k) - V_i^*)]_{q_{i,I}^{\min}}^{q_{i,I}^{\max}},$$

where K_i is the gain to be determined and $[x]_a^b$ is the projection of x into the interval $[a,b]$. This type of controller actually compiles with the IEEE 1547.8 standard [18]. However, it has been shown in [15] that this droop like controller may result in instable trajectories of voltage profile for inappropriate choices of K_i's and the conditions guaranteeing stable voltage profile is somewhat restrictive.

More recently, an incremental controller have been proposed in [16], [17], which can be considered as the algorithm to solve the following optimization problem,

$$\underset{\boldsymbol{q}_I \in [\boldsymbol{q}_I^{\min}, \boldsymbol{q}_I^{\max}]}{\text{Minimize}} \quad \|\boldsymbol{V} - \boldsymbol{V}^*\|^2_{\boldsymbol{X}^{-1}} + C(\boldsymbol{q}_I) \tag{12a}$$

$$\text{subject to} \quad (11),, \tag{12b}$$

where $\|\cdot\|^2_{\boldsymbol{X}^{-1}}$ is the Euclidean norm scaled by matrix $\boldsymbol{X}^{-1}$, and $C(\cdot)$ is commonly a quadratic function capturing the cost of reactive power injection. It has been shown that the above problem naturally admits fully decentralized algorithms, which can be selected as the control policy. This type of controller allows for a less restrictive convergent condition and provides a much faster convergence speed.

VII. Conclusions

In this paper, a novel theoretical framework was proposed for integrating DERs into distribution systems. This framework provides a scalable and flexible way to manage the large-scale integration of DERs. It relates short-term scheduling and real-time control together through hierarchical coordination and control taking into account power flow constraints of distribution system. Market-based approaches are adopted for energy management of various DERs in order to respect their local objectives and reward their participation. On the other hand, the real-time Volt/VAR control using PV inverters is considered to complement the active power control so that the nodal voltage profiles are within acceptable ranges in the presence of system operational uncertainties.

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