

A GAME-THEORETIC RATIONALE FOR VAGUENESS

1. INTRODUCTION

This paper presents a game-theoretic rationale for vagueness, in showing how vagueness can assure that communication still takes place between a speaker and a listener with conflicting interests.¹ For the related phenomena of generality and ambiguity, similar game-theoretic rationales have already been provided, and are relatively straightforward. A game-theoretic rationale for generality is found in Crawford and Sobel (1982). To understand this rationale, one can think of a speaker who has private information, and of a listener who would like to let his actions depend on the speaker's information. The speaker prefers the listener to take the action the listener takes when the listener receives no information at all from the speaker, creating a conflict of interest between speaker and listener. The speaker can now assure that the listener takes this action by making a general utterance that encompasses any private information that the speaker may have. It should be noted that Crawford and Sobel's account of strategic generality is more involved, and shows how the degree of generality of the speaker's utterances increases with the degree of conflict between speaker and listener. A game-theoretic rationale for ambiguity² is found in Aragonès and Zeeman (2000), and is best understood by an example.

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¹ Lipman (2001) shows that in a standard speaker-listener (or sender-receiver) game without any conflict of interest, there is no rationale for vagueness as vagueness is then suboptimal. Lipman argues that solving conflicts of interest does not seem to be the real reason why language is vague, and concludes that only bounded rationality of individuals can account for the phenomenon of vagueness. The ambition of my paper is not to account for vagueness as such, but rather to show how *pre-existing* vagueness can be exploited to solve conflicts of interest.

² A game-theoretic rationale for ambiguity where, contrary to what is the case in Aragonès and Zeeman (2000), there is no conflict of interest between speaker and listener, is



During an election campaign, a politician faces the dilemma that on the one hand, in order not to damage his reputation, he does not want to tell lies, and on the other hand he wants to appeal to as large an audience as possible, including voters with opposing interests. The politician can solve this dilemma by making an ambiguous utterance of a particular form. The politician's utterance is ambiguous in that it has several possible contents. Each voter finds it probable enough that this utterance possesses his most preferred content, and this to such an extent that each voter is willing to vote for the politician. Voters who experience after the elections that the politician's utterance had a different content can still not catch the politician on a lie, as his utterance was ambiguous.

A game-theoretic rationale for vagueness is less straightforward. This is both because strategic vagueness requires a different equilibrium concept than is required for strategic generality and strategic ambiguity (correlated equilibrium rather than Nash equilibrium, as we will see below), and because there is a lot of controversy concerning non-strategic views of vagueness in the first place. A good basis for my treatment is Lewis's philosophy of language (1969, 1975), as it is also game-theoretically founded. A short review of his philosophy of language is given in Section 2, and is interpreted in the light of recent developments in game theory. Different views of vagueness are also reviewed in this section, with reference to the signalling game described. In Section 3, Lewis's treatment is extended to the concept of correlated equilibrium. This allows us to give a first rationale for vagueness in the context of a game between two players seeking to meet each other at the same place. It is shown how this rationale for vagueness can be linked to the epistemic view of vagueness (Williamson, 1994). In Section 4, the rationale for vagueness is extended to a signalling game. Section 5 provides a conclusion.

2. USE OF LANGUAGE AS A COORDINATION PROBLEM

Lewis (1969) starts his classical book in the philosophy of language by claiming that "[u]se of language belongs to a class of situations with a conspicuous common character: situations ... [he] call[s] coordination

found in Parikh (2000). Parikh assumes that ambiguous utterances are more costly to make than unambiguous ones. His argument is that the speaker saves unambiguous, relatively expensive utterances for surprising situations. When a unsurprising situation occurs, the speaker makes an ambiguous, cheap, utterance. By putting himself in the speaker's shoes, the listener infers the content of the speaker's ambiguous utterance, in understanding that this utterance should be given its most likely interpretation, namely that it refers to an unsurprising situation.

	C1	C2
R1	(1, 1)	(0, 0)
R2	(0, 0)	(1, 1)

Figure 1. Coordination problem.

problems” (p. 5). What a coordination problem is, is best understood by an example. The canonical example is that of two persons seeking to meet each other, without one person directly observing where the other person is heading to. Figure 1 is a game-theoretic representation of such a coordination problem.

It is assumed in Figure 1 that there are only two places where the two players, call them Row and Column, could possibly meet, namely P1 and P2. Ri (resp. Ci) represents Row’s (resp. Column’s) action of going to Pi, with $i = 1, 2$. The expressions between brackets represent the players’ payoffs, the first number being Row’s payoff, and the second number Column’s payoff. These numbers stylise the fact that it does not matter to Row and Column whether they go to P1 or P2, as long as they go to the same place. To come to predictions about the outcome of such a game, the solution concept most commonly applied by game theorists is the Nash equilibrium. A Nash equilibrium is a combination of strategies such that each player maximises his payoff, given the strategies of all other players. In other words, in a Nash equilibrium, each player chooses his *best response* to the other player’s action, and no player prefers to take a different action given the actions of the other players. Thus, the action combinations R1-C1 and R2-C2 (or, meeting at P1 or meeting at P2) are Nash equilibria of the game described in Figure 1. Lewis’s own equilibrium concept of a coordinated equilibrium in the context of this game is more restrictive, in that not only should a player not want to change his action given the other players’ actions, but he should also not want the other players to change their actions, given his action. Lewis defines a coordination problem as a game that has at least two coordination equilibria (since otherwise there is nothing to coordinate on).

What premises justify the prediction that one of the Nash equilibria (or coordination equilibria) would be the solution to the game? Let us start by assuming that one of the players somehow knows where the other player is heading to. Why would a player now go to a certain place? First of all, he would have to know that he has a reason to go there, i.e., he would have

to know his own preferences. Second, he would have to be rational, in that he takes decisions in line with his preferences. Thus e.g., if Row would know that Column goes to P1, Row would also go to P1. But now, how could Row know Column's choice? For Row to know Column's choice, Row somehow has to be able to put himself in Column's shoes. Thus, Row must know Column's preferences, and must know that Column is rational. But that is not enough, as Row should realise that, to come to a certain choice, Column should also know Row's preferences and know that Row is rational. Thus, Row must know that Column knows that Row has certain preferences and that he is rational. When such a process, of knowing that another person knows that you know that the other person knows ... a certain state of affairs, goes on indefinitely, we say that this state of affairs is *common knowledge*. Thus, for one of the action combinations R1-C1, R2-C2 to be the unique solution to the game in Figure 1, it should be common knowledge which preferences each of the players have, and that the players are rational. This is what is required by the Nash equilibrium concept, the concept most often used in game theory.

However, as pointed out by Aumann (1987), and as already noted by Lewis, this is not sufficient as an argument for a unique outcome. Indeed, in the game in Figure 1, for players to come to a unique solution, there must also be a state of affairs that produces common knowledge of where each of the players will go. Lewis (1969) provides three such states of affairs. A first state of affairs that could produce such common knowledge is communication between the players, i.e., an *agreement* on which place to go to. But such an agreement would require language, and it is precisely use of language, which as argued by Lewis involves a coordination problem of its own, that Lewis is aiming to explain. A second state of affairs that could produce such common knowledge is *salience* of a certain combination of strategies. That is, some combination of strategies may possess a special feature, and one player knows that the other player knows that the first player knows this, and so on. In a version of the game in Figure 1, suppose players are told that they have to meet each other tomorrow in a certain town, but are not told when and where. It turns out that most players will decide to go to the central railway station at noon (Schelling, 1960). Somehow this choice, due to its centrality both in time and place, has a special feature that causes the players to expect each other to make it. The third state of affairs that could produce common knowledge is a variant of salience, namely *precedent*. In this case, in the game in Figure 1, Row and Column go to one of the two places now because that place possesses the special feature that they have met there *in the past*. Lewis calls such a regularity arising from repeatedly solving a coordination problem as in

Figure 1 a *convention*. Summarising, the fact of having met at a certain location in the past creates mutual expectations that the next meeting will take place at this same location. Players realise that they could as well meet at another location, but that a convention has arisen to meet at a certain place.

The analogy of a conventional meeting place with conventional language is now clear. Just as in the meeting game, it does not matter to players where they meet as long as it is in the same place, in a language game it does not matter to speaker and listener what language is spoken as long as they understand each other. But the meeting game in Figure 1 is more than a metaphor for a language game, as we will now see. Indeed, in Figure 1, Row could just as well be a player with private information, who lets the actions he takes (e.g., making certain sounds, or making marks on paper) depend on the private information he possesses; and Column could just as well be a player who adopts a different action for every different action that Row takes. The payoffs in such a game with contingency plans are represented in Figure 2. The column-player may now be thought of as a general who obtains a payoff of 1 if he attacks when his enemy is unprepared, and a payoff of $-M$ if he attacks when the enemy is prepared; if he does not attack, he obtains payoff 0, whether or not the enemy is prepared. The general does not know whether or not the enemy is prepared, but an informant (the row-player) does. This informant's interests perfectly coincide with those of the general. It is common knowledge that the enemy is prepared with probability δ , and unprepared with probability $1 - \delta$. The row strategies consist of all the actions that the informant could possibly adopt, depending on whether he observed the enemy to be prepared or unprepared; though there is no natural limit on the number of such actions, to keep the analysis tractable, I consider only two actions of the informant, namely a and b . The column strategies in Figure 2 then represent the general's actions contingent on whether he observes the informant taking action a or b .

The payoffs in Figure 2 now denote the players expected payoffs for each combination of contingency plans. If the general manages to attack when the enemy is unprepared, and not to attack otherwise, then both players' expected payoffs equal $(1 - \delta) * 1 + \delta * 0 = (1 - \delta)$. If the general always attack, whatever the enemy's condition, both players' expected payoffs equal $(1 - \delta) * (1) + \delta * (-M)$. If the general attacks when the enemy is prepared, and does not attack otherwise, the expected payoffs equal $(1 - \delta) * 0 + \delta * (-M) = -\delta * M$. If the general never attacks, expected payoffs equal $(1 - \delta) * 0 + \delta * 0 = 0$. It is assumed that the general, when not obtaining any information, is better off not attacking,

	C1	C2	C3	C4
R1	(1- δ), (1- δ)	$-\delta * M$, $-\delta * M$	0, 0	$-\delta * M + (1-\delta)$, $-\delta * M + (1-\delta)$
R2	$-\delta M$, $-\delta M$	(1- δ), (1- δ)	0, 0	$-\delta * M + (1-\delta)$, $-\delta * M + (1-\delta)$
R3	$-\delta M + (1-\delta)$, $-\delta M + (1-\delta)$	0, 0	0, 0	$-\delta * M + (1-\delta)$, $-\delta * M + (1-\delta)$
R4	0, 0	$-\delta * M + (1-\delta)$, $-\delta * M + (1-\delta)$	0, 0	$-\delta * M + (1-\delta)$, $-\delta * M + (1-\delta)$

R1: Action *a* when unprepared, action *b* when prepared;
R2: Action *a* when prepared, action *b* when unprepared;
R3: Always action *a*
R4: Always action *b*

C1: Attack when action *a*, do not attack when action *b*;
C2: Attack when action *b*, do not attack when action *a*;
C3: Never attack
C4: Always attack

Figure 2. Signalling game as a coordination problem.

i.e., that $(1 - \delta) * (1) + \delta * (-M) < 0$. A sufficient condition for this is that M , the cost of attacking when the enemy is prepared, is large.

Clearly, the game corresponds to Lewis's definition of a coordination problem, in that it has two coordination equilibria, namely the combinations R1-C1, and R2-C2. So far, I have followed Lewis, but now I will consider Lewis's treatment of games such as Figure 2 in the perspective of recent developments in game theory. Indeed, untreated by Lewis, the two coordination equilibria are not the only Nash equilibria to this game: there are also two Nash equilibria without communication, where the informant always takes the same action, and the general never attacks.³ When wanting to make a prediction that one of the combinations R1-C1, or R2-

³ Upon reflection, there is even a continuum of Nash equilibria where the informant randomises among taking actions *a* or *b* in such a way that it is not worth for the general to let his decision on whether or not to attack depend on the informant's action. Given that the general does not respond to the informant's signals, any randomisation of signals is a best response for the informant. The informant's randomisation among actions in turn justifies the general's response of ignoring the informant's signals when deciding what action to take, as long as the information that can be inferred from these actions is insufficient. For instance, consider a class of Nash equilibria where the informant always takes action *a* when the enemy is unprepared, and takes action *a* with probability $\text{Prob}(a/p)$ and action *b* with probability $\text{Prob}(b/p)$ (where of course $\text{Prob}(a/p) + \text{Prob}(b/p) = 1$) when the enemy is prepared. Then the general who observes action *b* knows that the enemy is prepared, since such an action is only taken by an informant who observes the enemy to be prepared, and does not attack. The general will not attack when observing

C2 is the unique solution to this game, and not one of the Nash equilibria without communication, one is now faced with two problems. First, taking as one's starting point an action combination without communication, if the informant now takes a single action only for a single state of the enemy, why would this mean anything to the general? Indeed, the general is justified in expecting that such an action does not carry any meaning, since expecting this to be the case makes it a best response for the informant not to let actions carry any meaning. Second, even if – again taking as one's starting point an action combination without communication – a new action taken by the informant does carry a meaning to the general, why would the general trust that this action would be taken only for a single state of the enemy, and not for both enemy states? Again, the general is justified in not trusting the informant's action to correspond to one particular state of the enemy, because his belief again makes it a best response for the informant not to be truthful.

There have been two approaches in game theory to solve these two problems, namely the *axiomatic approach* and the *evolutionary approach*. I argue that both of these approaches have their counterparts in the philosophy of language. The axiomatic approach in game theory is due to Farrell (1993). Farrell solves the problem of why the actions of the informant would mean anything to the general by assuming that there is a pre-existing, rich common language. Starting from the situation where there would be no communication, if the informant takes the action of expressing in this pre-existing language that the enemy is prepared respectively unprepared, then at least this will be understood by the general. To solve the problem of credibility, Farrell points to the salience of the action combination where the informant uses the action with a literal meaning

action a either, as long as the informant takes action a sufficiently often when the enemy is prepared. In particular, using Bayes' rule, we can calculate from the perspective of a general who observes action a the conditional probability that the enemy is unprepared as $(1 - \delta) * [(1 - \delta) + \delta * \text{Prob}(a/p)] - 1$, and the conditional probability that the enemy is prepared as $\delta * \text{Prob}(a/p) * [(1 - \delta) + \delta * \text{Prob}(a/p)] - 1$. The general will not strongly prefer to attack when observing action a as long as $(1 - \delta) * [(1 - \delta) + \delta * \text{Prob}(a/p)] - 1 * 1 + \delta * \text{Prob}(a/p) * [(1 - \delta) + \delta * \text{Prob}(a/p)]^{-1} * (-M) \leq 0$, i.e., as long as $\text{Prob}(a/p) \geq (1 - \delta) * (\delta * M)^{-1}$, or equivalently $\text{Prob}(b/p) \leq 1 - (1 - \delta) * (\delta M)^{-1}$. Therefore, there is a continuum of Nash equilibria where the informant always takes action a when the enemy is unprepared, and takes action a when the enemy is prepared with probability $(1 - \delta) * (\delta M)^{-1} \leq \text{Prob}(a/p) \leq 1$, and takes action b when the enemy is prepared with the complementary probability. Note that, except for the case where $\text{Prob}(a/p) = 1$, these represent equilibria where communication does take place, but insufficient communication for the general to diversify his actions. In the body of the text, for simplicity, we only confront communication equilibria with equilibria where communication does not take place at all.

truthfully, and where the general trusts it to be truthful. Concretely, if the informant takes the action of uttering that the enemy is unprepared, and if the general believes this utterance to be true, then the informant who observes the enemy to be prepared does not have any incentive to utter that the enemy is unprepared, given the players' common interests. This fact makes truthfulness of the informant and trustfulness of the general salient. Farrell's axiomatic approach can be linked to Grice (1975). Grice puts forward that "[o]ur talk exchanges do not normally consist of a succession of disconnected remarks, and would not be rational if they did. They are characteristically, to some degree at least, cooperative efforts", and labels this the Cooperative principle. Thus, Grice puts forward an axiom saying that any utterance by the informant is an attempt to cooperate, and will thus be truthful, and will in turn be trusted by the general. However, Grice does not assume a fixed meaning to any utterance, or at least not a fixed non-literal meaning. Concretely, Grice decomposes his Cooperative principle in four maxims, which regard among others the clarity and the relevance of a speaker's utterances. Though these maxims are generally met, if the speaker violates them anyway the listener will understand what the speaker implies by violating the maxims. A good example is irony, where the (axiomatic) literal meaning of the speaker's utterance is exactly the opposite of its non-literal meaning, which the listener understands because of the axioms of truthfulness and trustfulness.

The evolutionary approach for eliminating equilibria without communication (e.g., Wärneryd, 1993) does not put forward any axioms about a fixed meaning, or about truthfulness and trustfulness. It starts from the assumption that any game is played repeatedly and anonymously by a population of speakers and listeners. Strategies that in one period yield players a higher payoff are played more frequently in the population in the next period. The players may thus come to a so-called evolutionary stable equilibrium, where the actions of which this equilibrium is composed remain optimal responses even if a small proportion of the population starts taking alternative actions. In the context of the game in Figure 2, a population of informants could thus evolve towards taking different actions depending on whether the enemy is prepared or not, and a population of generals could evolve towards letting their decision on whether or not to attack depend on the informants' actions.⁴ But then the informants' actions

⁴ At first sight, the Nash equilibria R3-C3 and R4-C3 would thus seem to be evolutionary unstable. However, this is only the case if there is always a new action that the informant could take. Indeed, if the informant randomises among any conceivable action, then there is no deviating action that can make the informant better off. Thus, to argue that the Nash equilibria without communication are evolutionary unstable, it should always be possible

may be said to have acquired a meaning, in that they have become signals of whether or not the enemy is prepared. The generals have evolved towards trusting that these actions are signals of whether or not the enemy is prepared, and the informants have evolved towards using these signals truthfully. Thus, rather than meaning, trustfulness and truthfulness being axioms that, allowing for communication between the two players, assure a unique solution, they are now regularities that arise from playing games as the one in Figure 2 repeatedly.

Though the evolutionary approach seems at first sight very similar to Lewis's (1969) argument that precedent can lead to a Nash equilibrium with communication, there are also important differences with Lewis. Indeed, an important aim of the evolutionary approach is to relax the assumption of rationality of players; players are assumed to be boundedly rational, in that they learn to play strategies that do better. In Lewis, on the contrary, "verbal activity is, for the most part, rational" (1975, p. 134), and Lewis does not seek to explain the evolution of language. Rather, precedent of a language having been used in the past (i.e., a convention), and precedent of truthfulness in this language in the past (and trustfulness of the listener), breeds present common knowledge that this language will be used today as well, even though players realise that, had another language been used in the past, there would now have been a convention to talk in this other language.

Extending the signalling game in Figure 2 to a situation with many possible states to be signalled by the informant, both the axiomatic and evolutionary approaches predict that the informant will send a different signal for each state, and that each signal will be true. However, looking at a more realistic situation with a continuum of degrees of preparedness of the enemy, borderline cases will arise where the informant is not able to tell whether it is true or false that the enemy is prepared. In other words, natural language utterances such as 'prepared' or 'unprepared' are vague. But the fact that an utterance would be neither true nor false is counter to standard logic. Several approaches have been proposed to this problem.

for the informant to take one more action. And indeed, there would seem to be an infinite number of actions that the informant could take. Moreover, such a new action should not be punished by the general. This is required because, given that the action is never taken, it is a weak best response for the general to punish the action. Thus, one should allow for evolutionary drift, where the population of generals can learn not to punish the action, even though they are indifferent between punishing or not, given that the action is never taken. Finally, it should be noted that by these arguments, each of the candidate equilibria R1-C1 and R2-C2 are also unstable, in that informants and generals can then learn to use one signalling system in a communicative way rather than another. A solution to this is to consider R1-C1, R2-C2, and any further communication equilibrium as an evolutionary stable set (see Rubinstein, 2000, Chapter 2, Section 2.4).

One approach is to say that there is no problem at all. Such an idea has been expressed by Parikh (1994) in this journal. Vagueness arises in this case because of people's failure ever to come to a consensus about the states of the world to which e.g., adjectives such as 'prepared' or 'unprepared' refer. However, as long as using such adjectives is useful to people, they will continue to employ them. Other approaches to vagueness on the contrary directly try to deal with the challenge vagueness poses to logic. A first such approach has been to extend standard logic, with its bivalence of truth and falsity, to a multi-valued logic (Zadeh, 1975). For instance, in a three-valued logic, one could say that whether the enemy is unprepared is either true, neutral, or false. In so-called fuzzy logic, one could assign many degrees of truth between 0 (false) or 1 (true). In the second approach of this type, supervaluationism (Fine, 1975), in the context of the informant-general example one could think of all the circumstances that the enemy could be in. If, for all such circumstances, the enemy is truly unprepared, then unpreparedness is referred to as supertrue. If on the contrary for all such cases, the enemy is prepared, then unpreparedness is referred to as superfalse. If for some circumstances, the enemy is prepared, and for others, he is not, unpreparedness and preparedness are neither true nor false. The third approach that tries to deal with the challenge posed by vagueness to standard logic, termed the epistemic view of vagueness (Williamson, 1994), holds that whether the enemy is unprepared is always either true or false, but that we may simply be ignorant of what is the case. No modification of standard logic is then required. In this view, we may make systematic mistakes in agreeing or disagreeing with a proposition, so that meaning does not strictly follow from use as is the case in Parikh (1994).

Multi-valued logic and supervaluationism are criticised in the epistemic view of vagueness, in that these approaches assume that one can observe when it is vague whether the enemy is prepared or not. The essence of vagueness, however, is that one cannot observe where the borderline between preparedness and unpreparedness lies. This may be seen by looking at the well-known Sorites paradox. Assume that the enemy is a large army in a valley overlooked by the informant, that it is early morning, and that preparedness depends on the number of soldiers who have emerged from their tents. Then, starting from the case where no soldiers are seen awake, and where therefore it is true that the enemy is unprepared, then surely one extra soldier awake will not make any difference. But if one additional soldier awake does not make any difference, then ultimately one should conclude that it is true that the enemy is unprepared when *all* soldiers are awake, which is false. The paradox arises because we are

	C1	C2
R1	(5, 1)	(0, 0)
R2	(0, 0)	(1, 5)

Figure 3. Taking turns.

unable to tell where the borderline lies between truth and falsity. Nor are we for that matter able to tell where the borderlines lie between facts that are clearly true, neither clearly true nor false, and clearly false. In general, we are unable to observe borderlines for any degree of clarity, which is known as the phenomenon of *higher-order vagueness*.

While the view of vagueness that I will describe in this paper can be linked to the epistemic view of vagueness, at the same time it differs from any of the views of vagueness described above. I describe a strategic view of vagueness, where expressing oneself more vague than one could have done assures that communication can still take place in a game where there is some degree of conflict between speaker and listener. As well, the account I give differs from both the axiomatic and evolutionary approaches to signalling games described above, in that it relies on approximate common knowledge rather than on full common knowledge. Before describing the strategic view of vagueness, given that it relies on the concept of correlated equilibrium, I must first introduce this concept, which I now do using Lewis's framework.

3. CORRELATED EQUILIBRIA

To introduce the concept of correlated equilibrium (Aumann, 1987), I provide in Figure 3 a simple variant of the game in Figure 1, where players now are no longer indifferent about which of the coordination equilibria would be the solution to the game. In the example of a meeting game, Row prefers to meet at P1, and Column prefers to meet at P2, though both players prefer meeting to not meeting. To Lewis (1969), it does not matter that one player prefers one coordination equilibrium over another. What matters is that if precedent has produced a regularity to meet at e.g., P1, even though Column would prefer to meet at P2, it is in Column's interest to abide by the convention which has thereby arisen. Thus, players here coordinate their strategies by taking into account a single state of affairs which has occurred in the past, namely that most of the time, they have

	C1	C2
R1	(5, 1)	(0, 0)
R2	(4, 4)	(1, 5)

Figure 4. Chicken game.

met at P1. But in a simple extension to Lewis's analysis, also found in Vanderschraaf (1995), the players may coordinate their actions by letting them depend on more than one state of affairs. For instance, a regularity may happen to have arisen where the players toss a coin, and meet at P1 when the coin shows heads, and at P2 when the coin shows tails; moreover, such a regularity may additionally be salient in that it seems fair to let each of the players obtain their highest payoff in turn. One way to look at this is that a different game is being played when the coin shows heads or tails, and to apply Lewis's analysis to each of these games. But Lewis himself provides an alternative view, in treating this as a signalling game with Nature taking the actions of deciding whether the coin shows heads or tails, and the two players observing these actions, and letting their actions depend on them. Nature is then sending signals to the two players, with the "heads" signal meaning to both of them that they should go to P1, and the "tails" signal meaning that they should go to P2. This is the simplest type of correlated equilibrium where the players' actions are perfectly correlated.

However, Aumann's concept of a correlated equilibrium (1987) is not restricted to cases where the players' actions are perfectly correlated. In the modified game in Figure 4 (see Myerson, 1991),⁵ it may be in the players' interest not to let their actions be perfectly correlated. Clearly, both players' payoffs will increase if, starting from the example where a coin was tossed, the Row player would for some reason sometimes go to P2 when "heads" showed up and both players were supposed to go to P1, and when the Column player for some reason would sometimes go to P1 when "tails" showed up, and both players were supposed to go to P2. This is where vagueness could come in. Assume that, instead of tossing a coin, the players have come to coordinate on going to P1 in the afternoon, and going to P2 in the evening (assuming for simplicity that these are the

⁵ This game is known as the Chicken game because of the story that has been linked to it of two opponents who may either be aggressive to each other, or may behave as 'chicken'. If both players behave as chickens, the payoffs (4,4) are obtained. However, if one player behaves as a chicken, it pays for the other player to be aggressive.

only two eligible meeting times). Now afternoon and evening are vague concepts. When deciding to go to P1 or P2, each player will sometimes be faced with times at which it is not clearly afternoon and not clearly evening, and where he needs to guess where the other player will go. Assume it as given that some cases that are considered by the Row player as evening are considered by the Column player as afternoon, but that no cases that are considered by the Row player as afternoon are considered by the Column player as evening. Let us pool then, in a continuum of states of the world leading players to guess that it is afternoon or evening, in the discrete state x_1 all the states where both players guess that it is afternoon, in the discrete state x_2 all the states where Row guesses that it is evening but Column guesses that it is afternoon, and in the discrete state x_3 all the states where both Row and Column guess that it is evening. Concretely, this can be symbolized by $P_R = \{\{x_1\}, \{x_2, x_3\}\}$; $P_C = \{\{x_1, x_2\}, \{x_3\}\}$, where P_R and P_C denotes Row's and Column's partition of the states of the world, and where two states of the worlds in the same brackets (i.e., in the same partition cell) denotes that a player cannot distinguish between these two states of the world. Let us assume that states x_1 and x_3 each occur with probability $0.5 * (1 - \epsilon)$, and that state x_2 occurs with the complementary probability ϵ . Let us also assume that these probabilities are common knowledge among the players. We check now whether there is an equilibrium where each of the players goes to P1 when he believes it to be afternoon, and goes to P2 when he believes it to be evening. To do this, we apply Bayes' rule to calculate the probability that Column believes it is evening when Row believes it is afternoon. Denoting this probability by $\text{Prob}(e_C/a_R)$ (to be read as the probability (*Prob*) that Column (*subscript C*) believes it to be evening (*e*), given that (*I*) Row (*subscript R*) believes it to be afternoon (*a*)), one finds that $\text{Prob}(e_C/a_R) = \frac{\epsilon}{\epsilon + 0.5 * (1 - \epsilon)}$. Literally, this probability is equal to the proportion of cases where Column believes it to be afternoon (ϵ), out of the proportion of cases where Row believes it to be evening ($\epsilon + 0.5 * (1 - \epsilon)$). In the same way, one can calculate $\text{Prob}(a_C/a_R) = \frac{0.5 * (1 - \epsilon)}{\epsilon + 0.5 * (1 - \epsilon)}$, $\text{Prob}(e_R/a_C) = \frac{\epsilon}{\epsilon + 0.5 * (1 - \epsilon)}$, $\text{Prob}(a_R/a_C) = \frac{0.5 * (1 - \epsilon)}{\epsilon + 0.5 * (1 - \epsilon)}$. Row's expected utility of going to P2 when he believes it is evening, given that Column goes to P1 when he believes it is afternoon and goes to P2 when he believes it is evening, can now be calculated as $\frac{\epsilon}{\epsilon + 0.5 * (1 - \epsilon)} * 4 + \frac{0.5 * (1 - \epsilon)}{\epsilon + 0.5 * (1 - \epsilon)} * 1$. Row's expected utility of going to P1 in the same case is $\frac{\epsilon}{\epsilon + 0.5 * (1 - \epsilon)} * 5 + \frac{0.5 * (1 - \epsilon)}{\epsilon + 0.5 * (1 - \epsilon)} * 0$. Comparing the two expected utilities, it follows that Row will not prefer to go to P1 when it is evening as long as $\epsilon \leq 1/3$. Row's expected utility of going to P1 when he believes it is afternoon, conditional on Column going to P1 when he thinks it is afternoon, is 5, since Column certainly believes it

is afternoon when Row believes it is afternoon; Row's expected utility of going to P2 in the same case is 0. Therefore, if Column goes to P1 when he believes it is afternoon, then so does Row. Applying the same argument to Column, we obtain that there is indeed an equilibrium as described above, on the condition that $\epsilon \leq 1/3$. Each of the players' expected utility, before obtaining their information on whether it is afternoon or evening, is then $0.5 * (1 - \epsilon) * 5 + \epsilon * 4 + 0.5 * (1 - \epsilon) * 1 = 3 + \epsilon$. Therefore, the larger ϵ , or the less the meanings the players attribute to the signals "it is afternoon", or "it is evening" overlap, the better for the players. However, ϵ is constrained to be smaller or equal than $1/3$. The reason why such an alternative correlated equilibrium makes the players better off is evidently the way the payoffs have been chosen in the game in Figure 4, with $4 > 0.5 * (5 + 1)$.

Several comments are now due concerning the way in which this fits in with Lewis's concept of convention, and what view of vagueness this example implies. Concerning the view of vagueness implied, it should not be the case that the players' partition cells overlap is due to failure of precedent to have lead to consensus, as is the case in Parikh's view of vagueness (1994). Clearly, if he is able to, the Row player has every interest to learn that in the state of the world x_2 , Column considers that it is afternoon and goes to P1, so that Row can also go to P1 instead of to P2, and in this way obtain payoff 5 instead of 4. Of course, the partitions set out above could also describe a situation where the players have not yet learned each others' concepts of afternoon and evening. But the point is that eventually they will. As well, even accepting the argument that we are considering a particular cross-section of time, there is no reason why the players' partition cells would overlap in the particular way described. Nor does the view of vagueness implied in the example fit in with multi-valued logic (Zadeh, 1975), or with supervaluationism (Fine, 1975). If it would, then the required imperfect correlation in the players' actions could not be achieved, since players would agree on the cases which are afternoon or evening to a certain degree of truth (as in multi-valued logic), or which are neither truly afternoon nor truly evening (as in supervaluationism).

The view of vagueness in the example can instead be linked to the epistemic view of vagueness (Williamson, 1994). The states x_i should be interpreted as states that make the players guess that it is afternoon or evening. Row makes a systematic mistake in being unable to distinguish among states x_2 and x_3 , and Column a systematic mistake in being unable to distinguish among states x_1 and x_2 . It may be objected to this that surely – given Row's interest in learning to distinguish among x_2 and x_3 , and if one interprets the states of the world underlying the concepts of afternoon

and evening as degrees to which there is light – Row will infer that for the cases in the partition cell $\{x_2, x_3\}$ where there is less light, Column will believe that it is afternoon. In this view, there is no reason why the Row player would be unable to discriminate between cases where it is clear to the Column player that it is evening and cases where this is not clear. But as argued in the previous section with reference to the Sorites paradox, this again raises the question of how Row would be able to identify the borderline between (Column's) clear and unclear cases.

Since it is not common knowledge among the players when they will go to P1 or P2, the analysis of Figure 3 does not fit in with Lewis's original account of a convention. Fortunately, Lewis's concept of convention can be extended to cases of approximate common knowledge (Monderer and Samet, 1989). Therefore, in the example, instead of the Row player who observes that the tossed coin showed "tails" knowing that the Column player knows that the Row player knows ... etc. that a meeting will take place at P2, the Row player who observes it to be evening now believes that it is probable enough that the Column player believes that it is probable enough that the Row player believes ... etc. that it is evening, for the Row player to go to P2. It goes without saying that the conventional aspects of the example lie in the fact that players could have used other devices for their imperfect coordination. But on top of these conventional aspects of players' coordination, players must also coordinate on states regarding which they have asymmetric information, and where this asymmetric information takes a particular form. With respect to Grice's axiomatic approach, there is a more fundamental difference, in that players who agree to meet at P1 in the afternoon and at P2 in the evening are violating the Cooperative Principle, in that they are not being as clear as they could. However, the fact of making a vague agreement may on the contrary imply to the players that they are cooperating to the best of their abilities.

Still, there is no reason why the imperfect coordination in the example would require communication. Simply, the players may come to tacitly coordinate on a regularity that takes the required form. From the Row player's perspective, this regularity may be whether it is dark or light, and from the Column player's perspective, this regularity may be whether it is cloudy or not. As long as these events are correlated in the way described above, and as long as each player is epistemically constrained not to find out what states the other player observes, such a correlated equilibrium as described may exist. To come to a full account of strategic vagueness, I must apply this analysis to a true signalling game. This is done in the next section.

		Attack	Do not attack
Attack		(1, 1)	(-M, 0)
Do not attack		(0, -M)	(0, 0)

Figure 5. The enemy is unprepared (probability $(1 - \delta)$).

4. VAGUE SIGNALS AS A COORDINATION DEVICE IN A SIGNALLING GAME

In this section, I treat a modified version of the coordinated attack problem (Morris and Shin, 1997).⁶ For the assumptions of the game, I refer to the game described in Figure 2. The difference between the coordinated attack problem and the game in Figure 2 is that the informant is now himself a general, and that the payoff of 1 when the enemy is unprepared can now only be obtained if both players attack together. Whether or not the enemy is prepared, a general who attacks by himself obtains a payoff of $-M$ (with this M again assumed large), and a general who does not attack obtains payoff 0. If both generals attack when the enemy is prepared, in the coordinated attack problem, both players equally well obtain $-M$. In my modified version of this problem, I introduce a conflict of interest by assuming that the first, informed, general instead obtains $0 < x \leq 1$ when both players attack when the enemy is prepared. If there would now be a first-general signal that would mean to the second general that the enemy is unprepared, and if the second general would trust this signal to be truthful and attack when observing it, then the first general would send this signal whether or not the enemy is prepared, and the second general would not trust this signal anymore. I exclude an equilibrium where both generals always attack, by assuming that $\delta * (-M) + (1 - \delta) * 1 < 0$ (again, it suffices for this that M is large). Because I would otherwise have to draw quite a big table, I follow Morris and Shin (1997) in summarising the game in Figures 5 and 6, excluding any possible signalling stage.

⁶ My reasons for constructing this particular example are two-fold. First, it is a variant of a game which is well-known well beyond the fields of game theory and economics (though in the context not of vagueness, but in the analysis of common knowledge). Second, to construct an example of strategic vagueness in a pure signalling game, where the signaller only sends signals but does not undertake actions, one needs three actions and two states, making the calculations more involved (see De Jaegher and Jegers, 2001).

	Attack	Do not attack
Attack	$(x, -M)$	$(-M, 0)$
Do not attack	$(0, -M)$	$(0, 0)$

Figure 6. The enemy is prepared (probability δ).

It is easy to check that this game has a single pure-strategy Nash equilibrium,⁷ where no information is transmitted from the first to the second general, where neither of the players ever attacks, and where each player earns a payoff of 0. If signals are either precise or imprecise, that is the end of the story; if the first general sends any signal at all, then it is one where the first general says: "I cannot tell you whether or not the enemy is prepared" (cf. Crawford and Sobel, 1982). But I now show that this game does have communication equilibria, i.e., equilibria where the second general's decision is contingent on the first general's signals. It is easy to see that, if any such communication equilibrium exists, it must be a correlated equilibrium. Again, if there is a signal of which the first general knows that when he sends it, it results in the second general always attacking, the first general will always send this signal, whether or not the enemy is unprepared. Therefore, for communication still to be possible, when the

⁷ It can be checked that there is also a mixed equilibrium where the second general who receives a signal not to attack always follows this advice, and where the second general who receives a signal to attack does not follow this advice with probability $x * (M + x)^{-1}$, and follows it with the complementary probability (this in order to make the first general who observes the enemy to be prepared indifferent between lying and telling the truth). The first general who observes the enemy to be unprepared always sends a signal to attack and attacks himself, whereas a first general who observes the enemy to be prepared sends a signal to attack and attacks himself with probability $(1 - \delta) * (\delta * M)^{-1}$, and does not send such a signal and does not attack with the complementary probability (this in order to make the second general who observes a signal to attack indifferent between attacking and not attacking). In the class of correlated equilibria where signals are precise, and where the players let their truthfulness and trustfulness depend on privately observed states of the world, the mixed equilibrium describes the limit case where there is zero correlation between the states of the world observed by the first and second general. The mixed equilibrium yields the first general an expected payoff of $(1 - \delta) * (1 - x) * M * (x + M)^{-1}$. However, the second general obtains expected payoff 0, and is therefore equally well off as in case no communication occurs. Though a mixed equilibrium introduces noise into the information transmission, it cannot be related to vagueness, as the first general's signals to attack or not attack are not vague.

first general sends an attack signal, there must be something that the first general does not observe, and that induces the second general sometimes to attack, and sometimes not to attack – or in other words, any possible communication equilibrium must be a correlated equilibrium.

In fact, there is a whole set of correlated communication equilibria, which I could now go on to describe. But instead of doing so, I will concentrate on some of these equilibria. First of all, I will not treat the type of correlated equilibria where the first general decides to be truthful or untruthful depending on the state of the world he observes, and the second general similarly decides whether to be trustful or untrustful depending on the state of the world he observes from his side. Instead, I will concentrate on equilibria where the conventions of truthfulness and trustfulness that are likely to arise from coordination problems (arguably still the majority of language games played), are not violated. Concretely, I look at equilibria where the first general is truthful concerning the signals he sends, and the second general is trustful concerning the signals he perceives, but where signals sent and perceived are not always one and the same. Second, within this class of correlated equilibria, I only treat equilibria where it is possible that the second general does not attack given that the first general sends a signal to attack, but where it is not possible that the second general attacks given that the first general sends a signal not to attack. In order to make a communication equilibrium possible, attacking should become less attractive to the first general, and this can be achieved if the second general sometimes fails to attack when the first general attacks. If the second general sometimes attacks when the first general does not attack, then this will only make the second general worse off.

In particular, assume that the first general can send one of two signals,⁸ namely a or b . If the first general sends signal a , then his knowledge of the states of the world is described by the partition $\{\{x_1\}\}$. In other words, when sending signal a , the first general knows for certain that state of the world x_1 occurs. If the first general sends signal b , then his knowledge about the states of the world is described by the partition $\{\{x_2, x_3\}\}$. That is, when sending signal b , the first general knows that one of the two states of the world x_2 and x_3 occurs, but does not know which. Upon the first general having sent a signal, the second general's knowledge is described by the partition $\{\{x_1, x_2\}, \{x_3\}\}$. In other words, as partition cell $\{x_3\}$ can only be generated if the first general sends signal b , when observing this partition cell, the second general knows for certain that the first general sent signal b .

⁸ More correctly, a and b should be dealt with as *actions*, that may become *signals*. However, in this context confusion then arises between the first general's actions a and b , and his actions to attack or not.

However, when observing partition cell $\{x_1, x_2\}$, the second general does not know for certain whether the first general sent signal a or b . When sending signal a , given that this always generates state of the world x_1 , the first general knows that the second general will observe partition cell $\{x_1, x_2\}$. When sending signal b , given that this generates either state of the world x_2 or x_3 , between which the first general cannot distinguish, the first general does not know for certain whether the second general will observe partition cell $\{x_1, x_2\}$ or partition cell $\{x_3\}$.

Assume that upon the signal b , states of the world x_2 and x_3 are generated with probabilities of respectively ϵ and $(1 - \epsilon)$. I will now show that for some levels of ϵ , a correlated equilibrium exists where the first general sends signal a and does not attack when the enemy is prepared, and sends signal b and attacks when the enemy is unprepared. From his side, the second general does not attack when observing partition cell $\{x_1, x_2\}$, and attacks when observing partition cell $\{x_3\}$. Intuitively, this signalling system introduces noise into the first general's signal to attack (signal b), as it does not always result in the second general attacking, and this noise could reduce the first general's payoff from a joint attack when the enemy is prepared in such a way that the first general prefers not to jointly attack in this case.

We first show that, given that the first general sends signal a when the enemy is prepared and signal b when the enemy is unprepared, the player's actions as described in the candidate equilibrium are mutual best responses under certain conditions. Evidently, if the first general sends signal a when the enemy is unprepared, he knows for certain that the second general will observe partition cell $\{x_1, x_2\}$, and will not attack. Hence, his optimal strategy, once having send signal a , is not to attack. If the first general has sent signal b when the enemy is unprepared, given that with probability $(1 - \epsilon)$, the second general observes partition cell $\{x_3\}$ and also attacks, and that with probability ϵ the second general observes partition cell $\{x_1, x_2\}$ and does not attack, the first general does not prefer not to attack as long as $\epsilon * (-M) + (1 - \epsilon) * 1 \geq 0$. When observing partition cell $\{x_3\}$, the second general's best response is to attack, given that he knows that the first general observed the enemy to be unprepared, and given that the first general attacks. When observing partition cell $\{x_1, x_2\}$, the first general can calculate that the probability that he observes this partition cell is equal to the probability that the enemy is prepared, namely δ , plus the probability that the enemy is unprepared multiplied by the probability that the first general's signal b generates state of the world x_2 , namely $(1 - \delta) * \epsilon$. Using Bayes' rule, the second general can now calculate that δ times out of the $\delta + (1 - \delta) * \epsilon$ times that he observes partition cell $\{x_1, x_2\}$, the enemy is

prepared and the first general does not attack, i.e., that the probability that the enemy is prepared given that he observes $\{x_1, x_2\}$ is $\frac{\delta}{\delta + (1-\delta) * \epsilon}$. Similarly, the probability that the enemy is unprepared and that the first general attacks, given that the second general observes $\{x_1, x_2\}$, is $\frac{(1-\delta) * \epsilon}{\delta + (1-\delta) * \epsilon}$. Therefore, when observing $\{x_1, x_2\}$, the second general does not prefer to attack as long as $\frac{\delta}{\delta + (1-\delta) * \epsilon} * (-M) + \frac{(1-\delta) * \epsilon}{\delta + (1-\delta) * \epsilon} * 1 \leq 0$. Eliminating the denominators, this condition reduces to $\delta * (-M) + (1 - \delta) * \epsilon * 1 \leq 0$.

Second, we check under which conditions it is a best response for the first general to send the signals in the way described in the candidate equilibrium, given that the players take the actions as described in the candidate equilibrium. As the second general will certainly observe the state of the world to belong to partition cell $\{x_1, x_2\}$ when the first general sends signal a , and as the second general will then not attack, the first general never attacks when sending signal a , whatever the state for which he sends this signal. Therefore, the first general will not prefer to send signal a when the enemy is unprepared as long as $\epsilon * (-M) + (1 - \epsilon) * 1 \geq 0$. Whether the first general will not prefer to send signal b when the enemy is prepared depends on what the first general does, once he has sent signal b when the enemy is prepared. The first general then strongly prefers to attack if $\epsilon * (-M) + (1 - \epsilon) * x > 0$. But if this is the case, then the first general will never prefer to send signal a when the enemy is prepared, as this yields him a zero payoff. Therefore, the first general will not prefer to send signal b when the enemy is prepared as long as $\epsilon * (-M) + (1 - \epsilon) * x \leq 0$.

Summarising, we have the following restrictions on ϵ for a correlated equilibrium of the desired form to exist: $\frac{x}{M+x} \leq \epsilon \leq \frac{1}{M+1}$; $\epsilon \leq \frac{\delta * M}{1-\delta}$. Given that $\epsilon < 1$, the latter restriction is always met, given the assumption that a second general without any information prefers not to attack ($\delta * (-M) + (1 - \delta) * 1 < 0$, or $1 < \frac{\delta * M}{1-\delta}$). The former restriction can only be met if $x \leq 1$. This shows that communication under a conflict of interest is only possible if the payoff to the first general of a joint attack when the enemy is prepared is not too high. Evidently, it is in both players' interest that ϵ is as small as possible. Therefore, among the equilibria considered, the payoff to both players is highest if $\epsilon = \frac{x}{M+x}$, which is the minimum ϵ for which the first general is still willing to be truthful.⁹ As well, this minimum ϵ is smaller, and the maximum attainable payoff to both players is higher, the smaller is x , i.e., the smaller is the degree of conflict between the generals. Therefore, in the equilibrium that I have in this way shown

⁹ Concretely, for minimal ϵ , the first general's payoff is $(1 - \delta) * (1 - \epsilon)$, whereas the second general's payoff is $(1 - \delta) * (1 - x) * M * (x + M)^{-1}$. Therefore, the second general is better off than with the mixed equilibrium (see Footnote 7), while the first general is equally well off.

to exist, when the enemy is prepared, none of the players attack; when the enemy is prepared, the first general always attacks, but the second general sometimes fails to attack, and this often enough to keep the first general willing to send signal *a* when the enemy is prepared.

Can this example now be interpreted in the light of the game-theoretic rationale for vagueness suggested in the previous section? This seems at first sight problematic, given the discrete states in the model ('enemy is prepared' and 'enemy is unprepared'). The vagueness would then have to lie in the signals sent or received, rather than in the states of the world. Still, the discrete states of preparedness and unpreparedness could be interpreted as a simplification of a more intuitive continuum of states, starting from cases where the first general observes the enemy to be clearly prepared, and ending with cases where he observes the enemy to be clearly unprepared, with intermediate cases where he cannot tell whether the enemy is prepared or unprepared. The first general then is likely to sometimes make a wrong guess about whether the enemy is prepared or unprepared. But in a common-interest version of this game, these wrong guesses, as long as they do not occur too often, do not matter to the second general, and simply, he would like to find out the first general's guess about the enemy's state, which could then be expressed by a simple imperative as "Attack" or "Don't Attack". Such utterances would also meet the Gricean maxim that one should be as relevant as one can. The occasional wrong guesses by the first general may then be considered as implicit in the payoffs. In the version of the game with conflicting interests on the contrary, instead of following the Gricean maxim of relevance and using imperatives, the first general may instead use the indicative mood,¹⁰ that is may try and express his private information on the extent to which the enemy is prepared, and leave the decision to attack or not to the second general's discretion. In the multi-valued-logic view of vagueness, or in supervaluationism, this would not make a difference. The first general could express in the former case the degrees of truth that he perceives, and in the latter case whether it is supertrue that the enemy is prepared, superfalse, or neither true nor false. But in the epistemic view of vagueness, I argue that when the first general is trying to express his information on borderline cases, this may leave the first general himself uncertain as to what the second general will infer from his expressions. Similarly, the second general may be left uncertain as to what the first general means. Moreover, given that not attacking is the safe action, and attacking the risky action, misunderstandings are more likely to take the form of the second general inferring that no attack should take place, while the first general meant that an attack should take place, putting

¹⁰ Lewis (1969) extends his analysis of his signalling game to such moods.

misunderstandings in the right form for the correlated equilibrium that I have described. From the perspective of Grice's analysis (1975), instead of following the Gricean maxim of relevance, by being irrelevant the first general is implying that he still wants to cooperate to the extent that he credibly can. Credibility would be lost if the first general would try and express himself in more precise terms. The interpretation of the signalling system described above in terms of an extension of Lewis's concept of convention to the case of approximate common knowledge is the same as in the previous section.

5. CONCLUSION

Game theorists are, most of the time, naturally drawn to analysing problems of conflict, rather than problems of coordination as analysed by Lewis (1969). Thus, insofar as situations of conflict are relevant for language games, game theory has something to contribute. As well, given their utilitarian approach, game theorists have a tendency to explain a phenomenon by looking for a way in which it may contribute to people's utility. And indeed, a signalling game was constructed above where vagueness is useful, in that it may solve conflicts of interest. This of course does not imply that any vagueness has such a function. Rather, the claim of this paper is that vagueness that pre-existed for epistemic reasons may be exploited to solve conflicts of interest.

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REFERENCES

- Aragonès, E. and Neeman, Z.: 2000, 'Strategic Ambiguity in Electoral Competition', *Journal of Theoretical Politics* **12**, 183–204.
- Aumann, R.: 1987, 'Correlated Equilibria as an Expression of Bayesian Rationality', *Econometrica* **55**, 1–18.
- Benthem, J. van: 2000, 'Ariel Rubinstein: "Economics and Language"', in A. Rubinstein, *Five Essays on: Economics and Language*, Cambridge University Press.
- Crawford, V. P. and Sobel, J.: 1982, 'Strategic Information Transmission', *Econometrica* **50**, 1431–1451.

- De Jaegher, K. and Jegers, M.: 2001, 'The Physician-Patient Relationship as a Game of Strategic Information Transmission', *Health Economics* **10**, 651–668.
- Farrell, J.: 1993, 'Meaning and Credibility in Cheap-Talk Games', *Games and Economic Behavior* **5**, 514–231.
- Fine, K.: 1975, 'Vagueness, Truth and Logic', *Synthese* **30**, 265–300.
- Grice, H. P.: 1975, 'Logic and Conversation', in P. Cole and J. Morgan (eds.), *Syntax and Semantics*, Vol. 3, Academic Press.
- Lewis, D.: 1969, *Convention – A Philosophical Study*, Harvard University Press.
- Lewis, D.: 1975 'Languages and Language', in K. Gunderson (ed.), *Language, Mind and Knowledge*, University of Minnesota Press.
- Lipman, B. L.: 2001, 'Why Is Language Vague', working paper, University of Wisconsin.
- Monderer, D. and Samet, D.: 1989, 'Approximating Common Knowledge with Common Beliefs', *Games and Economic Behavior* **1**, 170–190.
- Morris, S. and Shin, H. S.: 1996, 'Approximate Common Knowledge and Co-ordination: Recent Lessons from Game Theory', *Journal of Logic, Language and Information* **6**, 171–190.
- Myerson, R. B.: 1991, *Game Theory – Analysis of Conflict*, Harvard University Press.
- Parikh, R.: 1994, 'Vagueness and Utility: the Semantics of Common Nouns', *Linguistics and Philosophy* **17**, 521–535.
- Parikh, P.: 2000, 'Communication, Meaning and Interpretation', *Linguistics and Philosophy* **23**, 185–212.
- Rubinstein, A.: 2000, *Five Essays on: Economics and Language*, Cambridge University Press.
- Schelling, T. C.: 1960, *The Strategy of Conflict*, Harvard University Press.
- Vanderschraaf, P.: 1995, 'Convention as Correlated Equilibrium', *Erkenntnis – An International Journal of Analytical Philosophy* **42**, 65–88.
- Wärneryd, K.: 1993, 'Cheap Talk, Coordination and Economic Stability', *Games and Economic Behavior* **5**, 532–546.
- Williamson, T.: 1994, *Vagueness*, Routledge.
- Zadeh, L.: 1975, 'Fuzzy Logic and Approximate Reasoning', *Synthese* **30**, 407–428.

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