## Coursework 1

### Solving differential equations with Euler's method

The following questions are to help you practise your Python, they are not to be submitted and will not be marked, instead you can compare to the model graphs available in the same github folder.

1. Solve numerically in Python using Euler's method the differential equation

$$\frac{df}{dt} = f^2 - 3f + e^{-t} \tag{1}$$

   on the interval $[0, 3]$ with time step $\delta t = 0.01$ and graph the solution, taking care to label the axes. Although Python has good libraries for solving differential equations numerically it would be useful educationally not to use them for these question.

2. For the problem above try solve with $\delta t = 0.01$, 0.1, 0.5 and one. Plot all the curves on one graph. What is a good value of $\delta t$ for this equation.

### Integrate and fire neurons

The questions in this section are to be submitted for marking and will make up 10% of your final mark. In this and next assignments, all the plots should have axes labels, and if there are multiple graphs on a plot, the legend (or key) should be included. For each missing label or legend, 1% of mark will be subtracted.

Write a brief report, no longer than three pages including the figures and the comments specified above; submissions exceeding the page limit will be rejected, I will take a dim view of super-narrow margins or tiny fonts. Submit it in the pdf format together with the Python code by the deadline. Remember, provided it is in good time, I am happy to answer questions about Python and to help debug faulty code.

1. Simulate an integrate and fire model with the following parameters for 1 s: $\tau_m = 10$ms, $E_L = V_r = -70$ mV, $V_t = -40$ mV, $R_m = 10$ M$\Omega$, $I_e = 3.1$ nA. Use Euler's method with timestep $\delta t = 1$ ms. Here $E_L$ is the leak potential, $V_r$ is the reset voltage, $V_t$ is the threshold, $R_m$ is the membrane resistance, that is one over the conductance, and $\tau_m$ is the membrane time constant. Plot the voltage as a function of time. For simplicity assume that the neuron does not have a refractory period after producing a spike. [20% of marks]. You do not need to plot spikes - once membrane potential exceeds threshold, simply set the membrane potential to $V_r$.

2. Compute analytically the minimum current $I_e$ required for the neuron with the above parameters to produce an action potential. [10% of marks].

3. Simulate the neuron for 1 s for the input current with amplitude $I_e$ which is 0.1 [nA] lower than the minimum current computed above, and plot the voltage as a functions of time. [15% marks].

4. Simulate the neuron for 1s for currents ranging from 2 [nA] to 5 [nA] in steps of 0.1 [nA]. For each amplitude of current count the number of spikes produced, that is the firing rate. Plot the firing rate as the function of the input current. [15% of marks]. It

is possible to calculate this curve analytically; there is no requirement that you do this, but you might find it interesting to try.

5. Simulate two neurons which have synaptic connections between each other, that is the first neuron projects to the second, and the second neuron projects to the first. Both model neurons should have the same parameters: $\tau_m = 20$ ms, $E_L = -70$ mV $V_r = -80$ mV $V_t = -54$ mV $R_m I_e = 18$ mV and their synapses should also have the same parameters: $R_m \bar{g}_s = 0.15$, $P = 0.5$, $\tau_s = 10$ ms; don't get confused by being given $R_m \bar{g}_s$ rather than $\bar{g}_s$ on its own, to get $\tau_m$ rather than the capacitance on the left hand side of the integrate and fire equation everything is multiplied by $R_m$. For simplicity take the synaptic conductance to satisfy
$$\tau_s \frac{ds}{dt} = -s \tag{2}$$
with a spike arriving causing $s$ to increase by $P$. This is equivalent to the simple synapse model in the lectures. Simulate two cases: a) assuming that the synapses are excitatory with $E_s = 0$ mV, and b) assuming that the synapses are inhibitory with $E_s = -80$ mV. For each simulation set the initial membrane potentials of the neurons $V$ to different values chosen randomly from between $V_r$ and $V_t$ and simulate 1 s of activity. For each case plot the voltages of the two neurons on the same graph (with different colours). [20% of marks].

6. In many real neurons the firing rate falls off after the first few spikes. This can be simulated with a slow potassium current. For the neuron described in the first question add a slow potassium current. This current should have reversal potential $E_K = -80$ mV, its conductance should increase by 0.005 $(\mathrm{M}\Omega)^{-1}$ every time there is a spike, otherwise it should decay towards zero with time constant $\tau = 200$ ms. Plot the voltage of this neuron for one second. [10% of marks].

7. This is a slightly more open-ended question. Consider the model with coupled neurons, but use an alpha function to model $s(t)$ rather than the single exponential model. Does this change the behaviour? [10% of marks].