## The full posterior:

$$P(\text{Absences}_i, \text{Higher-yes}_i, \beta, \sigma^2, \alpha_0, \alpha_1, \gamma_0, \gamma_1 \mid y, X_{rest})$$

$$\propto \mathcal{L}(y \mid \text{Absences}_i, \text{Higher-yes}_i, \beta, \sigma^2, \alpha_0, \alpha_1, \gamma_0, \gamma_1) \prod_{i=1}^{n} \pi(\text{Absences}_i \mid \text{Age}_i)$$

$$\prod_{i=1}^{n} \pi(\text{Higher-yes}_i \mid \text{Age}_i) \; \pi(\beta, \sigma^2) \quad \pi(\alpha_0)\, \pi(\alpha_1)\, \pi(\gamma_0)\, \pi(\gamma_1)$$

$$\propto \left[ (\sigma^2)^{-n/2} e^{-\frac{1}{2\sigma^2}(y - X\beta)^T (y - X\beta)} \right] \prod_{i=1}^{n} \left(1 - \frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}}\right)^{1-\text{higher\_yes}_i}$$

$$\left( \frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}} \right)^{\text{higher\_yes}_i} \prod_{i=1}^{n} \frac{\left(e^{\gamma_0 + \gamma_1 \text{Age}_i}\right)^{\text{Absences}_i} e^{-\left(e^{\gamma_0 + \gamma_1 \text{Age}_i}\right)}}{\text{Absences}_i!}$$

$$\left(\frac{1}{\sigma^2}\right) \quad e^{-\frac{\alpha_0^2}{2 \cdot 100}} \quad e^{-\frac{\alpha_1^2}{2 \cdot 100}} \quad e^{-\frac{\gamma_0^2}{2 \cdot 100}} \quad e^{-\frac{\gamma_1^2}{2 \cdot 100}}$$

## Full conditionals

$$P(\sigma^2 \mid \text{everything else}) \sim IG\left(\frac{n}{2}, \frac{1}{2}(y - X\beta)^T (y - X\beta)\right)$$

$$p(\beta \mid \text{everything else}) \sim MVN\left((X^T X)^{-1} X^T y, \; \sigma^2 (X^T X)^{-1}\right)$$

↑ derivation example 11.8 (course notes)

For $i = 1, \dots, n$:

$$p(\text{Absences}_i \mid \text{everything else}) \sim \text{Poisson}\left(e^{\gamma_0 + \gamma_1 \text{Age}_i}\right)$$

For $i = 1, \dots, n$:

$$p(\text{Higher-yes}_i \mid \text{everything else}) \sim \text{Bernoulli}\left(\frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}}\right)$$

$$p(\alpha_0 \mid \text{everything else}) \propto \prod_{i=1}^{n} \left(1 - \frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}}\right)^{1-\text{higher\_yes}_i} \left( \frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}} \right)^{\text{higher\_yes}_i} e^{-\frac{\alpha_0^2}{2 \cdot 100}}$$

$$p(\alpha_1 \mid \text{everything else}) \propto \prod_{i=1}^{n} \left(1 - \frac{e^{\alpha_0 + \alpha_1 Age_i}}{1 + e^{\alpha_0 + \alpha_1 Age_i}}\right)^{1 - higher\_yes_i} \left(\frac{e^{\alpha_0 + \alpha_1 Age_i}}{1 + e^{\alpha_0 + \alpha_1 Age_i}}\right)^{higher\_yes_i} e^{-\frac{\alpha_1^2}{2 \cdot 100}}$$

$$p(\gamma_0 \mid \text{everything else}) \propto \prod_{i=1}^{n} \frac{\left(e^{\gamma_0 + \gamma_1 Age_i}\right)^{Absences_i} e^{-(e^{\gamma_0 + \gamma_1 Age_i})}}{Absences_i\,!} e^{-\frac{\gamma_0^2}{2 \cdot 100}}$$

$$p(\gamma_1 \mid \text{everything else}) \propto \prod_{i=1}^{n} \frac{\left(e^{\gamma_0 + \gamma_1 Age_i}\right)^{Absences_i} e^{-(e^{\gamma_0 + \gamma_1 Age_i})}}{Absences_i\,!} e^{-\frac{\gamma_1^2}{2 \cdot 100}}$$