SCENARIO ONE

The full posterior:

$$P(\text{Absences}_i, \text{Higher-yes}_i, \beta, \sigma^2, \alpha_0, \alpha_1, \gamma_0, \gamma_1 \mid y, X_{rest})$$

$$\propto \mathcal{L}(y \mid \text{Absences}_i, \text{Higher-yes}_i, \beta, \sigma^2, \alpha_0, \alpha_1, \gamma_0, \gamma_1) \prod_{i=1}^{n} \pi(\text{Absences}_i \mid \text{Age}_i)$$

$$\prod_{i=1}^{n} \pi(\text{Higher-yes}_i \mid \text{Age}_i) \; \pi(\beta, \sigma^2) \quad \pi(\alpha_0)\, \pi(\alpha_1)\, \pi(\gamma_0)\, \pi(\gamma_1)$$

$$\propto \left[(\sigma^2)^{-n/2} e^{-\frac{1}{2\sigma^2}(y - X\beta)^T (y - X\beta)} \right] \prod_{i=1}^{n} \left(1 - \frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}}\right)^{1-\text{higher\_yes}_i}$$

$$\left(\frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}} \right)^{\text{higher\_yes}_i} \prod_{i=1}^{n} \frac{(e^{\gamma_0 + \gamma_1 \text{Age}_i})^{\text{Absences}_i} \, e^{-(e^{\gamma_0 + \gamma_1 \text{Age}_i})}}{\text{Absences}_i !}$$

$$\left(\frac{1}{\sigma^2}\right) \quad e^{-\frac{\alpha_0^2}{2 \cdot 100}} \quad e^{-\frac{\alpha_1^2}{2 \cdot 100}} \quad e^{-\frac{\gamma_0^2}{2 \cdot 100}} \quad e^{-\frac{\gamma_1^2}{2 \cdot 100}}$$

Full conditionals

$$p(\sigma^2 \mid \text{everything else}) \sim IG\left(n/2 , \tfrac{1}{2} (y - X\beta)^T (y - X\beta) \right)$$

$$p(\beta \mid \text{everything else}) \sim MVN\left((X^T X)^{-1} X^T y, \; \sigma^2 (X^T X)^{-1} \right)$$

↑ derivation example 11.8 (course notes)

For $i = 1, \dots, n$:

$$p(\text{Absences}_i \mid \text{everything else}) \sim \text{Poisson}\left(e^{\gamma_0 + \gamma_1 \text{Age}_i} \right)$$

For $i = 1, \dots, n$:

$$p(\text{Higher-yes}_i \mid \text{everything else}) \sim \text{Bernoulli}\left(\frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}} \right)$$

$$p(\alpha_0 \mid \text{everything else}) \propto \prod_{i=1}^{n} \left(1 - \frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}}\right)^{1-\text{higher\_yes}_i} \left(\frac{e^{\alpha_0 + \alpha_1 \text{Age}_i}}{1 + e^{\alpha_0 + \alpha_1 \text{Age}_i}} \right)^{\text{higher\_yes}_i} e^{-\frac{\alpha_0^2}{2 \cdot 100}}$$

$$p(\alpha_1 \mid \text{everything else}) \propto \prod_{i=1}^{n} \left(1 - \frac{e^{\alpha_0 + \alpha_1 Age_i}}{1 + e^{\alpha_0 + \alpha_1 Age_i}}\right)^{1-higher\_yes_i} \left(\frac{e^{\alpha_0 + \alpha_1 Age_i}}{1 + e^{\alpha_0 + \alpha_1 Age_i}}\right)^{higher\_yes_i} e^{-\frac{\alpha_1^2}{2 \cdot 100}}$$

$$p(\gamma_0 \mid \text{everything else}) \propto \prod_{i=1}^{n} \frac{\left(e^{\gamma_0 + \gamma_1 Age_i}\right)^{Absences_i} e^{-(e^{\gamma_0 + \gamma_1 Age_i})}}{Absences_i!} e^{-\frac{\gamma_0^2}{2 \cdot 100}}$$

$$p(\gamma_1 \mid \text{everything else}) \propto \prod_{i=1}^{n} \frac{\left(e^{\gamma_0 + \gamma_1 Age_i}\right)^{Absences_i} e^{-(e^{\gamma_0 + \gamma_1 Age_i})}}{Absences_i!} e^{-\frac{\gamma_1^2}{2 \cdot 100}}$$

SCENARIO TWO:

Using 61 or 62 as the other variable with missing data so:

$$61_i \mid Age_i \text{ are } N(e^{\gamma_0 + \gamma_1 Age_i}, 0.5)$$

something small

makes sense for the mean to be positive

The full posterior:

$$P(61_i, \text{Higher-yes}_i, \beta, \sigma^2, \alpha_0, \alpha_1, \gamma_0, \gamma_1 \mid y, X_{rest})$$

$$\propto \mathcal{L}(y \mid 61_i, \text{Higher-yes}_i, \beta, \sigma^2, \alpha_0, \alpha_1, \gamma_0, \gamma_1) \prod_{i=1}^{n} \pi(61_i \mid Age_i)$$

$$\prod_{i=1}^{n} \pi(\text{Higher-yes}_i \mid Age_i)\ \pi(\beta, \sigma^2)\ \pi(\alpha_0)\pi(\alpha_1)\pi(\gamma_0)\pi(\gamma_1)$$

$$\propto \left[(\sigma^2)^{-n/2} e^{-\frac{1}{2\sigma^2}(y - X\beta)^T(y - X\beta)} \right] \prod_{i=1}^{n} \left(1 - \frac{e^{\alpha_0 + \alpha_1 Age_i}}{1 + e^{\alpha_0 + \alpha_1 Age_i}}\right)^{1-\text{higher\_yes}_i}$$

$$\left(\frac{e^{\alpha_0 + \alpha_1 Age_i}}{1 + e^{\alpha_0 + \alpha_1 Age_i}} \right)^{\text{higher\_yes}_i} \prod_{i=1}^{n} e^{-\frac{(61_i - \gamma_0 + \gamma_1 Age_i)^2}{2 \cdot 0.5}}$$

$$\left(\frac{1}{\sigma^2}\right) e^{-\frac{\alpha_0^2}{2 \cdot 100}} e^{-\frac{\alpha_1^2}{2 \cdot 100}} e^{-\frac{\gamma_0^2}{2 \cdot 100}} e^{-\frac{\gamma_1^2}{2 \cdot 100}}$$

Full conditionals

$$P(\sigma^2 \mid \text{everything else}) \sim IG\left(n/2, \tfrac{1}{2}(y - X\beta)^T(y - X\beta)\right)$$

$$P(\beta \mid \text{everything else}) \sim MVN\left((X^TX)^{-1}X^Ty, \sigma^2(X^TX)^{-1}\right)$$

derivation example 11.8 (course notes)

For $i = 1, \dots, n$:

$$P(61_i \mid \text{everything else}) \sim N(e^{\gamma_0 + \gamma_1 Age_i}, 0.5)$$

For $i=1,\dots,n$:

$$P(\text{Higher\_yes}_i \mid \text{everything else}) \sim \text{Bernoulli}\left(\frac{e^{\alpha_0+\alpha_1 Age_i}}{1+e^{\alpha_0+\alpha_1 Age_i}} \right)$$

$$p(\alpha_0 \mid \text{everything else}) \propto \prod_{i=1}^{n} \left(1-\frac{e^{\alpha_0+\alpha_1 Age_i}}{1+e^{\alpha_0+\alpha_1 Age_i}}\right)^{1-\text{higher\_yes}_i} \left(\frac{e^{\alpha_0+\alpha_1 Age_i}}{1+e^{\alpha_0+\alpha_1 Age_i}}\right)^{\text{higher\_yes}_i} e^{-\frac{\alpha_0^2}{2\cdot 100}}$$

$$p(\alpha_1 \mid \text{everything else}) \propto \prod_{i=1}^{n} \left(1-\frac{e^{\alpha_0+\alpha_1 Age_i}}{1+e^{\alpha_0+\alpha_1 Age_i}}\right)^{1-\text{higher\_yes}_i} \left(\frac{e^{\alpha_0+\alpha_1 Age_i}}{1+e^{\alpha_0+\alpha_1 Age_i}}\right)^{\text{higher\_yes}_i} e^{-\frac{\alpha_1^2}{2\cdot 100}}$$

$$p(\gamma_0 \mid \text{everything else}) \propto \prod_{i=1}^{n} e^{-\frac{(G1_i-\gamma_0+\gamma_1 Age_i)^2}{2\cdot 0.5}} \quad e^{-\frac{\gamma_0^2}{2\cdot 100}}$$

$$p(\gamma_1 \mid \text{everything else}) \propto \prod_{i=1}^{n} e^{-\frac{(G1_i-\gamma_0+\gamma_1 Age_i)^2}{2\cdot 0.5}} \quad e^{-\frac{\gamma_1^2}{2\cdot 100}}$$