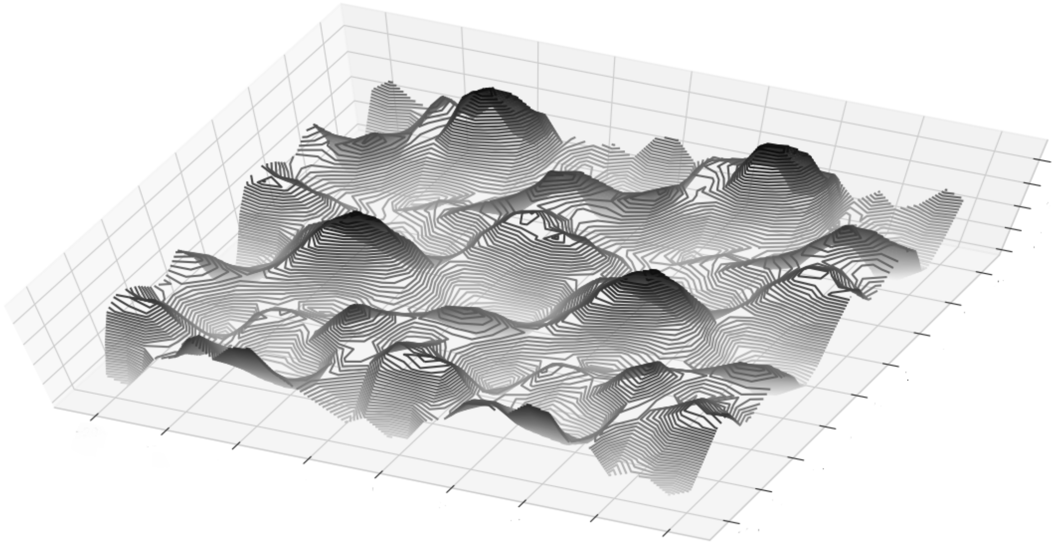
# Modélisation, construction d'un réseau routier

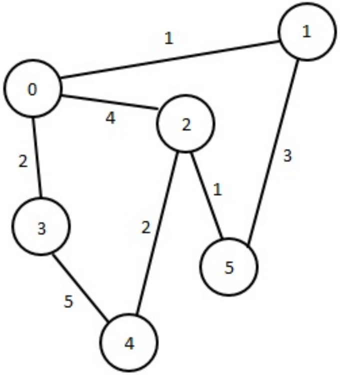
Numéro 32203

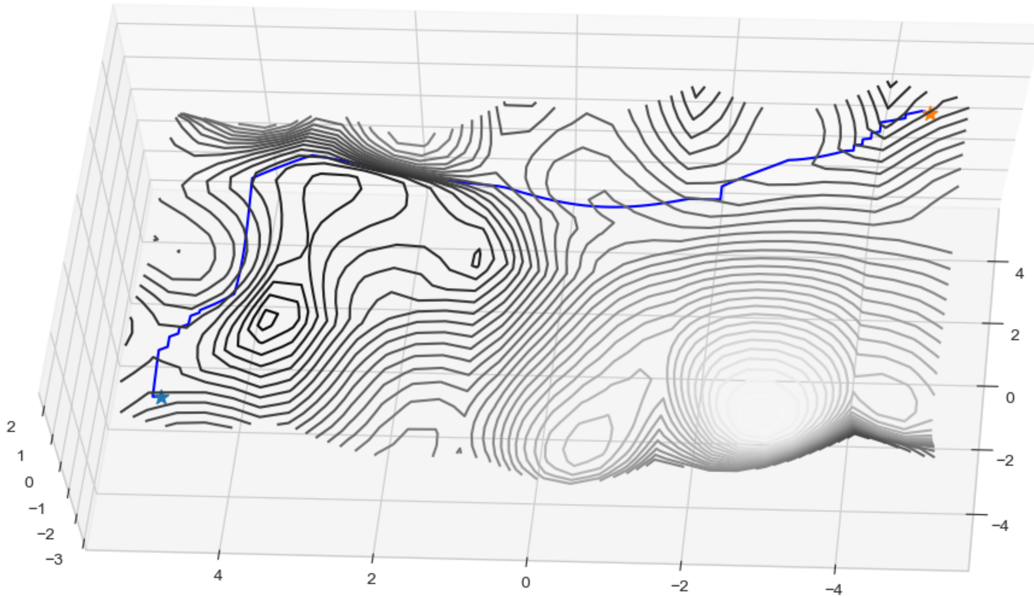
8 juin 2019

# Cadre de l'étude



# Construction par plus courts chemins

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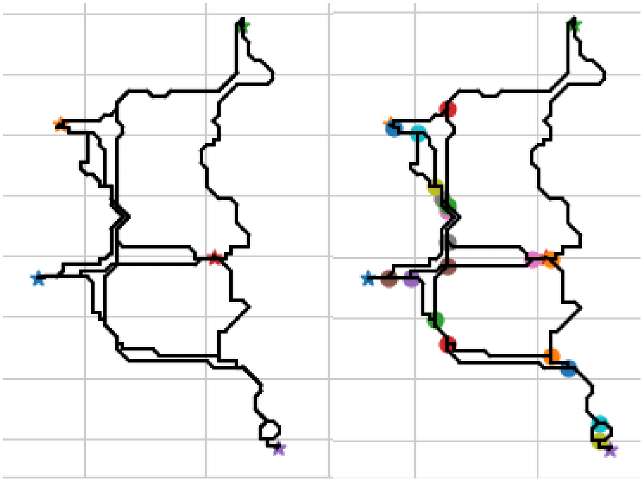
# Problème des jonctions

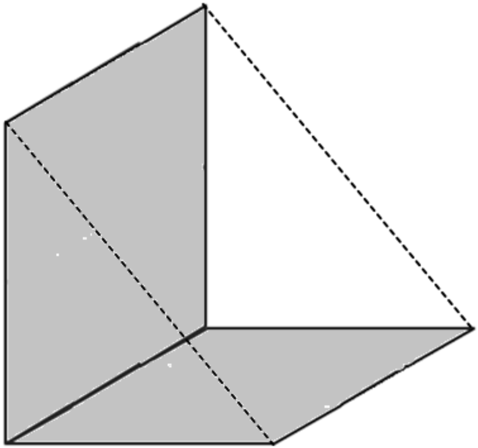


```
Entree : N = (V,R) le graphe precedent
Pour r,s deux routes de R :
I = les sommets communs a r,s
  Tant que I n'est pas vide :
    Retirer un sommet v de I
    Si il s'agit d'une jonction :
      Raccorder r,s a la jonction
    Sinon:
      Creer une jonction en ce sommet
```

Il faut ensuite retirer les sommets vides, les boucles et routes en double éventuelles et mettre à jour la structure de données...

# Principe

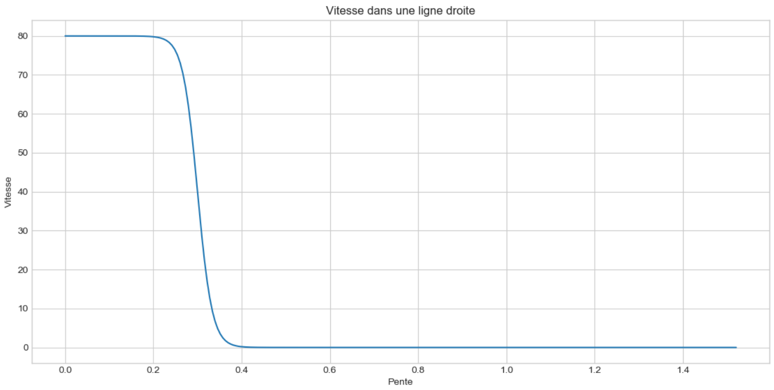
$$C_c(r) = \alpha L(r) + \beta V_{deplace}(r)$$



# Modèle

$$C_{u,r} = \Delta E_m \times \Delta t$$

$$\Delta E_m = \Delta E_c + \Delta E_{p,grav} + E_{roulement} + E_{frott} = mg(1 + c_r)\Delta z + S\rho v^3 \Delta t$$



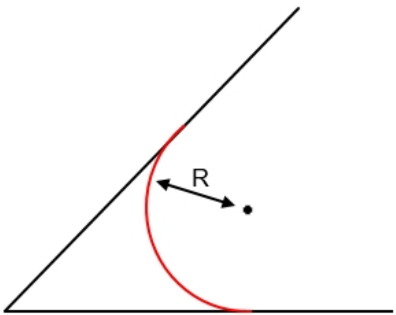
# Prise en compte des virages

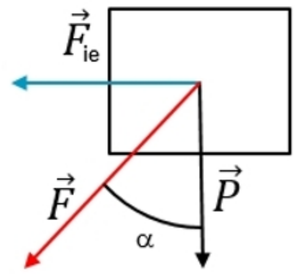
$F_{ie} = m\Omega^2 R$

$\tan\alpha = \frac{F_{ie}}{mg} = \frac{v^2}{Rg}$

Soit

$$v < \sqrt{Rg \tan\alpha_{max}}$$



# Synthèse

$$C_u = \sum_{i<j} C_u(i,j)$$

$$C_u(i,j) = \int_{i \leftrightarrow j} E(t)dt = \sum_{r \in i \leftrightarrow j} C_{u,r} + C_{vir}$$

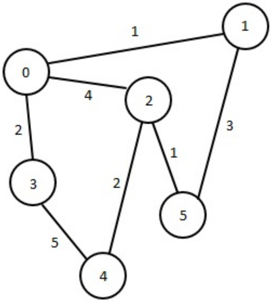



Figure – 0,1 est emprunté quatre fois : $0 \leftrightarrow 1, 0 \leftrightarrow 5, 3 \leftrightarrow 1, 3 \leftrightarrow 5$

# Un compromis

$$C = f(C_c, C_u)$$

On cherche à minimiser C
La fonction retenue est la moyenne géométrique

# Suppression d'arcs inutiles

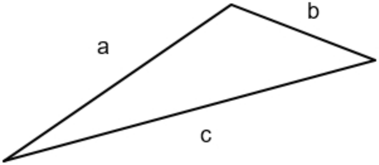
```
Entree : N = (V,R)
Pour r une route de R :
  Si r ne fait pas partie d'un plus court chemin :
    Retirer r
```

Complexité : $O(|R|^2)$

# Suppression des $K_3$

```
Entree : N = (V,R)
Pour tout sous-graphe triangulaire de N :
  Si le plus long cote est assez grand devant les deux autres :
    Le retirer
```

On se donne un critère que l'on fait varier
$x \in [0,1], (1+x)c_c c_u \leq (a_c + b_c)(a_u + b_u)$



# Recherche du minimum par backtracking

```
Entree : N = (V,R)
Pour une route r de R :
  P = N prive de r
  Si C(P) < C(N) :
    N = P
```

Complexité : $O(|R|!)$, améliorable en $O(2^{|R|})$

# Le recuit simulé

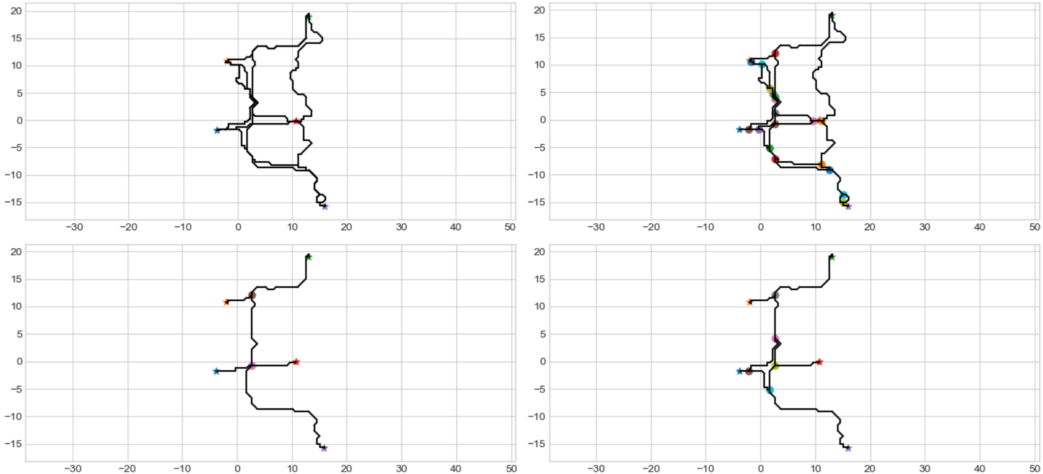
Principe : descente du gradient probabiliste
Analogie thermodynamique : si deux états $E_1, E_2$ sont distants d'une énergie $\Delta E$ à une température T, l'algorithme passe du premier au second si $\Delta E < 0$ avec probabilité $1$ ou avec probabilité $\exp(\frac{\Delta E}{k_B T})$ sinon. T diminue durant l'exécution.
Avantage : permet de s'extraire des minima locaux contrairement à la descente du gradient Inconvénient : ne converge que si T diminue peu rapidement

# Probabilisation de l'algorithme

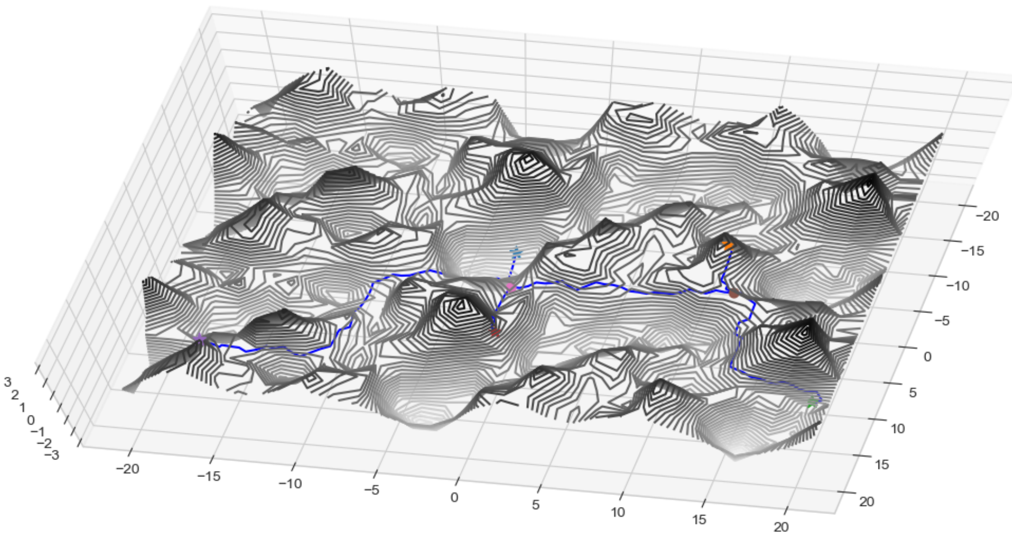
`Entree : N = (V,R) Pour une route aleatoire r de R ne brisant pas la connexite : Diminuer T P = N prive de r Delta = C(P) - C(R) Si Delta < 0 ou random() < exp(Delta/T): N = P`

Complexité : $O(|R|^2)$ par tour de boucle

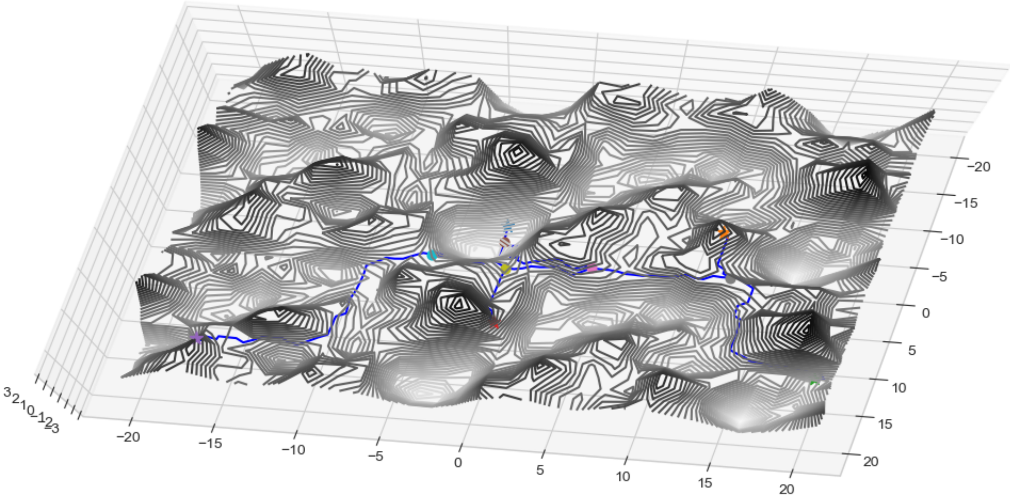
# Conclusion



# Conclusion

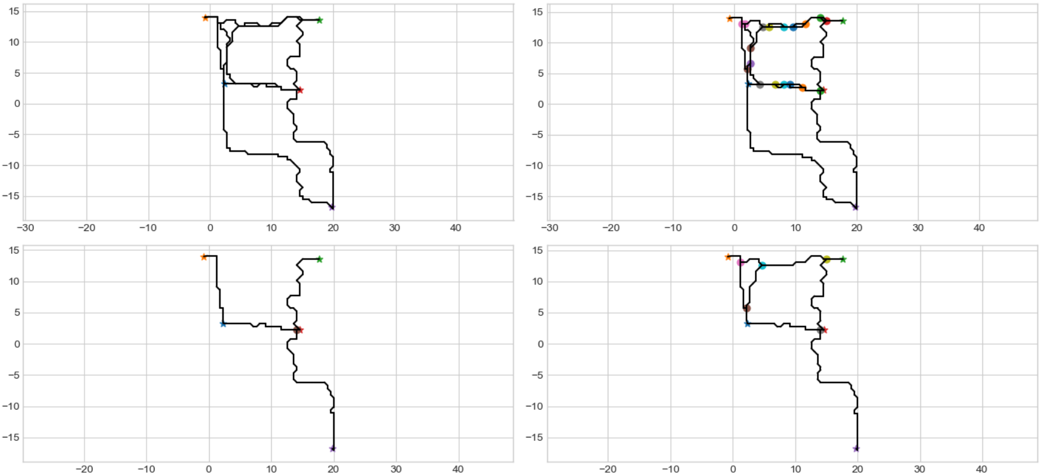


# Conclusion



4 Galerie

5 Algorithmes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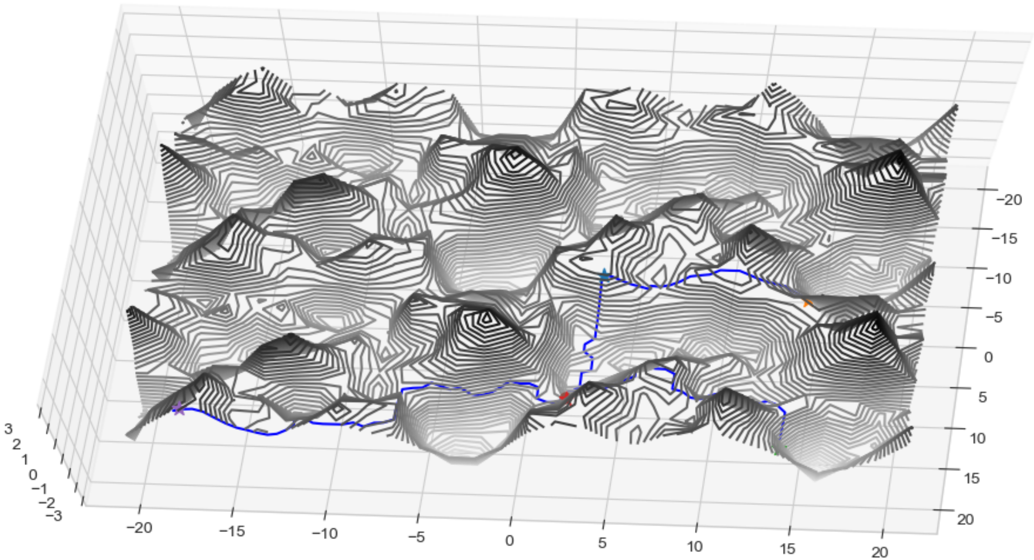
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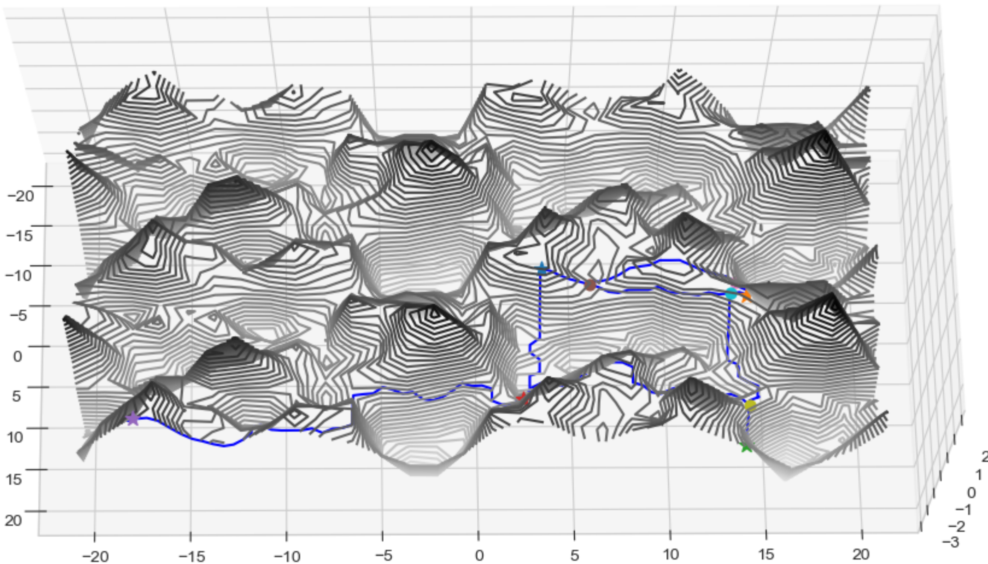
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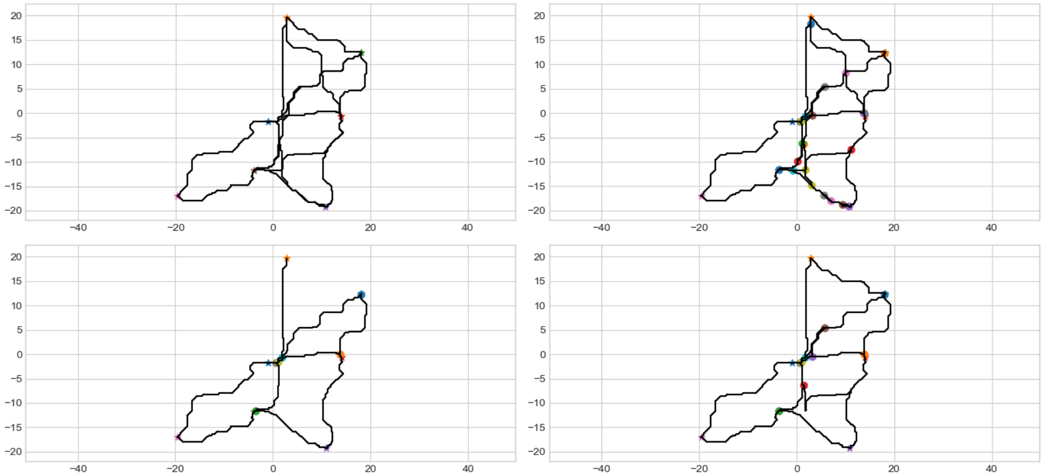
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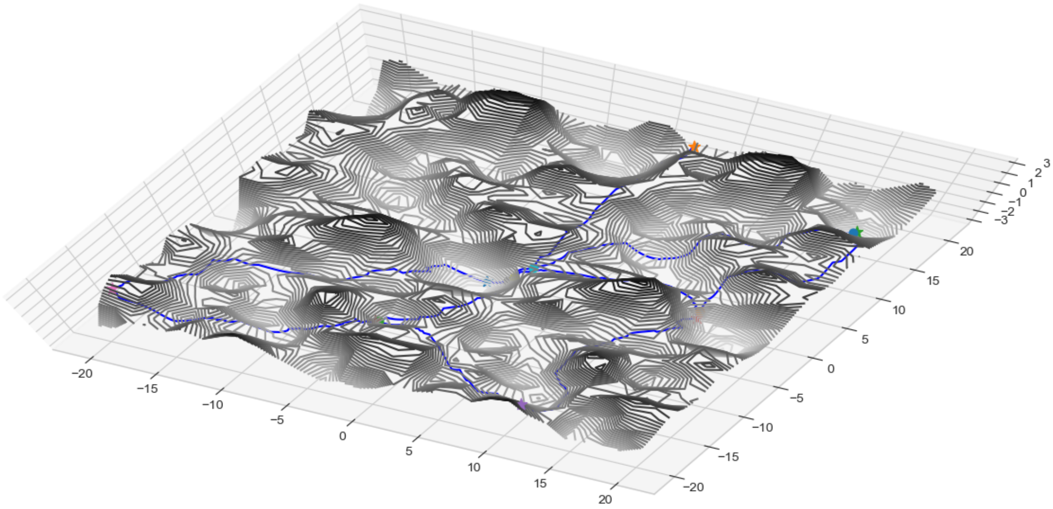
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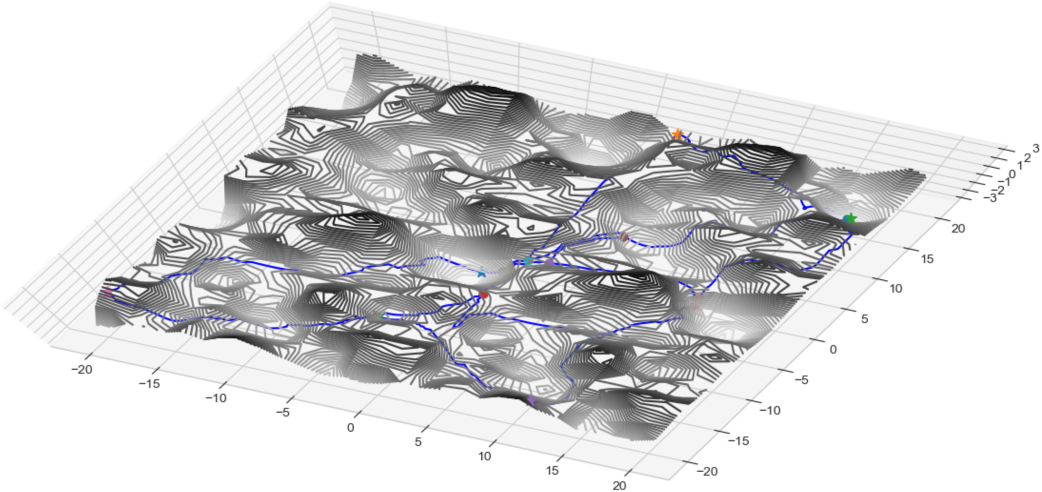
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```python
import numpy as np
import matplotlib.pyplot as plt
from time import time
from copy import deepcopy
plt.style.use('seaborn-whitegrid')
from mpl_toolkits import mplot3d

carte = [4, 3, 2, 0, 1, 4, 1, 0, 2, 1, 3, 0, 0, 3, 0, 4, 0, 1, 4, 1, 0, 2, 3, 4]

def f(x,y,tab=carte) :
    '''Fonction donnant l'altitude d'un point de coordonnees (x,y)'''
    x /= 3
    y /= 3
    return (np.sin(x-y+tab[0]) ** tab[1] + np.sin(y * tab[2]) * np.cos(x-tab[3]) + tab[4] * np.cos(tab[5]*y-tab[6]) ** tab[7]  + np.sin(np.cos(x) * y)/tab[8] - tab[9] * np.sin(x+tab[10]) ** tab[11] + np.sin(x) * np.cos(y-tab[12]) + tab[13] * np.cos(tab[14]*y+tab[15]) ** 2) - (abs(np.sin(-x-y+tab[16]) ** tab[17]) + abs(np.sin(y * tab[18]-1)) * abs(np.cos(x+tab[19])) + abs(tab[20] * np.cos(tab[21]*x+tab[22]) ** tab[23]))
```

# Discrétisation

```
def discretise(f,s0,sf,h) :   # Complexite : quadratique en la resolution
    (x0,y0) = s0
    (xf,yf) = sf
    n = int(abs(xf-x0) // h) + 1
    p = int(abs(yf-y0) // h) + 1
    h = 0.5 * (abs(xf-x0) / n + abs(yf-y0) / p)
    n += 1
    p += 1
    S = [(x0+i*h,y0+j*h,f(x0+i*h,y0+j*h)) for i in range(n) for j in range(p)]
    A = [[] for i in range(n) for j in range(p)]

    for x in range(n*p) :
        for a in [-1,0,1] :
            for b in [-1,0,1] :
                jx,ix = divmod(x,n)
                y = a + ix + n * (b + jx)
                if y != x and 0 <= a + ix < n and 0 <= b + jx < p :
                    c = cout_construction([S[x],S[y]])
                    A[y].append((x,c))
                    A[x].append((y,c))
    return S,A,n,p,h
```

```python
class Heap :    # Structure de tas binaire

 def tasse(self,i) :
   n = len(self.heap) - 1
   if 2 * i + 1 >= n :
     self.percole(i)
   else :
     self.tasse(2*i)
     self.tasse(2*i+1)
     self.percole(i)

 def __init__(self,l=[],compare=lambda a,b:a<b) :
   self.heap = [len(l)] + l
   self.comp = compare
   self.tasse(1)

 def remonte(self,i) :
   if i // 2 > 0 :
     if self.comp(self.heap[i],self.heap[i//2]) :
       self.heap[i],self.heap[i//2] = self.heap[i//2],self.heap[i]
       self.remonte(i//2)
```

```python

def add(self,x) :
    self.heap.append(x)
    self.heap[0] += 1
    self.remonte(self.heap[0])

def take(self) :
    self.heap[0] -= 1
    x = self.heap.pop()
    return x

def percole(self,i) :
    if 2 * i + 1 >= len(self.heap) - 1 :
        if 2 * i < len(self.heap) - 1 :
            j = 2 * i
            if self.comp(self.heap[j],self.heap[i]) :
                self.heap[i],self.heap[j] = self.heap[j],self.heap[i]
                self.percole(j)
    else :
        if self.comp(self.heap[2*i],self.heap[2*i+1]) :
            j = 2 * i
        else :
            j = 2 * i + 1
        if self.comp(self.heap[j],self.heap[i]) :
            self.heap[i],self.heap[j] = self.heap[j],self.heap[i]
            self.percole(j)

def take_min(self) :
    self.heap[1],self.heap[-1] = self.heap[-1],self.heap[1]
    x = self.take()
    self.percole(1)
    self.heap[0] -= 1
    return x
```

# Algorithme de Dijkstra

```
def dijkstra(S,A,n,p,h,s0,sf) :   # Complexite : O(|R|log|V|)
    deb = pos(s0,S,h)
    fin = pos(sf,S,h)
    pred = [-1 for _ in S]
    dist = [np.infty for _ in S]
    dist[fin] = 0
    pq = Heap([(0,fin)],lambda a,b : a[0] < b[0])
    deja_vu = [False for s in S]

    while len(pq.heap) > 1 :
        u = pq.take_min()[1]
        if deja_vu[u] :
            continue
        deja_vu[u] = True
        for v,w in A[u] :
            if dist[v] > dist[u] + w :
                dist[v] = dist[u] + w
                pred[v] = u
                pq.add((dist[v],v))

    C = [s0]
    s = deb
    while s != fin :
        C.append(S[s])
        s = pred[s]
    C.append(sf)
    return C
```

```
class Map :   # Structure comprenant les elements essentiels de la modelisation

    def __init__(self,pt,s0=(-20,-20),sf=(20,20),h=0.5,graph=-1,dmin = 30,cc=-1,cu=-1) :
        if graph == -1 :
            (S,A,n,p,h) = discretise(f,s0,sf,h)
            self.graph = (S,A,n,p,h,s0,sf)
        else :
            self.graph = graph
        self.network = []
        self.towns = pt
        self.cc = cc
        self.cu = cu

        self.init()

    def init(self,dmin=30) :    # Construit le graphe initial par plus courts chemins
        l = len(self.towns)
        (S,A,n,p,h,s0,sf) = self.graph

        for i in range(l) :
            t = Node(self.towns[i],tow=True,nb = i)
            self.network.append(t)
            self.network[i].roads = []

        ct = 0
        for i in range(l) :
            for j in range(i + 1, l) :
                if abs(self.towns[i][0] - self.towns[j][0]) < dmin and abs(self.towns[i][1] - self.towns[j][1]) < dmin :
                    ct += 1
                    C = dijkstra(S,A,n,p,h,self.towns[i],self.towns[j])
                    r = Road(i,j,C,False)
                    self.network[i].roads += [r]
                    self.network[j].roads += [r.retourne()]
```

```python
def __repr__(self) :    # Outil de trace permettant de visualiser le reseau en 2D
    N = self.network
    for s in N :
        (x,y,z) = s.coord
        for r in s.roads :
            if len(N) > r.end > s.id :
                u = N[r.end]
                (x2,y2,z2) = u.coord
                if r.tunn :
                    for i in range(len(r.path) - 1) :
                        (x1,y1,z1) = r.path[i]
                        (x2,y2,z2) = r.path[i+1]
                        plt.plot([x1,x2],[y1,y2],'--',color='black')
                else :
                    for i in range(len(r.path) - 1) :
                        (x1,y1,z1) = r.path[i]
                        (x2,y2,z2) = r.path[i+1]
                        plt.plot([x1,x2],[y1,y2],color='black')
        if s.town :
            plt.scatter(x,y,marker='*',s=40)
        elif len(s.roads) > 2 :
            plt.scatter(x,y,s=40)
    plt.axis('equal')
    plt.grid()
    return ""
```

```
def plot_3D(self) :     # Outil de trace permettant de visualiser le reseau en 3D
    N = self.network

    (x0,y0,z0) = self.graph[0][0]
    (xf,yf,zf) = self.graph[0][-1]

    x = np.linspace(x0, xf, 30)
    y = np.linspace(y0, yf, 30)

    X, Y = np.meshgrid(x, y)
    Z = np.vectorize(f)(X, Y)

    fig = plt.figure()
    ax = plt.axes(projection='3d')
    ax.contour3D(X, Y, Z, 50, cmap='binary')

    for i in range(len(N)) :
        if len(N[i].roads) > 0 :
            (xi,yi,zi) = N[i].coord
            if N[i].town :
                ax.scatter(xi,yi,zi,marker='*',s=80)
            elif len(N[i].roads) > 2 :
                ax.scatter(xi,yi,zi,s=40)
            for r in N[i].roads :
                j = r.end
                if i < j :
                    C = r.path
                    xs = [c[0] for c in C]
                    ys = [c[1] for c in C]
                    zs = [c[2] for c in C]
                    ax.plot3D(xs,ys,zs,color='b')
    plt.show()
```

```python
def copy(self) :          # Complexite : O(|V|)
    P = []
    pt = self.towns

    for s in self.network :
        t = Node(s.coord,[],s.town,s.id)
        P.append(t)

    for i in range(len(self.network)) :
        for r in self.network[i].roads :
            P[i].roads.append(Road(r.start,r.end,r.path,r.tunn,r.length,r.cc,r.cu,r.flux)

    M = Map(pt,graph=self.graph)
    M.network = P
    M.cc,M.cu = self.cc,self.cu
    return M
```

```
def normalize(self) :   # Complexite : au plus en O(r^2 |R|^5)
    cree_jonctions(self)
    fusion(self.network)    #Fusion des tronηons
    tailladeur(self.network)    #Suppression des premiers sommets vides
    supprime_nuls(self.network)     #Suppression des boucles
    supprime_doubles(self.network)  #Suppression des routes en double
    P = maj_indices(self.network)
    tailladeur(P)   #Suppression des derniers vides
    self.network = P
    self.update_fl()    #Mise a jour des coΰts

def update_fl(self) :   # Complexite : O(|R|^2)
      for s in self.network :
          for r in s.roads :
              r.flux = 0
      self.cc,self.cu = attr_flux(self)

def longueur(L) :
    return sum([distance(L[i],L[i+1]) for i in range(len(L) - 1)])
```

```
def jonct(r1,r2) :        # Complexite : O(|c1||c2|)
    c1 = r1.path
    c2 = r2.path
    J = []
    for j in range(len(c2)) :
        for i in range(len(c1)) :
            if c1[i] == c2[j] :
                J.append(c1[i])

    return J

def attr_flux(N) :        # Complexite : O(|R|^2)
    G = [dijkstra_generalise(N.network,i) for i in range(len(N.network))]
    n = len(N.towns)
    cu = 0
    cc = 0
    for s in N.network :
        for r in s.roads :
            cc += r.cc
            r.flux = len([1 for (i,j) in [(i,j) for i in range(n) for j in
                range(n)] if r in G[i][j] + G[j][i]])
            cu += sum([r.cu for (i,j) in [(i,j) for i in range(n) for j in
                range(n)] if r in G[i][j] + G[j][i]])
    return cc,cu
```

```
def supprime_nuls(N) :  # Complexite : O(|R|)
    for s in N :
        for r in s.roads :
            if r.start == r.end or r.length == 0 :
                s.roads.remove(r)

def supprime_doubles(N) : # Complexite : O(|R|)
    for s in N :
        D = []
        for r in s.roads :
            if r.end not in D :
                D.append(r.end)
        E = [np.infty for d in D]
        R = [0 for d in D]
        for i in range(len(R)) :
            for r in s.roads :
                if r.end == D[i] :
                    R[i] = r
                    break
            E[i] = r.cu
            for r in s.roads :
                if r.end == D[i] and r.cu < E[i] :
                    R[i] = r
                    E[i] = r.cu
        s.roads = R
```

```
def tailladeur(N) :   # Complexite : O(|V||R|)
    i = 0
    while i < len(N) - 1 :
        i += 1
        s = N[i]
        if not s.town and len(s.roads) == 1 :
            for r in s.roads :
                u = N[r.end]
                u.rem(s)
            s.roads = []
            i = 0
```

```python
def cree_jonctions(M) :   # Complexite : O(r^2 |V|^5)
    h = M.graph[4]
    for s in M.network :
        for u in M.network :
            if s.id < u.id :
                for rs in s.roads :
                    t = M.network[rs.end]
                    for ru in u.roads :
                        v = M.network[ru.end]
                        L = jonct(rs,ru)

                        while L != [] :
                            (x,y,z) = L.pop()
                            b = False

                            for w in M.network :
                                (xw,yw,zw) = w.coord
                                if 0.001 < abs(xw-x) < h/2 and 0.001 < abs(yw-y) < h/2 :
                                    b = True
                                    normalise_jonction(M.network,w,True)
                                elif abs(xw-x) < h/2 and abs(yw-y) < h/2 :
                                    b = True
                                    normalise_jonction(M.network,w)

                            if not b :
                                w = Node((x,y,z),tow=False,nb = len(M.network))
                                M.network.append(w)
                                normalise_jonction(M.network,w)
```

```python
def normalise_jonction(N,t,existait=False) :     # Complexite : O(|V||R|)
    for s in N :
        for r in s.roads :
            C = r.path
            u = N[r.end]
            for i in range(1, len(C) - 1) :
                if C[i] == t.coord :
                    s.rem(u)
                    u.rem(s)
                    cst = C[:i + 1]
                    ctu = C[i:]

                    if existait :
                        cst.append(t.coord)
                        ctu = [t.coord] + ctu

                    rst = Road(s.id,t.id,cst,traffic=r.flux)
                    rtu = Road(t.id,u.id,ctu,traffic=r.flux)
                    rts = rst.retourne()
                    rut = rtu.retourne()

                    s.roads.append(rst)
                    t.roads += [rts,rtu]
                    u.roads.append(rut)
```

```python
def fusion(N) :   # Complexite : O(|R|)
    for s in N :
        r = s.roads
        if len(r) == 2 and not s.town :
            [rt,ru] = r
            t = N[rt.end]
            u = N[ru.end]
            cts = rt.path[::-1]
            csu = ru.path

            if cts[-1] == csu[0] :
                cts.pop()

            ctu = cts + csu

            t.rem(s)
            u.rem(s)
            s.roads = []
            rtu = Road(t.id,u.id,ctu)
            rut = rtu.retourne()
            t.roads.append(rtu)
            u.roads.append(rut)
```

```
def maj_indices(N) :  # Complexite : O(|R|^2)
    P = []
    i = 0
    j = 0
    for s in self.network :     #Mise a jour des indices
        if s.roads == [] :
            j += 1
        else :
            s.id = i
            for u in self.network :
                for r in u.roads :
                    if r.start == j :
                        r.start = i
                    if r.end == j :
                        r.end = i
            for u in P :
                for r in u.roads :
                    if r.start == j :
                        r.start = i
                    if r.end == j :
                        r.end = i
            P.append(s)
            i += 1
            j += 1
    for s in P :
        for r in s.roads :
            if r.end >= len(P) :
                s.roads.remove(r)
```

```python
class Node :  # Structure representant les sommets du graphe

    def __init__(self,coordinates=(0.5,0.5,0),roads=[],tow=False,nb=-1) :
        self.coord = coordinates
        self.roads = []
        self.town = tow
        self.id = nb

    def rem(self,v) :
        for r in self.roads :
            if r.end == v.id :
                self.roads.remove(r)

    def __repr__(self) :
        (x1,y1,z1) = self.coord
        x = round(x1,3)
        y = round(y1,3)
        z = round(z1,3)
        if self.town :
            aff ="Ville localisee en {}".format((x,y,z))
        else :
            aff = "Jonction localise en {}".format((x,y,z))
        return aff
```

```python
class Road :  # Structure representant les arκtes du graphe

    def __init__(self,start,end,path,is_tunnel=False,length=-1,cc=-1,cu=-1,traffic=0,i=1)
        self.start = start
        self.end = end
        self.path = path
        self.flux = traffic
        if length == -1 :
            self.length = longueur(path)
        else :
            self.length = length
        self.tunn = is_tunnel
        if cc == -1 :
            self.cc = cout_construction(path)
        else :
            self.cc = cc
        if cu == -1 :
            self.cu = cout_usage(path,6)
        else :
            self.cu = cu

    def retourne(self) :
        return Road(self.end,self.start,self.path[::-1],self.tunn,self.length,self.cc,self.c
```

```python
def cout_construction(C,i=1) :  # Complexite : O(|C|)
    S = 0
    delta = 10
    for i in range(len(C)-1) :
        dz = abs(C[i+1][2] - C[i][2])
        d = distance(C[i],C[i+1])
        S += delta * (d + dz * 1 * 0.5 * 36 * d ** 2)
    return S

def pente(i,j) :
    (xi,yi,zi),(xj,yj,zj) = i,j
    d = np.sqrt((xj-xi)**2 + (yj-yi)**2 + (zj-zi)**2)

    if xj == xi :
        if yj == yi :
            p = 0
        else :
            p = abs((zj-zi)/(yj-yi))
    else :
        if yj == yi :
            p = abs((zj-zi)/(xj-xi))
        else :
            p = abs((zj-zi)/(xj-xi) + (zj-zi)/(yj-yi))
    return p
```

```python
def rayon(angle,l) :
    R = 0
    if abs(angle - 135) < 0.01 :
        R = 16 * l
    if abs(angle - 90) < 0.01 :
        R = 4 * l
    if abs(angle - 45) < 0.01 :
        R = l
    return R

def distance(i,j,k=1) :
    (xi,yi,zi),(xj,yj,zj) = i,j
    return ((xj-xi)**2 + (yj-yi)**2 + (zj-zi)**2) ** 0.5

def angle(a,b,c) :
    (xa,ya,za) = a
    (xb,yb,zb) = b
    (xc,yc,zc) = c
    u = (xa-xb,ya-yb)
    v = (xc-xb,yc-yb)
    q = (u[0] * v[0] + u[1] * v[1]) / np.sqrt(u[0] ** 2 + u[1] ** 2) /
        np.sqrt(v[0] ** 2 + v[1] ** 2)
    return np.arccos(round(q,5))
```

```python
def cout_usage(C,l,i=1) :
    S = 0
    acc = 1
    alpha = np.tan(np.pi/14)
    g = 9.81
    cr = 0.01
    m = 2000
    ro = 1.3
    Sp = 4
    if len(C) < 3 :
        [a,b] = C
        dz = a[2] - b[2]
        p = pente(a,b)
        vmax = 80/(1 + np.exp(60 * p - 18)) / 3.6 + 1
        t = distance(a,b,i) / vmax
        Epp = m * (1 + cr) * g * dz
        Er = 0.5 * m * vmax ** 2
        Et = Sp * ro * vmax ** 3 * t
        if Epp + Et < 0 :
            S += 0
        else :
            S += t * (Er + Epp + Et)
    else :
        for i in range(len(C) - 2) :
            a = C[i]
            b = C[i+1]
            c = C[i+2]
            dz = C[i+1][2] - C[i][2]
            beta = angle(a,b,c)
            p = pente(a,b)
            d = distance(a,b,i)
            vmax = 80/(1 + np.exp(60 * p - 18)) / 3.6 + 1
            if abs(beta - np.pi) < 0.1 :
                t = 0
```

```
        else :
            R = rayon(beta,l)
            L = R * abs(beta)
            vir = min(np.sqrt(R * g * alpha),vmax) + 0.00001
            tmax = (vmax - vir) / acc
            D = 0.5 * acc * tmax ** 2 + vir * tmax
            tadd = L * (1/vir - 1/vmax) + 2 * (tmax - D/vmax)
            t = tadd
        t += distance(a,b,i) / vmax
        Epp = m * (1 + cr) * g * dz
        Er = 0.5 * m * vmax ** 2
        Et = Sp * ro * vmax ** 3 * t
        if Epp + Et < 0 :
            S += 0
        else :
            S += t * (Er + Epp + Et)
        return S / 10000000
```

```
def dijkstra_generalise(N,i) :
    dist = [np.infty for _ in N]
    dist[i] = 0
    pq = Heap([(0,i)],lambda a,b : a[0] < b[0])
    deja_vu = [False for _ in N]
    pred = [[] for _ in N]
    while len(pq.heap) > 1 :
        u = pq.take_min()[1]
        if deja_vu[u] :
            continue
        deja_vu[u] = True
        for r in N[u].roads :
            if dist[r.end] > dist[u] + r.cu :
                dist[r.end] = dist[u] + r.cu
                pred[r.end] = pred[u] + [r]
                pq.add((dist[r.end],r.end))
    return pred
```

```python
def est_connexe(N) :  # Test de connexite (lineaire)
    n = len(N)
    deja_vu = [False for _ in range(n)]
    explore(0,N,deja_vu)
    for i in range(n) :
        if N[i].town and not deja_vu[i]:
            return False
    return True

def explore(i,N,deja_vu) :
    deja_vu[i] = True
    s = N[i]
    for r in s.roads :
        j = r.end
        if not deja_vu[j] :
            explore(j,N,deja_vu)
```

```python
def deep_copy(N) :
    P = []
    for s in N :
        t = Node(s.coord,[],s.town,s.id)
        P.append(t)

    for i in range(len(N)) :
        for r in N[i].roads :
            P[i].roads.append(Road(r.start,r.end,r.path,r.tunn,r.length,r.cc,r.cu))

    return P

def elimination(N) :
    N.update_fl()
    P = N.copy()
    for s in P.network :
        for r in s.roads :
            if r.flux == 0 :
                s.roads.remove(r)
            i += 1
    P.normalize()
    return P
```

```python
def cherche_triangles(N) :  # Complexite : O(|R|^3)
    n = len(N)
    A = aretes(N)
    T = []
    for i in range(len(A)) :
        for j in range(i+1,len(A)) :
            for k in range(j+1,len(A)) :
                r1,r2,r3 = A[i],A[j],A[k]
                if r1.end == r2.start and r2.end == r3.start and r3.end == r1.start :
                    T.append((r1,r2,r3))
                if r1.end == r3.start and r3.end == r2.start and r2.end == r1.start :
                    T.append((r1,r2,r3))
    return T

def nettoie_triangles(T,j,k) :  # Complexite : O(|R|^3)
    U = []
    for t in T :
        (a,b,c) = t
        if a.start != j.id :
            if b.start != k.id and c.start != k.id :
                U.append(t)
        elif b.start != j.id :
            if c.start != k.id :
                U.append(t)
    return U
```

```python
def compromis(N) :
    return N.cu * N.cc

def rem_road(r,N) :
    s = N[r.start]
    u = N[r.end]

    for v in s.roads:
        if v.end == r.end :
            s.roads.remove(v)

    for w in u.roads :
        if w.end == r.start :
            u.roads.remove(w)

def critere(crit,rij,rik,rjk) :
    return (rij.cc + rik.cc) * (rij.cu + rik.cu) >= crit * rjk.cc * rjk.cu
```

```
def detriangularisation(M,crit) :   # Complexite : O(|R|^6) en theorie, O(|R|^3) sinon
    N = M.network
    P = M.copy()
    T = cherche_triangles(P.network)

    while T != [] :
        (rij,rik,rjk) = T.pop()
        if critere(crit,rij,rik,rjk) :
            rem_road(rjk,P.network)
            P.update_fl()
            T = nettoie_triangles(T,N[rjk.start],N[rjk.end])
        elif critere(crit,rij,rjk,rik) :
            rem_road(rik,P.network)
            P.update_fl()
            T = nettoie_triangles(T,N[rik.start],N[rik.end])
        elif critere(crit,rjk,rik,rij) :
            rem_road(rij,P.network)
            P.update_fl()
            T = nettoie_triangles(T,N[rij.start],N[rij.end])
    tailladeur(P.network)
    return P
```

```python
def compare_couts(P) :
    n = len(P.network)
    K = np.linspace(0,1,60)
    L = []
    M = []
    C = []
    cun = P.cu
    ccn = P.cc
    cn = compromis(P)
    for k in K :
        D = detriangularisation(P,k)
        L.append(D.cc)
        M.append(D.cu)
        C.append(compromis(D))
    i = min_l(C)
    return K[i],L[i],M[i]

def aretes(N) :
    A = []
    for s in N :
        for r in s.roads :
            if r.end > r.start :
                A.append(r)

    return A
```

```python
def sol_opt(N,i) :    # Complexite theorique : O(|R|!)
    A = aretes(N.network)
    m = compromis(N)
    P = N.copy()

    if m == np.infty or i >= len(A) - 1 :
        return P

    for j in range(i+1,len(A)-1) :
        a = A[j]
        B = N.copy()
        rem_road(a,B.network)
        B = sol_opt(B,j)
        c = compromis(B)
        if c < m :
            m = c
            P = B

    return P
```

```python
def retire_chemin(N,L) :
    k = np.random.randint(len(N))
    s = N[k]
    r = s.roads
    while r == [] :
        k = np.random.randint(len(N))
        s = N[k]
        r = s.roads
    i = np.random.randint(len(r))
    u = r[i].end
    L.append(r[i])
    s.rem(N[u])
    N[u].rem(s)

def ajoute_chemin(N,L) :
    r = L.pop(np.random.randint(len(L)))
    s = r.start
    u = r.end
    N[s].roads.append(r)
    N[u].roads.append(r.retourne())
```

```python
def recuit(N) :   # Complexite : O(n|R|^2)
    P = elimination(N.copy())
    G = N.copy()
    n = 100
    L = []
    e = compromis(P)
    g = compromis(G)
    k = 0
    T = [100000 / x for x in range(2,n+2)]
    while k < n :
        M = P.copy()
        print(k)
        if np.random.random() < 0.5 or L == [] :
            retire_chemin(M.network,L)
        else :
            ajoute_chemin(M.network,L)
        M.update_fl()
        E = compromis(M)
        if (E < e or np.random.random() < np.exp((e - E) / T[k])) and est_connexe(M.network) :
            P = M.copy()
            e = E
            if g > e :
                g = e
                G = P.copy()

        k += 1
    return G
```