# SIW004: Homework 1

H1: Looking at Multivariate data

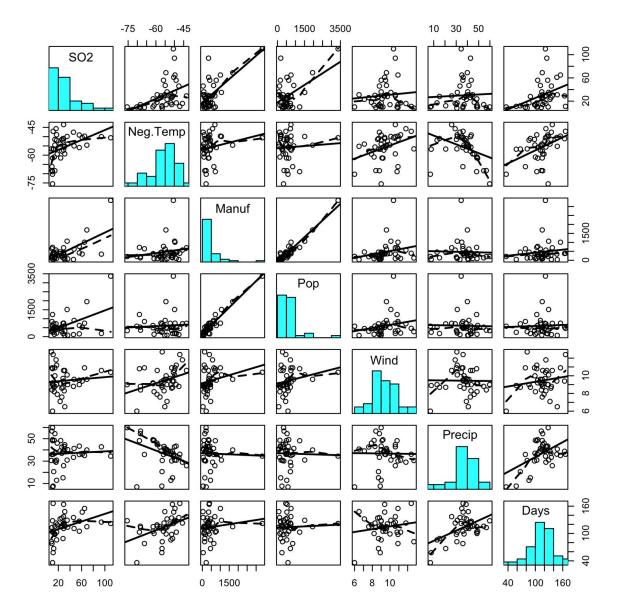
Denny, Alaa, and Lucas

Deadline: send work by email to mateu@uji.es by January 8, 2017

Note: the answers to each of these questions should be written clearly reflecting your understanding of the statistical procedure followed, and you should include any R code you have written.

1) Present a complete description using the concepts learned in Chapter 2 in Everitt of the dataset: usair.dat

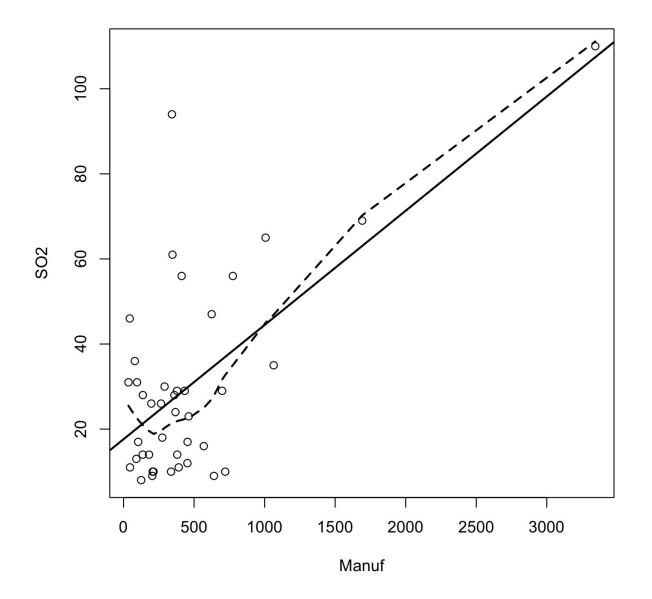
We begin with a scatterplot matrix to show an overview of how the data is correlated. In our graphs we show two lines a solid line representing the least squares fit (using the lsfit method of R) and a dotted line representing a locally-weighted polynomial regression, i.e. a smoothed line fitted locally (using the lowess method of R).



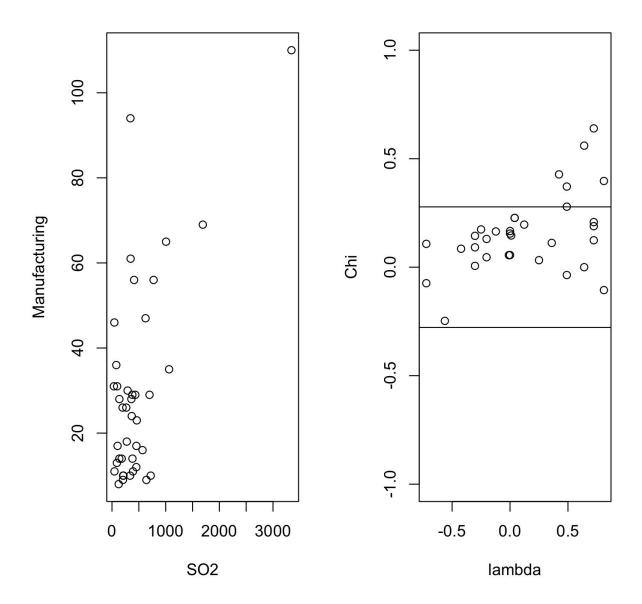
Some of these graphs are meaningless, such as population graphed against wind, but we still find it helpful to see a complete picture. Other graphs are uninteresting because they are obvious, such as the two graphs showing high correlation between population and manufacturing. It is not surprising that high-population cities contain more manufacturing.

Other graphs show correlation that wasn't necessarily expected, such as SO2 against manufacturing, which we can now visualize in a larger format.

```
attach(usair.dat)
plot(Manuf, SO2)
abline(lsfit(Manuf, SO2), lwd=2)
abline(lines(lowess(Manuf, SO2), lty=2, lwd=2))
```



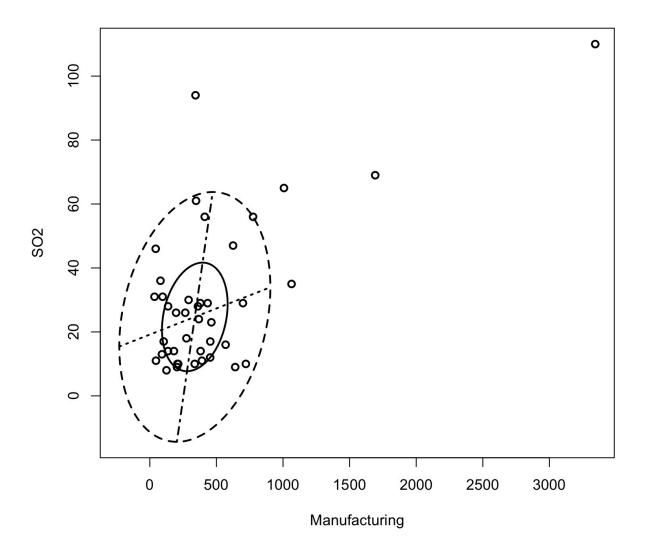
It seems from looking at this larger scatterplot like these two variables (SO2 and Manufacturing) might actually not be correlated, but let's make a chiplot just to double check.



This chiplot shows most of the values in the center, which indicates that indeed the values are *independent*, so it turns out our original hypothesis of correlation was false.

A bivariate boxplot shows the same conclusion.

```
source(paste(getwd(), "/functions.txt", sep="")) # Load functions from file
bvbox(cbind(Manuf, SO2), xlab="Manufacturing", ylab="SO2")
```



In this plot, the acute angle between the regression lines is large, which means that the absolute value of correlation between the two variables is small.

- 2) Using the matrix data of usair.dat, calculate the set of following distances:
- a) Euclidean distances for original data

```
dis<-dist(usair.dat)
dis.matrix<-dist2full(dis)
round(dis.matrix,digits=2)</pre>
```

```
[,1]
               [,2]
                      [,3]
                             [,4]
                                   [,5]
                                           [,6]
                                                  [,7]
                                                          [,8]
                                                                  [,9]
[1,]
        0.00 \quad 472.55 \quad 277.31 \quad 255.93 \quad 481.20 \quad 527.10 \quad 294.36 \quad 131.96 \quad 268.94
     472.55
             0.00 688.47 529.17 326.29
                                           60.56 713.27 399.98 236.21
[3,] 277.31 688.47
                    0.00 202.16 564.94 739.46 67.82 373.02 459.66
[4,] 255.93 529.17 202.16
                            0.00 365.17 575.32 245.84 322.84 313.01
[5,] 481.20 326.29 564.94 365.17
                                   0.00 341.92 600.30 464.97 276.47
[6,] 527.10
             60.56 739.46 575.32 341.92
                                           0.00 764.01 453.47
                    67.82 245.84 600.30 764.01
[7,] 294.36 713.27
                                                  0.00 376.04 480.66
[8,] 131.96 399.98 373.02 322.84 464.97 453.47 376.04
                                                           0.00 207.16
[9,] 268.94 236.21 459.66 313.01 276.47 287.87 480.66 207.16
[10,] 198.69 458.59 241.71 99.77 343.81 507.05 268.52 234.59 229.91
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[2,]	864.09	474.24	842.63	234.02	134.22	72.03	451.98	491.88	1108.27	
[3,]	258.25	223.98	340.78	459.60	623.44	730.28	279.28	271.93	564.76	
[4,]	339.78	78.47	341.93	321.26	490.69	575.31	116.96	82.15	607.51	
[5,]	653.40	354.09	589.62	303.10	386.61	370.75	311.58	301.14	840.60	
[6,]	908.27	523.12	881.64	286.95	181.32	56.76	496.81	534.37	1145.49	
[7,]	267.26	256.30	367.81	482.30	646.39	750.98	302.91	306.09	575.91	
[8,]	604.40	247.58	647.58	190.66	308.24	425.79	269.62	335.91	901.54	
[9,]	641.84	248.35	639.21	56.51	205.33	278.12	230.36	282.90	904.49	
[10,]	414.62	28.34	427.30	241.44	415.67	501.44	69.45	105.87	690.25	
[11,]	3727.41	4120.70	3761.52	4375.71	4545.01	4634.25	4144.49	4100.82	3509.79	
[12,]	338.74	241.27	433.40	438.64	597.05	705.66	288.69	310.17	647.99	
[13,]	806.37	412.87	793.15	165.18	88.73	112.71	393.76	438.92	1059.69	
[14,]	742.45	344.69	737.83	91.42	91.92	191.93	335.47	383.90	1005.61	
[15,]	435.78	127.94	485.95	271.45	433.20	537.68	171.02	221.34	734.83	
[16,]	627.90	231.19	630.10	44.39	210.27	297.41	221.54	275.87	895.90	
[17,]	181.02	468.18	320.52	714.07	881.70	980.19	504.52	481.56	414.69	
[18,]	851.32	1216.52	937.27	1463.10	1628.77	1729.79	1247.96	1219.12	765.17	
[19,]	0.00	399.01	150.56	654.03	826.73	907.62	418.46	375.77	310.89	
[20,]	399.01	0.00	412.63	256.24	428.89	518.49	81.21	103.25	676.15	
[21,]	150.56	412.63	0.00	655.99	824.23	889.65	422.57	358.28	270.35	
[22,]	654.03	256.24	655.99	0.00	176.00	273.04	249.49	302.54	922.70	
[23,]	826.73	428.89	824.23	176.00	0.00	155.83	423.74	472.59	1092.56	
[24,]	907.62	518.49	889.65	273.04	155.83	0.00	492.98	537.50	1153.85	
[25,]	418.46	81.21	422.57	249.49	423.74	492.98	0.00	80.78	682.24	
[26,]	375.77	103.25	358.28	302.54	472.59	537.50	80.78	0.00	623.03	
[27,]	310.89	676.15	270.35	922.70	1092.56	1153.85	682.24	623.03	0.00	
[28,]	478.87	125.28	517.27	214.40	378.02	479.05	151.12	214.50	771.74	
[29,]	1562.97	1950.45	1613.93	2203.67	2372.22	2465.86	1977.32	1938.12	1381.48	
[30,]	418.76	76.52	442.05	249.27	421.96	505.38	89.81	139.32	699.32	
[31,]	671.31	340.82	620.32	248.51	324.17	309.63	302.63	307.20	877.43	
[32,]	384.31	125.99	440.65	318.74	482.72	588.69	181.52	214.31	686.22	
[33,]	518.17	123.46	531.07	140.77	315.11	406.11	127.31	187.79	794.58	
[34,]	133.64	426.35	265.12	677.70	845.76	944.12	464.52	433.83	389.80	
[35,]	492.20	801.90	615.37	1037.91	1198.98	1307.29	840.47	822.68	566.03	
[36,]	800.59	412.17	779.39	178.00	119.35	122.60	391.88	429.89	1045.78	
							337.90			
[38,]	671.57	278.51	662.92	56.55	174.69	240.60	259.88	307.03	929.33	
							71.61			
							530.33			
[41,]	134.20	283.14	231.39	536.76	708.59	798.69	313.25	285.42	443.26	

	[,28]	[ <b>,</b> 29]	[,30]	[,31]	[,32]	[,33]	[,34]	[,35]	[,36]
[1,]	125.26	2017.41	194.89	442.71	154.13	174.33	504.45	829.50	417.36
[2,]	445.69	2423.18	469.96	270.29	550.11	366.21	900.08	1268.54	74.90
[3,]	266.32	1750.37	242.35	557.76	155.57	326.95	228.44	584.23	626.31
[4,]	197.31	1896.35	132.95	365.46	165.47	196.90	379.57	767.26	464.27
[5,]	410.17	2202.28	368.44	81.57	474.94	323.25	726.63	1119.92	281.03
[6,]	496.70	2469.12	516.36	287.19	602.39	416.97	949.01	1319.61	117.44
[7,]	275.48	1734.20	257.12	588.94	166.19	347.86	228.03	556.41	653.34
[8,]	134.46	2108.03	220.07	415.10	222.80	161.43	596.72	915.46	356.91
[9,]	216.55	2194.98	240.78	225.48	318.21	134.12	671.27	1035.09	186.06
[10,]	113.34	1966.30	60.28	326.98	131.86	105.42	443.52	816.52	397.88
[11,]	4181.45	2178.17	4135.44	4379.78	4072.80	4237.89	3700.57	3384.30	4526.40
[12,]	227.24	1795.24	230.32	571.15	127.41	310.50	300.69	606.45	613.72
[13,]	377.17	2362.99	405.85	253.30	483.49	301.42	838.35	1202.69	47.82
[14,]	303.58	2293.32	339.78	258.01	407.76	231.25	766.74	1126.97	105.26
[15,]	60.21	1950.86	100.39	422.18	62.24	146.51	433.75	771.69	446.73
[16,]	193.10	2177.88	224.35	245.66	295.00	113.68	652.63	1013.83	204.16
[17,]	512.94	1493.66	476.50	780.41	404.25	576.45	81.65	344.20	878.14
[18,]	1258.44	766.03	1225.22	1517.57	1149.14	1325.69	793.76	444.98	1627.51
[19,]	478.87	1562.97	418.76	671.31	384.31	518.17	133.64	492.20	800.59
[20,]	125.28	1950.45	76.52	340.82	125.99	123.46	426.35	801.90	412.17
[21,]	517.27	1613.93	442.05	620.32	440.65	531.07	265.12	615.37	779.39
[22,]	214.40	2203.67	249.27	248.51	318.74	140.77	677.70	1037.91	178.00
[23,]	378.02	2372.22	421.96	324.17	482.72	315.11	845.76	1198.98	119.35
[24,]	479.05	2465.86	505.38	309.63	588.69	406.11	944.12	1307.29	122.60
[25,]	151.12	1977.32	89.81	302.63	181.52	127.31	464.52	840.47	391.88
[26,]	214.50	1938.12	139.32	307.20	214.31	187.79	433.83	822.68	429.89
[27,]	771.74	1381.48	699.32	877.43	686.22	794.58	389.80	566.03	1045.78
[28,]	0.00	2005.94	91.42	375.49	115.94	94.75	486.51	830.01	389.41
[29,]	2005.94	0.00	1963.43	2226.43	1896.84	2065.57	1527.40	1208.65	2359.68
[30,]	91.42	1963.43	0.00	343.39	124.79	114.80	446.24	808.25	409.06
[31,]	375.49	2226.43	343.39	0.00	453.53	287.93	735.37	1123.27	221.05
[32,]	115.94	1896.84	124.79	453.53	0.00	187.34	376.49	720.00	492.49
[33,]	94.75	2065.57	114.80	287.93	187.34	0.00	541.01	903.09	308.30
[34,]	486.51	1527.40	446.24	735.37	376.49	541.01	0.00	398.13	837.49
[35,]	830.01	1208.65	808.25	1123.27	720.00	903.09	398.13	0.00	1208.40
[36,]	389.41	2359.68	409.06	221.05	492.49	308.30	837.49	1208.40	0.00
[37,]	288.44	2290.17	331.60	285.99	398.15	227.66	765.75	1116.68	144.17
[38,]	251.54	2227.71	271.46	201.27	354.46	168.46	704.23	1071.21	142.16
[39,]	117.32	1934.31	49.65	361.89	120.30	141.33	417.91	783.75	436.78
[40,]	523.04	2505.76	547.63	333.28	632.00	448.09	985.04	1350.40	159.99
[41,]	351.38	1668.68	301.29	588.91	251.98	399.13	154.45	539.17	693.42

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[24,] 201.05 240.60 534.44 51.16 798.69
[25,] 337.90 259.88 71.61 530.33 313.25
[26,] 394.17 307.03 118.52 573.22 285.42
[27,] 1014.41 929.33 666.47 1186.79 443.26
[28,] 288.44 251.54 117.32 523.04 351.38
[29,] 2290.17 2227.71 1934.31 2505.76 1668.68
[30,] 331.60 271.46 49.65 547.63 301.29
[31,] 285.99 201.27 361.89 333.28 588.91
[32,] 398.15 354.46 120.30 632.00 251.98
[33,] 227.66 168.46 141.33 448.09 399.13
[34,] 765.75 704.23 417.91 985.04 154.45
[35,] 1116.68 1071.21 783.75 1350.40 539.17
[36,] 144.17 142.16 436.78 159.99 693.42
[37,] 0.00 101.61 363.63 246.91 625.87
[38,] 101.61
            0.00 299.03 281.70 560.02
[39,] 363.63 299.03 0.00 574.65 269.58
[40,] 246.91 281.70 574.65 0.00 838.79
[41,] 625.87 560.02 269.58 838.79 0.00
```

#### b) Euclidean distances for normalized data

```
usair.dat <- data.matrix(usair.dat)
std<-sd(usair.dat)
usair.dat.std<-sweep(usair.dat,2,std,FUN="/")
dis<-dist(usair.dat.std)
dis.matrix<-dist2full(dis)
round(dis.matrix,digits=2)</pre>
```

```
[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12]
[1,] 0.00 1.23 0.72 0.66 1.25 1.37 0.76 0.34 0.70 0.52 10.89 0.62
 [2,] 1.23 0.00 1.79 1.37 0.85 0.16 1.85 1.04 0.61 1.19 11.92 1.74
 [3,] 0.72 1.79 0.00 0.53 1.47 1.92 0.18 0.97
                                               1.19 0.63 10.20
[4,] 0.66 1.37 0.53 0.00 0.95 1.49 0.64 0.84 0.81 0.26 10.55
[5,] 1.25 0.85 1.47 0.95 0.00 0.89 1.56 1.21 0.72 0.89 11.30
                                                              1.53
[6,] 1.37 0.16 1.92 1.49 0.89 0.00 1.98 1.18 0.75 1.32 12.04 1.88
[7,] 0.76 1.85 0.18 0.64 1.56 1.98 0.00 0.98 1.25 0.70 10.16 0.19
[8,] 0.34 1.04 0.97 0.84 1.21 1.18 0.98 0.00 0.54 0.61 11.13 0.81
[9,] 0.70 0.61 1.19 0.81 0.72 0.75 1.25 0.54 0.00 0.60 11.34 1.14
[10,] 0.52 1.19 0.63 0.26 0.89 1.32 0.70 0.61 0.60 0.00 10.75 0.65
[11,] 10.89 11.92 10.20 10.55 11.30 12.04 10.16 11.13 11.34 10.75 0.00 10.32
[12,] 0.62 1.74 0.29 0.66 1.53 1.88 0.19 0.81 1.14 0.65 10.32 0.00
[13,] 1.05 0.19 1.62 1.22 0.82 0.33 1.68 0.86 0.46 1.03 11.77 1.57
[14,] 0.84 0.39 1.42 1.06 0.83 0.54 1.49 0.67 0.30 0.86 11.60 1.37
[15,] 0.33 1.31 0.55 0.49 1.17 1.44 0.57 0.44 0.71 0.32 10.72 0.44
[16,] 0.62 0.66 1.14 0.78 0.77 0.80 1.19 0.47 0.08 0.56 11.30 1.08
[17,] 1.38 2.44 0.68 1.11 2.02 2.57 0.63 1.61 1.84 1.25 9.53 0.80
[18,] 3.29 4.39 2.61 3.04 3.91 4.52 2.56 3.51 3.79 3.20 7.64
                                                              2.70
[19,] 1.36 2.24 0.67 0.88 1.70 2.36 0.69 1.57
                                               1.67 1.08
                                                         9.68
[20,] 0.51 1.23 0.58 0.20 0.92 1.36 0.67 0.64 0.65 0.07 10.70 0.63
[21,] 1.48 2.19 0.89 0.89 1.53 2.29 0.96 1.68 1.66 1.11 9.77 1.13
[22,] 0.64 0.61 1.19 0.83 0.79 0.75 1.25 0.50 0.15 0.63 11.37 1.14
[23,] 0.98 0.35 1.62 1.27 1.00 0.47 1.68 0.80 0.53 1.08 11.81 1.55
[24,] 1.32 0.19 1.90 1.49 0.96 0.15 1.95 1.11 0.72 1.30 12.04 1.83
[25,] 0.66 1.17 0.73 0.30 0.81 1.29 0.79 0.70 0.60 0.18 10.77 0.75
[26,] 0.78 1.28 0.71 0.21 0.78 1.39 0.80 0.87 0.73 0.28 10.65
[27,] 2.14 2.88 1.47 1.58 2.18 2.98 1.50 2.34 2.35 1.79 9.12
[28,] 0.33 1.16 0.69 0.51 1.07 1.29 0.72 0.35 0.56 0.29 10.86
                                                               0.59
[29,] 5.24 6.29 4.55 4.93 5.72 6.41 4.50 5.48 5.70 5.11 5.66
[30,] 0.51 1.22 0.63 0.35 0.96 1.34 0.67 0.57 0.63 0.16 10.74
[31,] 1.15 0.70 1.45 0.95 0.21 0.75 1.53 1.08 0.59 0.85 11.38 1.48
[32,] 0.40 1.43 0.40 0.43 1.23 1.56 0.43 0.58 0.83 0.34 10.58 0.33
[33,] 0.45 0.95 0.85 0.51 0.84 1.08 0.90 0.42 0.35 0.27 11.01 0.81
[34,] 1.31 2.34 0.59 0.99 1.89 2.47
                                    0.59 1.55 1.74 1.15 9.61 0.78
[35,] 2.15 3.30 1.52 1.99 2.91 3.43 1.45 2.38 2.69 2.12 8.79
                                                               1.58
[36,] 1.08 0.19 1.63 1.21 0.73 0.31 1.70 0.93 0.48 1.03 11.76 1.59
[37,] 0.81 0.46 1.42 1.08 0.91 0.59 1.46 0.59 0.31 0.86 11.60 1.33
[38,] 0.77 0.52 1.28 0.88 0.67 0.65 1.34 0.62 0.13 0.68 11.42 1.24
[39,] 0.57 1.29 0.58 0.29 0.98 1.41 0.62 0.65 0.69 0.16 10.66 0.57
[40,] 1.44 0.25 2.01 1.60 1.01 0.15 2.06 1.22 0.83 1.41 12.14 1.95
[41,] 1.02 1.96 0.34 0.61 1.51 2.09 0.37 1.23 1.37 0.78 9.97 0.55
```

```
[,13] [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24]
 [1,] 1.05 0.84 0.33 0.62 1.38 3.29 1.36 0.51 1.48 0.64 0.98 1.32
 [2,] 0.19 0.39 1.31 0.66 2.44 4.39 2.24 1.23 2.19 0.61 0.35 0.19
 [3,] 1.62 1.42 0.55 1.14 0.68 2.61 0.67 0.58 0.89 1.19 1.62 1.90
 [4,] 1.22 1.06 0.49 0.78 1.11 3.04 0.88 0.20 0.89 0.83 1.27 1.49
[5,] 0.82 0.83 1.17 0.77 2.02 3.91 1.70 0.92 1.53 0.79 1.00 0.96
[6,] 0.33 0.54 1.44 0.80 2.57 4.52 2.36 1.36 2.29 0.75 0.47 0.15
[7,] 1.68 1.49 0.57 1.19 0.63 2.56 0.69 0.67 0.96 1.25 1.68 1.95
[8,] 0.86 0.67 0.44 0.47 1.61 3.51 1.57 0.64 1.68 0.50 0.80 1.11
[9,] 0.46 0.30 0.71 0.08 1.84 3.79 1.67 0.65 1.66 0.15 0.53 0.72
[10,] 1.03 0.86 0.32 0.56 1.25 3.20 1.08 0.07 1.11 0.63 1.08 1.30
[11,] 11.77 11.60 10.72 11.30 9.53 7.64 9.68 10.70 9.77 11.37 11.81 12.04
[12,] 1.57 1.37 0.44 1.08 0.80 2.70 0.88 0.63 1.13 1.14 1.55 1.83
[13,] \quad 0.00 \quad 0.21 \quad 1.13 \quad 0.50 \quad 2.28 \quad 4.22 \quad 2.09 \quad 1.07 \quad 2.06 \quad 0.43 \quad 0.23 \quad 0.29
[14,] 0.21 0.00 0.94 0.32 2.09 4.03 1.93 0.90 1.92 0.24 0.24 0.50
[15,] 1.13 0.94 0.00 0.65 1.19 3.12 1.13 0.33 1.26 0.71 1.13 1.40
[16,] 0.50 0.32 0.65 0.00 1.79 3.74 1.63 0.60 1.64 0.12 0.55 0.77
[17,] 2.28 2.09 1.19 1.79 0.00 1.95 0.47 1.22 0.83 1.85 2.29 2.55
[18,] 4.22 4.03 3.12 3.74 1.95 0.00 2.21 3.16 2.43 3.80 4.23 4.49
[19,] 2.09 1.93 1.13 1.63 0.47 2.21 0.00 1.04 0.39 1.70 2.15 2.36
[20,] \quad 1.07 \quad 0.90 \quad 0.33 \quad 0.60 \quad 1.22 \quad 3.16 \quad 1.04 \quad 0.00 \quad 1.07 \quad 0.67 \quad 1.11 \quad 1.35
[21,] 2.06 1.92 1.26 1.64 0.83 2.43 0.39 1.07 0.00 1.70 2.14 2.31
[22,] \quad 0.43 \quad 0.24 \quad 0.71 \quad 0.12 \quad 1.85 \quad 3.80 \quad 1.70 \quad 0.67 \quad 1.70 \quad 0.00 \quad 0.46 \quad 0.71
[23,] 0.23 0.24 1.13 0.55 2.29 4.23 2.15 1.11 2.14 0.46 0.00 0.40
[24,] \quad 0.29 \quad 0.50 \quad 1.40 \quad 0.77 \quad 2.55 \quad 4.49 \quad 2.36 \quad 1.35 \quad 2.31 \quad 0.71 \quad 0.40 \quad 0.00
[25,] 1.02 0.87 0.44 0.58 1.31 3.24 1.09 0.21 1.10 0.65 1.10 1.28
[26,] 1.14 1.00 0.57 0.72 1.25 3.17 0.98 0.27 0.93 0.79 1.23 1.40
[27,] 2.75 2.61 1.91 2.33 1.08 1.99 0.81 1.76 0.70 2.40 2.84 3.00
[28,] 0.98 0.79 0.16 0.50 1.33 3.27 1.24 0.33 1.34 0.56 0.98 1.24
[29,] 6.14 5.96 5.07 5.66 3.88 1.99 4.06 5.07 4.19 5.72 6.16 6.41
[30,] 1.05 0.88 0.26 0.58 1.24 3.18 1.09 0.20 1.15 0.65 1.10 1.31
[31,] 0.66 0.67 1.10 0.64 2.03 3.94 1.74 0.89 1.61 0.65 0.84 0.80
[32,] 1.26 1.06 0.16 0.77 1.05 2.98 1.00 0.33 1.14 0.83 1.25 1.53
[33,] 0.78 0.60 0.38 0.30 1.50 3.44 1.35 0.32 1.38 0.37 0.82 1.05
[34,] 2.18 1.99 1.13 1.70 0.21 2.06 0.35 1.11 0.69 1.76 2.20 2.45
[35,] 3.12 2.93 2.00 2.63 0.89 1.16 1.28 2.08 1.60 2.70 3.11 3.40
 [ 36, ] \quad 0.12 \quad 0.27 \quad 1.16 \quad 0.53 \quad 2.28 \quad 4.23 \quad 2.08 \quad 1.07 \quad 2.02 \quad 0.46 \quad 0.31 \quad 0.32 
[37,] 0.29 0.16 0.90 0.32 2.07 4.02 1.93 0.91 1.94 0.25 0.28 0.52
[38,] 0.36 0.22 0.80 0.17 1.93 3.88 1.74 0.72 1.72 0.15 0.45 0.62
[39,] 1.13 0.96 0.30 0.65 1.17 3.11 1.00 0.18 1.07 0.72 1.18 1.39
[40,] \quad 0.40 \quad 0.61 \quad 1.51 \quad 0.88 \quad 2.66 \quad 4.60 \quad 2.46 \quad 1.45 \quad 2.40 \quad 0.82 \quad 0.52 \quad 0.13
[41,] 1.81 1.63 0.79 1.33 0.52 2.44 0.35 0.74 0.60 1.39 1.84 2.07
```

```
[,25] [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36]
[1,] 0.66 0.78 2.14 0.33 5.24 0.51 1.15 0.40 0.45 1.31 2.15 1.08
 [2,] 1.17 1.28 2.88 1.16 6.29 1.22 0.70 1.43 0.95 2.34 3.30 0.19
 [3,] 0.73 0.71 1.47 0.69 4.55 0.63 1.45 0.40 0.85 0.59 1.52 1.63
[4,] 0.30 0.21 1.58 0.51 4.93 0.35 0.95 0.43 0.51 0.99 1.99 1.21
[5,] 0.81 0.78 2.18 1.07 5.72 0.96 0.21 1.23 0.84 1.89 2.91 0.73
[6,] 1.29 1.39 2.98 1.29 6.41 1.34 0.75 1.56 1.08 2.47 3.43 0.31
[7,] 0.79 0.80 1.50 0.72 4.50 0.67 1.53 0.43 0.90 0.59 1.45 1.70
[8,] 0.70 0.87 2.34 0.35 5.48 0.57 1.08 0.58 0.42 1.55 2.38 0.93
[9,] 0.60 0.73 2.35 0.56 5.70 0.63 0.59 0.83 0.35 1.74 2.69 0.48
[10,] 0.18 0.28 1.79 0.29 5.11 0.16 0.85 0.34 0.27 1.15 2.12 1.03
[11,] 10.77 10.65 9.12 10.86 5.66 10.74 11.38 10.58 11.01 9.61 8.79 11.76
[12,] 0.75 0.81 1.68 0.59 4.66 0.60 1.48 0.33 0.81 0.78 1.58 1.59
[13,] \quad 1.02 \quad 1.14 \quad 2.75 \quad 0.98 \quad 6.14 \quad 1.05 \quad 0.66 \quad 1.26 \quad 0.78 \quad 2.18 \quad 3.12 \quad 0.12
[14,] 0.87 1.00 2.61 0.79 5.96 0.88 0.67 1.06 0.60 1.99 2.93 0.27
[15,] 0.44 0.57 1.91 0.16 5.07 0.26 1.10 0.16 0.38 1.13 2.00 1.16
[16,] 0.58 0.72 2.33 0.50 5.66 0.58 0.64 0.77 0.30 1.70 2.63 0.53
[17,] 1.31 1.25 1.08 1.33 3.88 1.24 2.03 1.05 1.50 0.21 0.89 2.28
[18,] 3.24 3.17 1.99 3.27 1.99 3.18 3.94 2.98 3.44 2.06 1.16 4.23
[19,] 1.09 0.98 0.81 1.24 4.06 1.09 1.74 1.00 1.35 0.35 1.28 2.08
[20,] 0.21 0.27 1.76 0.33 5.07 0.20 0.89 0.33 0.32 1.11 2.08 1.07
[21,] 1.10 0.93 0.70 1.34 4.19 1.15 1.61 1.14 1.38 0.69 1.60 2.02
[22,] 0.65 0.79 2.40 0.56 5.72 0.65 0.65 0.83 0.37 1.76 2.70 0.46
[23,] 1.10 1.23 2.84 0.98 6.16 1.10 0.84 1.25 0.82 2.20 3.11 0.31
[24,] 1.28 1.40 3.00 1.24 6.41 1.31 0.80 1.53 1.05 2.45 3.40 0.32
[25,] \quad 0.00 \quad 0.21 \quad 1.77 \quad 0.39 \quad 5.14 \quad 0.23 \quad 0.79 \quad 0.47 \quad 0.33 \quad 1.21 \quad 2.18 \quad 1.02
[26,] \quad 0.21 \quad 0.00 \quad 1.62 \quad 0.56 \quad 5.03 \quad 0.36 \quad 0.80 \quad 0.56 \quad 0.49 \quad 1.13 \quad 2.14 \quad 1.12
[27,] 1.77 1.62 0.00 2.00 3.59 1.82 2.28 1.78 2.06 1.01 1.47 2.72
[28,] 0.39 0.56 2.00 0.00 5.21 0.24 0.98 0.30 0.25 1.26 2.16 1.01
[29,] 5.14 5.03 3.59 5.21 0.00 5.10 5.78 4.93 5.37 3.97 3.14 6.13
[30,] 0.23 0.36 1.82 0.24 5.10 0.00 0.89 0.32 0.30 1.16 2.10 1.06
[31,] 0.79 0.80 2.28 0.98 5.78 0.89 0.00 1.18 0.75 1.91 2.92 0.57
[32,] 0.47 0.56 1.78 0.30 4.93 0.32 1.18 0.00 0.49 0.98 1.87 1.28
[33,] 0.33 0.49 2.06 0.25 5.37 0.30 0.75 0.49 0.00 1.41 2.35 0.80
[34,] 1.21 1.13 1.01 1.26 3.97 1.16 1.91 0.98 1.41 0.00 1.03 2.18
[35,] 2.18 2.14 1.47 2.16 3.14 2.10 2.92 1.87 2.35 1.03 0.00 3.14
[36,] 1.02 1.12 2.72 1.01 6.13 1.06 0.57 1.28 0.80 2.18 3.14 0.00
[37,] 0.88 1.02 2.63 0.75 5.95 0.86 0.74 1.03 0.59 1.99 2.90 0.37
[38,] 0.68 0.80 2.41 0.65 5.79 0.71 0.52 0.92 0.44 1.83 2.78 0.37
[39,] 0.19 0.31 1.73 0.30 5.02 0.13 0.94 0.31 0.37 1.09 2.04 1.13
[40,] 1.38 1.49 3.08 1.36 6.51 1.42 0.87 1.64 1.16 2.56 3.51 0.42
[41,] 0.81 0.74 1.15 0.91 4.33 0.78 1.53 0.65 1.04 0.40 1.40 1.80
```

```
[,37] [,38] [,39] [,40] [,41]
[1,] 0.81 0.77 0.57 1.44 1.02
 [2,] 0.46 0.52 1.29 0.25 1.96
 [3,] 1.42 1.28 0.58 2.01 0.34
[4,] 1.08 0.88 0.29 1.60 0.61
[5,] 0.91 0.67 0.98 1.01 1.51
[6,] 0.59 0.65 1.41 0.15 2.09
[7,] 1.46 1.34 0.62 2.06 0.37
[8,] 0.59 0.62 0.65 1.22 1.23
[9,] 0.31 0.13 0.69 0.83 1.37
[10,] 0.86 0.68 0.16 1.41 0.78
[11,] 11.60 11.42 10.66 12.14
[12,] 1.33 1.24 0.57 1.95 0.55
[13,] 0.29 0.36 1.13 0.40 1.81
[14,] 0.16 0.22 0.96 0.61 1.63
[15,] 0.90 0.80 0.30 1.51 0.79
[16,] 0.32 0.17 0.65 0.88 1.33
[17,] 2.07 1.93 1.17 2.66 0.52
[18,] 4.02 3.88 3.11 4.60 2.44
[19,] 1.93 1.74 1.00 2.46 0.35
[20,] 0.91 0.72 0.18 1.45 0.74
[21,] 1.94 1.72 1.07 2.40 0.60
[22,] 0.25 0.15 0.72 0.82 1.39
[23,] 0.28 0.45 1.18 0.52 1.84
[24,] 0.52 0.62 1.39 0.13 2.07
[25,] 0.88 0.68 0.19 1.38 0.81
[26,] 1.02 0.80 0.31 1.49 0.74
[27,] 2.63 2.41 1.73 3.08 1.15
[28,] 0.75 0.65 0.30 1.36 0.91
[29,] 5.95 5.79 5.02 6.51 4.33
[30,] 0.86 0.71 0.13 1.42 0.78
[31,] 0.74 0.52 0.94 0.87 1.53
[32,] 1.03 0.92 0.31 1.64 0.65
[33,] 0.59 0.44 0.37 1.16 1.04
[34,] 1.99 1.83 1.09 2.56 0.40
[35,] 2.90 2.78 2.04 3.51 1.40
[36,] 0.37 0.37 1.13 0.42 1.80
[37,] 0.00 0.26 0.94 0.64 1.63
[38,] 0.26 0.00 0.78 0.73 1.45
[39,] 0.94 0.78 0.00 1.49 0.70
[40,] 0.64 0.73 1.49 0.00 2.18
[41,] 1.63 1.45 0.70 2.18 0.00
```

## c) Mahalanobis distances for the original data

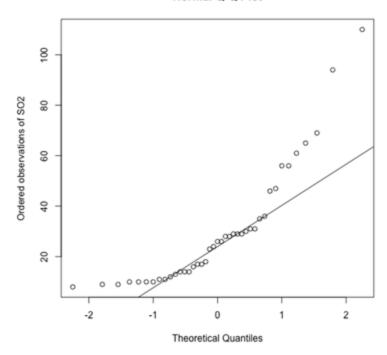
Sx <- cov(usair.dat)
mdis <- mahalanobis(usair.dat, colMeans(usair.dat), Sx)</pre>

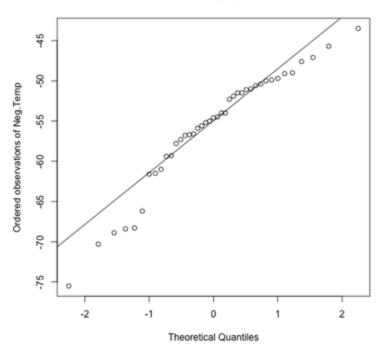
Phoenix	Little Rock	San Francisco
20.258912	6.207429	4.539210
Denver	Hartford	Wilmington
5.400965	7.760199	2.156830
Washington	Jacksonville	Miami
1.538300	5.286583	14.277037
Atlanta	Chicago	Indianapolis
1.448871	26.891450	4.270100
Des Moines	Wichita	Louisville
4.310154	9.060638	3.060861
New Orleans	Baltimore	Detroit
5.394046	3.421276	7.222633
Minneapolis-St. Paul	Kansas City	St. Louis
4.945830	3.060650	4.767204
Omaha	Alburquerque	Albany
3.052545	8.063093	4.081128
Buffalo	Cincinnati	Cleveland
12.880983	7.265585	11.489013
Columbus	Philadelphia	Pittsburgh
3.145760	6.722708	7.955753
Providence	Memphis	Nashville
18.176040	3.573384	2.564959
Dallas	Houston	Salt Lake City
6.368290	8.684843	4.431380
Norfolk	Richmond	Seattle
3.754730	2.492672	6.535345
Charleston	Milwaukee	
7.888784	5.593824	

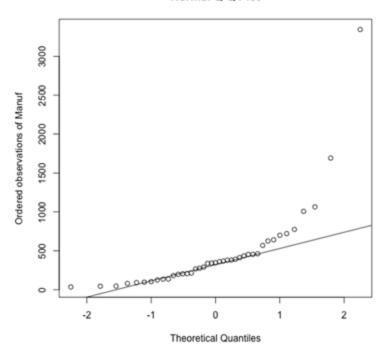
3) Show the Q-Q plots of individual variables for usair dataset and explain the results.

None of the Q-Q plots show extremely normal distribution of the observations, which would result in the points being aligned linearly. Only Wind and Neg.Temp look somewhat linear, as you might also expect if you look at the histograms in our original scatterplot matrix. The histograms for Wind and Neg.Temp look similar to a bell curve there, so it follows that the Q-Q plots for these variables do not give much indication of a departure from linearity.

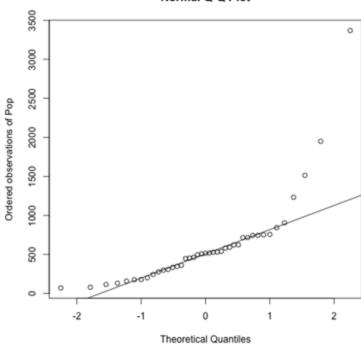
```
qqnorm(usair.dat[,1],ylab="Ordered observations of SO2")
qqline(usair.dat[,1])
dev.copy(png,'3.1.SO2_Neg.Temp.png')
dev.off()
qqnorm(usair.dat[,2],ylab="Ordered observations of Neg.Temp")
qqline(usair.dat[,2])
dev.copy(png,'3.2.SO2_Manuf.png')
dev.off()
qqnorm(usair.dat[,3],ylab="Ordered observations of Manuf")
qqline(usair.dat[,3])
dev.copy(png,'3.3.SO2_Neg.Temp.png')
dev.off()
qqnorm(usair.dat[,4],ylab="Ordered observations of Pop")
qqline(usair.dat[,4])
dev.copy(png,'3.4.SO2_Neg.Temp.png')
dev.off()
qqnorm(usair.dat[,5],ylab="Ordered observations of Wind")
qqline(usair.dat[,5])
dev.copy(png,'3.5.SO2_Neg.Temp.png')
dev.off()
qqnorm(usair.dat[,6],ylab="Ordered observations of Precip")
qqline(usair.dat[,6])
dev.copy(png,'3.6.SO2_Neg.Temp.png')
dev.off()
qqnorm(usair.dat[,7],ylab="Ordered observations of Days")
qqline(usair.dat[,7])
dev.copy(png,'3.7.SO2_Neg.Temp.png')
dev.off()
```

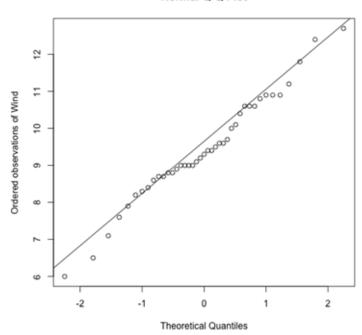


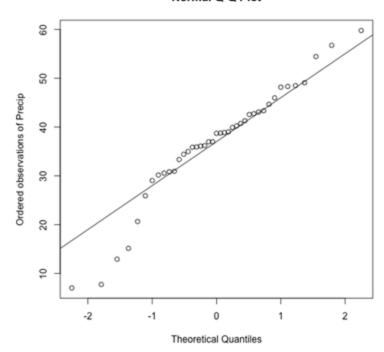


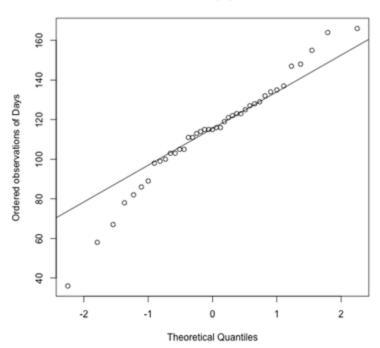










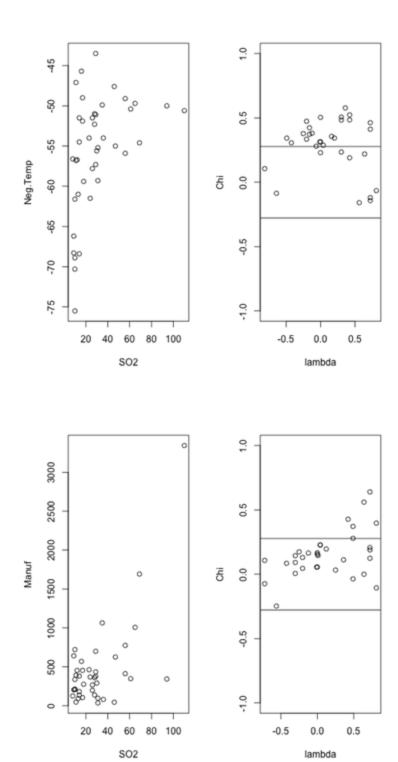


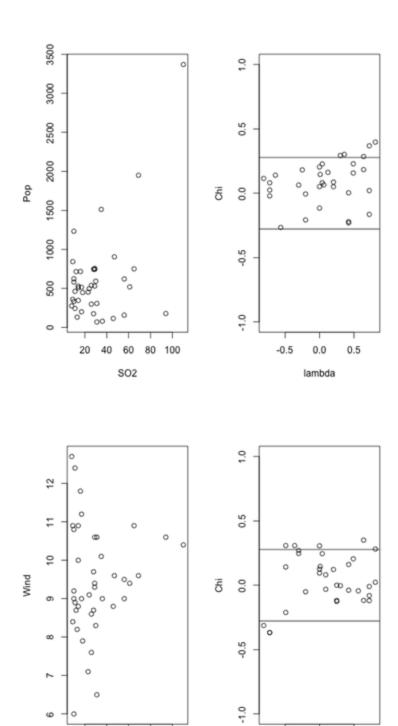
- 4) Solve exercise 2.6 in page 40 of Everitt's book.
- 2.6 Investigate the use of the chiplot function on all pairs of variables in the air pollution data.

Investigating the data using chiplots reveals dependence between only two of the variable pairs. The first is Manuf and Pop: all but one of the points lies outside of the central bars. The second is SO2 and Neg.Temp, although in this case several points are still within the central bars. All other variable pairs appear to be independent.

```
chiplot(SO2, Neg.Temp, vlabs=c("SO2", "Neg.Temp"))
dev.copy(png,'4.1.SO2_Neg.Temp.png')
dev.off()
chiplot(SO2, Manuf, vlabs=c("SO2", "Manuf"))
dev.copy(png,'4.2.SO2_Manuf.png')
dev.off()
chiplot(SO2, Pop, vlabs=c("SO2", "Pop"))
dev.copy(png,'4.3.S02_Pop.png')
dev.off()
chiplot(SO2, Wind, vlabs=c("SO2", "Wind"))
dev.copy(png,'4.4.SO2_Wind.png')
dev.off()
chiplot(SO2, Precip, vlabs=c("SO2", "Precip"))
dev.copy(png,'4.5.S02_Precip.png')
dev.off()
chiplot(SO2, Days, vlabs=c("SO2", "Days"))
dev.copy(png,'4.6.S02_Days.png')
dev.off()
chiplot(Neg.Temp, Manuf, vlabs=c("Neg.Temp", "Manuf"))
dev.copy(png,'4.7.Neg.Temp_Manuf.png')
dev.off()
chiplot(Neg.Temp, Pop, vlabs=c("Neg.Temp", "Pop"))
dev.copy(png,'4.8.Neg.Temp_Pop.png')
dev.off()
chiplot(Neg.Temp, Wind, vlabs=c("Neg.Temp", "Wind"))
dev.copy(png,'4.9.Neg.Temp_Wind.png')
dev.off()
chiplot(Neg.Temp, Precip, vlabs=c("Neg.Temp", "Precip"))
dev.copy(png,'4.10.Neg.Temp_Precip.png')
dev.off()
chiplot(Neg.Temp, Days, vlabs=c("Neg.Temp", "Days"))
dev.copy(png,'4.11.Neg.Temp_Days.png')
```

```
dev.off()
chiplot(Manuf, Pop, vlabs=c("Manuf", "Pop"))
dev.copy(png,'4.12.Manuf_Pop.png')
dev.off()
chiplot(Manuf, Wind, vlabs=c("Manuf", "Wind"))
dev.copy(png,'4.13.Manuf_Wind.png')
dev.off()
chiplot(Manuf, Precip, vlabs=c("Manuf", "Precip"))
dev.copy(png,'4.14.Manuf_Precip.png')
dev.off()
chiplot(Manuf, Days, vlabs=c("Manuf", "Days"))
dev.copy(png,'4.15.Manuf_Days.png')
dev.off()
chiplot(Pop, Wind, vlabs=c("Pop", "Wind"))
dev.copy(png,'4.16.Pop_Wind.png')
dev.off()
chiplot(Pop, Precip, vlabs=c("Pop", "Precip"))
dev.copy(png,'4.17.Pop_Precip.png')
dev.off()
chiplot(Pop, Days, vlabs=c("Pop", "Days"))
dev.copy(png,'4.18.Pop_Days.png')
dev.off()
chiplot(Wind, Precip, vlabs=c("Wind", "Precip"))
dev.copy(png,'4.19.Wind_Precip.png')
dev.off()
chiplot(Wind, Days, vlabs=c("Wind", "Days"))
dev.copy(png,'4.20.Wind_Days.png')
dev.off()
chiplot(Precip, Days, vlabs=c("Precip", "Days"))
dev.copy(png,'4.21.Precip_Days.png')
dev.off()
```





20 40 60 80 100

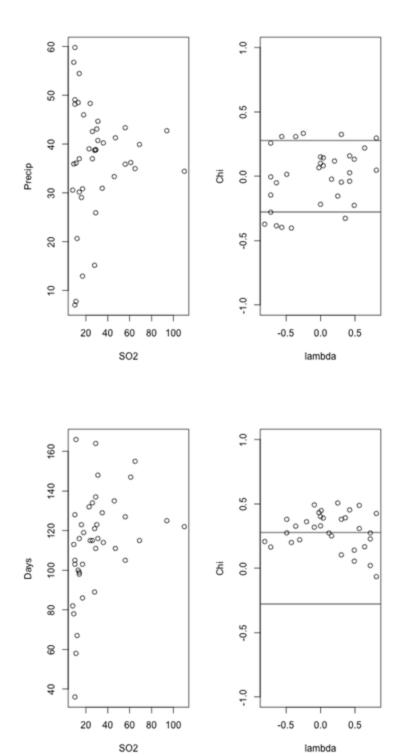
SO2

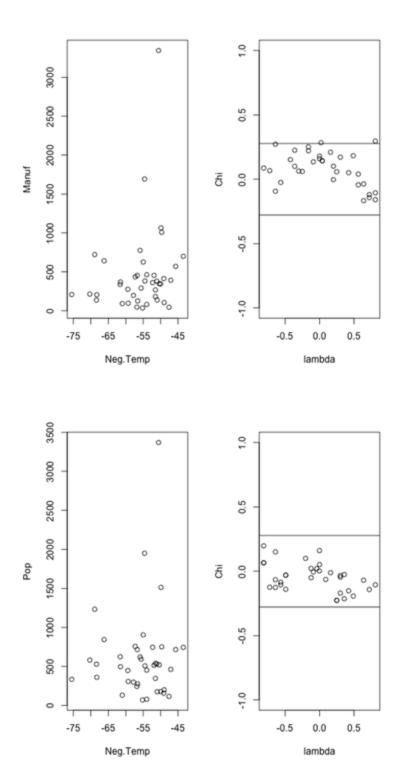
0.0

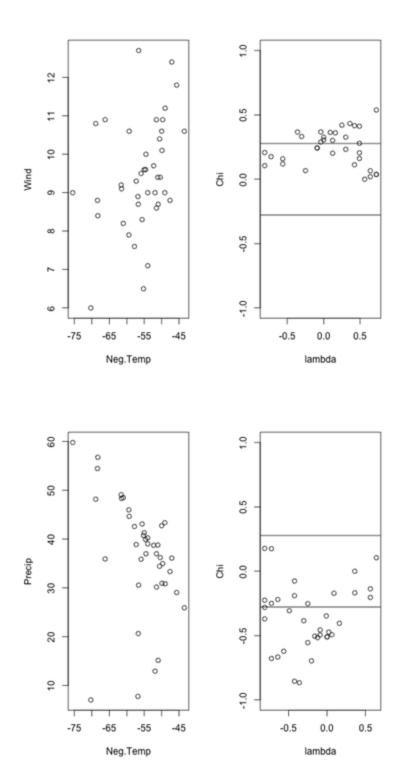
lambda

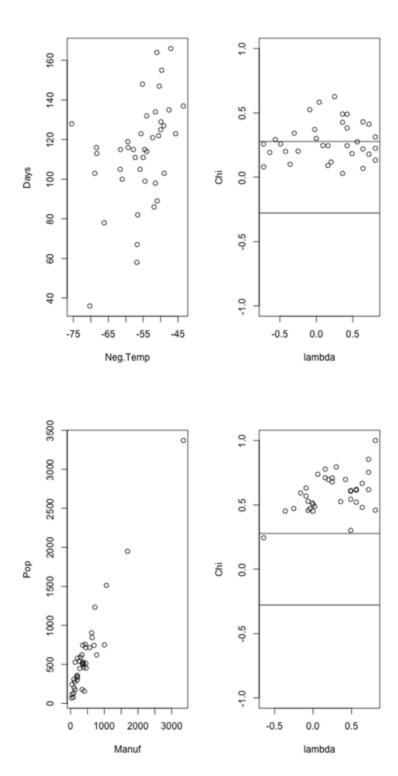
0.5

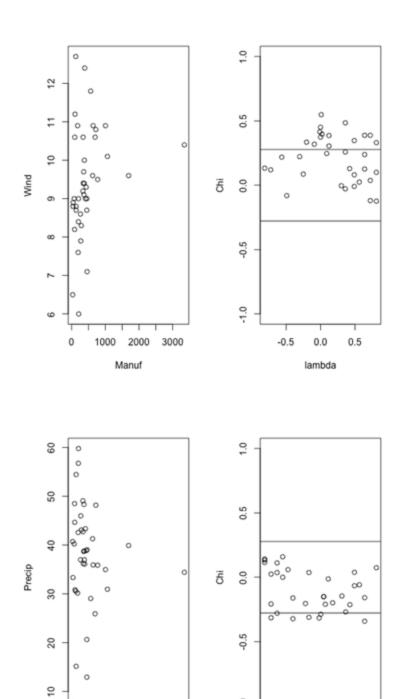
-0.5











-1.0

-0.5

0.0

lambda

0.5

0

1000 2000 3000

Manuf

