Problem 1 [30%]

To learn to use R (https://www.r-project.org/) or Python, read the labs in Chapter 2 of the textbook. We recommend that you use quarto notebooks with RStudio to typeset your solutions. Other options exist for Python

1. Download the advertising dataset (Advertising.csv) from https://www.statlearning.c

om/s/Advertising.csv and load it into R/Python.*italicized text*

```
import pandas as pd
df = pd.read_csv('https://www.statlearning.com/s/Advertising.csv')
df
```

	Unnamed: 0	TV	radio	newspaper	sales
0	1	230.1	37.8	69.2	22.1
1	2	44.5	39.3	45.1	10.4
2	3	17.2	45.9	69.3	9.3
3	4	151.5	41.3	58.5	18.5
4	5	180.8	10.8	58.4	12.9
...	...	...	...	...	...
195	196	38.2	3.7	13.8	7.6
196	197	94.2	4.9	8.1	9.7
197	198	177.0	9.3	6.4	12.8
198	199	283.6	42.0	66.2	25.5
199	200	232.1	8.6	8.7	13.4

200 rows × 5 columns

```
df.describe()
```

	Unnamed: 0	TV	radio	newspaper	sales
count	200.000000	200.000000	200.000000	200.000000	200.000000
mean	100.500000	147.042500	23.264000	30.554000	14.022500
std	57.879185	85.854236	14.846809	21.778621	5.217457
min	1.000000	0.700000	0.000000	0.300000	1.600000
25%	50.750000	74.375000	9.975000	12.750000	10.375000
50%	100.500000	149.750000	22.900000	25.750000	12.900000
75%	150.250000	218.825000	36.525000	45.100000	17.400000
max	200.000000	296.400000	49.600000	114.000000	27.000000

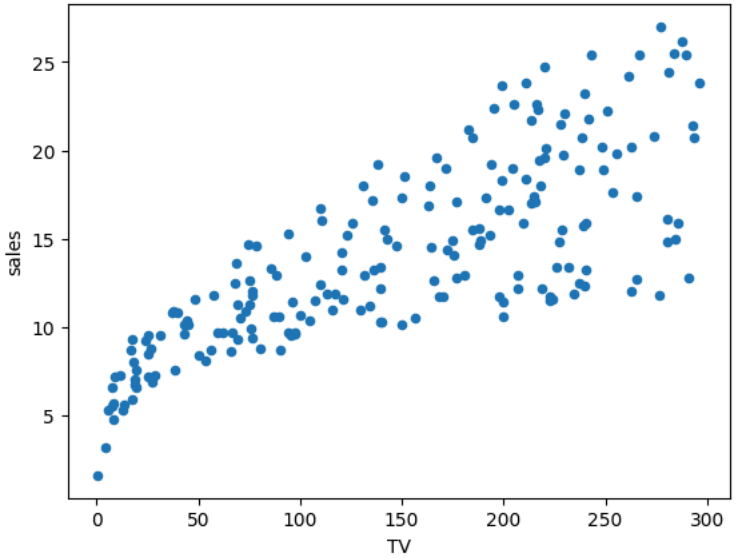
2. What are the minimum, maximum, and mean value of each feature?

	TV	Radio	Newspaper
Mean	147.0425	23.264	30.554
Min	0.7	0.0	0.3
Max	296.4	49.6	114.0

3. Produce a scatterplot matrix of all variables

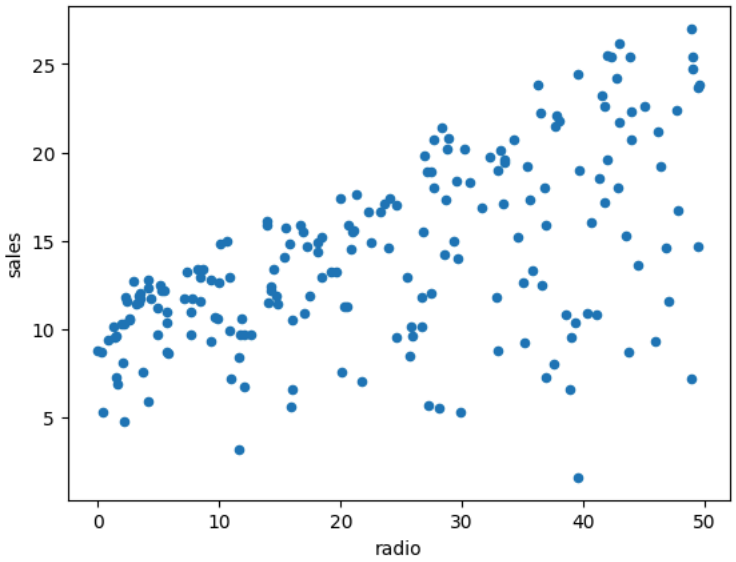
```
df.plot.scatter(x="TV", y="sales")
```

```
<Axes: xlabel='TV', ylabel='sales'>
```



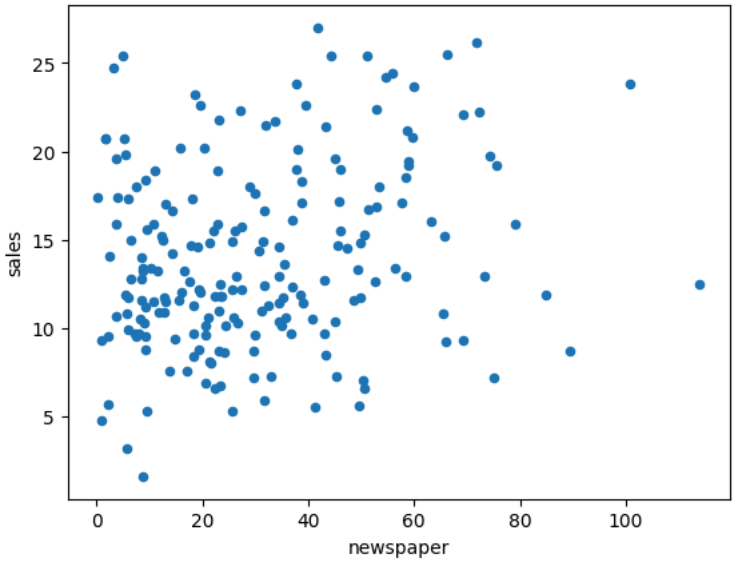
```
df.plot.scatter(x="radio", y="sales")
```

```
<Axes: xlabel='radio', ylabel='sales'>
```




```
df.plot.scatter(x="newspaper", y="sales")
```

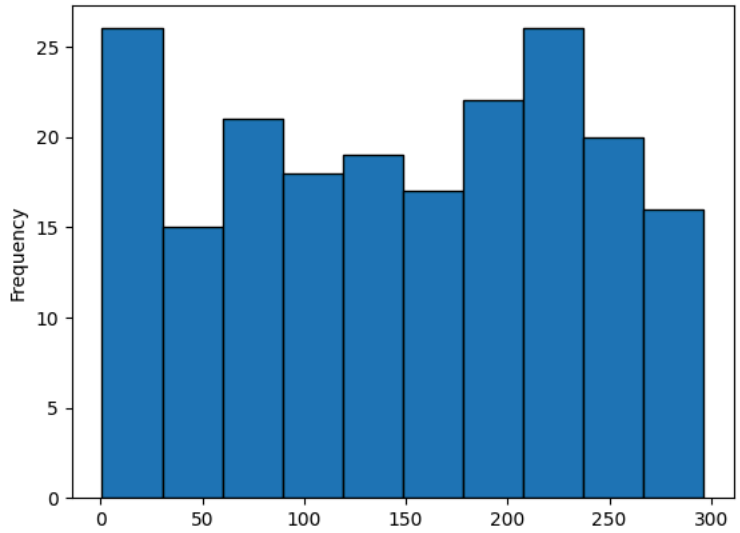
```
<Axes: xlabel='newspaper', ylabel='sales'>
```



4. Produce a histogram of TV advertising

```
df['TV'].plot.hist(edgecolor='black')
```

```
<Axes: ylabel='Frequency'>
```



Problem 2 [30%]

Consider the following bivariate distribution $p(x, y)$ of two discrete random variables X and Y :

	x1	x2	x3	x4	x5
y1	0.01	0.02	0.03	0.1	0.1
y2	0.05	0.1	0.05	0.07	0.2
y3	0.1	0.05	0.03	0.05	0.04

```
import pandas as pd

data = {
    "x1": [0.01, 0.05, 0.1],
    "x2": [0.02, 0.1,  0.05],
    "x3": [0.03, 0.05, 0.03],
    "x4": [0.1,  0.07, 0.05],
    "x5": [0.1,  0.2,  0.04]
}

index = ["y1", "y2", "y3"]

df = pd.DataFrame(data, index=index)

df
```

	x1	x2	x3	x4	x5
y1	0.01	0.02	0.03	0.10	0.10
y2	0.05	0.10	0.05	0.07	0.20
y3	0.10	0.05	0.03	0.05	0.04

Start coding or generate with AI.

1. Verify that this is a valid probability distribution.

```
#axix=1 sums columns into indices, axis=0 sums indices
sum = df.sum(axis=1).sum(axis=0)
sum
```

```
np.float64(1.0)
```

Since for all values of x in X, $0 < x < X$ and the sum of all p(x,y) in X,Y is 1, p(x,y) is a valid probability distribution

2. Compute the marginal distributions $p(x)$ and $p(y)$ for all values x and y

```
ymarg = df.sum(axis=1)
ymarg
```

	0
y1	0.26
y2	0.47
y3	0.27

dtype: float64

```
xmarg = df.sum(axis=0)
xmarg
```

	0
x1	0.16
x2	0.17
x3	0.11
x4	0.22
x5	0.34

dtype: float64

3. The conditional distributions $p(x \mid Y = y1)$ and $p(y \mid X = x3)$.

```
df.loc["y1"]
```

	y1
x1	0.01
x2	0.02
x3	0.03
x4	0.10
x5	0.10

dtype: float64

```
df['x3']
```

	x3
y1	0.03
y2	0.05
y3	0.03

dtype: float64

Problem 3 [30%]

Based on a true story, according to: The Drunkard's Walk: How Randomness Rules Our Lives, Leonard Mlodinow. A diagnosis of a rare disease does not always mean what you think.

Suppose that you applied for life insurance and underwent a physical exam. The bad news is that your application was rejected because you tested positive for HIV. The test's sensitivity is 99.7% and specificity is 98.5% [https://en.wikipedia.org/wiki/Diagnosis_of_HIV/AIDS#Accuracy_of_HIV_testing]. However, after studying the CDC website, you find that in your ethnic group (age, gender, race, ...) only one in 25,000 people is actually infected. What is the probability that you actually have HIV conditional on having a positive test?

In the statement above:

- Sensitivity refers to the probability of a positive test conditioned on truly having the condition.
- Specificity refers to the probability of a negative test conditioned on not having the condition.

Please complete the following steps:

1. Define the random variables that can be used to represent this problem

X is the probability that an HIV test is positive

Y is the probability that HIV is present

2. Express probabilities given in the statement of the problem

p(x=1 | y=1) = .997

p(x=1 | y=0) = 1-.985 = .015

p(y=1) = 1/25000 = .00004

3. Give the formula that answers the question of having HIV when a test is positive?

p(Y=y|X=x) = (p(X=x|Y = y) * p(Y=Y))/ (p(X=x))

4. Compute the probability

p(Y=1|X=1) = (p(X=1|Y=1) * p(Y=1))/ (p(X=1))

p(X=1) = p(X=1|Y=1) * p(Y=1) + p(X=1|Y=0) * p(Y=0)

p(X=1) = .997 * .00004 + .015 * .99996 = .01503

p(Y=1) = .00004 p(X=1|Y=1) = .997

p(X=x | Y=y) = (.997 * .00004) / .01503

p(X=x | Y=y) = .00265

Problem 4 [10%]

1. Give a simple example of a random variable which has a CDF but does not have a PDF.

The probability that a stone thrown by a shot putter lands at a given distance