Assignment 3
Machine Learning

Deadline: Wednesday 27 Nov 2019, 23:59

Introduction

In this assignment, you will further deepen your understanding of Mixture Models, Hidden Markov Models and Cross Validation. Please provide a latex based report in the PDF format.

Your report and code must be archived in a file named "firstname.lastname" and uploaded to the iCorsi website before the deadline expires. Late submissions will result in 0 points.

Where to get help

We encourage you to use the tutorials to ask questions or to discuss exercises with other students. However, do not look at any report written by others or share your report with others. Violation of that rule will result in 0 points for all students involved. For further questions you can send email to *krsto@idsia.ch*.

Tasks

1. **(15 points)** Poisson distribution is given with

   $$P(x; \lambda) = e^{-\lambda} \frac{\lambda^x}{x!}$$

   for $x = 0, 1, 2, \ldots$ (nonnegative integers) and $\lambda > 0$. Suppose you are given $\lambda_1, \ldots, \lambda_K$ and $\pi_1, \ldots, \pi_K$, $\pi_k \geq 0$, $\sum_k \pi_k = 1$ and the following generating process: sample $k \in \{1, \ldots K\}$ with probability π_k and then sample x from $P(x; \lambda_k)$.

   (a) **(5 points)** What is the distribution $p(x)$ under this generating process?

   (b) **(5 points)** Write the expression for responsibilities γ_{nk}.

   (c) **(5 points)** Write the expression for M-step of expectation maximisation algorithm assuming mixture of Poissons model.

2. **(35 points)** Your friend has two urns, labeled 1 and 2. Urn 1 contains 5 blue, 2 red and 4 yellow balls. Urn two contains 3 blue, 4 red and 3 yellow balls. She

covers your eyes with a tape. Then, she chooses one urn at random with equal probability. You pick one ball from that urn, she tells you its colour and then you return the ball to the urn you picked it from (you don't know which one as your eyes are covered). Your friend switches from urn 1 to urn 2 with probability 1/2 and from urn 2 to urn 1 with probability 3/4. You pick one ball again, she tells you its colour and the process repeats.

(a) **(5 points)** Describe the system as hidden Markov model. What are S, O, π, A, B?

(b) **(10 points)** What is the probability that initial urn was urn 1, then urn 2 and urn 1 again given that you picked yellow, red and blue balls respectively. Use dynamic programming! *Hint:* use Bayes formula!

(c) **(20 points)** What is the most probable sequence of urns given that you picked red, yellow and blue. Use Viterbi algorithm!

3. **(30 points)** Implement a Python function state_probability(pi, A, s, T) that takes initial state ditribution π, transition matrix A, state s and time T and outputs the probability of state s at time t. You can assume that the set of state is $S = \{0, 1, \ldots, N-1\}$. You can use file ex3.py provided online.

4. **(20 points)** You are given three data points $(x, y) : (-1, 0), (0, 1), (1, 0)$. We are using squared error loss function $\ell(y, \hat{y}) = (y - \hat{y})^2$.

(a) **(10 points)** What is leave-one-out cross-validation error of constant model $f(x) = c$? You can use the fact that for points $(x_1, y_1), (x_2, y_2)$ with $y_1 < y_2$, the constant model that achieves minimal square error is $f(x) = (y_2 - y_1)/2$ (i.e. the line in the middle between y_1 and y_2).

(b) **(10 points)** What is leave-one-out cross-validation error of linear model $f(x) = ax + b$?