1. **Integralai su elementariom funkcijom.**

A. Kavaliauskas. Aukštosios matematikos uždavinynas. Vilnius, 2005. 61psl.

579.

$$\int (x^4+3)dx = \int x^4 dx + \int 3dx = \frac{1}{5}x^5 + 3x + C$$

580.

$$\int \frac{x^2+1}{x}dx = \int \frac{x^2}{x}dx + \int \frac{1}{x}dx = \int xdx + \int \frac{1}{x}dx = \frac{x^2}{2} + \ln|x| + C$$

581.

$$\int \frac{(x-1)^2}{x^2}dx = \int \frac{x^2-2x+1}{x^2}dx = \int \frac{x^2}{x^2}dx - \int \frac{2x}{x^2}dx + \int \frac{1}{x^2}dx = x - 2\ln|x| - \frac{1}{x} + C$$

584.

$$\int \frac{x-1}{\sqrt{x}}dx = \int \frac{x}{\sqrt{x}}dx - \int \frac{1}{\sqrt{x}}dx = \int x^{\frac{1}{2}}dx - \int x^{-\frac{1}{2}}dx = \frac{2}{3}\sqrt{x^3} - 2\sqrt{x} + C$$

489.

$$\int \left(\frac{2}{1+x^2} - \frac{5}{\sqrt{1-x^2}}\right)dx = 2\int \frac{1}{1+x^2}dx - 5\int \frac{1}{\sqrt{1-x^2}}dx = 2arctgx - 5\,arcsinx + C$$

2. **Kintamojo keitimo integralai**

A. Kavaliauskas. Aukštosios matematikos uždavinynas. Vilnius, 2005. 63psl.

521.

$$\int \frac{dx}{(1+3x)^2} = \frac{1}{3}\int \frac{dy}{y^2} = \frac{1}{3}\int y^{-2}dy = \frac{1}{3}\frac{y^{-1}}{-1} = -\frac{1}{3(1+3x)} + C$$

$1+3x = y$

$x = \frac{y-1}{3}$

$dx = \frac{1}{3}dy$

522.

$$\int \frac{dx}{1+3x} = \frac{1}{3}\int \frac{dy}{y} = \frac{1}{3}\int y^{-1}dy = \frac{1}{3}\ln|x| + C = \frac{1}{3}\ln|1+3x| + C$$

$1+3x = y$

$x = \frac{y-1}{3}$

$dx = \frac{1}{3}dy$

533.

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int \frac{ye^y}{y}dy = 2\int e^y dy = 2e^y + C = 2e^{\sqrt{x}} + C$$

$\sqrt{x} = y$

$x = y^2$

$dx = 2ydy$

534.

$$\int \sqrt[3]{x^2+1}xdx = \int \frac{y^{\frac{1}{3}}\sqrt{y-1}}{2\sqrt{y-1}}dy = \frac{1}{2}\int y^{\frac{1}{3}}dy = \frac{1}{2}\frac{y^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{8}\sqrt[3]{y^4} + C = \frac{3}{8}\sqrt[3]{(x^2+1)^2} + C$$

$x^2 + 1 = y$
$x^2 = y - 1$
$x = \sqrt{y-1}$
$dx = \frac{1}{2\sqrt{y-1}}dy$

549.

$$\int \frac{x^2}{x^3+3}dx = \int \frac{(y-3)^{\frac{2}{3}}}{y} * \frac{1}{3(y-3)^{\frac{2}{3}}}dy = \frac{1}{3}\int \frac{1}{y}dy = \frac{1}{3}\int y^{-1}dy = \frac{1}{3}\ln|y| + C = \frac{1}{3}\ln|x^3+3| + C$$

$x^3 + 3 = y$
$x = \sqrt[3]{y-3}$
$dx = \frac{1}{3}(y-3)^{-\frac{2}{3}}dy$

3. Integravimas dalimis

A. Kavaliauskas. Aukštosios matematikos uždavinynas. Vilnius, 2005. 68psl.

594.

$$\int (1-3x)cosxdx = (1-3x)sinx - \int sinx(-3)dx = (1-3x)sinx + 3\int sinxdx = (1-3x)sinx -$$
$$-3cosx + C$$

$u = 1 - 3x \quad dv = coxdx$
$du = -3dx \quad v = \int cosxdx = sinx$

596.

$$\int x^3 lnxdx = \frac{x^4}{4}lnx - \int \frac{x^4}{4} * \frac{1}{x}dx = \frac{x^4}{4}lnx - \frac{1}{4}\int x^3dx = \frac{x^4}{4}lnx - \frac{x^4}{16} + C$$

$x = lnx \qquad dv = x^3$
$du = \frac{1}{x}dx \qquad v = \int x^3dx = \frac{x^4}{4}$

69psl:

600.

$$\int (e^{2x} + x)\,cosxdx = \int e^{2x}cosxdx + \int xcosxdx$$

$I = \int e^{2x}cosxdx$
$e = e^{2x} \qquad dv = cosxdx$
$du = 2e^{2x}dx \;\; v = sinx$
$I = e^{2x}sinx - 2\int e^{2x}sinxdx$
$u = e^{2x} \qquad dv = sinx$
$du = 2e^{2x}dx \quad v = -cosx$

$$I = e^{2x}sinx - 2\left(-e^{2x}cosx + 2\int cosxe^{2x}dx\right)$$
$$I = e^{2x}sinx - 2(-e^{2x}cosx + 2I)$$
$$I = e^{2x}sinx + 2e^{2x}cosx - 4I$$
$$5I = e^{2x}sinx + 2e^{2x}cosx$$
$$I = \frac{1}{5}e^{2x}(sinx + 2cosx)$$
$$\int xcosxdx = xsinx - \int 1sinxdx = xsinx + cosx$$
$$u = x \qquad dv = cosxdx$$
$$du = dx \quad v = sinx$$
$$I + xsinx + cosx + C = \frac{1}{5}e^{2x}(sinx + 2cosx) + xsinx + cpsx + C$$

601.

$$\int \frac{x-1}{\cos^2 x}dx = (x-1)tgx - \int tgxdx = (x-1)tgx - \int \frac{sinx}{cosx}dx = (x-1)tgx + \ln|cosx| + C$$
$$u = x - 1 \qquad dv = \frac{dx}{\cos^2 x}$$
$$du = dx \qquad v = tgx$$

604.

$$\int \sqrt{x}lnxdx = \frac{2}{3}\sqrt{x^3} - \frac{2}{3}\int x^{\frac{3}{2}} * x^{-1}dx = \frac{2}{3}\sqrt{x^3} - \frac{2}{3}\int x^{\frac{1}{2}}dx = \frac{2}{3}\sqrt{x^3} - \frac{4}{9}\sqrt{x^3} + C$$

$$u = lnx \qquad dv = \sqrt{x}dx$$
$$du = \frac{1}{x}dx \qquad v = \frac{2}{3}x^{\frac{3}{2}}$$

4. Trigonometriniai integralai

A. Kavaliauskas. Aukštosios matematikos uždavinynas. Vilnius, 2005. 66psl.

563.

$$\int \sin^2 5xdx = \int \frac{1 - cos10x}{2}dx = \frac{1}{2}\int dx - \frac{1}{2}\int cos10xdx = \frac{1}{2}x - \frac{1}{20}\int cos10xd(10x) =$$
$$= \frac{1}{2}x - \frac{1}{20}sin10x$$

564.

$$\int (1 - sin2x)^2 dx = \int (1 - 2sin2x + \sin^2 x)dx = \int dx - \frac{2}{2}\int \sin 2x\, d(2x) +$$
$$+ \int \sin^2 x\, dx = x - cos2x + \frac{1}{2}\int dx - \frac{1}{2}\int cos4xdx = x - cos2x + \frac{1}{2}x - \frac{1}{8}\int \cos(4x)\, d(4x) =$$
$$= \frac{3x}{2} - cos2x - \frac{sin4x}{8} + C$$

565.

$$\int \sin^2 x \cos^2 x\, dx = \int \frac{1 - cos2x}{2} * \frac{1 + cos2x}{2}dx = \int \frac{1 + cos2x - cos2x - \cos^2 2x}{4}dx =$$
$$= \int \frac{1 - \cos^2 2x}{4}dx = \frac{1}{4}\int dx - \frac{1}{4}\int \cos^2 2xdx = \frac{1}{4}x - \frac{1}{4}\int \frac{1 + cos4x}{2}dx =$$
$$= \frac{1}{4}x - \frac{1}{8}x - \frac{1}{32}\int cos4xd(4\,x) = \frac{1}{8}x - \frac{1}{32}sin4x + C$$

570.

$$\int sin3x\, sin5x\, dx = \int \frac{1}{2}(\cos(-2x) - cos8x)dx = \frac{1}{2}\int \cos(-2x)\, dx - \frac{1}{2}\int cos8xdx =$$
$$= -\frac{1}{4}\int \cos(-2x)\, d(-2x) - \frac{1}{16}\int cos8xd(8x) = -\frac{1}{4}\sin(-2x) - \frac{1}{16}sin8x + C$$

573.

$$\int \sin^4 x\, dx = \int \frac{1 - cos2x}{2} * \frac{1 - cos2x}{2} dx = \int \frac{1 - \cos^2 x - cos2x + \cos^2 2x}{4} dx =$$
$$= \frac{1}{4}\int dx - \frac{1}{4}\int cos2xdx - \frac{1}{4}\int cos2xdx + \frac{1}{4}\int \frac{1 + cos4x}{2} dx = \frac{3}{8}x - \frac{1}{4}sin2x + \frac{1}{32}sin4x + C$$

5. Racionalieji integralai

A. Kavaliauskas. Aukštosios matematikos uždavinynas. Vilnius, 2005. 72psl.

620.

$$\int \frac{2x + 3}{x^2 + 6x + 18} dx = \int \frac{2x + 3}{x^2 + 2 * 3x + 9 + 9} dx$$
$$I = \int \frac{2x + 3}{(x + 3)^2 + 9}$$
$$y = x + 3$$
$$x = y - 3$$
$$dx = dy$$
$$I = \int \frac{2(y - 3) + 3}{y^2 + 9} = \int \frac{2y - 3}{y^2 + 9} dy = \frac{1}{9}\int \frac{2y - 3}{\left(\frac{y}{3}\right)^2 + 1} dy$$
$$z = \frac{y}{3}$$
$$y = 3z$$
$$dy = 3dz$$
$$I = \frac{1}{9}\int \frac{(6z - 3)3}{z^2 + 1} dz = \frac{1}{3}\int \frac{3z - 3}{z^2 + 1} dz = \int \frac{z}{z^2 + 1} dz - \int \frac{dz}{z^2 + 1} = \int \frac{d(z^2 + 1)}{z^2 + 1} + \int \frac{dz}{z^2 + 1} =$$
$$= \ln|z^2 + 1| + arctgz + C$$
$$I = \ln\left|\frac{(x + 3)^2}{9} + 1\right| + arctg\left(\frac{x + 3}{3}\right) + C$$

521.

$$I = \int \frac{5x - 2}{x^2 + 2} dx = \int \frac{5x - 2}{(x + 1)^2 + 1} dx$$
$$y = x + 1$$
$$x = y - 1$$
$$dx = dy$$
$$I = \int \frac{5(y - 1) - 2}{y^2 + 1} dy = \int \frac{5y - 7}{y^2 + 1} dy = \int \frac{5y}{y^2 + 1} dy - \int \frac{7}{y^2 + 1} dy = 5\int \frac{d(y^2 + 1)}{y^2 + 1} dy -$$
$$-7\int \frac{1}{y^2 + 1} dy = 5\ln|y^2 + 1| - 7arctg(y^2 + 1) + C$$
$$I = 5\ln|(x + 1)^2 + 1| - 7arctg(x^2 + 2) + C$$