INTEGRALAI

Tiesioginis integravimas

1) $$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx=\int\left(x-2+\frac{1}{x}\right)dx=$$
$$=\frac{x^2}{2}-2x+\ln|x|+c$$

2) $$\int\frac{2-\sqrt{1-x^2}}{\sqrt{1-x^2}}dx=\int\left(\frac{2}{\sqrt{1-x^2}}-1\right)dx=$$
$$=2\arcsin x-x+c$$

3) $$\int\frac{dx}{\sin^2(3-4x)}=-\frac{1}{4}\int\frac{d(3-4x)}{\sin^2(3-4x)}=$$
$$=-\frac{1}{4}\cdot\left(-\mathrm{ctg}(3-4x)\right)+c=\frac{1}{4}\mathrm{ctg}(3-4x)+c$$

4) $$\int x^3\cos(x^4-3)\,dx=\frac{1}{4}\int\cos(x^4-3)\,d(x^4-3)=$$
$$=\frac{1}{4}\sin(x^4-3)+c$$

5) $$\int\frac{e^{2x}dx}{1-3e^{2x}}=-\frac{1}{6}\int\frac{d(1-3e^{2x})}{1-3e^{2x}}=$$
$$=-\frac{1}{6}\ln\left|1-3e^{2x}\right|+c$$

6) $$\int\frac{\sqrt{1+\ln x}}{x}dx=\int(1+\ln x)^{\frac{1}{2}}d(1+\ln x)=$$
$$=\frac{(1+\ln x)^{\frac{3}{2}}}{\frac{3}{2}}+c=\frac{2}{3}\sqrt{(1+\ln x)^3}+c$$

$$\left[\frac{dx}{x}=d(\ln x)\right]$$

7) $\int \frac{x^3 dx}{x^8+1} = \frac{1}{4}\int \frac{dx^4}{1+(x^4)^2} =$

$= \frac{1}{4} \operatorname{arctg} x^4 + C$

8) $\int \frac{\cos 4x}{\sqrt{1-\sin 4x}} dx = -\frac{1}{4}\int \frac{d(\sin 4x)}{\sqrt{1-\sin 4x}} =$

$= -\frac{1}{4}\int (1-\sin 4x)^{-\frac{1}{2}} d(\sin 4x) =$

$= -\frac{1}{2}\sqrt{1-\sin 4x} + C$

9) $\int \frac{\ln x}{x\sqrt{(1-5\ln^2 x)^3}} dx = \int \frac{\ln x}{\sqrt{(1-5\ln^2 x)^3}} d(\ln x)$

$= -\frac{1}{10}\int \frac{d(1-5\ln^2 x)}{\sqrt{(1-5\ln^2 x)^3}} = -\frac{1}{10} \frac{(1-5\ln^2 x)^{-\frac{1}{2}}}{-\frac{1}{2}} + C$

$= \frac{1}{5\cdot\sqrt{1-5\ln^2 x}} + C$

10) $\int \frac{3x + (\operatorname{arctg} 2x)^3}{1+4x^2} dx =$

Integravimas keičiant kintamąjį

1) $\int x^2 \sqrt{x+5}\,dx =$

$\sqrt{x+5} = t$

$x = t^2 - 5$

$dx = d(t^2-5) = (t^2-5)' dt = 2t\,dt$

$= \int (t^2-5)^2 t \cdot 2t\,dt = 2\int (t^4 - 10t^2 + 25) t^2\,dt =$

$= 2\int (t^6 - 10t^4 + 25t^2)\,dt = \frac{2}{7}t^7 - 4t^5 + \frac{50}{3}t^3 + c,$

kur $t = \sqrt{x+5}$

2) $\int \frac{x}{\sqrt{x-10}}\,dx =$

$\sqrt{x-10} = t$

$x = t^2 + 10$

$dx = d(t^2+10) = 2t\,dt$

$= \int \frac{(t^2+10)2\cancel{t}}{\cancel{t}}\,dt = 2\int (t^2+10)\,dt =$

$= \frac{2}{3}t^3 + 20t + c,\ \text{kai}\ t = \sqrt{x-10}$

3) $\int \frac{x}{\sqrt[3]{x+1}}\,dx =$

$\sqrt[3]{x+1} = t$

$x = t^3 - 1$ $\qquad dx = 3t^2\,dt$

$= \int \frac{t^3-1}{\cancel{t}}\,3t^{\cancel{2}}\,dt =$

$= 3\int (t^4 - t)\,dt = \frac{3}{5}t^5 - \frac{3}{2}t^2 + c,$

$\text{kai}\ t = \sqrt[3]{x+1}$

4) $\int \frac{x+3}{(x-7)^9}\,dx =$

$x - 7 = t$ $\qquad dx = dt$

$x = t + 7$

$= \int \frac{t+10}{t^9}\,dt = \int \left(\frac{1}{t^8} + \frac{10}{t^9}\right)dt =$

$= -\frac{1}{7t^7} - \frac{5}{4t^8} + c,\ \text{kai}\ t = x-7$

5) $\int (2x^2-5)(x-4)^{11}\,dx =$

$x-4=t \qquad dx=dt$

$x=t+4$

$= \int (2(t+4)^2-5)(t)^{11}\,dt =$

$= \int (2t^2+16t+27)t^{11}\,dt =$

$= \int (2t^{13}+16t^{12}+27t^{11})\,dt =$

$= \frac{1}{7}t^{14} + \frac{16}{13}t^{13} + \frac{9}{4}t^{12} + C$, kur $t=x-4$

6.) $\int \frac{e^{3x}\,dx}{e^x-1} =$

$e^x-1=t \qquad e^x\,dx=dt$

$e^x=t+1$

$= \int \frac{e^x \cdot e^x e^x\,dx}{e^x-1} = \int \frac{(t+1)(t+1)\,dt}{t} =$

$= \int \frac{t^2+2t+1}{t}\,dt = \int \left(t+2+\frac{1}{t}\right)dt =$

$\frac{t^2}{2} + 2t + \ln|t|$, kur $t=e^x-1$