Laboratorinis darbas Nr. 1

Gynimui darbus apiforminkite raštu. Turėti veikiančią programą kokio nors matematinio paketo aplinkoje. Darbe turi būti užduoties formulavimas, trumpas teorinis aprašymas, užduoties atlikimas, išvados. Šį darbą reikia apginti iki 2015 m. spalio 13 d. Ginantis vėliau bus mažinamas pažymys. Varianto numeris sutampa su numeriu grupės sąraše.

1. - 3. Išspręsti diferencialinę lygtį, patikrinti sprendinį, nubrėžti krypčių lauką ir keletą(3-5) integralinių kreivių.

4. - 5. Išspręsti Koši uždavinį, patikrinti sprendinį, nubrėžti krypčių lauką ir sprendinio grafiką.

1. 1) $3x^2y\,dy - 2xy^2\,dx = 4x\,dx - 6y\,dy$,
   2) $y' = y^2/x^2 + 2y/x + 1$,
   3) $y' = (x + 2y - 1)/(2x - y - 2)$,
   4) $y' - y/x = 2x^2, \quad y(1) = 0$,
   5) $y^2\,dx + (2x + e^{2/y})\,dy = 0, \quad y(e) = 2$.

2. 1) $x\sqrt{1+y^2} + yy'\sqrt{4+x^2} = 0$,
   2)$xy' = (3y^3 + yx^2)/(2y^2 + x^2)$,
   3. $y' = (x + y - 2)/(2x + y - 2)$,
   4. $y' - y\cos x = 4x\sin x, \quad y(\pi/2) = 0$,
   5. $(y^4e^y + 4x)y' = y, \quad y(0) = 1$.

3. 1. $\sqrt{4+y^2}\,dx - 2y\,dy = x^2y\,dy$,
   2. $y' = (x + 2y)/(x - 2y)$,
   3. $y' = (3y - 2x - 4)/(3x + y + 3)$,
   4. $y' + y\cos x = 2\sin(2x), \quad y(0) = 0$,
   5. $y^2\,dx + (xy - 2)\,dy = 0, \quad y(1) = e$.

4. 1. $\sqrt{3+y^2}\,dx - x\,dy = x^2\,dy$,
   2. $xy' = \sqrt{x^2+y^2} + 2y$,
   3. $y' = (2y - 2x)/(x + y - 2)$,
   4. $y' + y\tan x = \cos^2 x, \quad y(\pi/4) = 1/2$,
   5. $2(4y^2 + 4y - 2x)y' = 1, \quad y(0) = 0$.

5. 1. $6x\,dx - 6y\,dy = 3x^2y\,dy - 2xy^2\,dx$,
   2. $2y' = y^2/x^2 + 4y/x + 2$,
   3. $y' = (x + y - 2)/(3x - 2y - 2)$,
   4. $y' - y/(x + 2) = x^2 + 4x, \quad y(-1) = 3/2$,
   5. $(\cos 2y\cos^2 y - x)y' = 1, \quad y(1/4) = \pi/3$.

6. 1. $x\sqrt{3+y^2}\,dx + 2y\sqrt{1+x^2}\,dy = 0$,
   2. $xy' = (3y^3 + 4yx^2)/(2y^2 + 3x^2)$,
   3. $y' = (2x + y - 3)/(x + y - 1)$,
   4. $y' - y/x = xe^x, \quad y(1) = 0$,
   5. $(x\cos^2 y - y^2)y' = 1, \quad y(\pi) = \pi/4$.

7. 1. $(e^{2x} + 5)\,dx + ye^{2x}\,dy = 0$,
   2. $y' = (x + 2y)/(2x - y)$,
   3. $y' = (x + 7y - 8)/(9x - y - 7)$,
   4. $y' - y/x = \sin x/\cos x, \quad y(\pi/2) = 1$,
   5. $e^{y^2}(dx - 3xy\,dy) = y\,dy, \quad y(0) = 0$.

8. 1. $yy'\sqrt{(1-x^2)/(1-2y^2)} + 1 = 0$,
   2. $xy' = 2\sqrt{x^2 + y^2} + 2y$,
   3. $y' = (x + 3y + 4)/(3x + y - 6)$,
   4. $y' + y/x = \cos x, \quad y(1) = 1/\pi$,
   5. $(52y^3 - x)y' = 2y, \quad y(8) = 1$.

9. 1. $6x\,dx - 2y\,dy = 3x^2y\,dy - 2xy^2\,dx$,
   2. $3y' = y^2/x^2 + 4y/x + 2$,
   3. $y' = (3x + y + 3)/(2x + y - 1)$,
   4. $y' + y/2x = 2x^2, \quad y(1) = 1$,
   5. $dx + (xy - 3y^3)\,dy = 0, \quad y(-1) = 0$.

10. 1. $x\sqrt{5+y^2}\,dx + y\sqrt{2+x^2}\,dy = 0$,
    2. $xy' = (3y^3 + 4yx^2)/(2y^2 + x^2)$,
    3. $y' = (x + 2y - 3)/(4x + y - 3)$,
    4. $y' + 2xy/(1 + x^2) = x^2/(1 + x^2), \quad y(0) = 2/3$,
    5. $(3y\cos 2y - 2y^2\sin 2y - 2x)y' = y, \quad y(16) = \pi/4$.

11. 1. $y(e^x + 1)\,dx - e^x\,dy = 0$,
    2. $y' = (x^2 + 2xy - y^2)/(x^2 - xy)$,
    3. $y' = (x - 2y + 1)/(-2x - y + 2)$,
    4. $y' + (2x - 3)y/x^2 = 5, \quad y(2) = 4$,
    5. $4(2y^3 + xy - y)y' = 1, \quad y(0) = 0$.

12. 1. $\sqrt{2-x^2}y' + xy^2 + 2x = 0$,
    2. $xy' = \sqrt{x^2+2y^2} + 2y$,
    3. $y' = (x+8y-9)/(4x-y-9)$,
    4. $y' + y/x = (x+3)e^x/x, y(1) = e$,
    5. $(2\ln y - \ln^2 y)\,dy = y\,dx - x\,dy, \quad y(4) = e^2$

13. 1. $2x\,dx - 2y\,dy = 3x^2 y\,dy - xy^2\,dx$,
    2. $y' = y^2/x^2 + y/x + 6$,
    3. $y' = (2x+3y-5)/(5x+y-5)$,
    4. $y' - 2y/x = \ln x/x, \quad y(1) = 1$,
    5. $2(x+2y^4)y' = y, \quad y(-2) = -1$.

14. 1. $x\sqrt{1+y^2}\,dx + y\sqrt{5+x^2}\,dy = 0$,
    2. $xy' = (3y^3 + 6yx^2)/(2y^2 + 3x^2)$,
    3. $y' = (4y - x + 8)/(3x + 2y - 7)$,
    4. $y' - 2y/x = 12/x^3, \quad y(1) = 4$,
    5. $y^3(y-1)\,dx + xy^2(y-1)\,dy = (y+2)\,dy, \quad y(1/4) = 2$.

15. 1. $(e^x + 4)\,dx - 2ye^x\,dy = 0$,
    2. $y' = (x^2 + xy - y^2)/(x^2 - 2xy)$,
    3. $y' = (x+3y-4)/(5x-2y-4)$,
    4. $y' + y/x = x^3 - x, \quad y(1) = -5/6$,
    5. $y^2\,dx + (2x + e^{1/y})\,dy = 0, \quad y(e) = 1$.

16. 1. $\sqrt{5+y^2} + 2yy'\sqrt{1-x^2} = 0$,
    2. $xy' = 3\sqrt{x^2+2y^2} + y$,
    3. $y' = (y - 2x + 3)/(x + y - 1)$,
    4. $y' + 2y/x = 3x, \quad y(1) = 1$,
    5. $(xy + \sqrt{y})\,dy + 2y^2 dx = 0, \quad y(-1/2) = 4$.

17. 1. $6x\,dx - 2ydy = x^2 ydy - 3xy^2\,dx$,
    2. $2y' = 2y^2/x^2 + 8y/x + 4$,
    3. $y' = (x + 2y - 3)/(x + y - 1)$,
    4. $y' - xy/(1+x^2) = 1 + x^2, \quad y(1) = 3$,
    5. $\sin 2y\,dx = (\sin^2 2y - 2\sin^2 y + 2x\sin 2y)\,dy = 0, \quad y(-1/2) = \pi/4$.

18. 1. $y \ln y + 2xy' = 0$,
    2. $xy' = (3y^3 + 5yx^2)/(2y^2 + 5x^2)$,
    3. $y' = (3x + 2y - 1)/(x + y + 1)$,
    4. $y' + (1 - x)y/x^2 = 1 - x, \quad y(1) = 1$,
    5. $(2y^2 + y - x)y' = 1, \quad y(2) = 0$.

19. 1. $(1 + 2e^x)y' = ye^x$,
    2. $y' = (x^2 + xy - y^2)/(3x^2 - xy)$,
    3. $y' = (x + 5y + 5)/(4x + 3y - 1)$,
    4. $y' + y/x = 2/x^3, \quad y(1) = 1$,
    5. $y\sqrt{y}\,dx - (3x\sqrt{y} + 5)\,dy = 0, \quad y(-4) = 1$.

20. 1. $\sqrt{4 - x^2}y' + xy^2 + 2x = 0$,
    2. $xy' = \sqrt{x^2 + 2y^2} + y$,
    3. $y' = (x + 4y - 5)/(x - y - 5)$,
    4. $y' + xy = -x^3, \quad y(1) = 1/e$,
    5. $dx = (\sin y + 3\cos y + x)\,dy, \quad y(e^{\pi/2}) = \pi/2$.

21. 1. $x\,dx - 2y\,dy = x^2 y\,dy - 3xy^2\,dx$,
    2. $y' = y^2/x^2 + y/x + 6$,
    3. $y' = (x + y + 2)/(x - y + 1)$,
    4. $y' + xy/(1 - x^2) = x^2, \quad y(0) = 2/3$,
    5. $2(\sin y \cos 2y - x)y' = 1, \quad y(3/2) = 5\pi/4$.

22. 1. $y(1 + \ln y) + 2xy' = 0$,
    2. $xy' = (y^3 + 12yx^2)/(y^2 + 6x^2)$,
    3. $y' = (2x + y - 3)/(4x + y - 4)$,
    4. $y' + 2xy = -x^3 + x, \quad y(0) = 3$,
    5. $\sinh y\,dx = (1 + x \sinh y)\,dy, \quad y(1) = \ln 2$.

23. 1. $(1 + e^x)yy' = 2e^x$,
    2. $y' = (x^2 + 2xy - y^2)/(x^2 - xy)$,
    3. $y' = (2x + y - 3)/(x + 2y - 2)$,
    4. $y' - y/(x + 1) = e^x(x + 2)^2, \quad y(0) = 1$,
    5. $(3y^3 - x)y' = 3y, \quad y(5) = 1$.

24. 1. $\sqrt{2 + y^2} + \sqrt{5 - x^2}yy' = 0$,
    2. $xy' = \sqrt{3x^2 + 5y^2} + 2y$,
    3. $y' = (y - x)/(2x + 2y - 2)$,

4. $y' + xy = 2xe^{-x^2} \sin x, \quad y(0) = 1,$
5. $y^2(y^2+1)\,dx + xy(2y^2+5)\,dy = 2\,dy, \quad y(\pi/8) = 2.$

25. 1. $4x\,dx - 6y\,dy = 3x^2 y\,dy - xy^2\,dx,$
2. $y' = 2y^2/x^2 + 10y/x + 1,$
3. $y' = (x+y-6)/(7x-y-6),$
4. $y' - 2y/x = (x+1)^3, \quad y(1) = 1/2,$
5. $(2x + \ln y^2 - \ln y)y' = y, \quad y(2) = 1.$

26. 1. $\sqrt{3+y^2}\,dx + (x^2+4)\,dy = 0,$
2. $xy' = (y^3 + 7yx^2)/(y^2+x^2),$
3. $y' = (x+y-4)/(x+2y-2),$
4. $y' - y\cos x = 2\sin 2x, \quad y(0) = 3,$
5. $(xy + \sqrt{y})\,dy + y^2\,dx = 0, \quad y(-1/2) = 1.$

27. 1. $(4+e^x)yy' = 2e^x,$
2. $y' = (x^2 + 2xy - 5y^2)/(x^2 - 3xy),$
3. $y' = (2x+y-1)/(2x-y-2),$
4. $y' - xy = 4x^3 + x, \quad y(0) = -1/2,$
5. $y\,dx + (x - 2y\sin^2 y - y\sin 2y)\,dy = 0, \quad y(3/2) = \pi/4.$

28. 1. $(x^2 y + y)\,dy + \sqrt{1+y^2}\,dx = 0,$
2. $xy' = \sqrt{2x^2+y^2} + y,$
3. $y' = (3y - 2x + 1)/(3x + y + 3),$
4. $y' - y/x = \ln x/x, \quad y(1) = 1,$
5. $(y^3 - y + 2xy)\,dy = dx, \quad y(-2) = 0.$

29. 1. $x\,dx - y\,dy = 2yx^2\,dy - xy^2\,dx,$
2. $y' = y^2/x^2 + 10y/x + 1,$
3. $y' = (6y - x + 6)/(5x + 4y - 9),$
4. $y' - x^2 y = x^2(2+x^3)/3, \quad y(0) = 0,$
5. $(y + x\tan y - y^2 \tan y)\,dy = dx, \quad y(0) = \pi.$

30. 1. $x + 2xy^2 + \sqrt{1-x^2}y' = 0,$
2. $xy' = 4\sqrt{x^2+2y^2} + y,$
3. $y' = (x+y-7)/(8x-y-7),$
4. $y' - y\cos x = 2\sin 2x, \quad y(0) = -1,$
5. $y^2 dx + (2e^{1/2y} + x)dy = 0, \quad y(e) = 1/2.$