



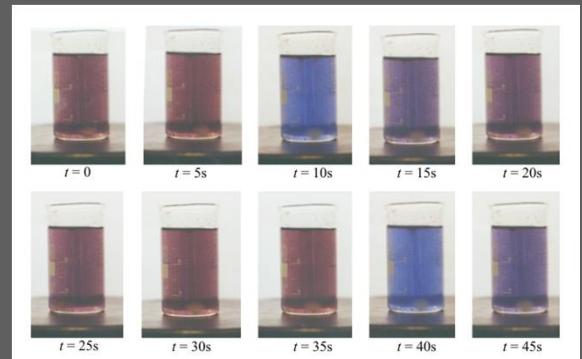
Sudétingumas

Sudėtingos sistemos

- ⇒ Šiuolaikinis požiūris į pasaulį remiasi **painiavos** (angl., *complexity*) supratimu.
Painų pasaulį padaro sudėtingos daugelio dalių sistemos ir įvairios šių dalių tarpusavio sąveikos. Tokios sistemos dominuoja mus supančiame pasaulyje. Taigi nenuostabu, nes tik tokios sistemos išsiskiria savaimine organizacija arba **savikūra**.
- ⇒ Paprasčiausiai **sudėtingas sistemas galėtume apibrėžti** kaip **sistemas sudarytas iš daugelio dalių ar elementų, arba komponentų, kurie gali būti tokie patys arba skirtini**.
Šios dalys ar komponentės gali būti sujungtos (susietos) paprastu ir/ar sudėtingesniu būdu.

Savikūra

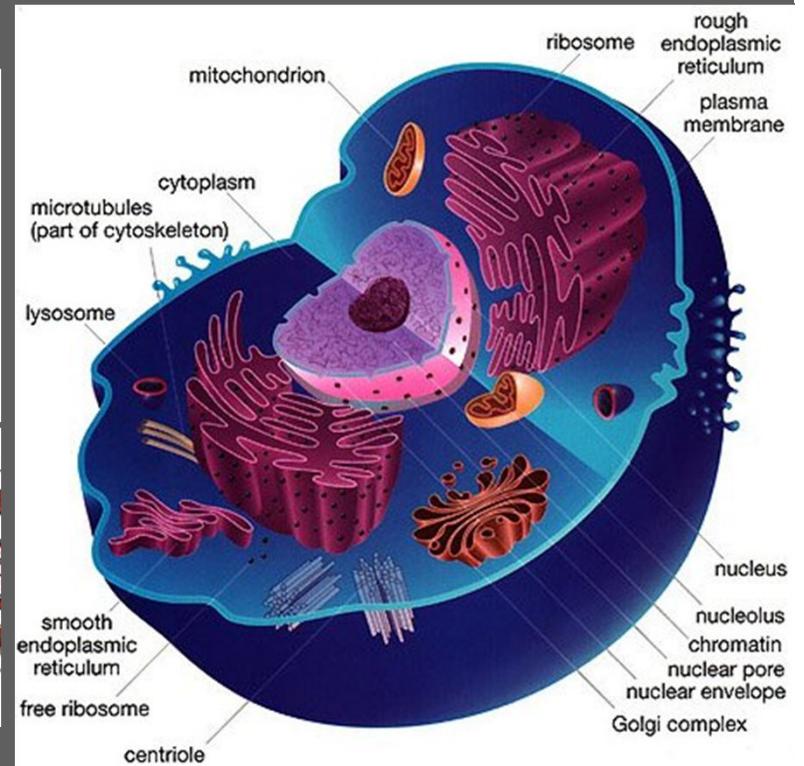
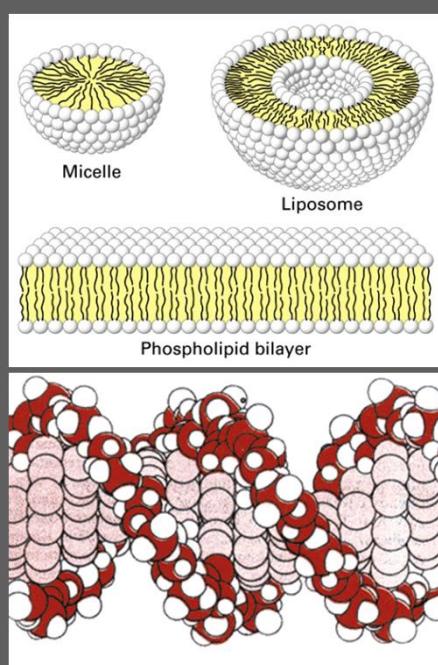
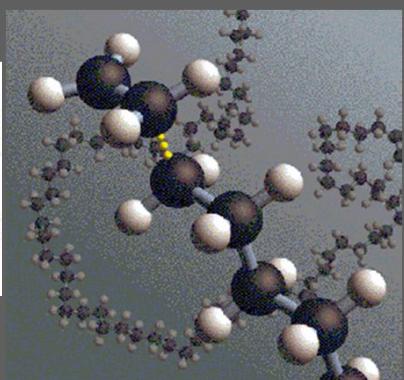
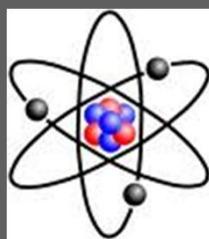
- ⇒ Savikūra (*angl. self-organization*) yra savaiminis
 - erdvinių
 - laikinių
 - funkcinių sistemų formavimasis.



Sudėtingų sistemų supratimas

- ⇒ Mikroskopinis – makroskopinis supratimas
- ⇒ Sistemai sudėtingėjant, t.y. pereinant iš mikroskopinio į makroskopinį lygį gimsta naujos sistemos savybės, kurių nėra mikroskopiniame lygyje

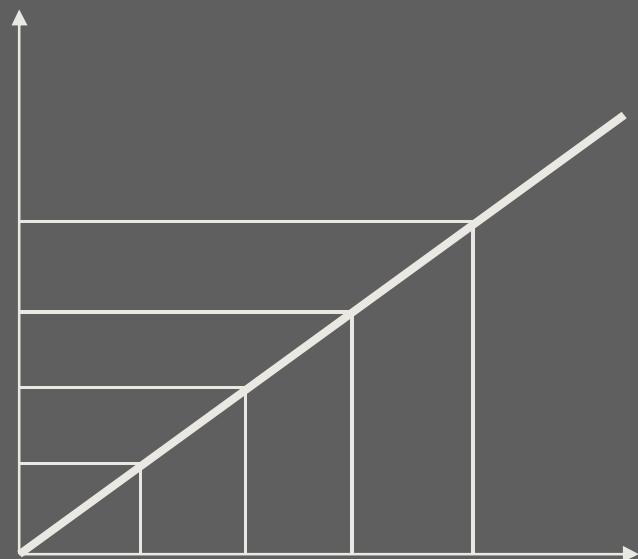
Sudètingumo skalès



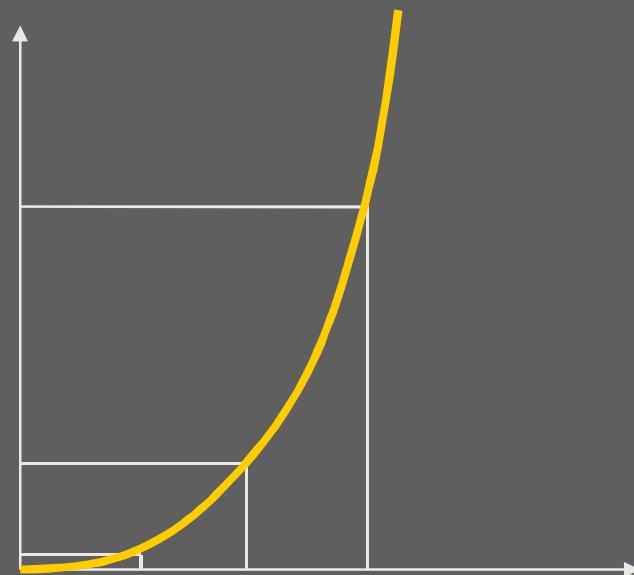
Netiesinė dinamika



⇒ Tiesinė dinamika

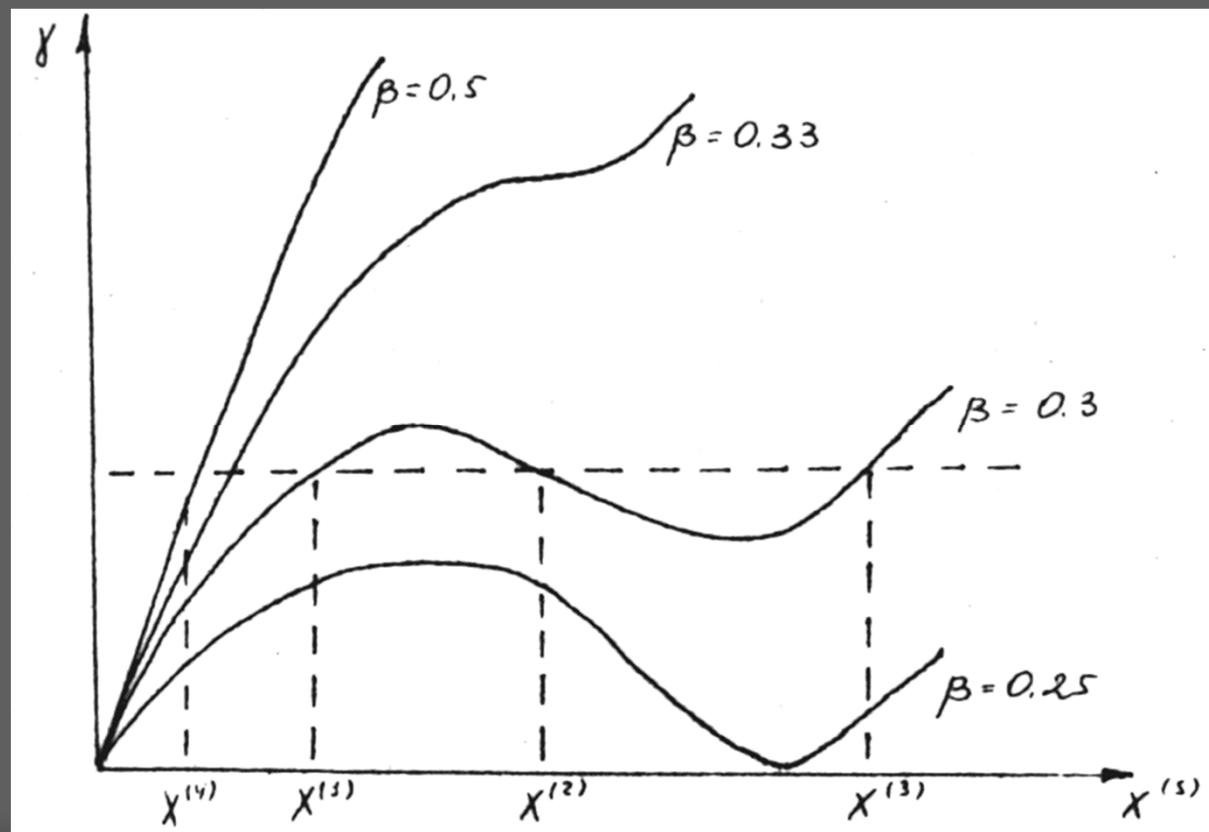


⇒ Netiesinė dinamika



Netiesinė dinamika – naujos būsenos

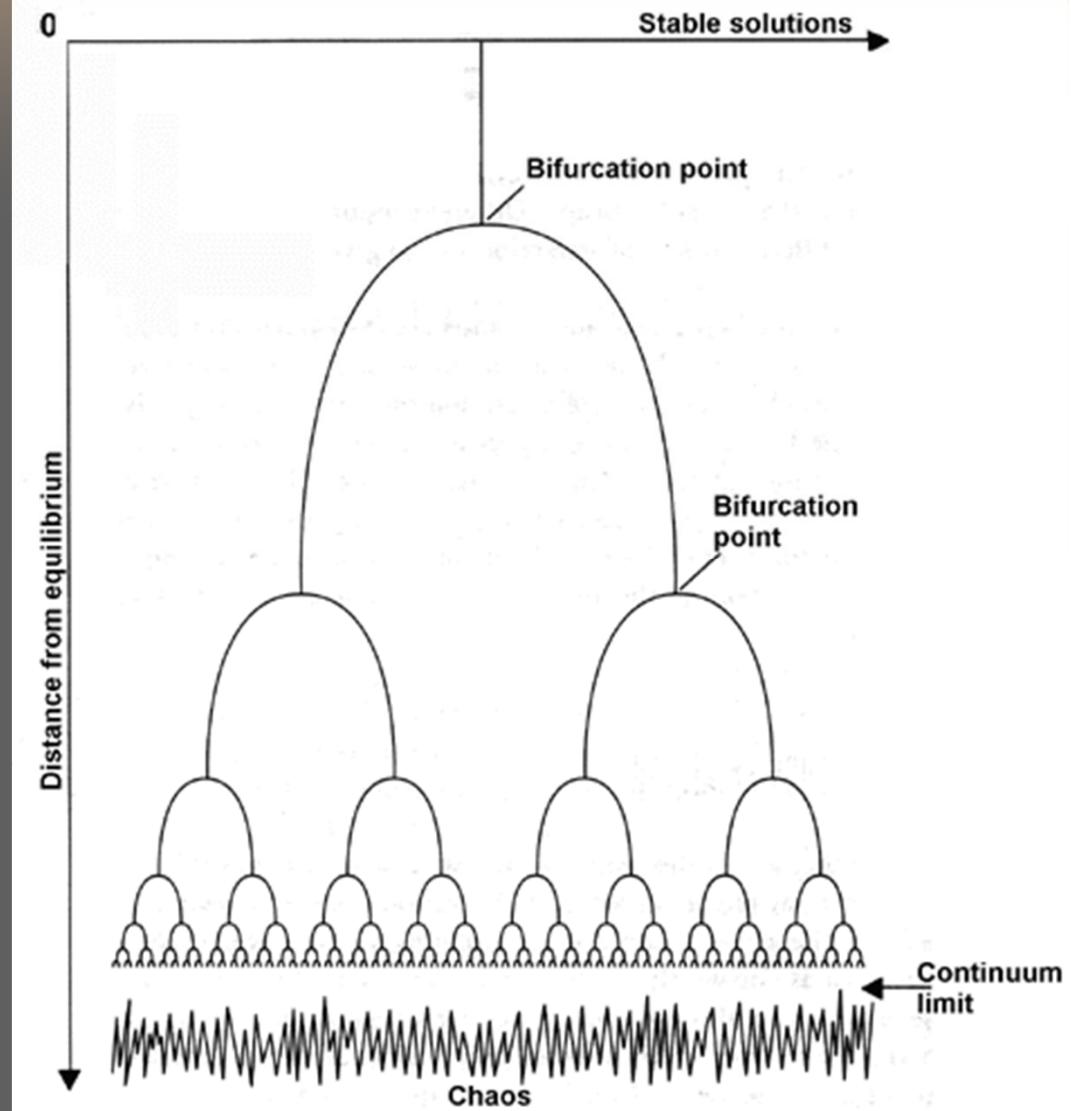
$$X^3 - X^2 + \beta X - \gamma = 0$$



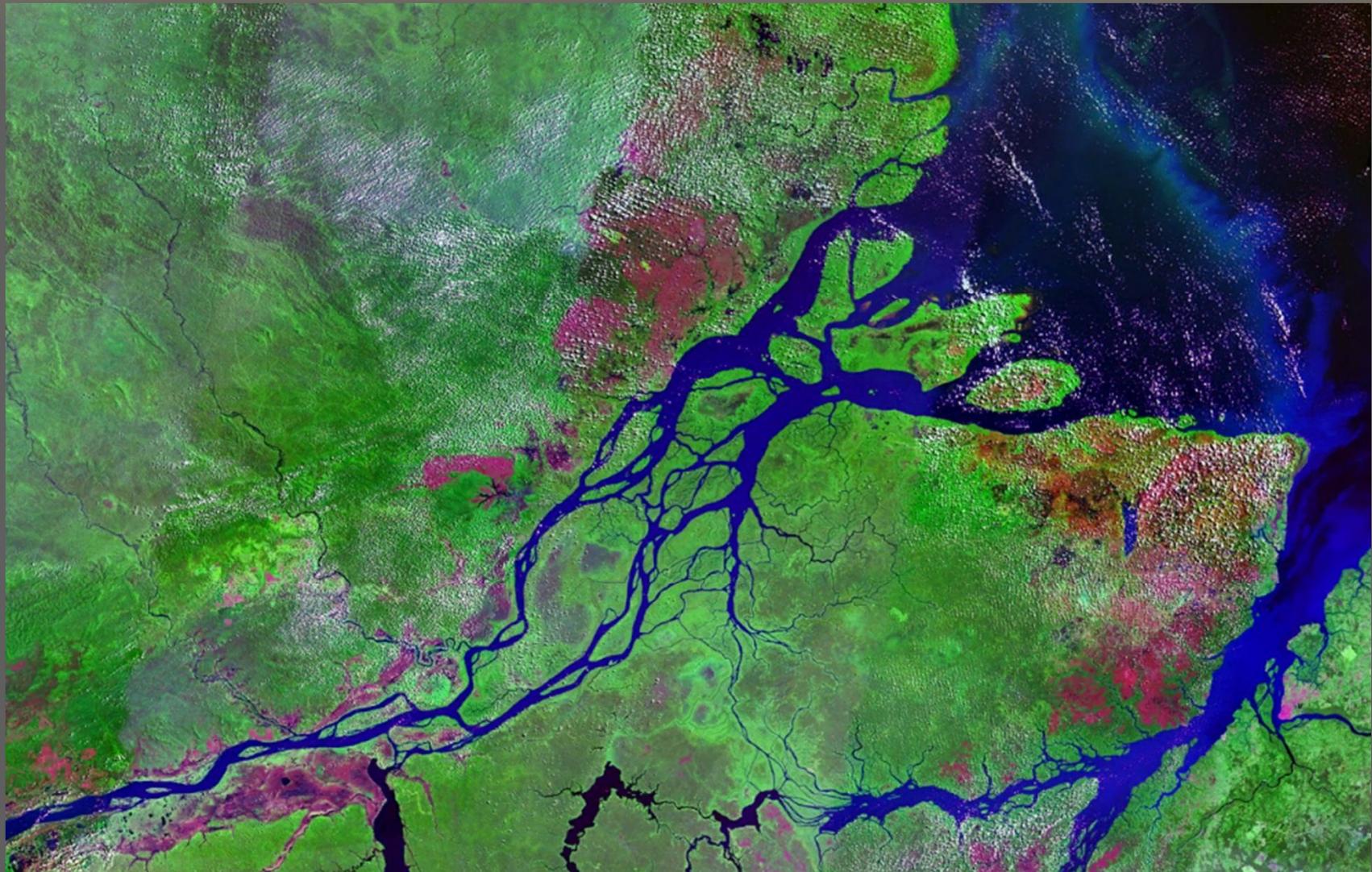
Dvejiniai

Bifurkacija tai yra staigus kokybinis netiesinės dinaminės sistemos elgesio arba topologinių savybių pokytis, kintant kokiam nors tą sistemą aprašančiam parametru.

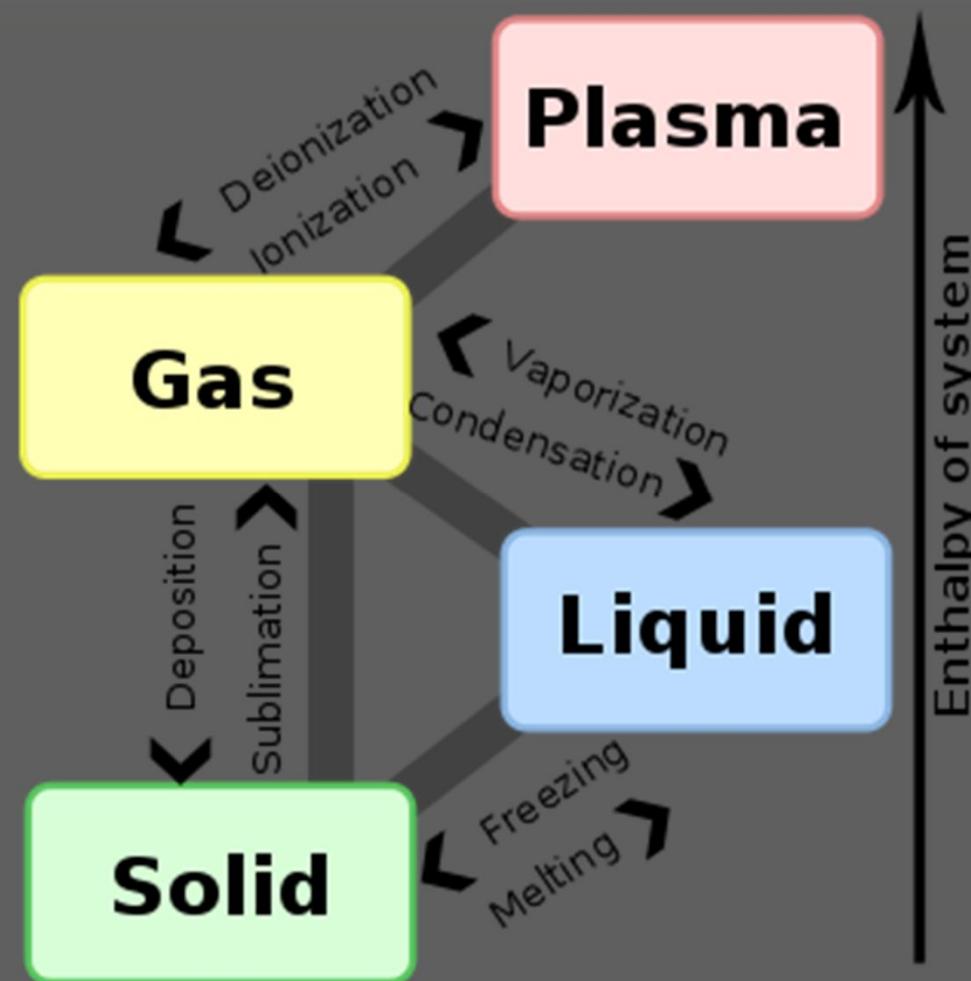
Kritinės to parametrao vertės, kurioms esant tai atsitinka yra vadinamos **bifurkacijos taškai**.



Amazoné



Faziniai virsmmai



Logistinė lygtis

$$x_{n+1} = rx_n(1 - x_n)$$

The logistic map for 100 generations as r moves from 0 to 4.

- ⊖ x_n is a number between zero and one, and represents the population at year n , and hence x_0 represents the initial population (at year 0)
- ⊖ r is a positive number, and represents a combined rate for reproduction and starvation.

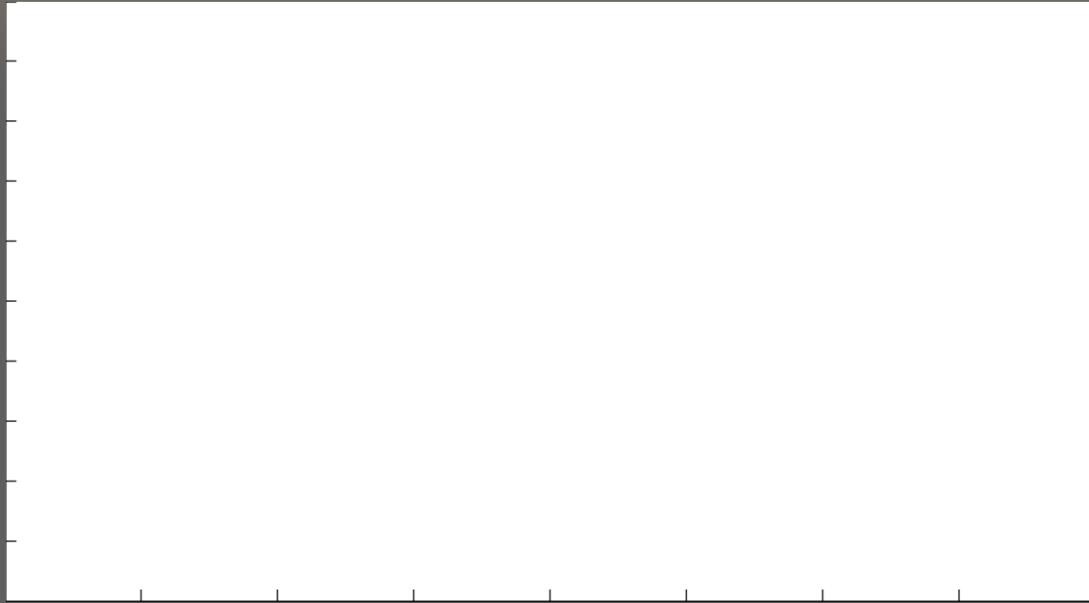
This nonlinear difference equation is intended to capture two effects

- ⊖ **reproduction** where the population will increase at a rate proportional to the current population when the population size is small.
- ⊖ **starvation** (density-dependent mortality) where the growth rate will decrease at a rate proportional to the value obtained by taking the theoretical "carrying capacity" of the environment less the current population.

Bifurkacijų diagrama

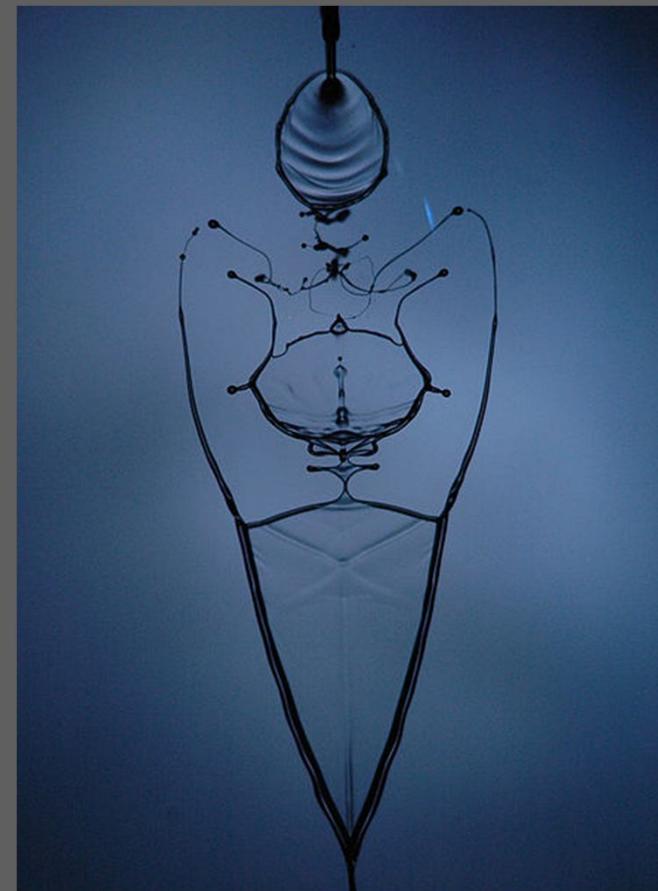
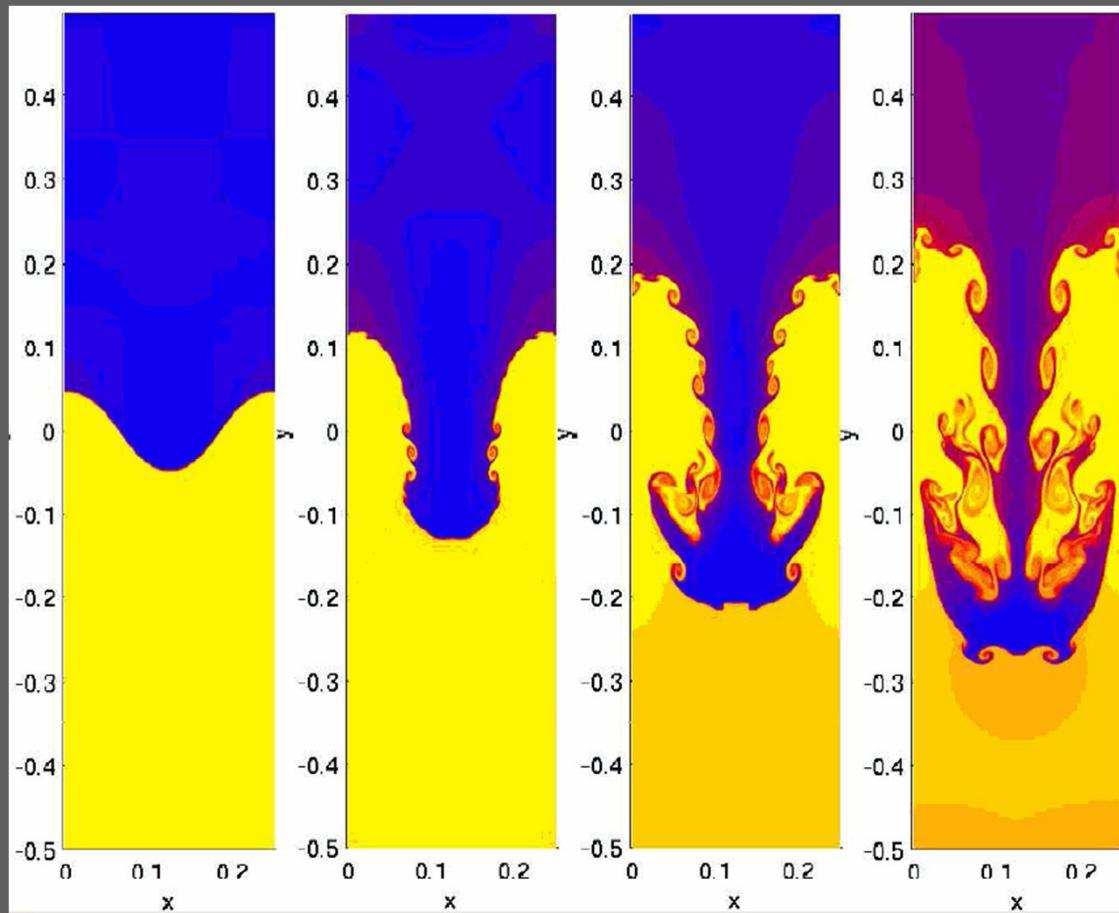
⇒ logistic map:

$$x_{n+1} = r x_n (1 - x_n)$$



- ⇒ The bifurcation parameter r is shown on the horizontal axis of the plot and the vertical axis shows the possible long-term population values of the logistic function. Only the stable solutions are shown here.
- ⇒ The bifurcation diagram shows the forking of the possible periods of stable orbits from 1 to 2 to 4 to 8 etc. Each of these bifurcation points is a period-doubling bifurcation.

Reilėjaus Teiloro nestabilumas susitrenkia du skysčiai



Dalelių ansamblis



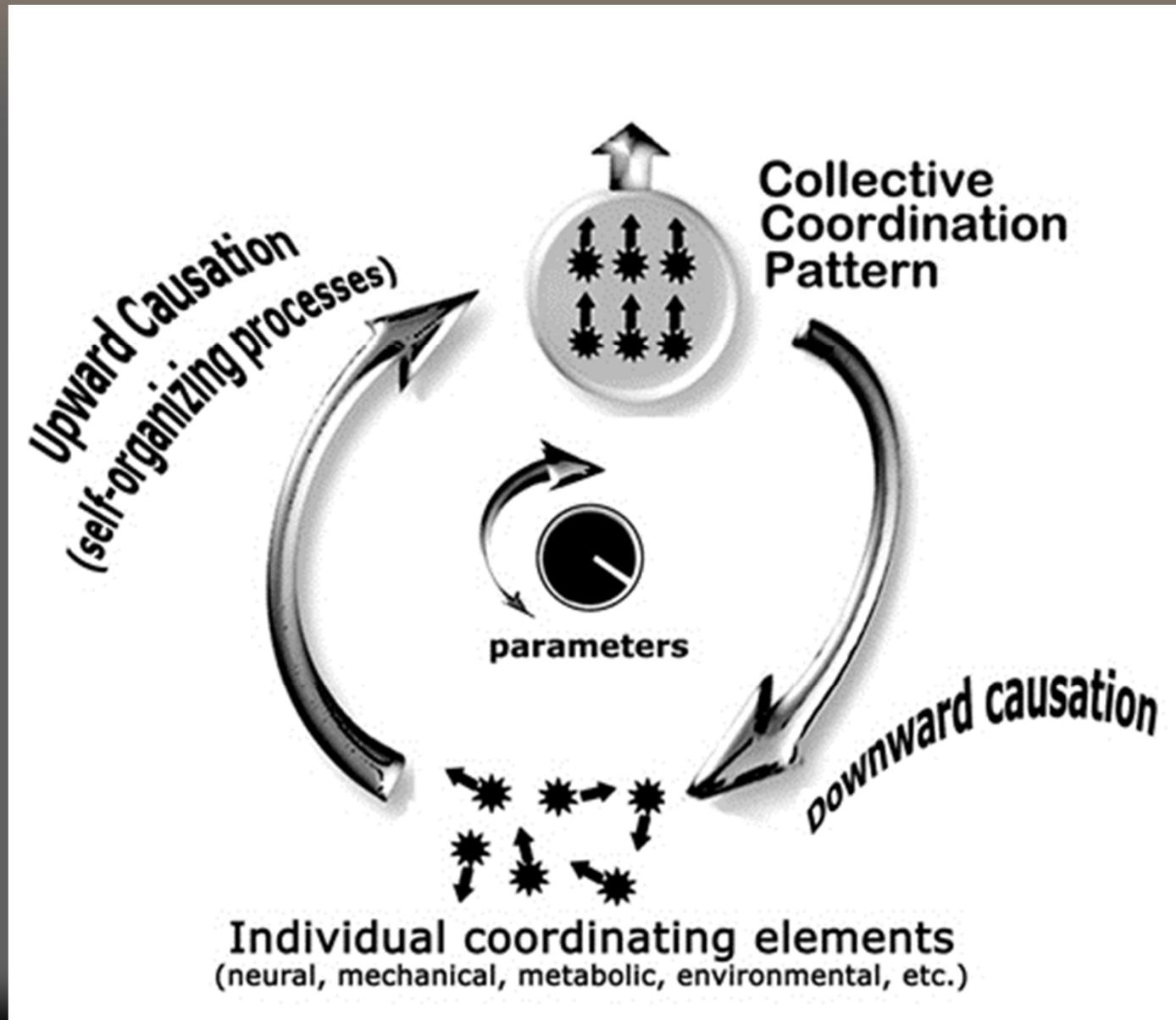
Sinergetika

H. Hakenas
(1927-)

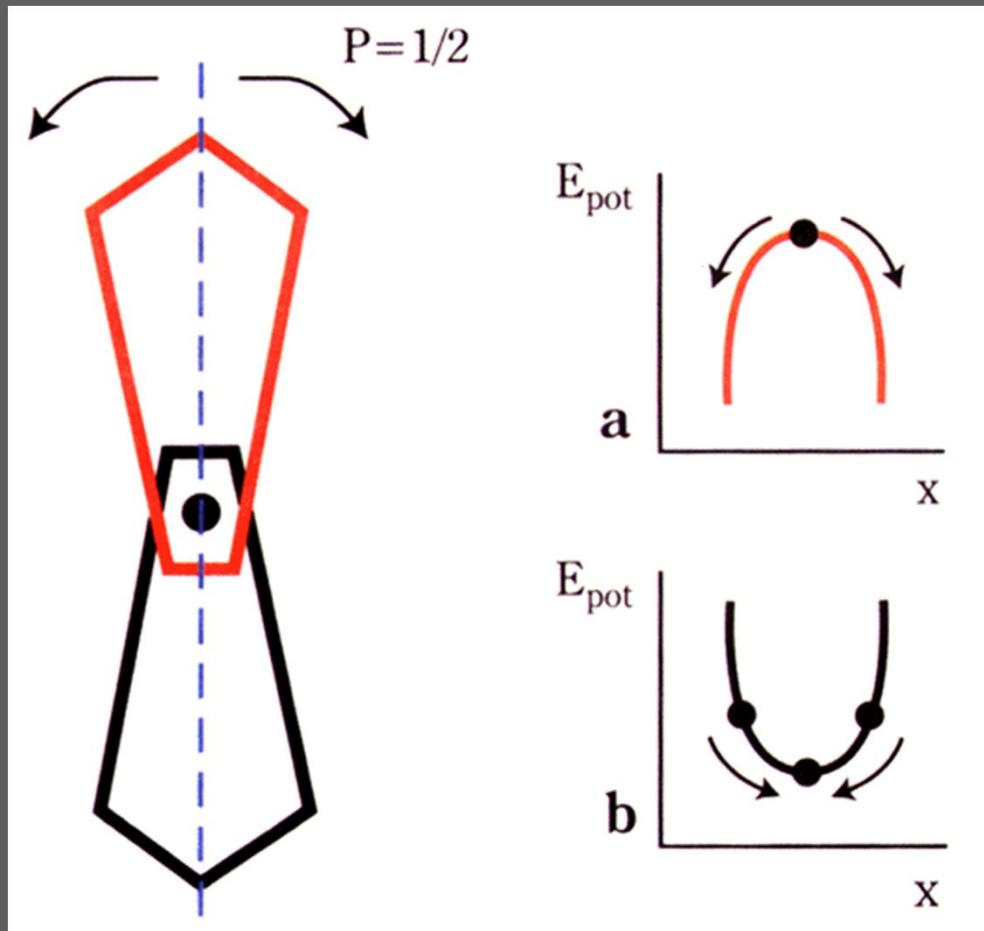


⇒ Sinergetika -
tarpdisciplininis mokslas,
aiškinantis
saviorganizacijos
reiškinius atvirose
termodinaminėse
sistemoje, esančiose toli
nuo termodinaminės
pusiausvyros

Savitarka



Nepusausvyrosios sistemos Klasikinė švytuoklė



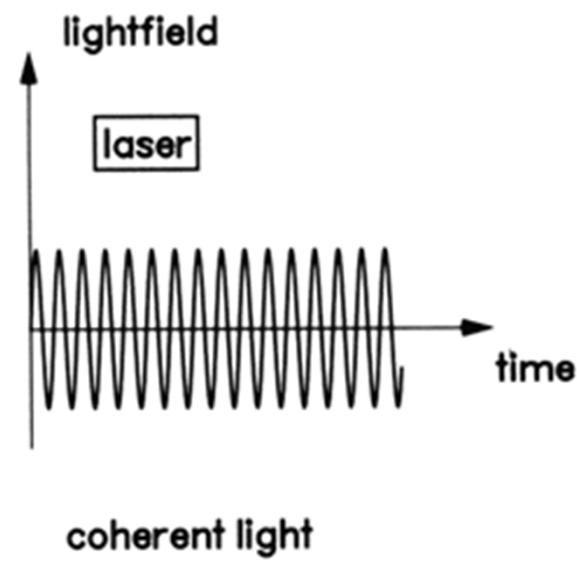
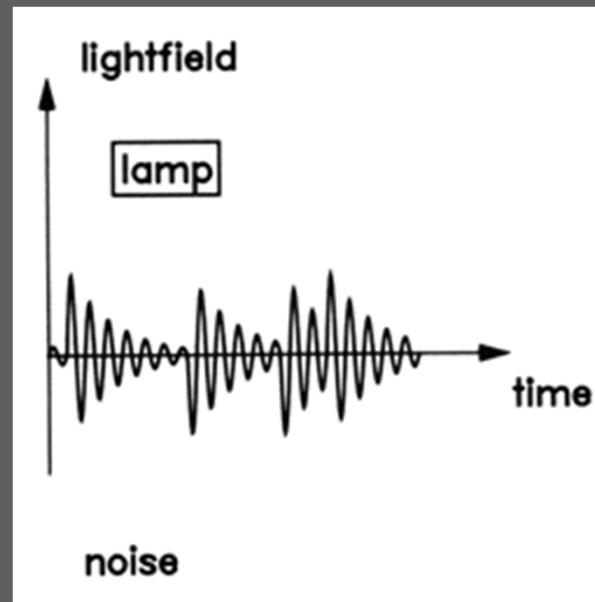
- ⇒ Švytuoklės nepusiausviroji ir pusiausviroji būsenos
- ⇒ Švytuoklės tikimybinio ir deterministinio judesio pavyzdžiai

Savitvarka – Lazeris

H. Haken

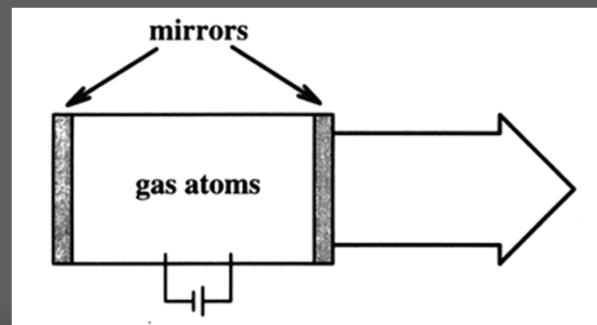


Laisvės
laipsniai

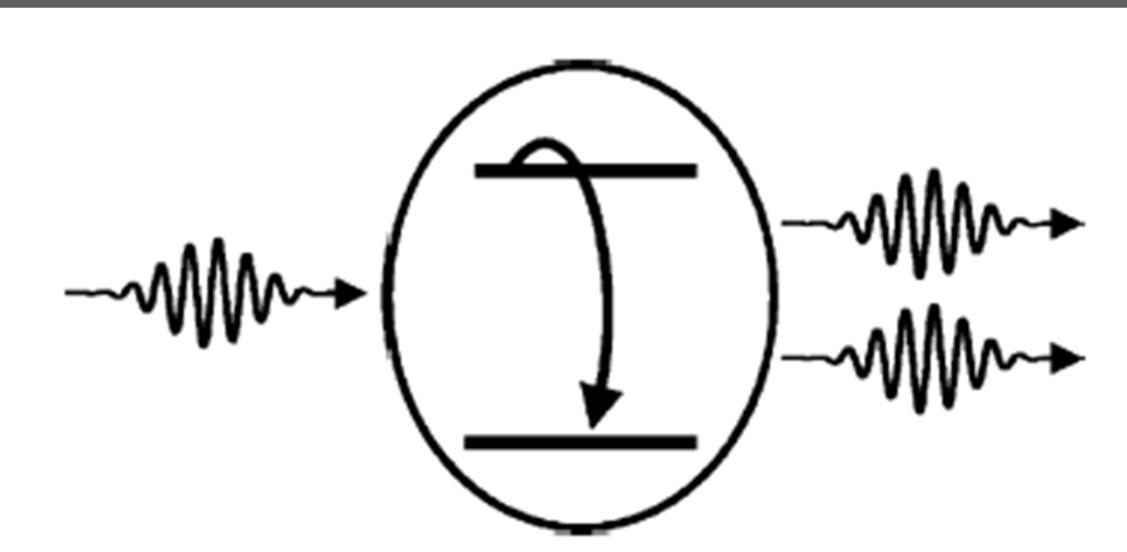
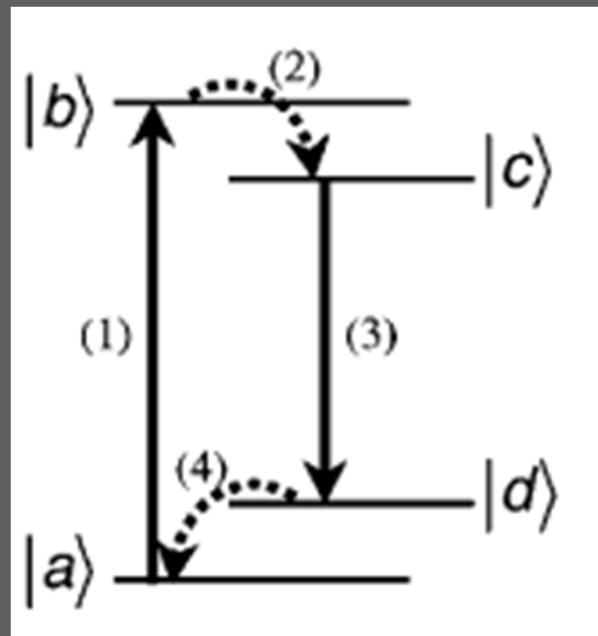


Laisvės
laipsniai

- ⌚ Nepusiausvira sistema
- ⌚ Tvarkos parametras
- ⌚ Netiesinis grįztamasis ryšys

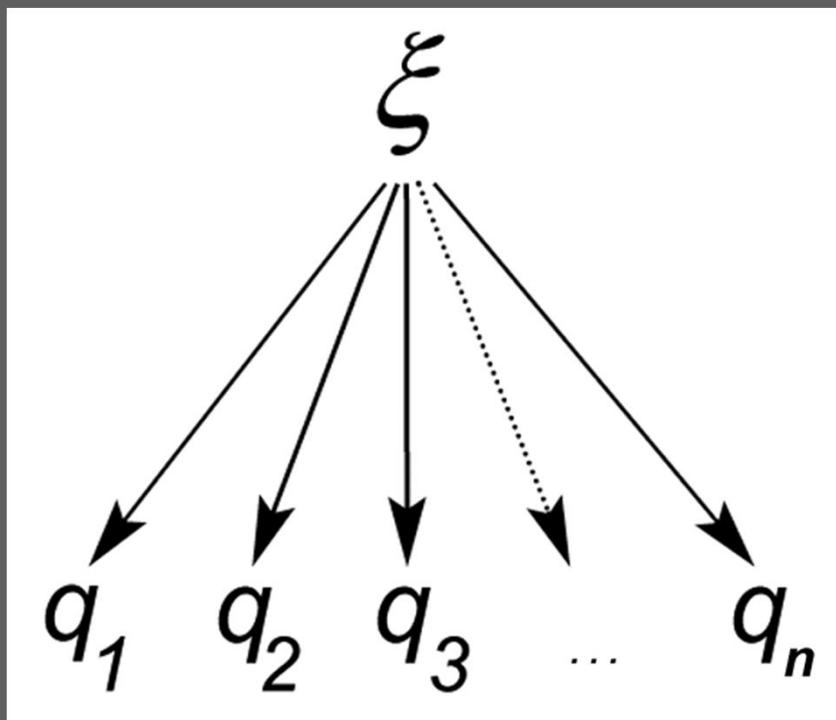


Priverstinė spinduliųotė

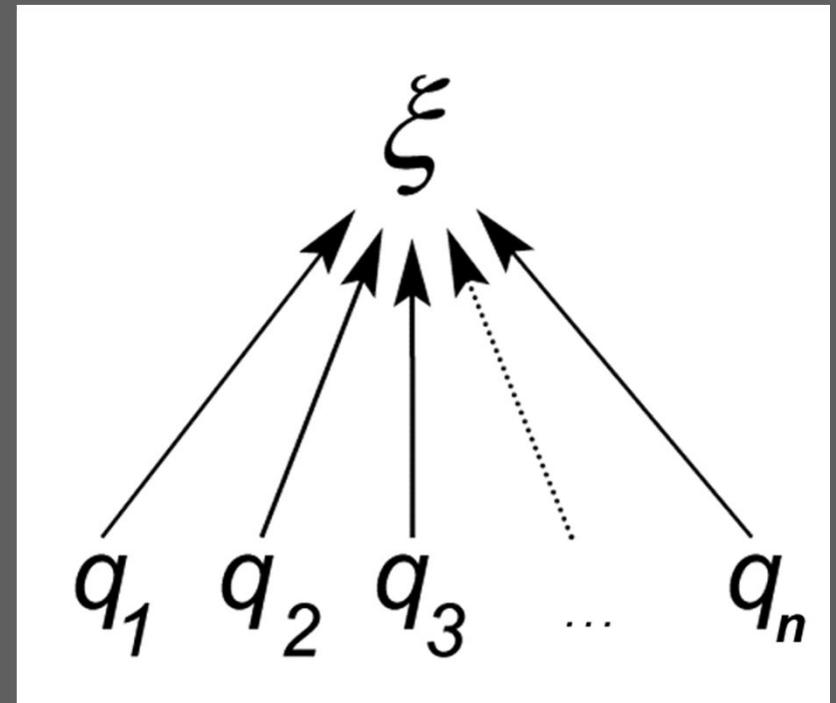


Baziniai sinergetikos principai

Pavergimas



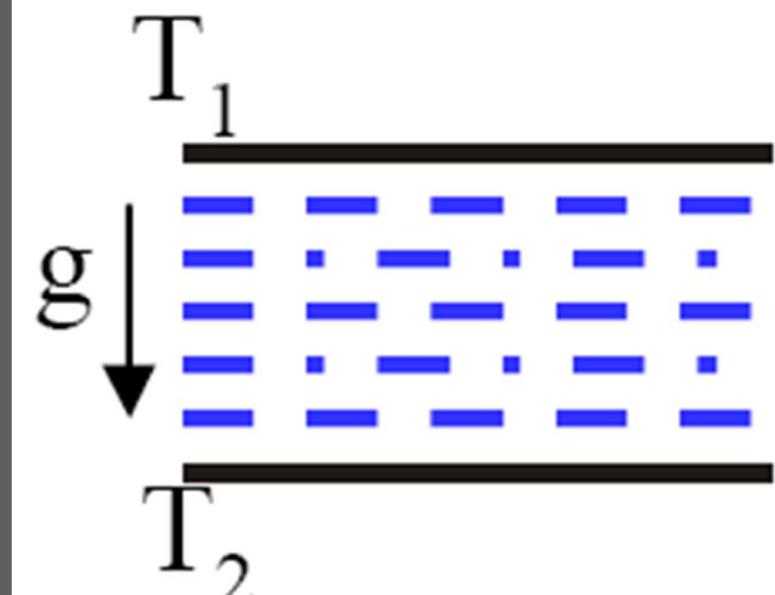
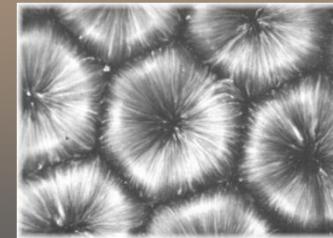
Tvarkos parametro generacija



Konsensusas - sutarimas

Saviorganizuotųjų sistemų demokratija

Bernaro celēs



Laisvės laipsniai



Tvarkos parametro koncepcija

- ⇒ Hakenas apibendrino tvarkos parametru koncepciją įvesdamas “pavergimo” principą, teigdamas, kad sistemos makroskopinius pokyčius lemia tik taip vadinamos lėtosios kitimo modos “tvarkos parametrai”. Spartaus kitimo komponentės spėja prisitaikyti prie lėto tvarkos parametru kitimo.
- ⇒ Todėl savitvarka reiškia dramatišką sistemos laisvės laipsnių (entropijos) sumažėjimą kuris makroskopiskai pasireiškia tvarkos atsiradimu.
- ⇒ Tai paaiškina daugelį savitvarkos reiškiniių fizikoje, chemijoje, biologijoje ir net socialinėse sistemoje.

Pavergimo tendencija ir Konsensuso suradimas

- ⇒ Jei dominuoja pavergimo principas sistema bus griežtai apibrėžta ir negalės vystytis.
- ⇒ Tipinėse sinergetinėse sistemoje vyrauja pusiausvira tarp pavergimo tendencijos ir individualių komponenčių laisvės laipsnių.
- ⇒ Taigi sinergetinėse sistemoje individualios komponentės pačios generuoja tvarkos parametrum, kuris jas ir pavergia bei reguliuoja bendrą sistemos elgesį.
- ⇒ Anot Hakeno pavergimo tendenciją subalansuoja konsensuso paiešką.

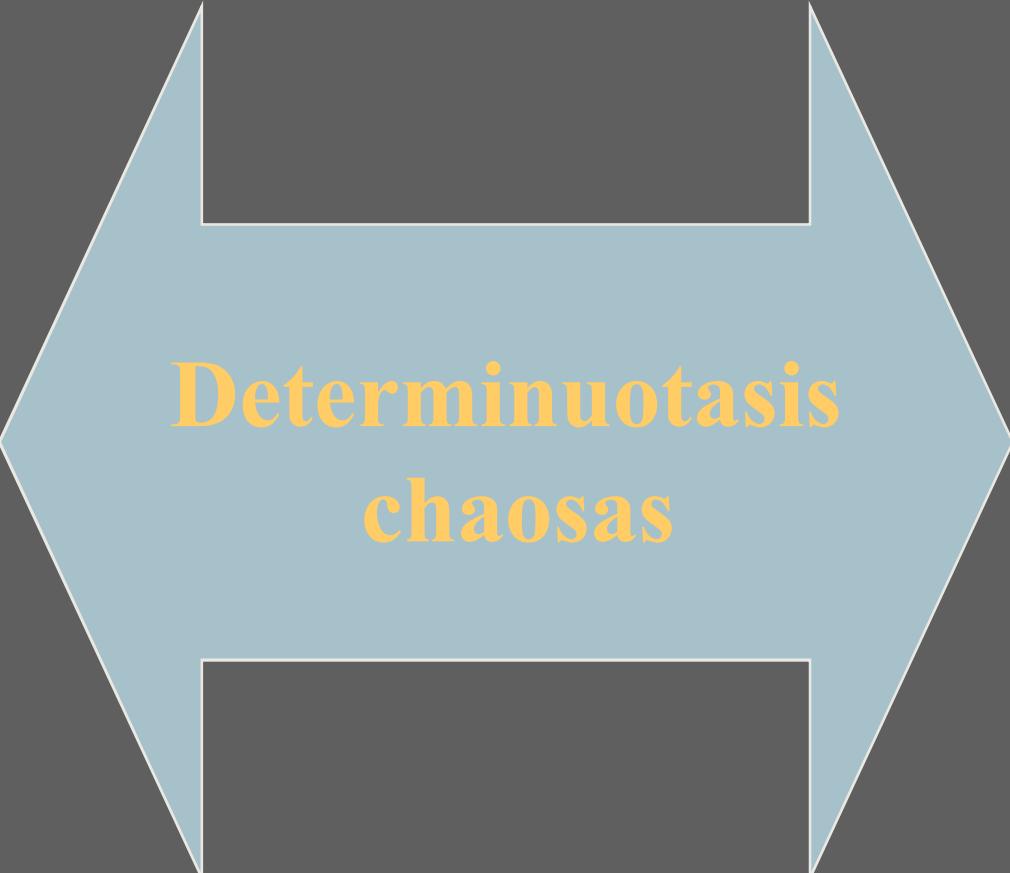
Savitvarka





Determinuotasis chaosas

Netvarka



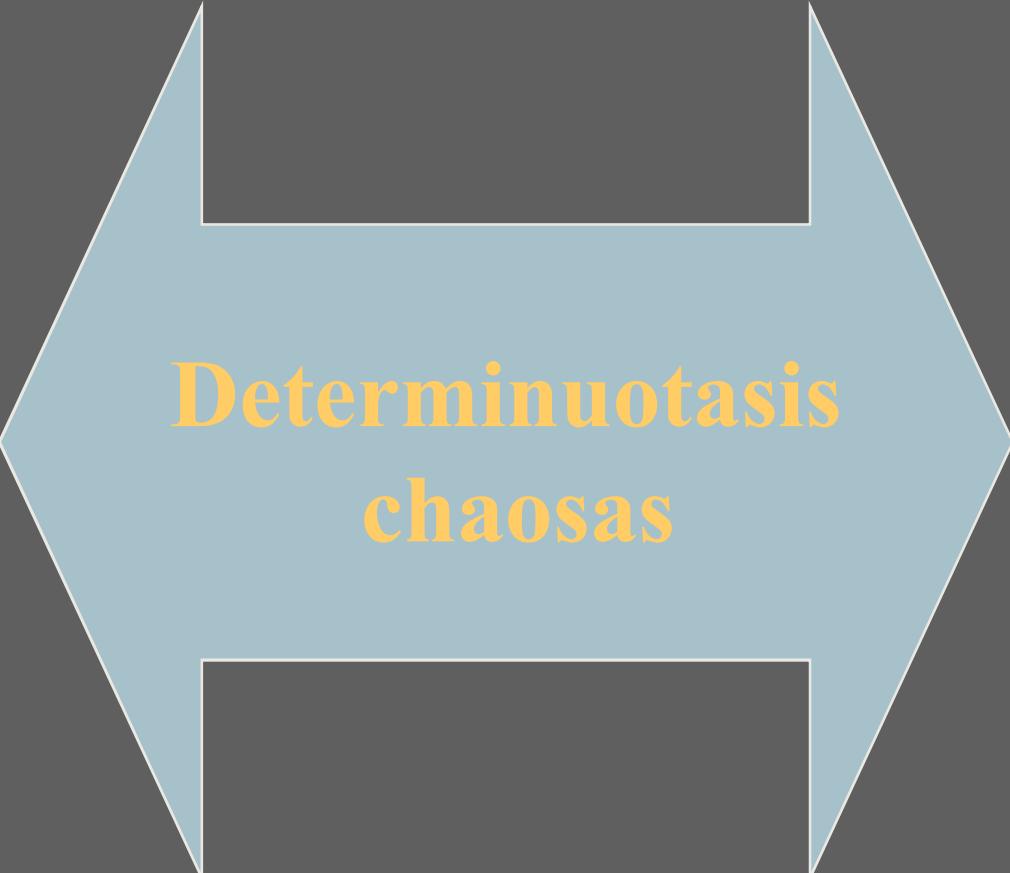
Determinuotasis
chaosas

Tvarka



Determinuotasis chaosas

Netvarka



Determinuotasis
chaosas

Tvarka

Anarchija – tvarkos motina!

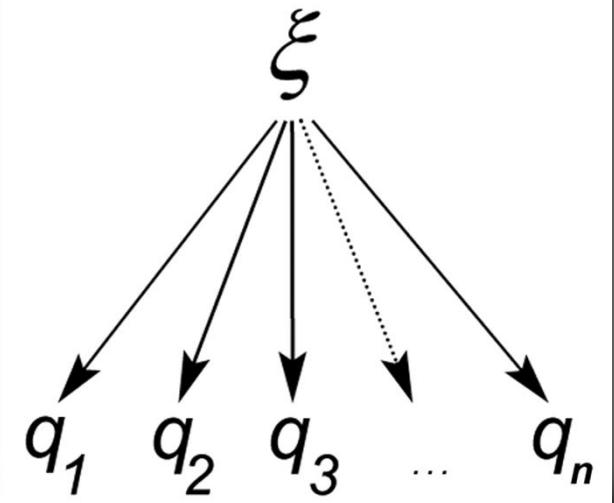
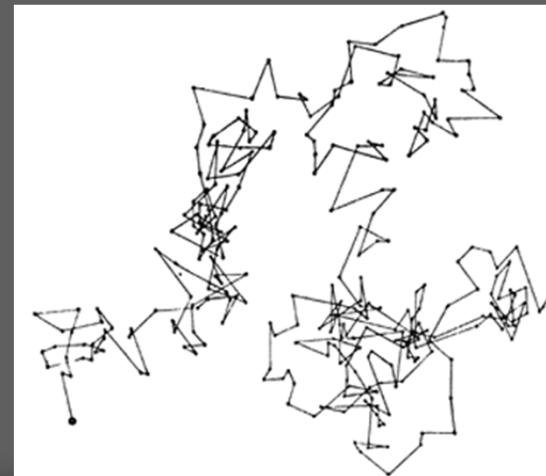
Determinuotas chaosas ir politika

Anarchija

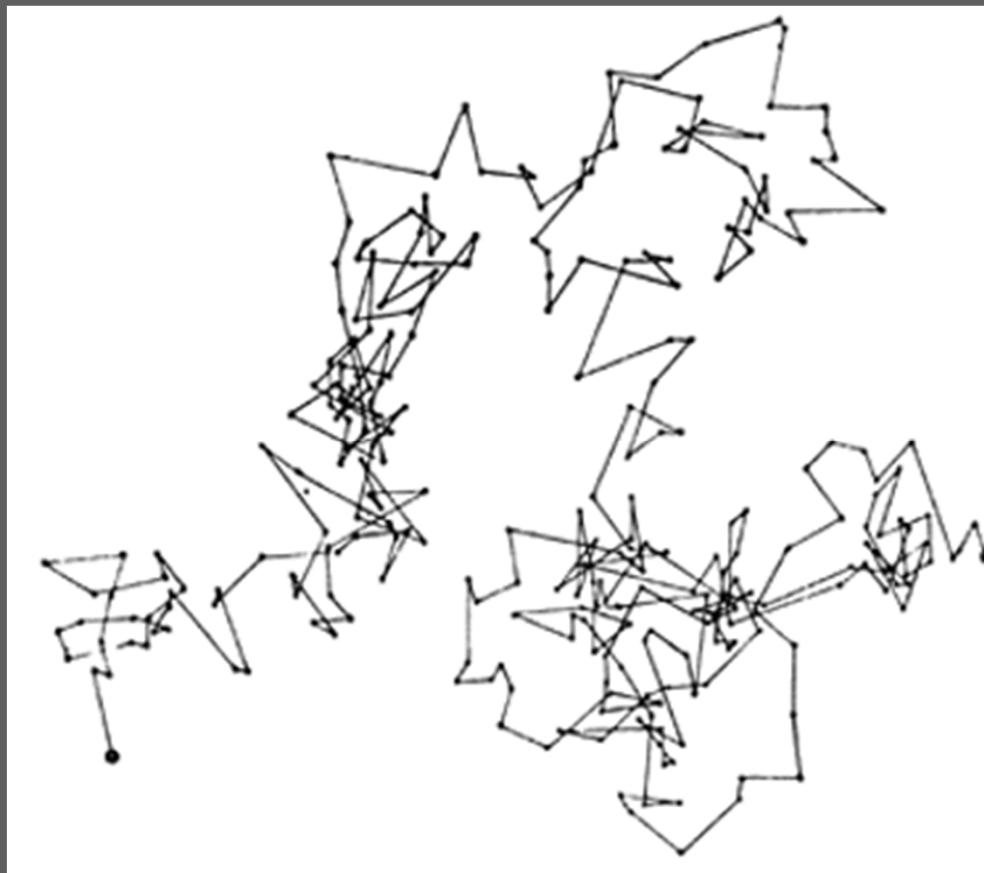
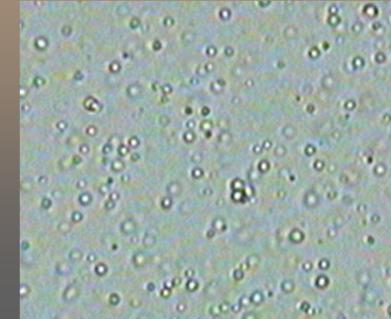
Demokratija

Diktatūra

Konsensusas



Chaosas mikropasaulyje - Brauno judėjimas



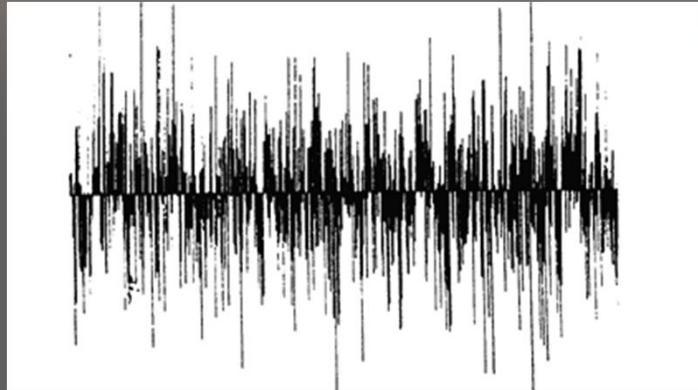
⇒ R. Braunas 1827

⇒ Žiedadulkių
judėjimo
trajektorijos
tirpale

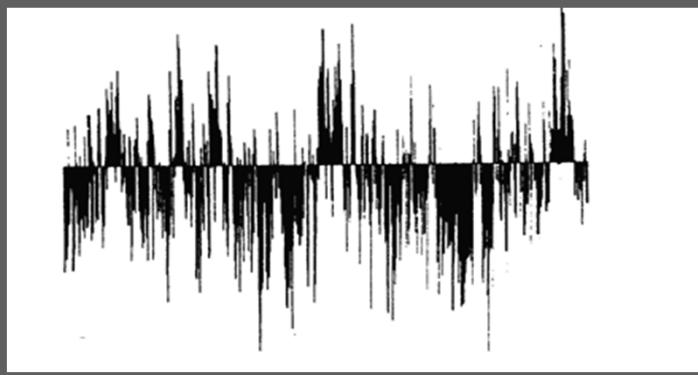
Chaoso rūšys

Triukšmas

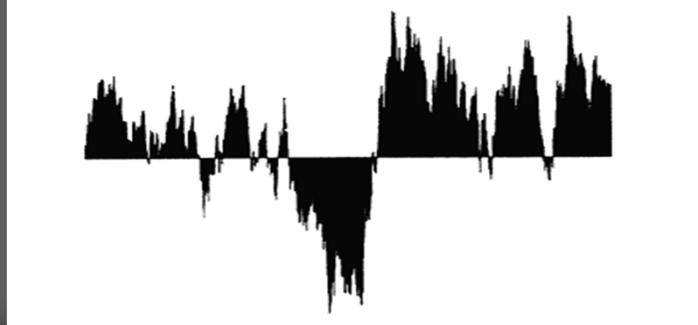
⇒ Baltasis triukšmas



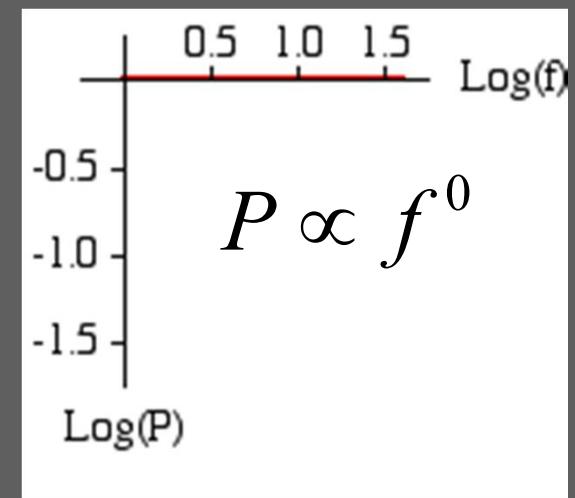
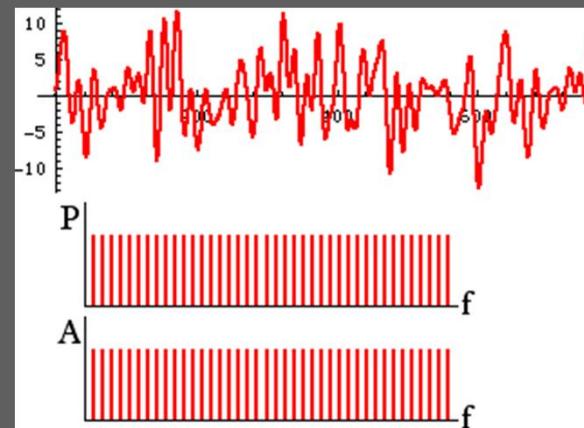
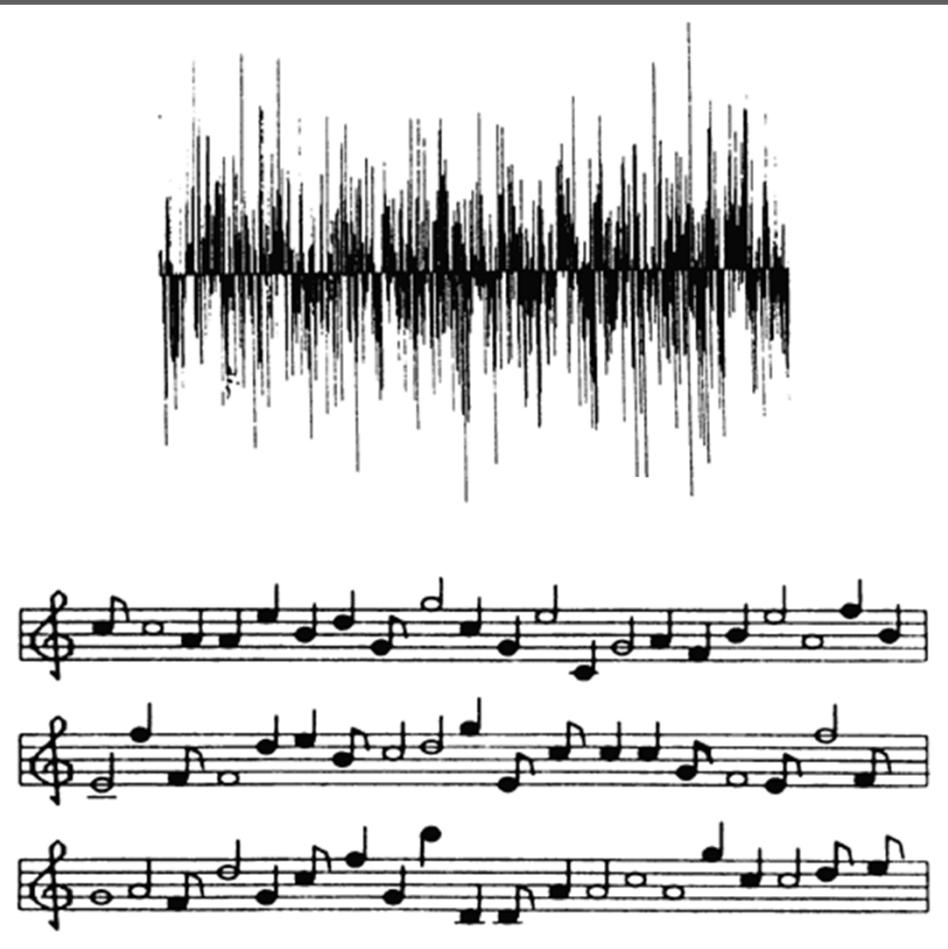
⇒ Rožinis triukšmas



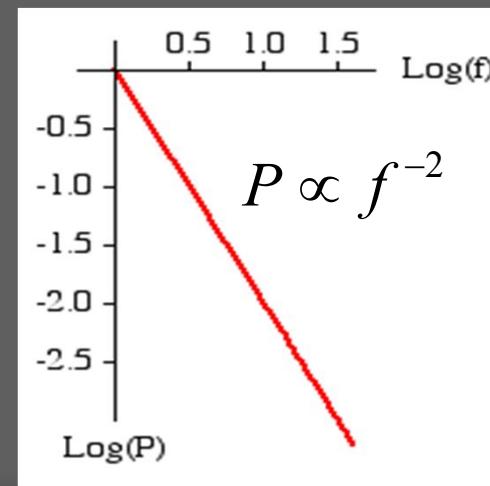
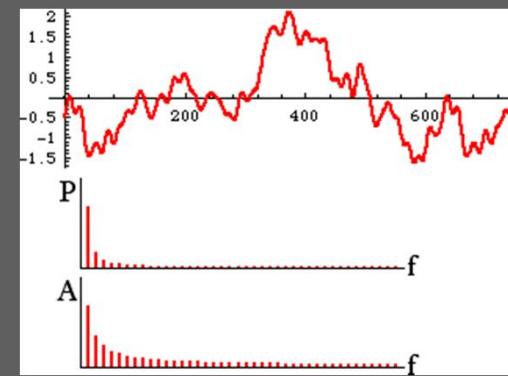
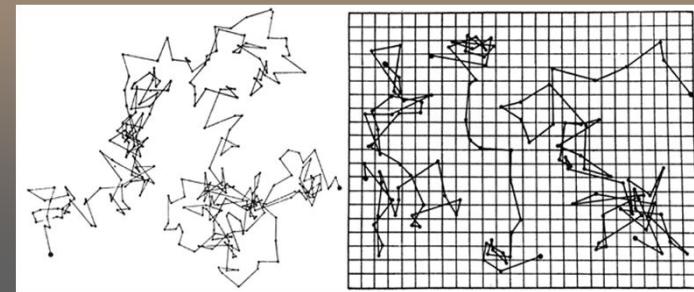
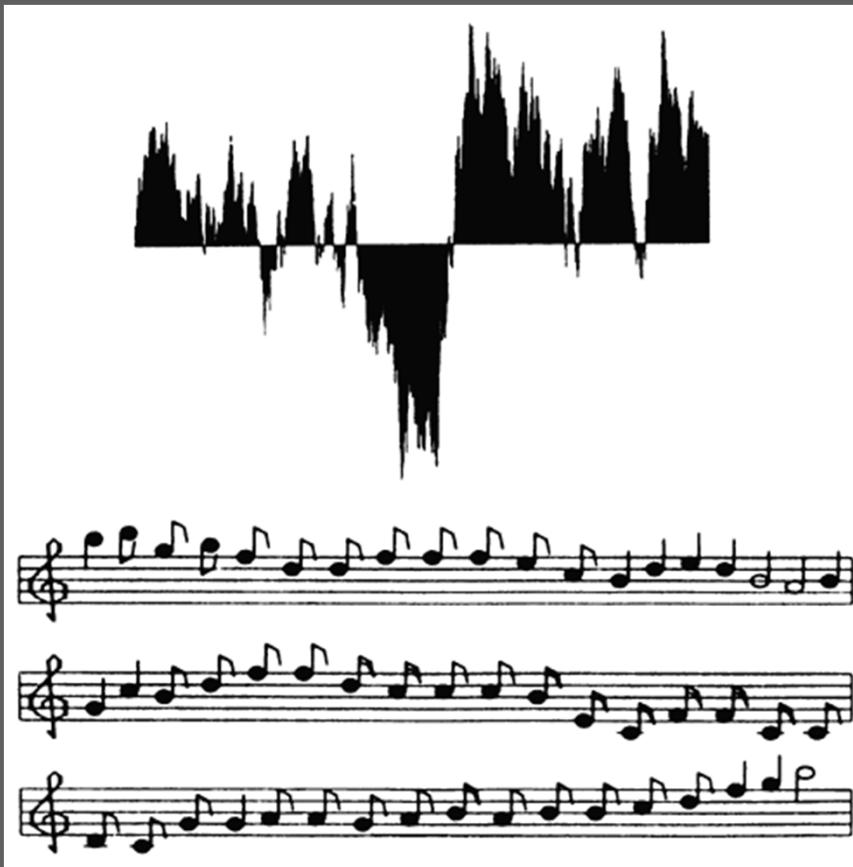
⇒ Rudasis triukšmas



Baltasis triukšmas

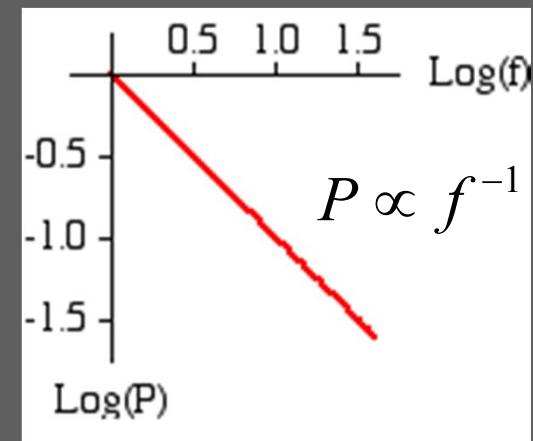
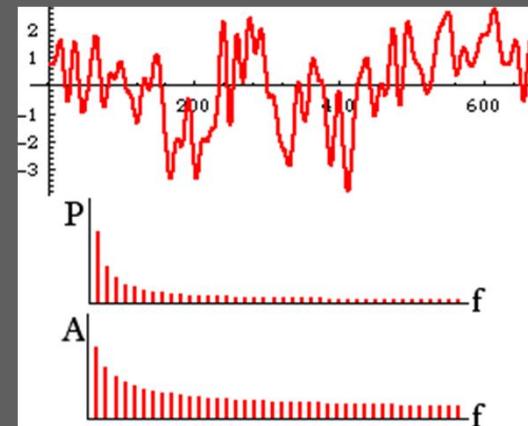
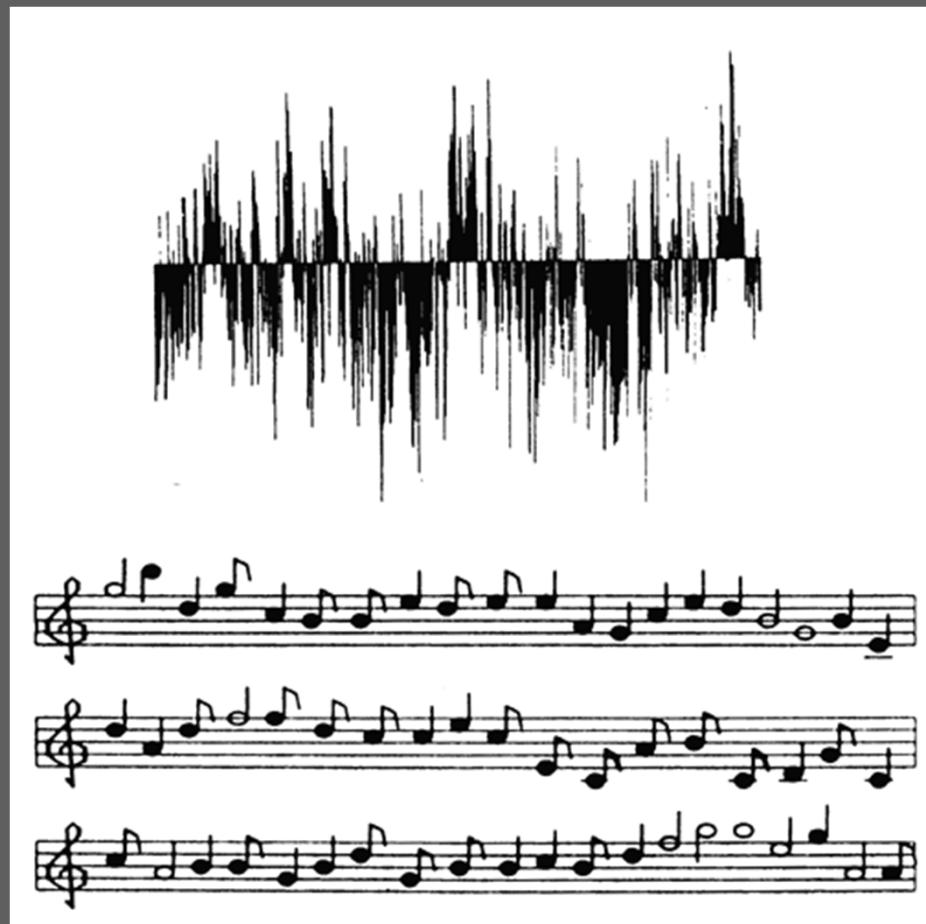
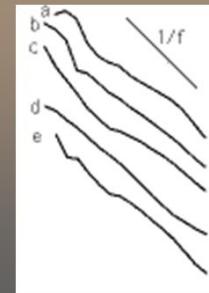


Rudasis triukšmas



Rožinis triukšmas

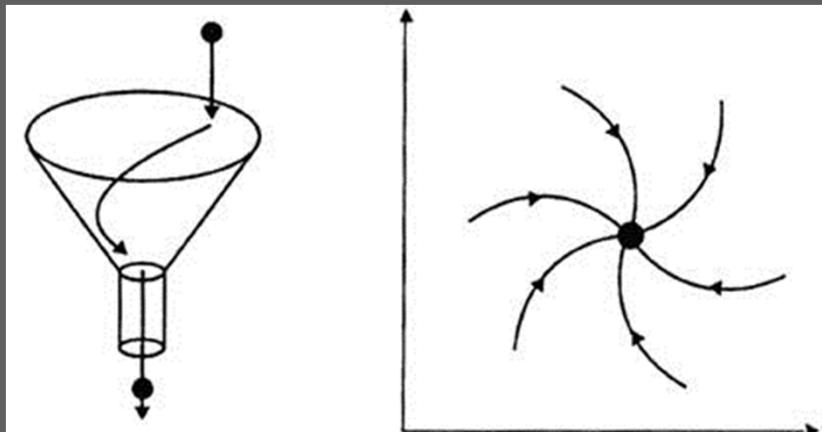
Ivairių tautų muzika
nuo senovės iki Bitlų



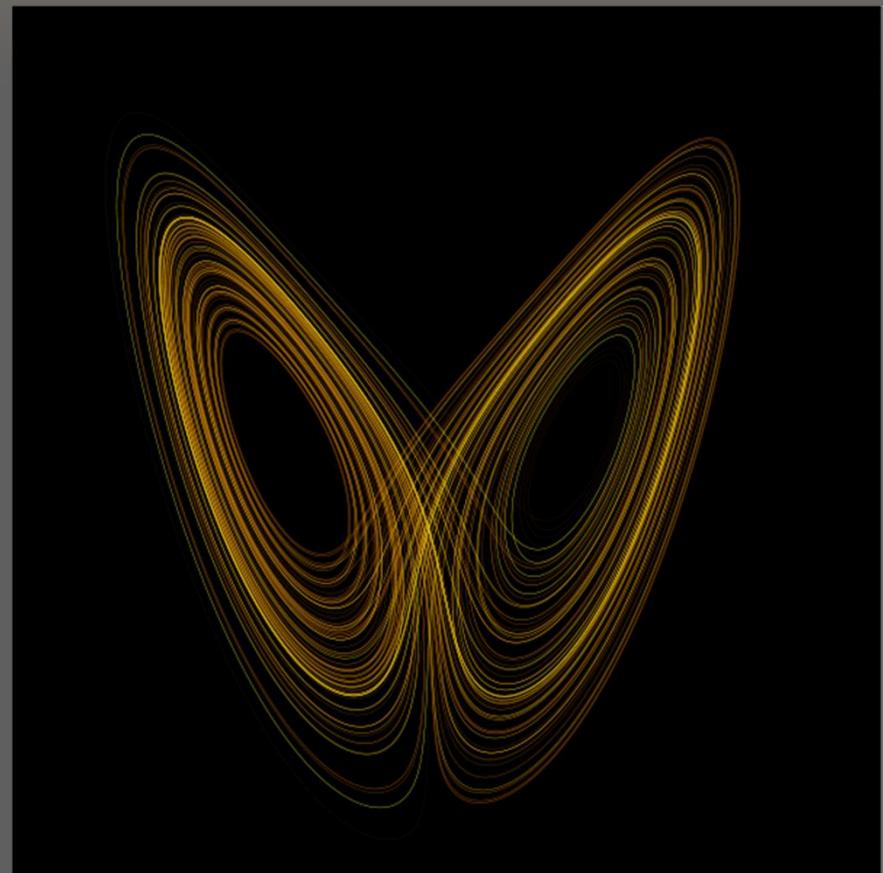
Atraktoriai

Chaotinės sistemos

Nuspėjamos sistemos



Kamuoliuko trajektorija piltuvėlyje imituoja taškinio atraktoriaus veikimą.
Kaip bemestume, kamuoliukas visada iškris per galą. Sprendinys visada nuspėjamas (apibrėžtas).



Lorenco keistasis atraktorius

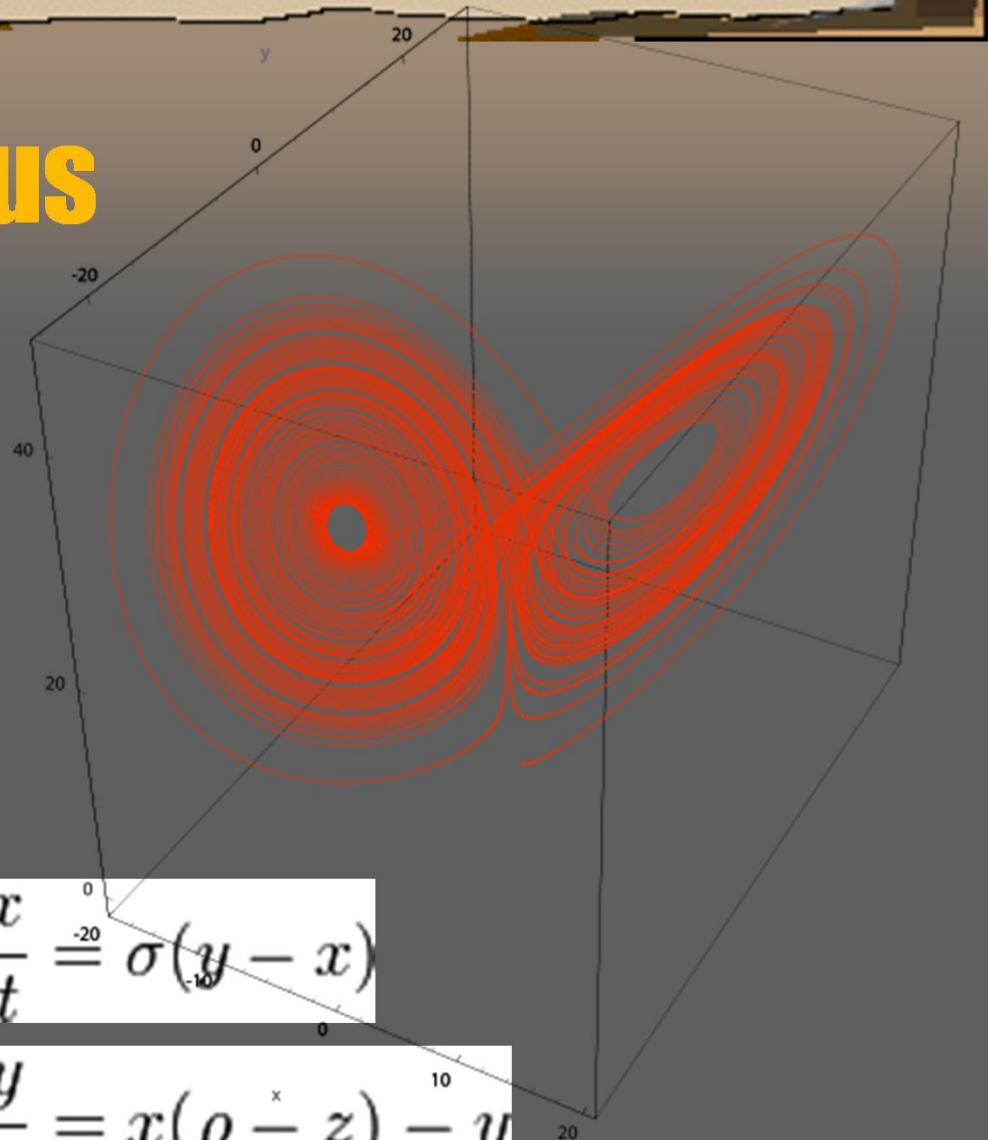
Lorenco atraktorius

Naudojamas sudarant orų prognozės modelius

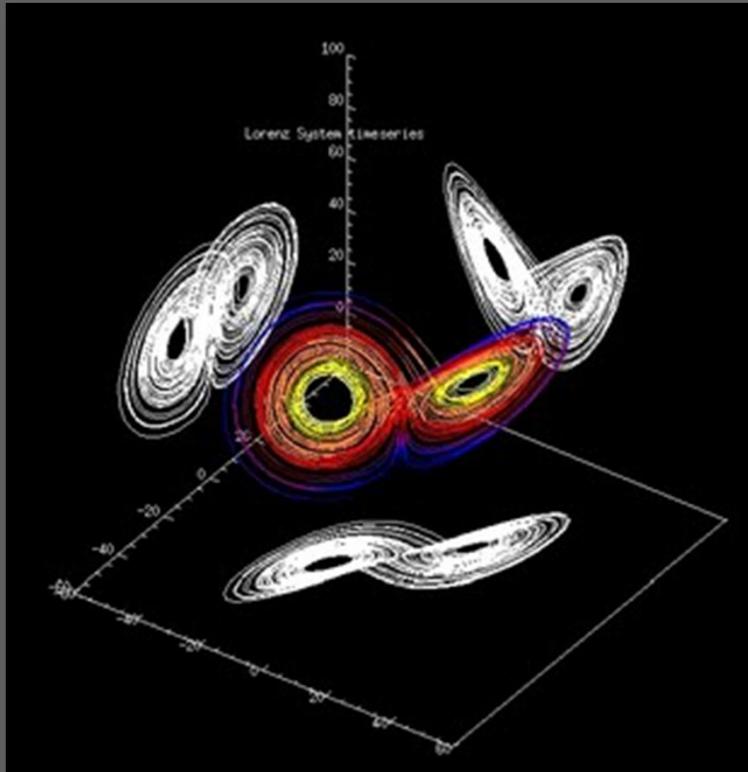
- ↪ Lorenco osciliatorius yra netiesinis, trimatis ir deterministinis. Tam tikroms parametru vertėms jis rodo chaotinę elgseną.

Nuspėjamumas
tik trumpu
periodu

$$\begin{aligned}\frac{dx}{dt} &= \sigma(y - x) \\ \frac{dy}{dt} &= x(\rho - z) - y \\ \frac{dz}{dt} &= xy - \beta z\end{aligned}$$

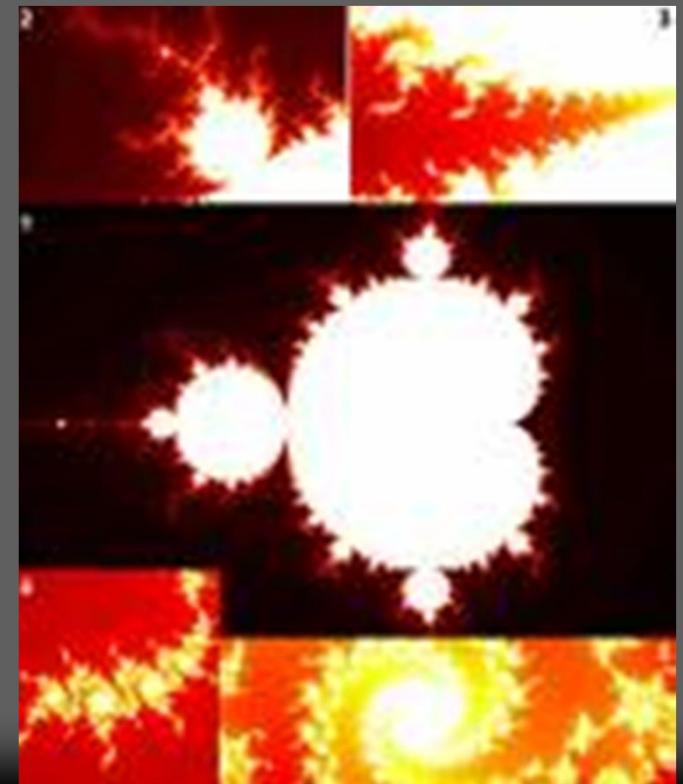


Netiesinės dinaminės struktūros



⇒ **Chaosas** – nereguliarumai laike

⇒ **Fraktalai** – nereguliarumai erdvėje



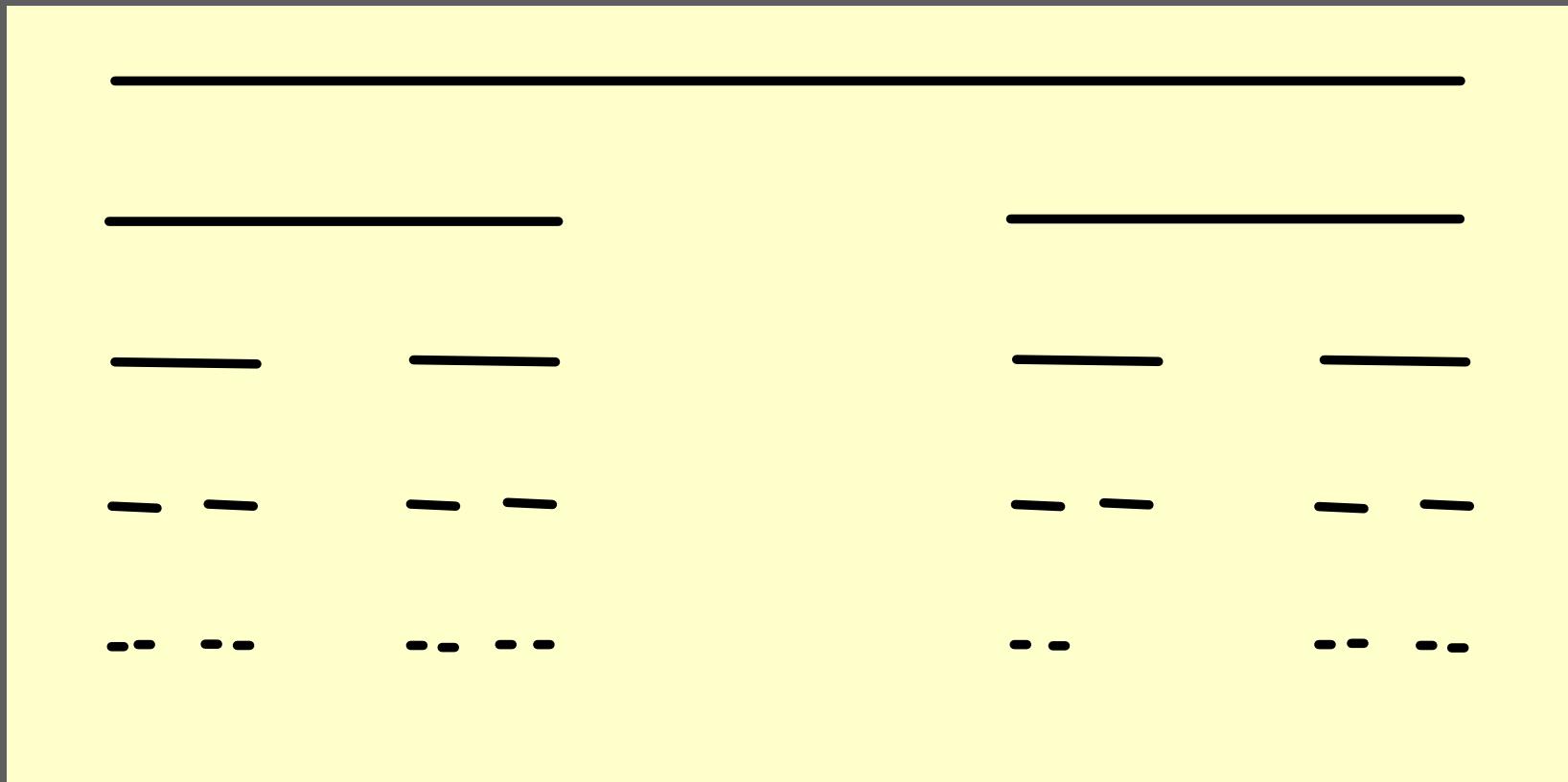
Fraktalai



G.Kantoro seka

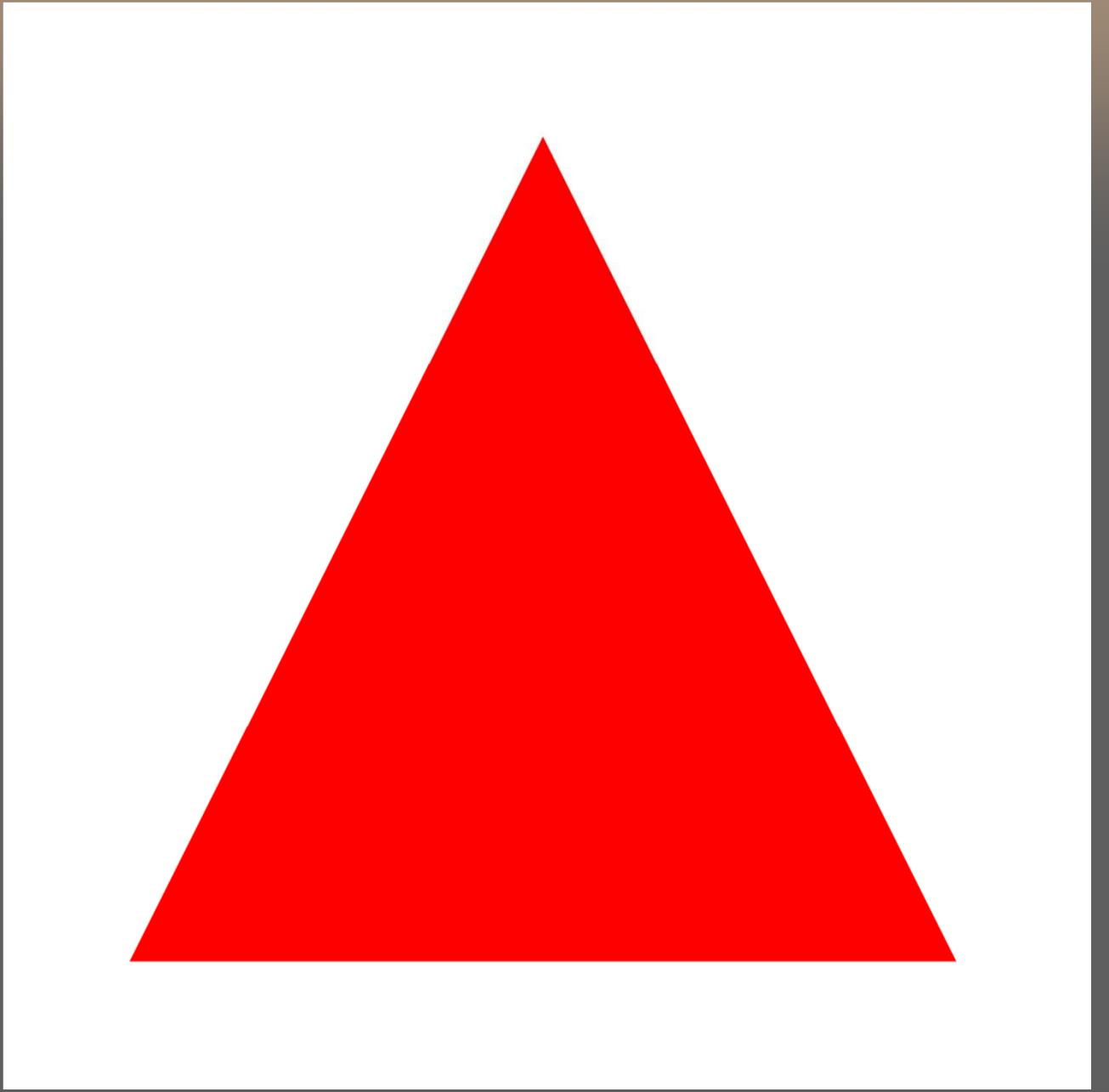
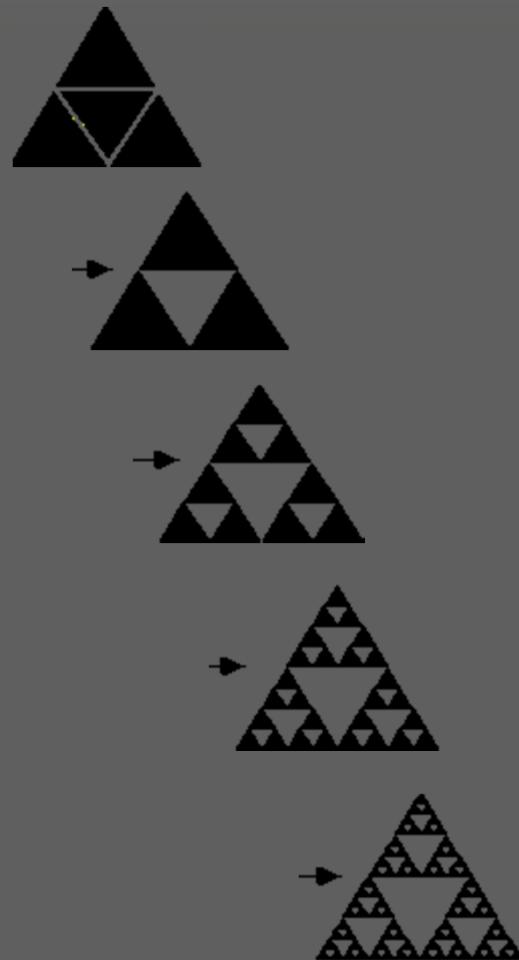


1845-1918

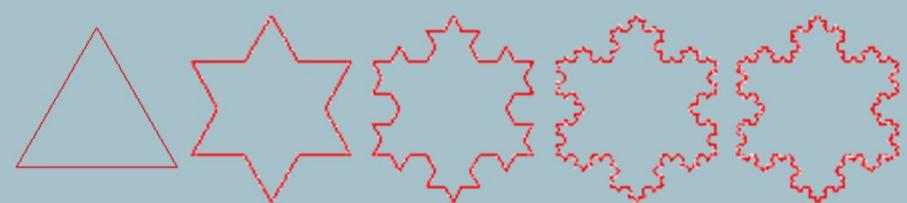
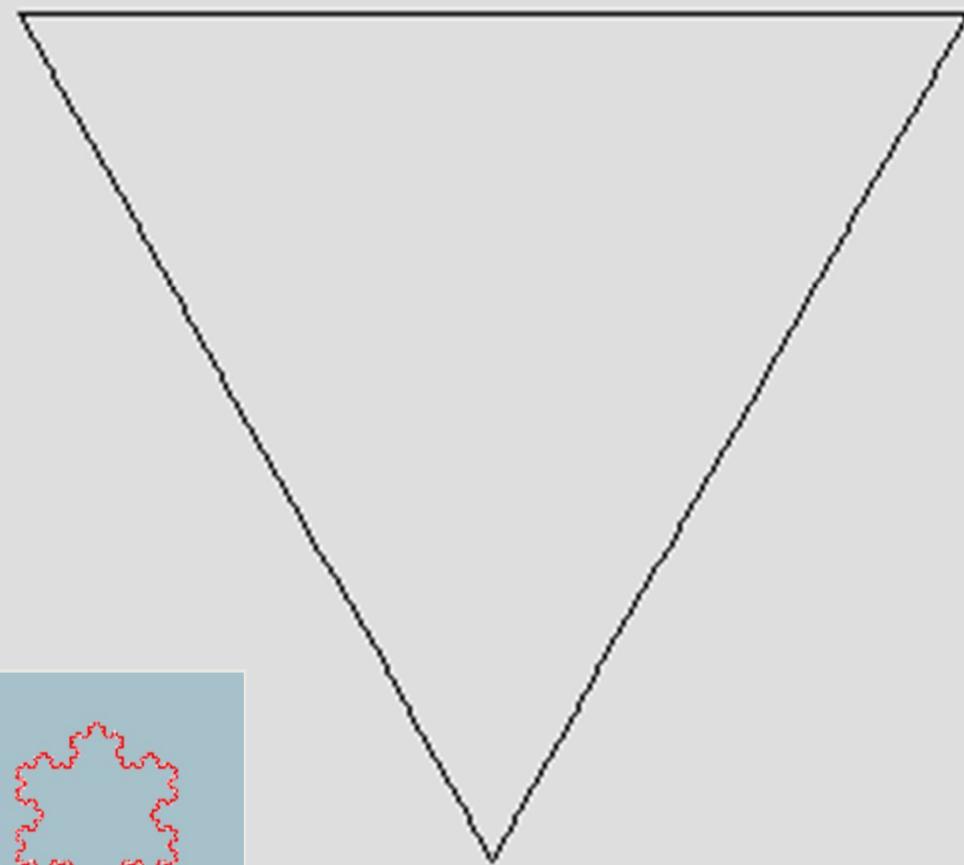


$0 < D=0.63 < 1$

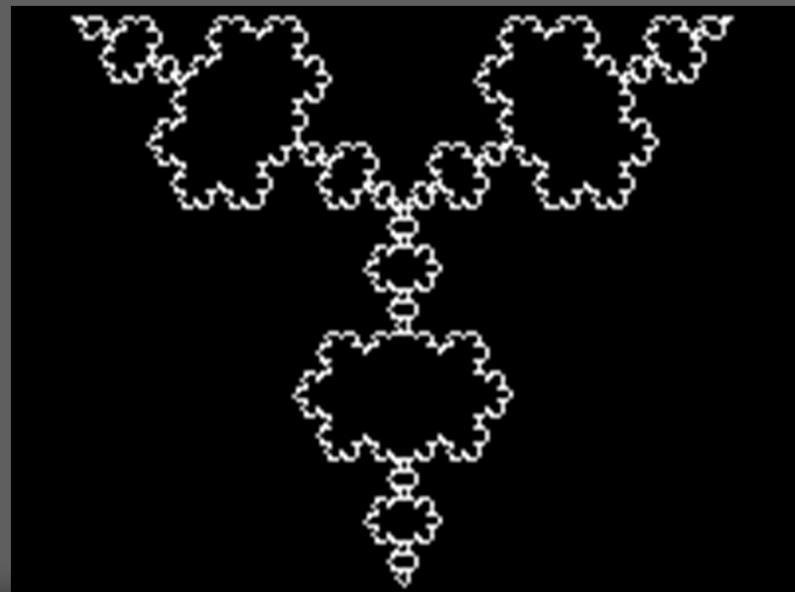
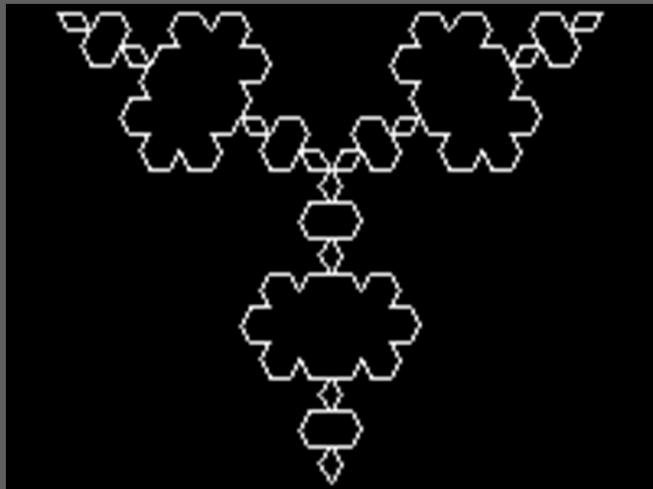
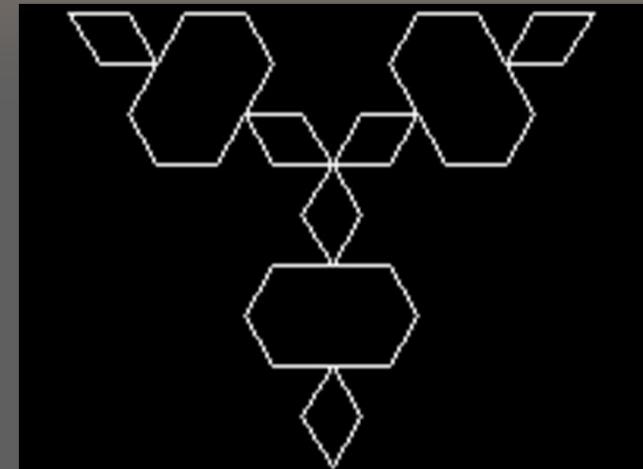
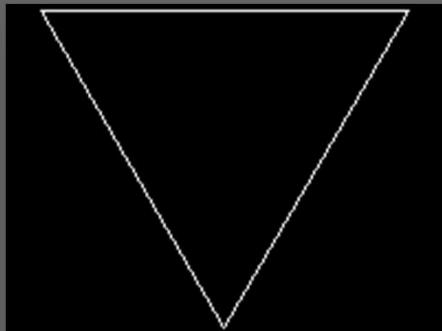
Sierpinskio trikampis



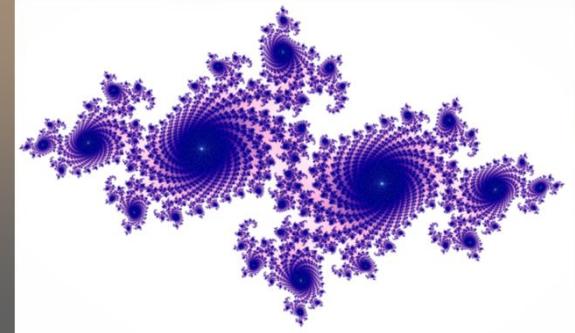
Kocho Snaigē



Kocho antisnaigé



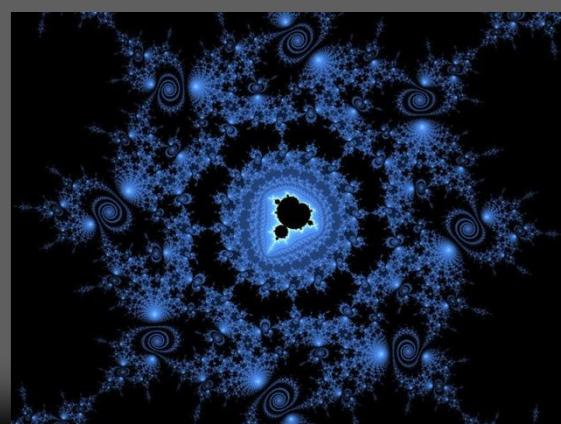
Julia ir Fatou seka



- ⦿ **Gaston Maurice Julia** (February 3, 1893 – March 19, 1978) was a French mathematician who devised the formula for the Julia set.
 - The article, which he published during 1918 at the age of 25, titled "*Mémoire sur l'itération des fonctions rationnelles*" described the iteration of a rational function.
- ⦿ **Pierre Joseph Louis Fatou** (28 February 1878,– 10 August 1929,) was a French mathematician working in the field of complex analytic dynamics.

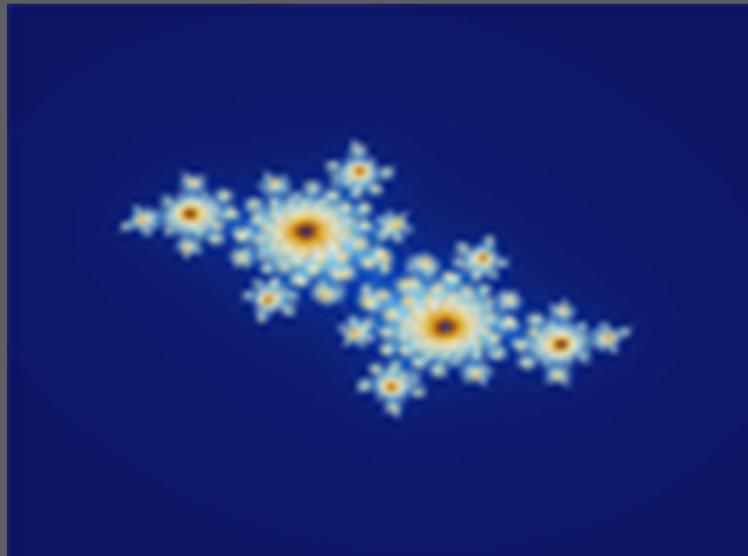


$$f_c(z) = z^2 + c$$

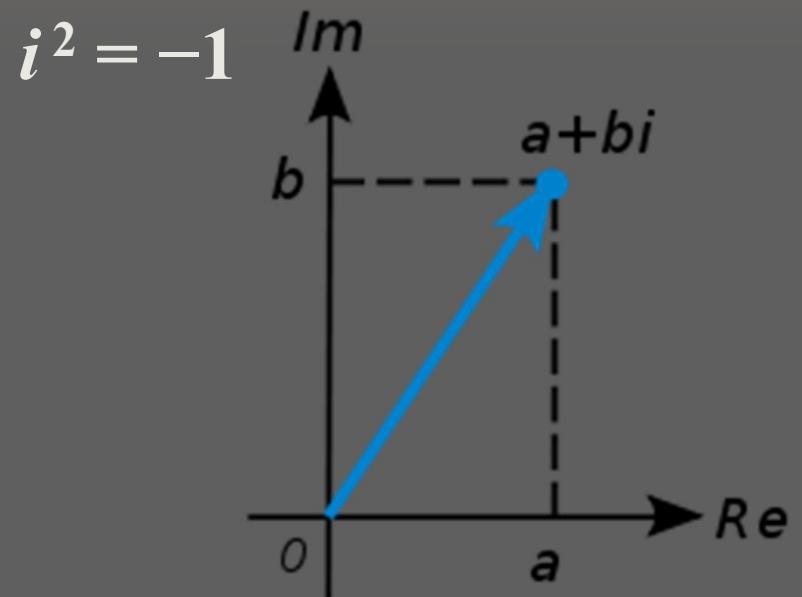


Julia seká

$$f_c(z) = z^2 + c$$



$$Z = x + iy$$



A complex number can be visually represented as a pair of numbers forming a vector on a diagram called an Argand diagram, representing the complex plane.

Wikipedia

Fraktalai



Benoît Mandelbrot
20 Lapkričio 1924

Gimė Varšuvoje
Lietuvos žydų šeimoje

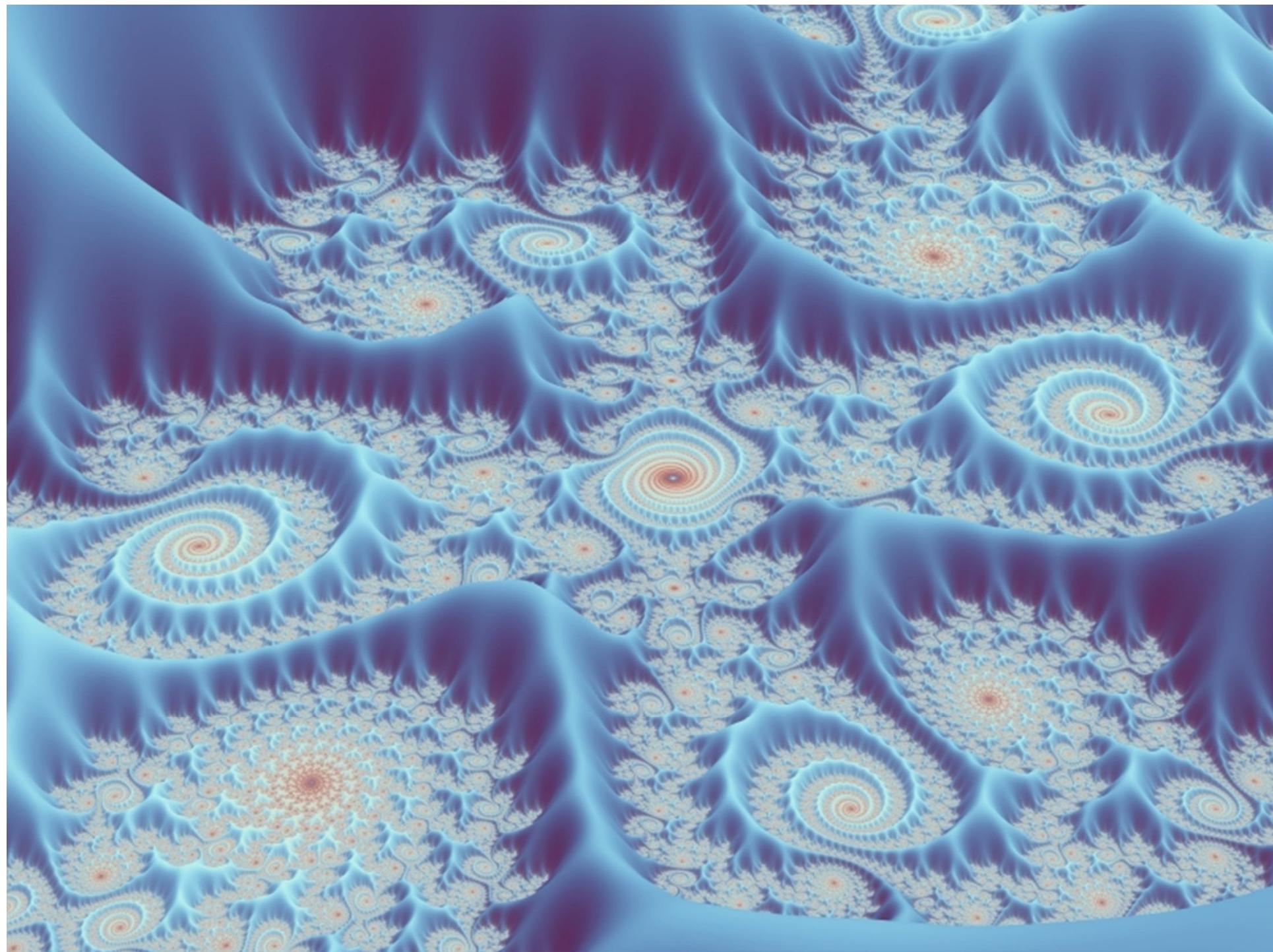
1979, Mandelbrot studijavo fraktalus vadinamus
Julia seka

In 1982, Mandelbrot perkėlė savo idėjas į
IBM

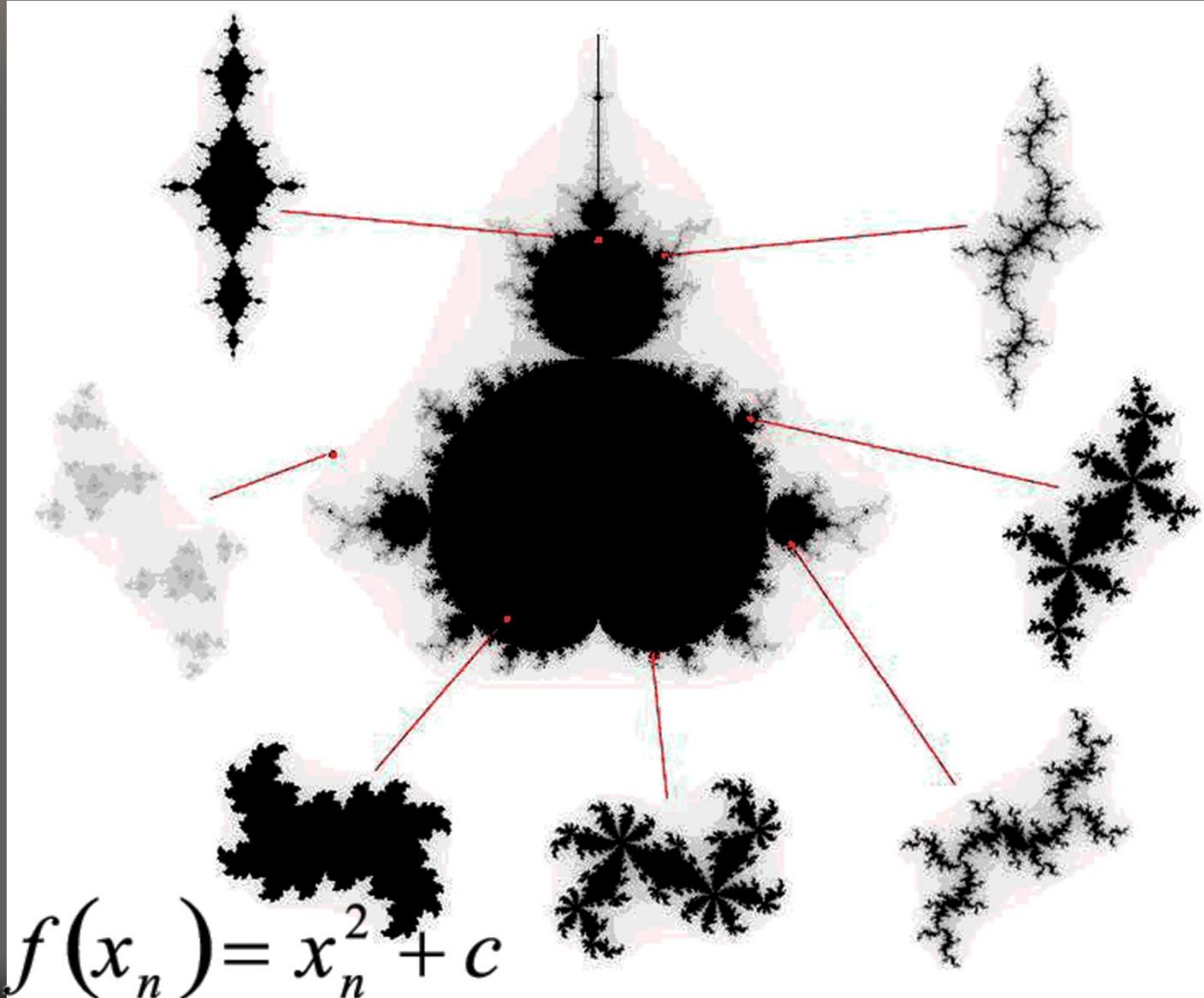
Fraktalinė Gamtos geometrija.

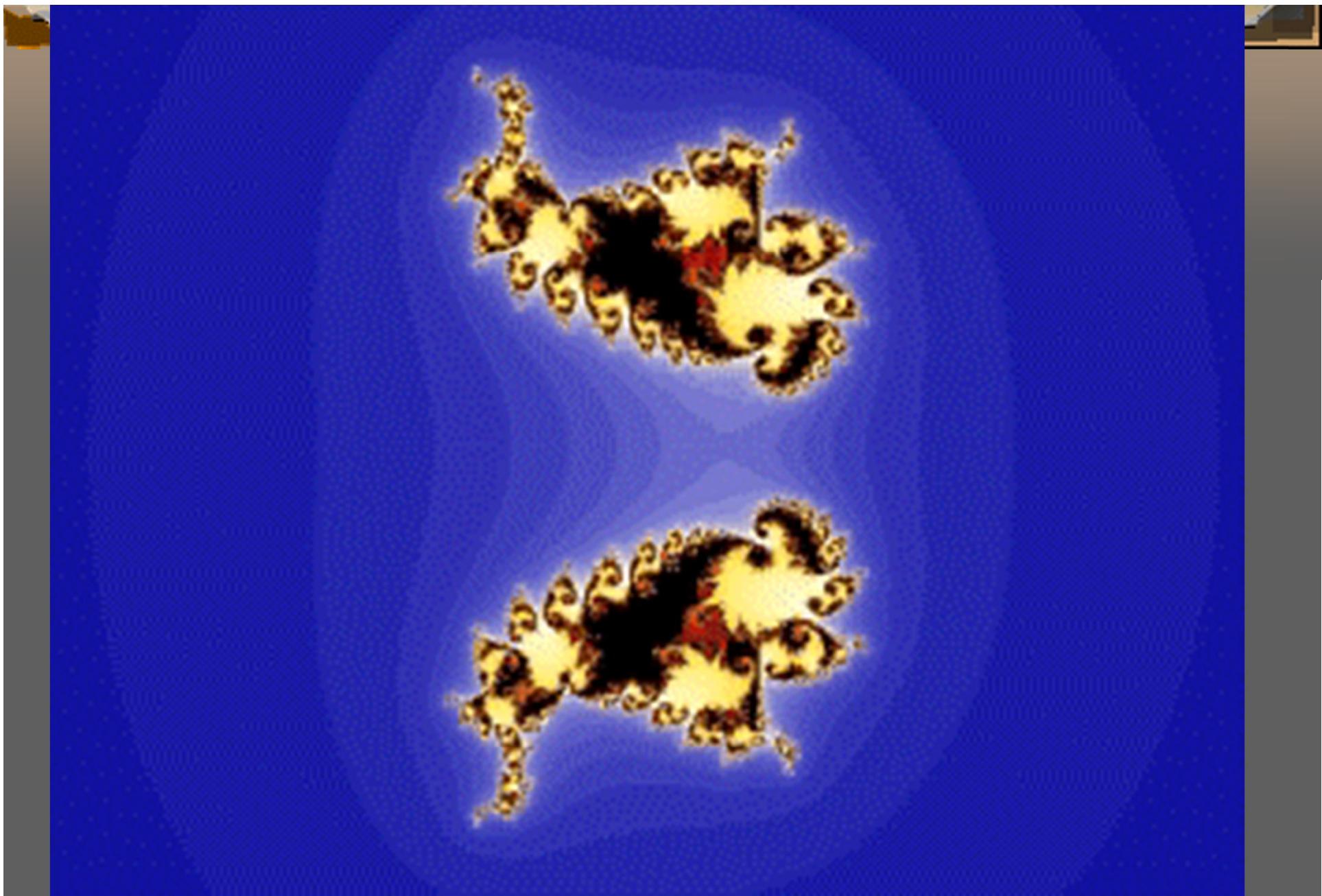


Mandelbrot kalba École
Polytechnique 2006



Mandelbro (Julia) seka

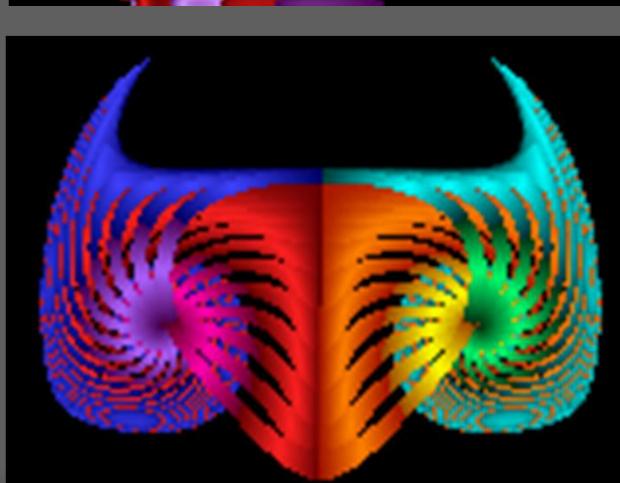
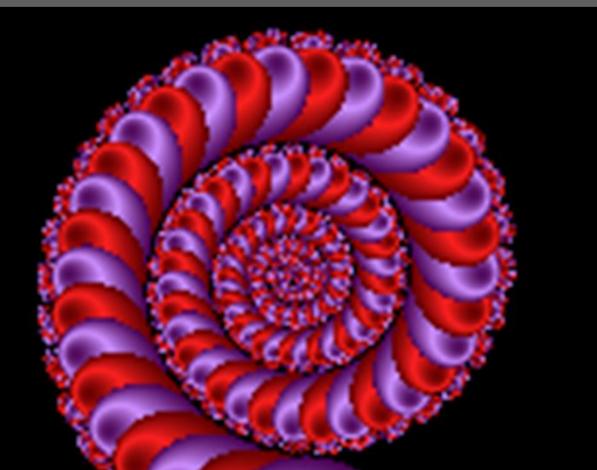
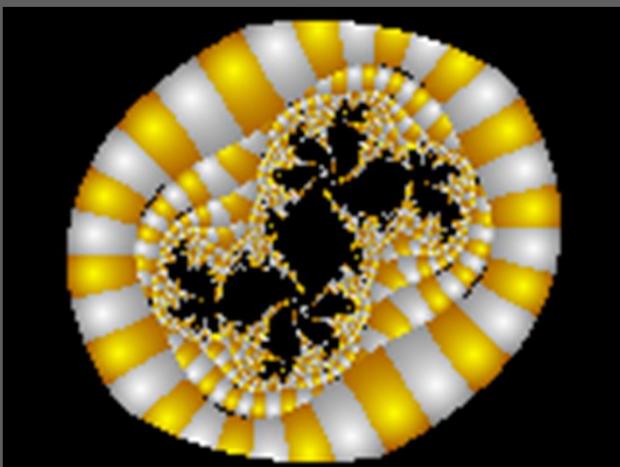




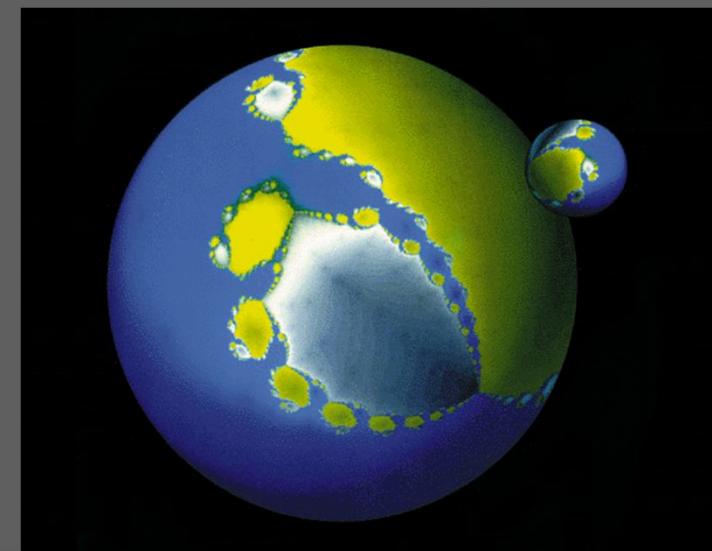
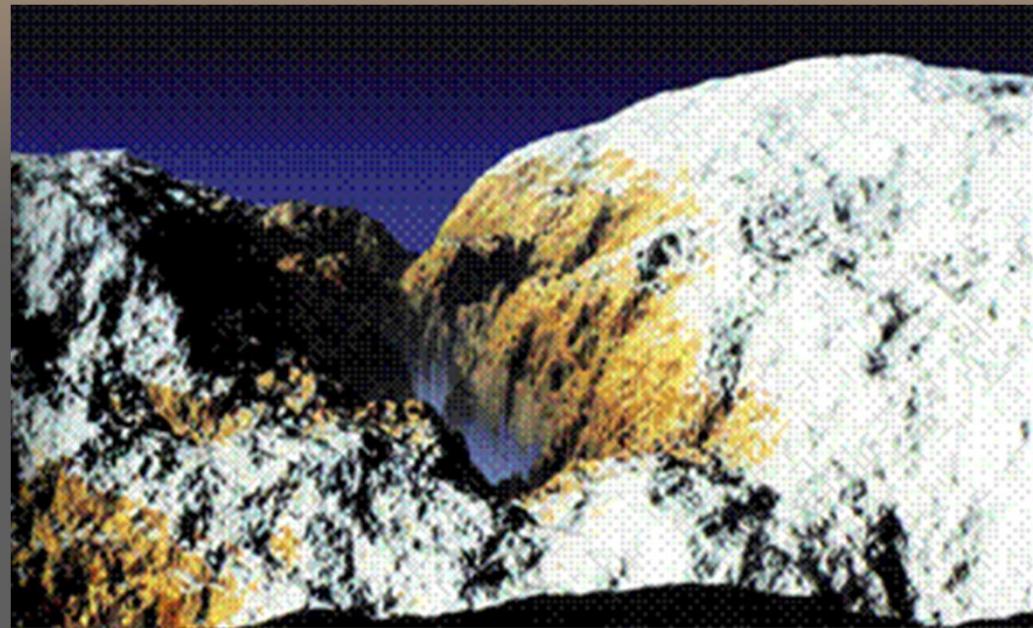
$$x_{n+1} = f(x_n) = x_n^2 + c \quad x_0 \rightarrow x_0^2 \rightarrow x_0^4 \rightarrow x_0^8 \rightarrow \dots$$

Netiesinių lygčių turtai

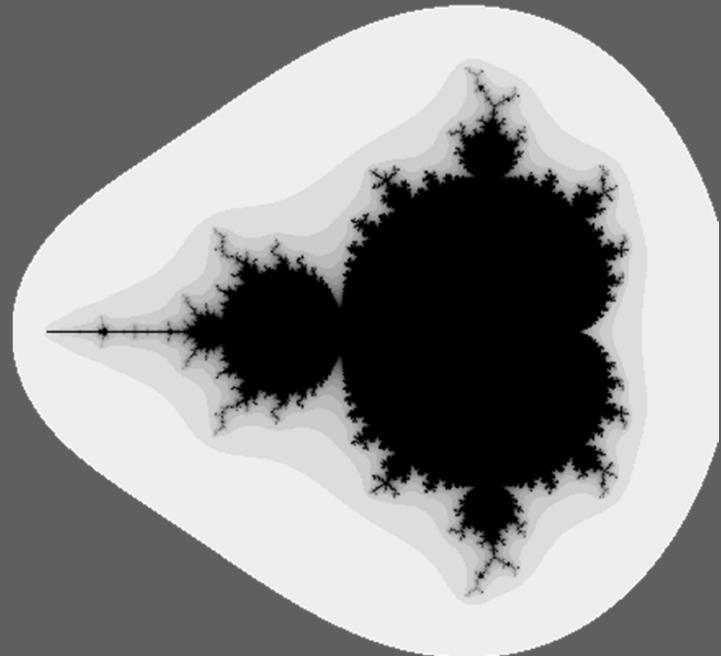
B.Mandelbro seka $x_{n+1} = f(x_n) = x_n^2 + c$



Fraktalų vaizdai



Nežinomas XIIa. prancūzų dailininkas



Panašumo simetrija



Šie stačiakampiai
nėra panašūs

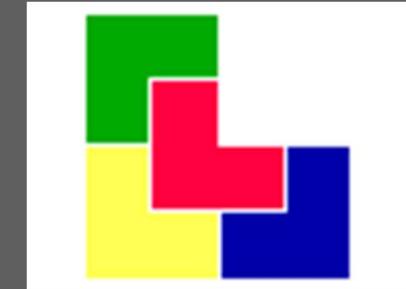
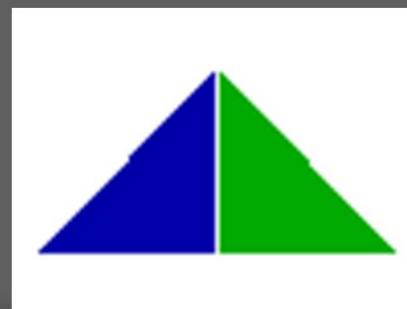
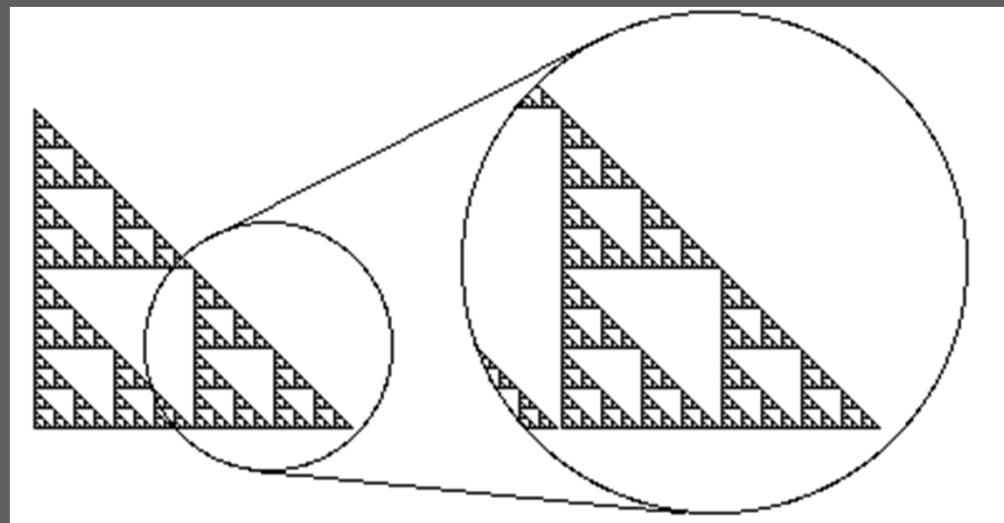
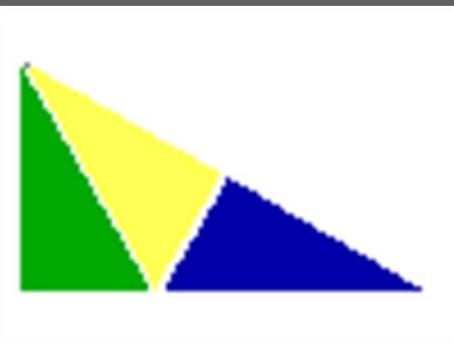


Šie kvadratai panašūs



Šie stačiakampiai panašūs

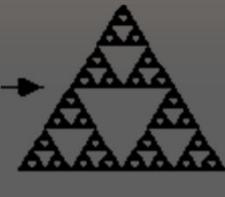
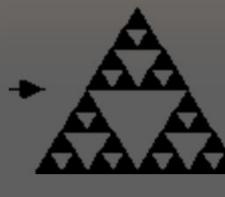
Mastelio simetrija



Fraktalų dimensija

0D

•A



1D

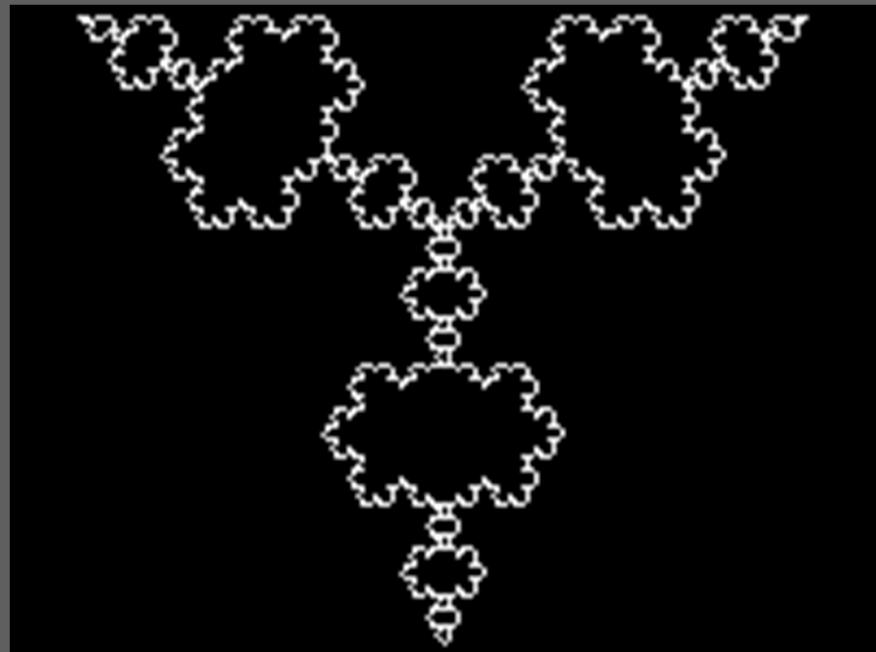
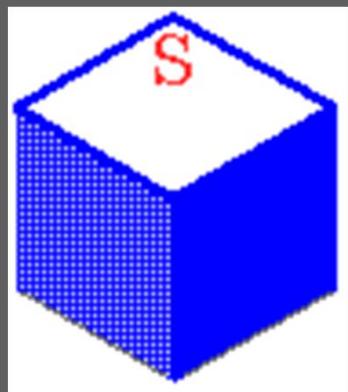


1,58D

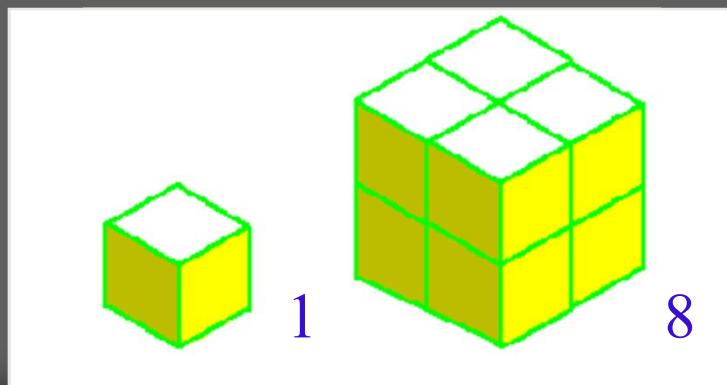
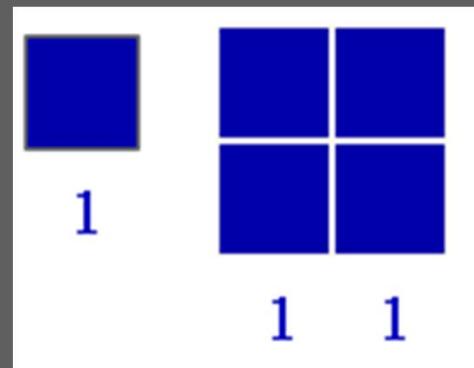
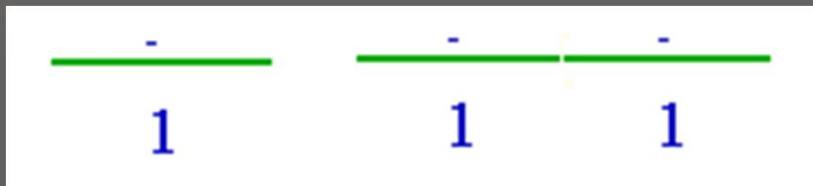
2D



3D



Mastelis ir matiškumas



Dvigubiname

Imkime panašias figūras ir digubinkime matmenis, pavyzdžiu i imkime atkarpa ir digubinkime ilgi. Reikės dviejų segmentų.

Kvadrato kraštinės dvigubinimas.
Gauname kvadratą, kuris sudarytas iš keturių mažesnių kvadratų.
Dvigubinimui reikia keturių kopijų.

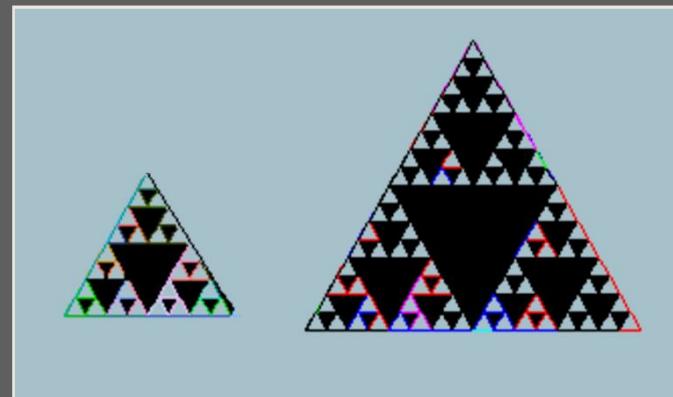
Kubo dvigubinimui reikia aštuonių kopijų.

Dimensija

Figūra	Dimensija	Kopijų skaičius
Atkarpa	1	$2 = 2^1$
Kvadratas	2	$4 = 2^2$
Kubas	3	$8 = 2^3$
Dvigubinimo panašumas	d	$n = 2^d$

Sierpinski trikampis

Figūra	Dimension	No. of Copies
Atkarpa	1	$2 = 2^1$
Sierpinski trikampis	?	$3 = 2^d$
Kvadratas	2	$4 = 2^2$
Kubas	3	$8 = 2^3$
Dvigubinimo panašumas	d	$n = 2^d$



Kraštinės dvigubinimas duoda tris kopijas, taigi $3 = 2^d$, čia d = dimensija.

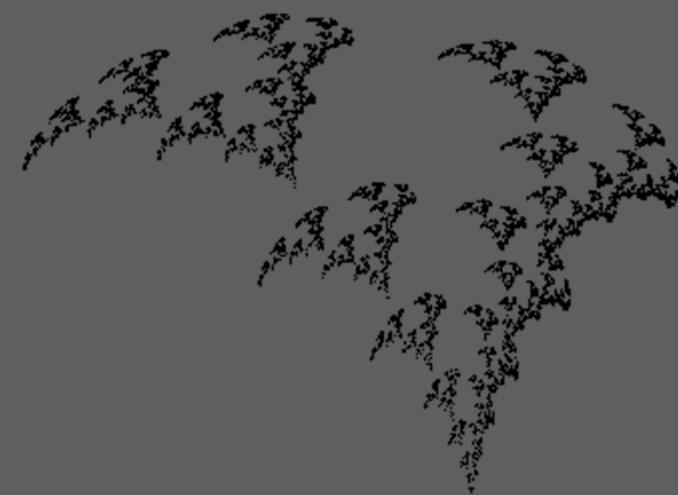
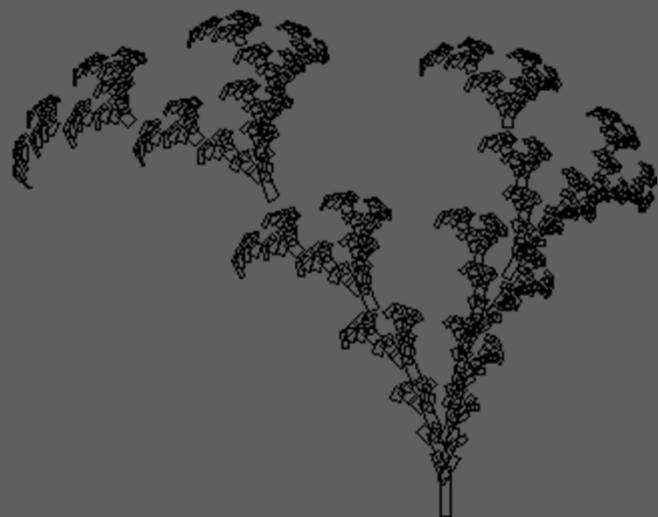
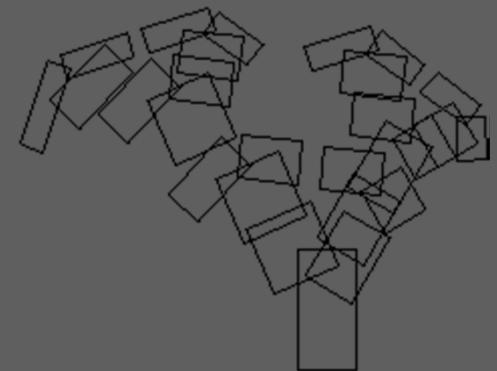
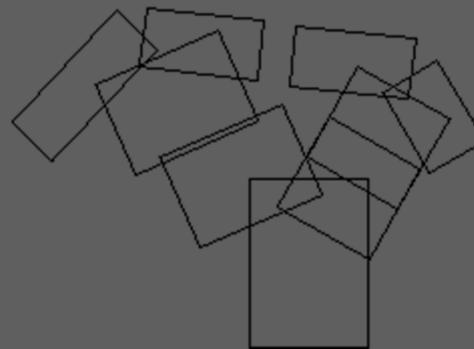
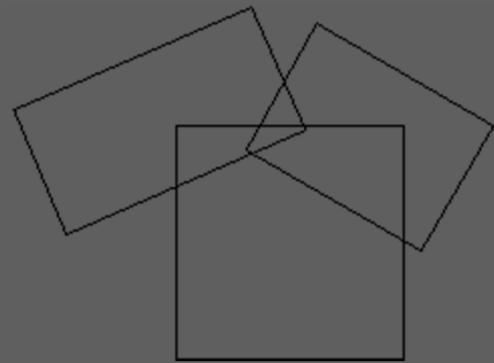
$$d=1.58$$



**2D -
3D**



Žaidimai



Žaidimai

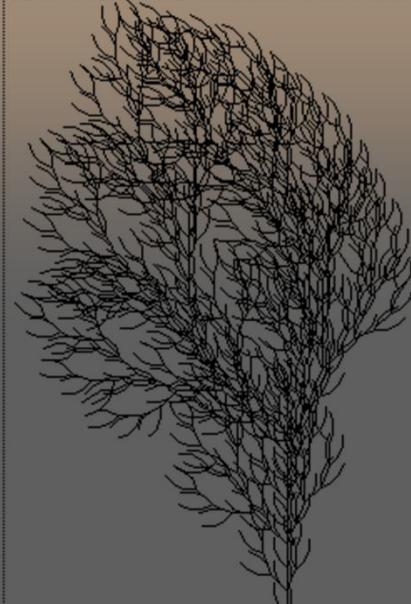
Axiom X
F → FF
X → F-
[(X)+X]+F[+FX]-X
 $\theta = 22.5$



L String length = 491
Turning angle = 20
Axiom (depth=0) = X
F → FF
X → F[+X]F[-X]+X



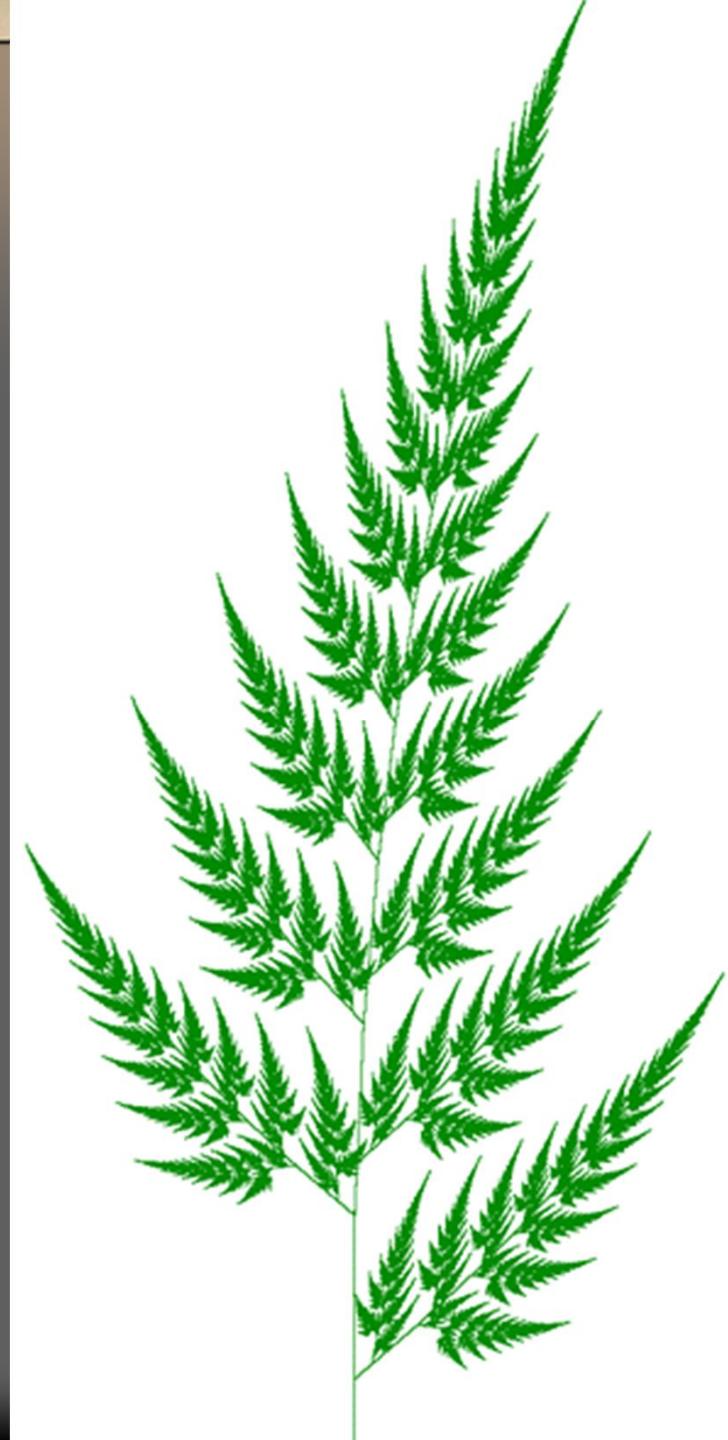
F → FF+[+F-F-F]-[-F+F+F]
Turning angle = 22.5
Axiom (depth=0) = F



Turning angle = 20
Axiom (depth=0) = VZFFF
F → F
V → [++W][---W]VV
W → +X[-W]Z
X → -W[+X]Z
Y → YZ
Z → [-FFF][+FFF]F

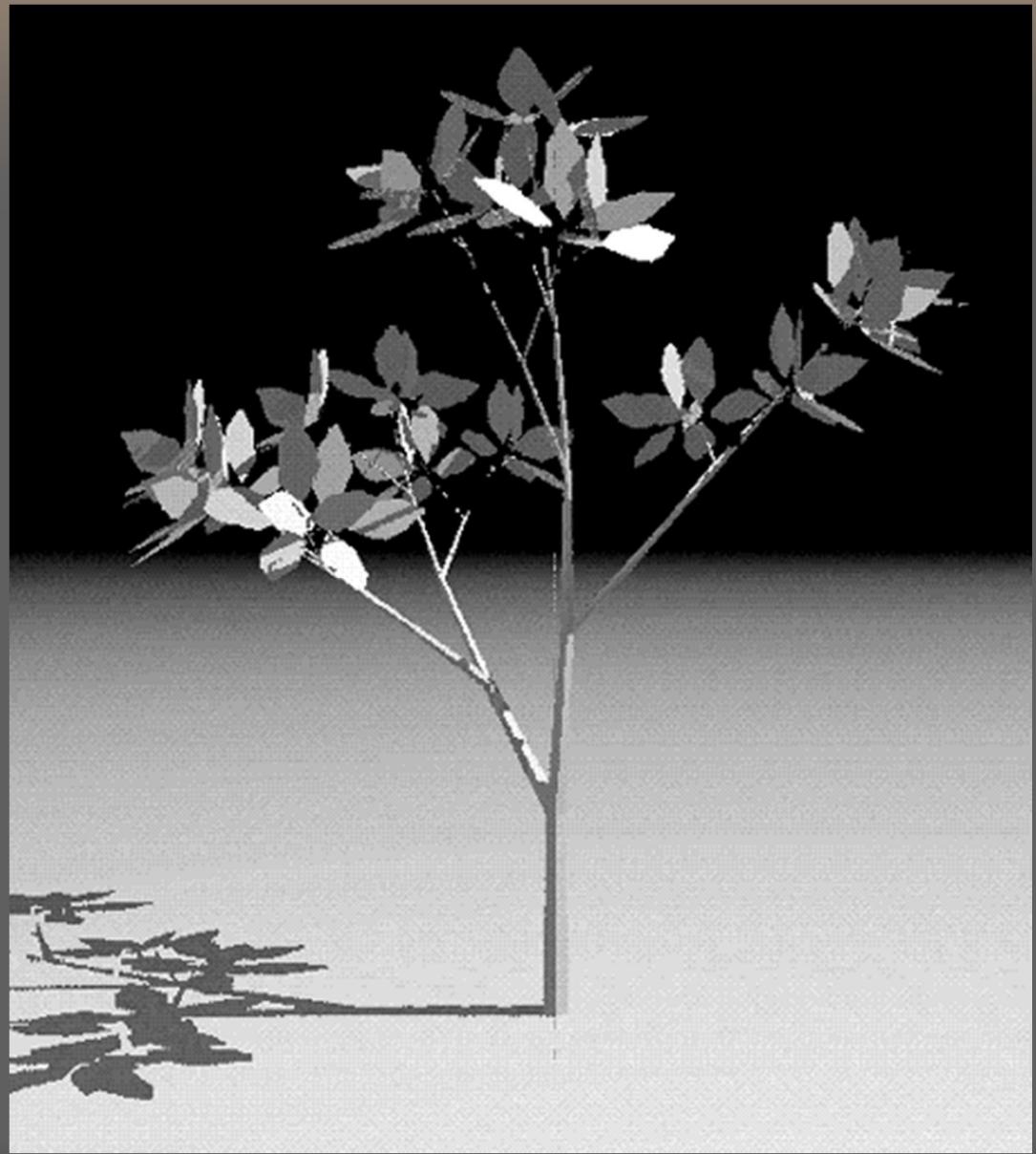


Dirbtinis papartis

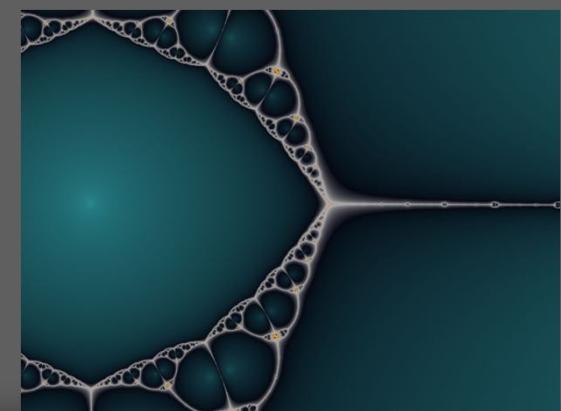
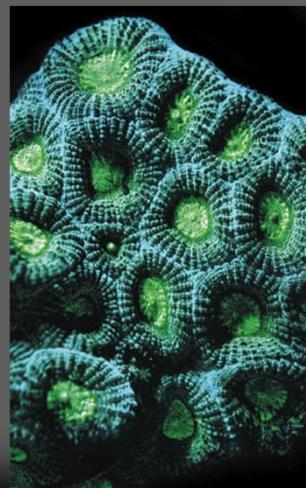
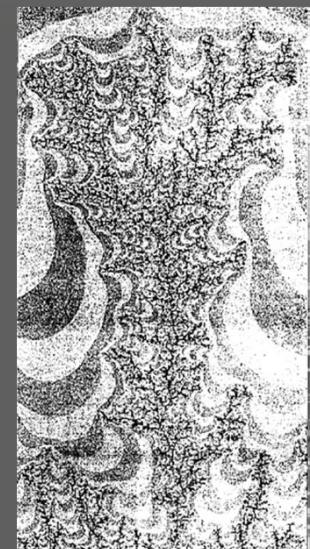
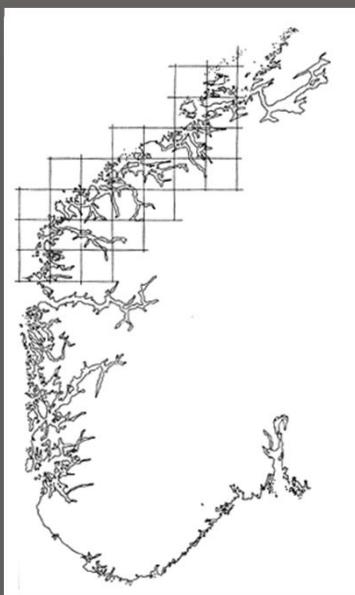


Žaidimai

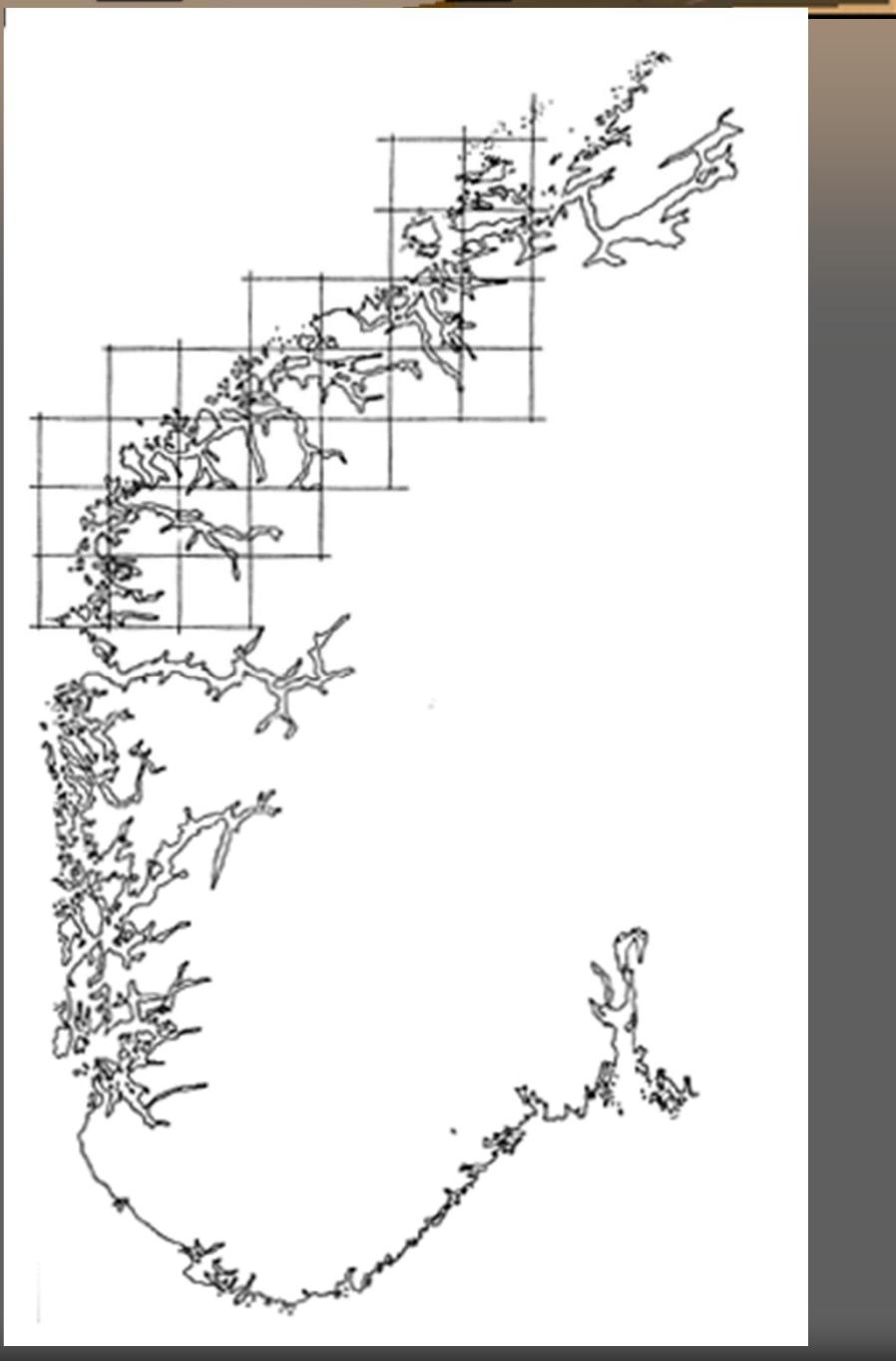
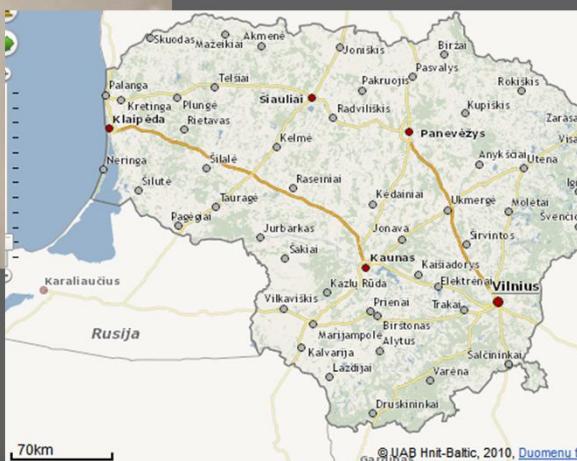
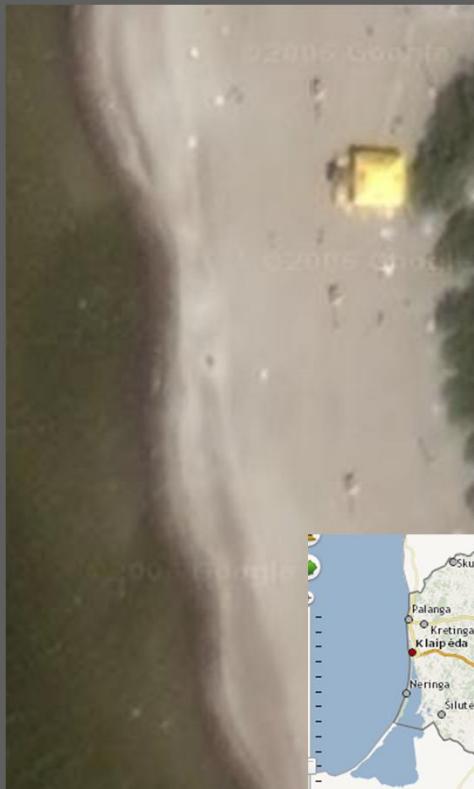
Featured on the cover
of the High
Performance
Computing magazine,
3 August 2001



Netiesinės dinaminės sistemos rodo fraktalų struktūras



Pakrantės ilgis



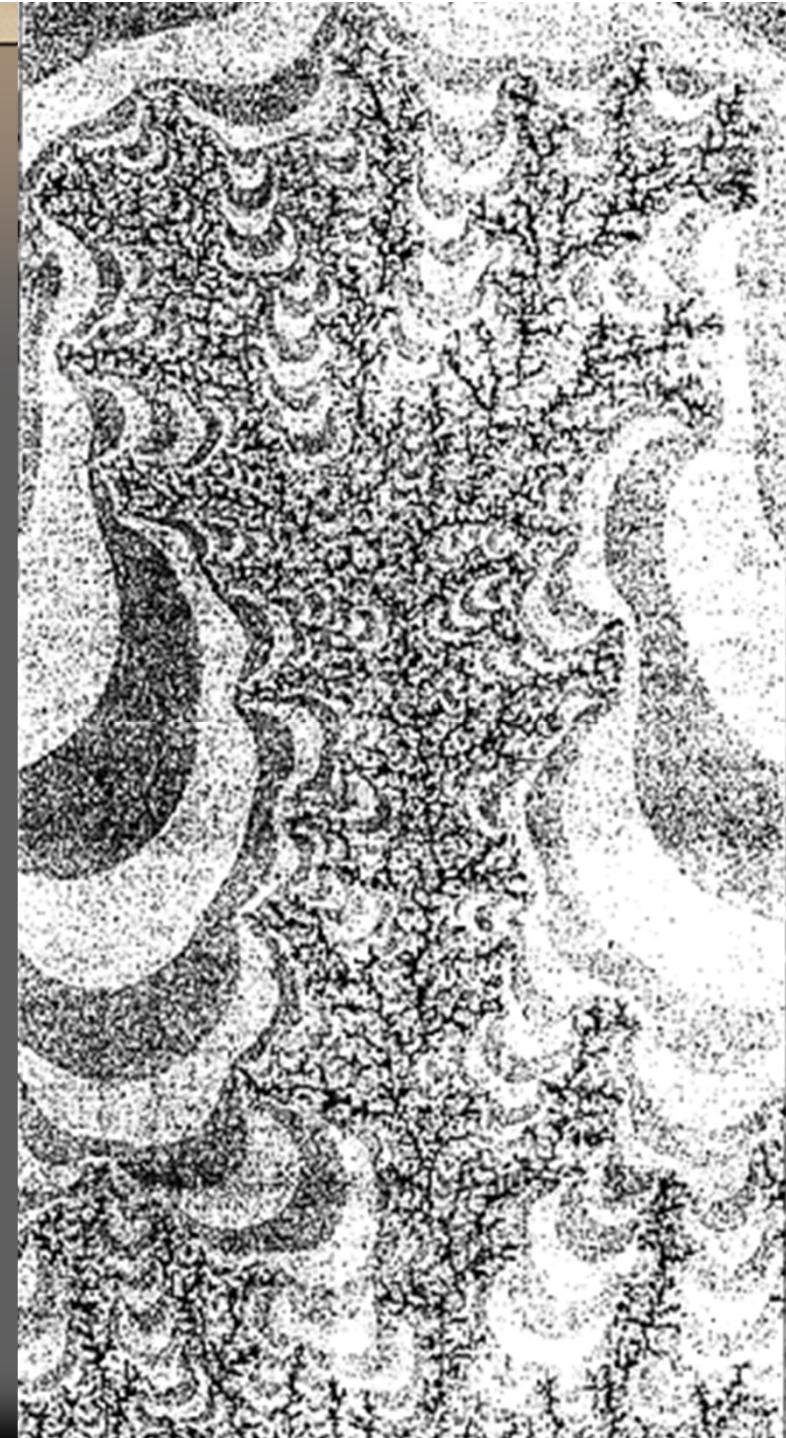
Difuzijos ribota agregacija



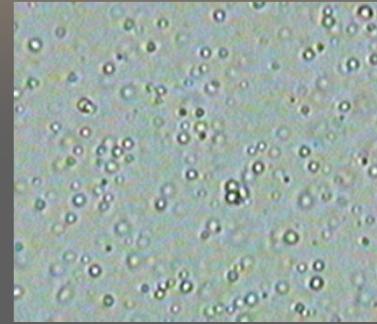
Kristalizacija



Korozija



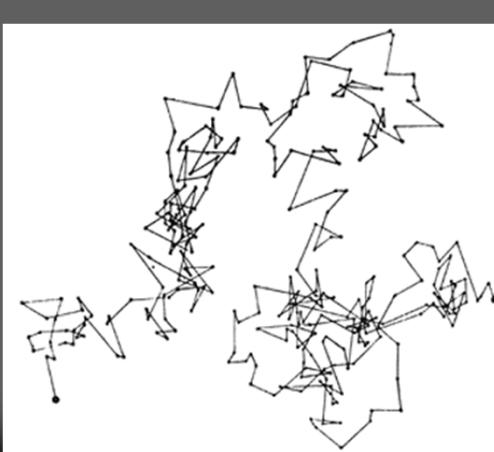
Chaos teorija



Netvarka



Determinuotas
Chaosas

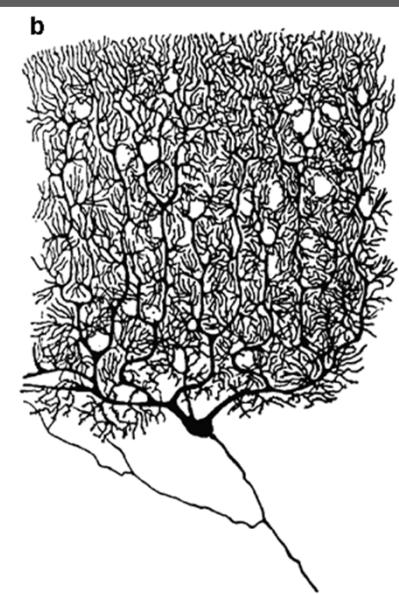
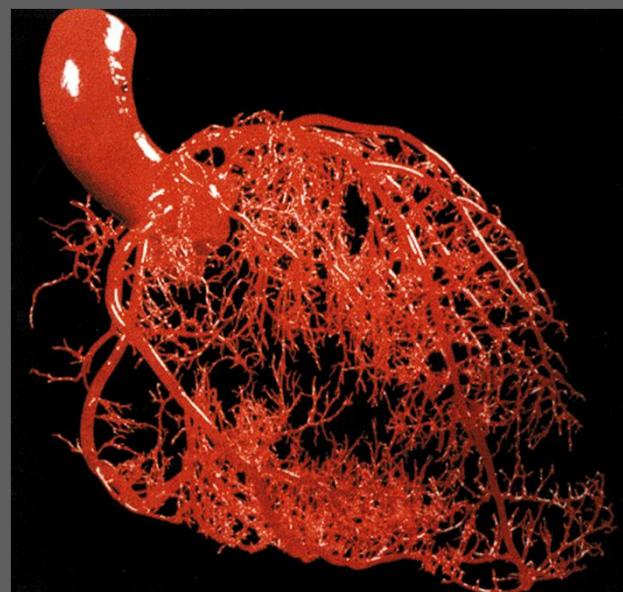
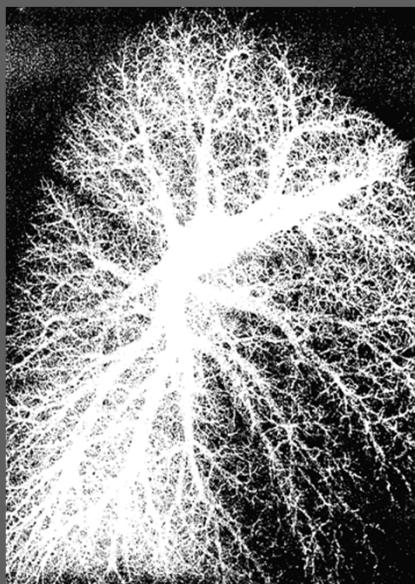
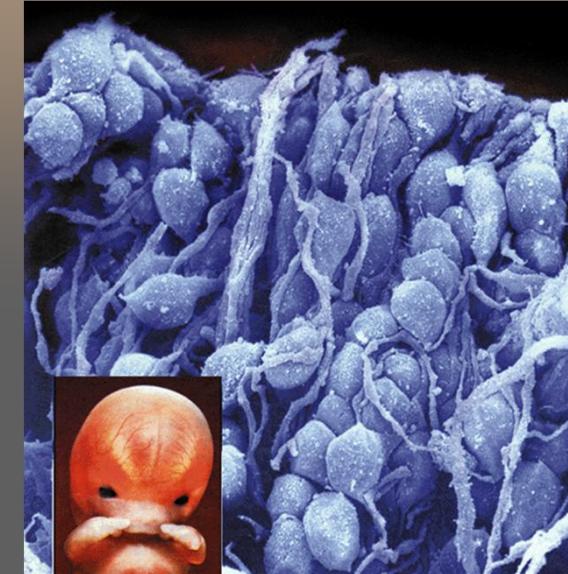


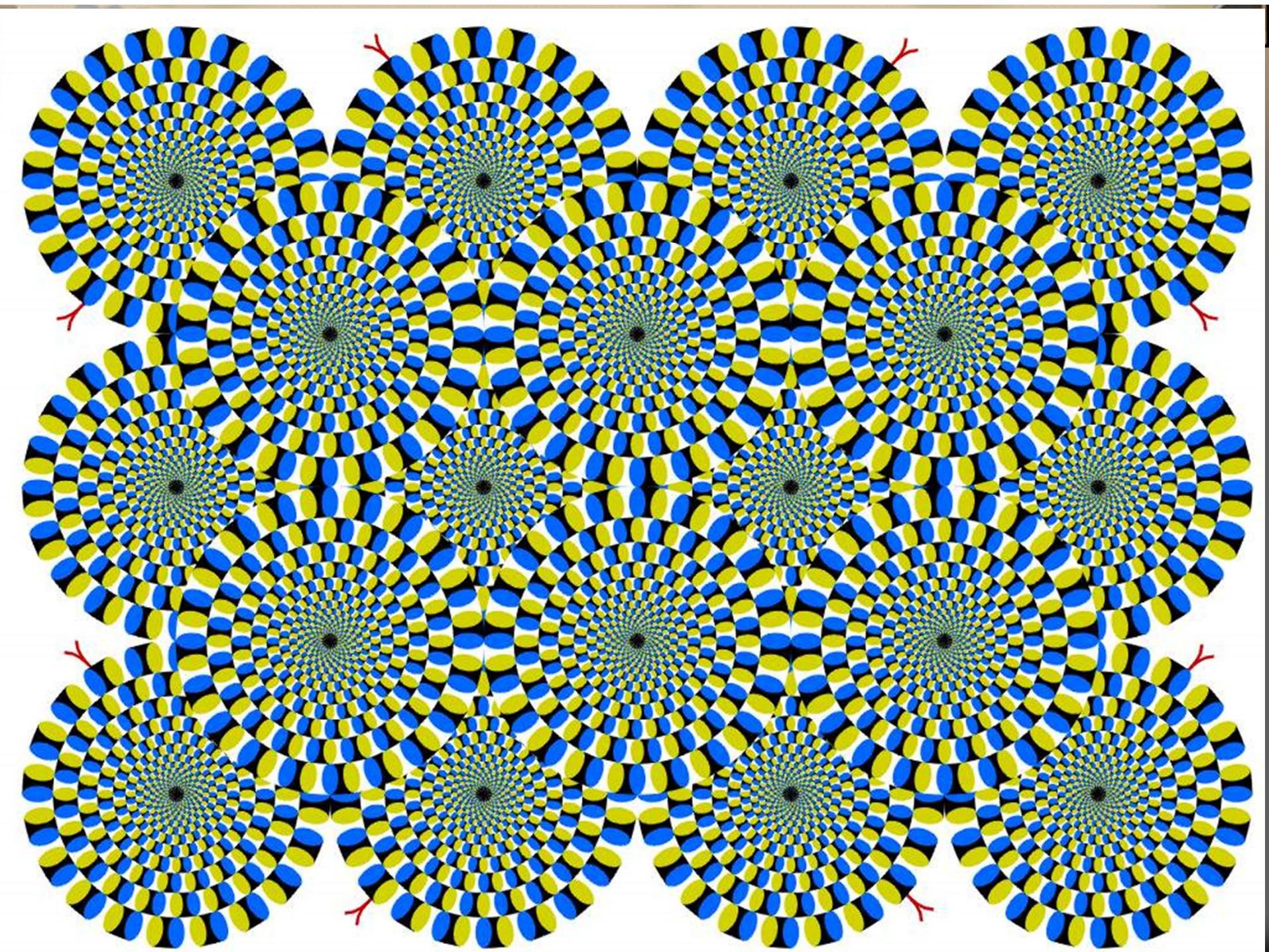
Tvarka



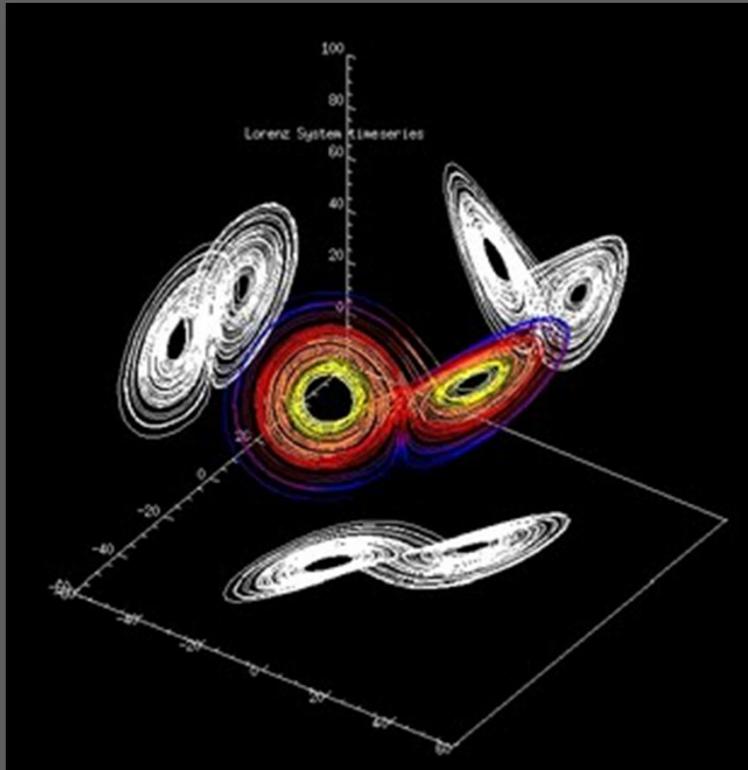
Gyvybė

→ Žmonių struktūra

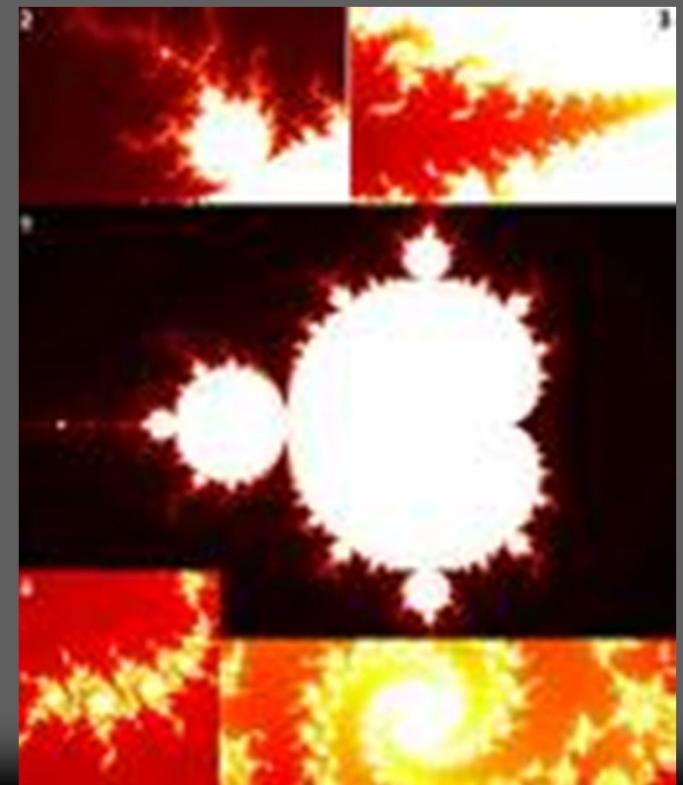




Netiesinės dinaminės struktūros



⇒ **Chaosas** – nereguliarumas laike



⇒ **Fraktalai** – nereguliarumas erdvėje

Gyvybė - dinaminė savitvarkės materijos būsena

- ⇒ **Bifurkacijos** – negrįžtamumas, išaldyti atsitiktinumai
- ⇒ **Sinergetika** –tvarkos parametras, gyvosiose sistemoje pavergimas yra balansuojanas konsensuso suradimu
- ⇒ **Chaosas** – nenusakomas, kintantis, determinuotasis chaosas
- ⇒ **Fraktalai** – panašumas į save, chaotinės gamtos geometrija

