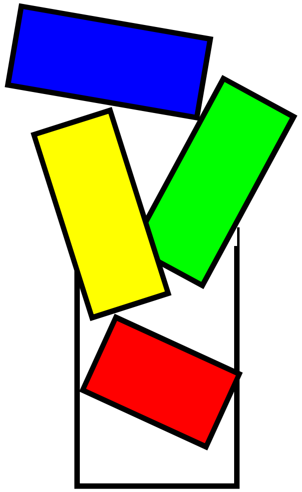
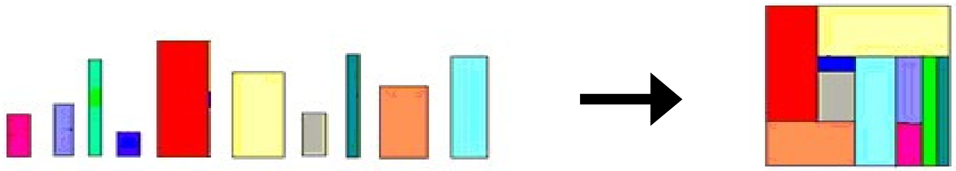


CAD of VLSI

Tutorial # 7

Optimization Methods in CAD of VLSI



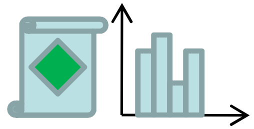
Introduction

- **C**omputer **A**ided **D**esign = Automation of the design process
- Design process consists of stages in each of which we solve *optimization problem*
  – Usually: minimize area, delay, power under different constrains
- Good design automation:
  – good enough (possibly optimal) solution of optimization problem
  – fast enough solution of optimization problem
- So, the main question in design automation is:

How to solve optimization problem in fast and exact way?

What is optimization

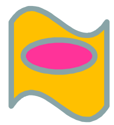
- Optimization, in general, is the process of decision making which leads to the best value of optimized objective
- In mathematics, optimization (or ***mathematical programming***) means minimizing or maximizing some function on certain (finite or infinite domain)
- If constrains are implied on function domain, then the optimization is ***constrained***, otherwise – ***unconstrained***.
- When number of possible values of objective function is finite, then it is ***combinatorial optimization*** problem.



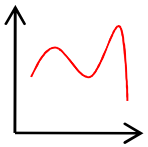
- Optimization problem always satisfies one of the following:
  - Infeasible
  - Has global optimum
  - Unbounded
- In many cases, optimization problem has also one or more local optima.

Optimization domains

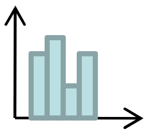
- If the objective function is continuous, then the optimization problem is ***continuous optimization problem***. Examples from CAD:
  - find sizes of inverters in buffer so that buffer power is minimized
  - minimize width of interconnect channel under delay constraints



- Continuous optimization problems in many cases may be solved analytically or numerically

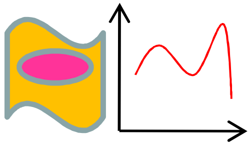


- Continuous problems with “good” properties may be provided to have a single optimum, i.e. each local optimum is also its global optimum.
- However, most of CAD optimization problems are ***constrained combinatorial optimization*** problems. Some examples:
  - Find the shortest path between two vertices in the graph
  - Find the placement of cells with minimum area
  - Find the smallest representation of Boolean function

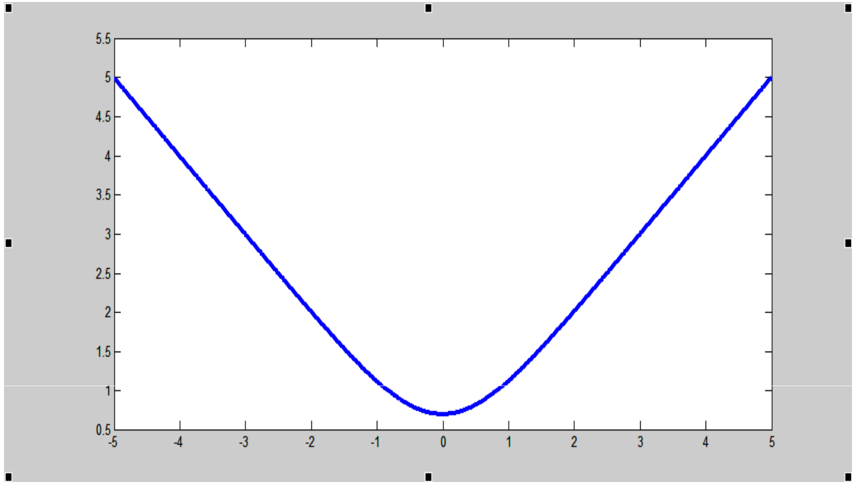


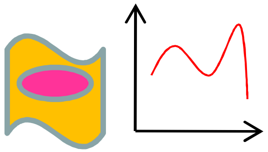
- The solution of combinatorial optimization problem can be obtained by simply checking all possible values of objective function, but usually the number of possibilities is very large (exponential)

Continuous optimization: convex vs. non-convex problems

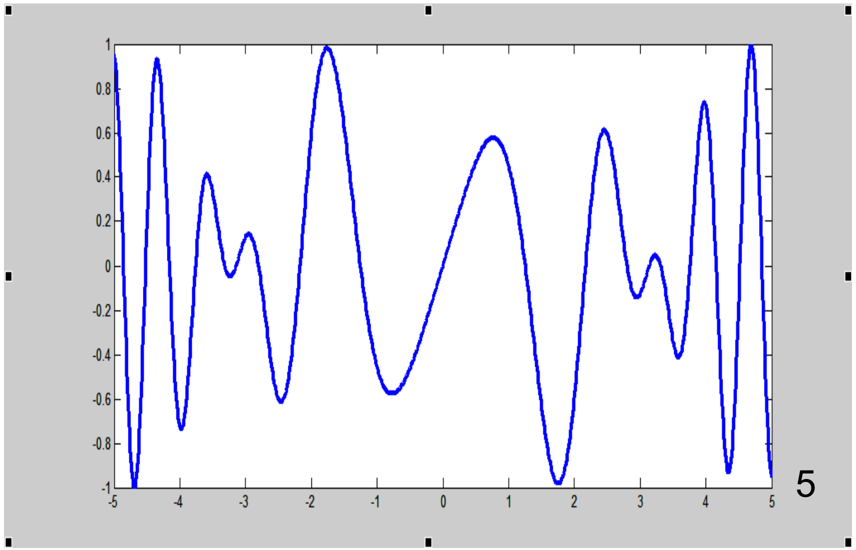


- Continuous problems with "good" properties are called ***convex***
  - Has single minimum
  - Efficiently solvable analytically or numerically (descent methods)



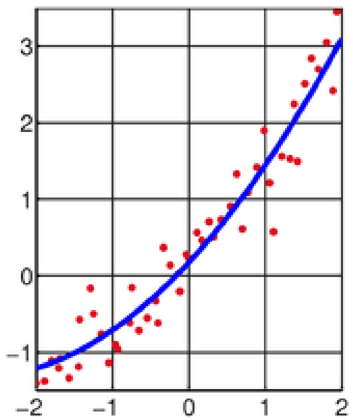


- ***Non-convex*** problems are hard to solve exactly
  - Usually solved numerically, using heuristic methods

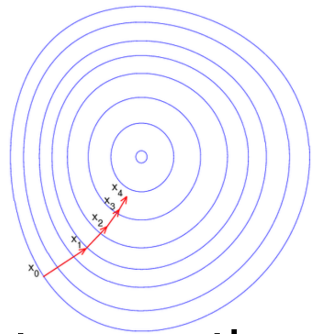


Analytical vs. numerical solution

- ***Analytical solution:*** set of equations and inequalities are solved
  - Obtained solution is exact
  - For example: least squares problem



- If problem is hard or impossible to solve analytically, then analytical investigation of the problem can help to develop numerical solution
- ***Numerical solution:*** non-exact. Usually differs from exact up to predefined accuracy



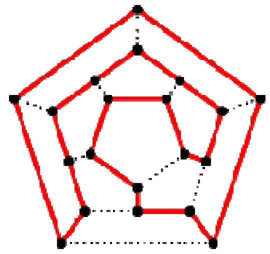
  - Example: steepest descent method, Newton method

Discrete optimization: order of growth

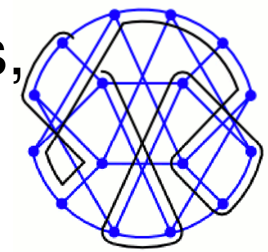
- Optimization domain of discrete problem depends on problem size
- For practicality, we want **polynomial-time** algorithms

T(n)=n	**10**	**20**	10^2	10^3	10^6
T(n)=3n	**30**	**60**	3×10^2	3×10^3	3×10^6
T(n)=nlogn	**10**	**26**	2×10^2	3×10^3	6×10^6
T(n)=n^2	10^2	4×10^2	10^4	10^6	10^{12}
T(n)=n^3	10^3	8×10^2	10^6	10^9	10^{18}
T(n)=2^n	10^3	10^6	10^{30}	10^{301}	$> 10^{500}$
T(n)=n!	3×10^6	2×10^{18}	9×10^{157}	$> 10^{500}$	$> 10^{500}$

Discrete optimization: **P** vs. **NP**



- ***P* – *“poly-find”*** – a class of problems solvable in polynomial time. Examples:
  - finding shortest path in a graph, finding minimum spanning tree



- ***NP-complete* – *“poly-verify”*** – a class of problems, which are not in P, but verifiable in polynomial time. Usually represent decision problems. Examples:
  - Are there any Hamiltonian paths in given graph?
  - Are there any variable assignments that satisfy given SOP?

Sum Of Products

- ***NP-hard*** – at least as hard as NP-complete. Usually represent optimization versions of appropriate decision problems.
  - What is the minimum length Hamiltonian path in given graph? (Travelling Salesperson Problem - TSP)

Exact and heuristic solutions

- ***Bad news:*** most of CAD problems are:
  NP-complete or NP-hard
  - Impossible to find exact solution by exhaustive search
- We use ***heuristic methods*** to get to optimal solution as close as possible
  - Heuristics are "rules of thumb", educated guesses, intuitive judgments or simply common sense
  - Sometimes can result in optimal solution!
  - Example : for TSP, start from minimum spanning tree, then try to convert it to Hamiltonian path

Optimization strategies for hard problems

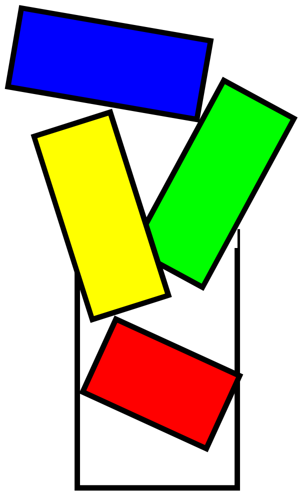
- We cannot afford spending exponential time to solve the problem.
- Solving the problem with high quality, i.e. use good heuristics.
  - Trade-off quality for run-time.
  - Might need to solve the problem at different steps, but have different requirements of solution quality at run-time.
  - Common case: approximate algorithms (of simpler complexity) that guarantee that the result is within some margin of the optimum.
- Solving a simpler (or restricted) version of the problem.
  - Reveal insight of the general problem.
  - Heuristic for solving original problem.

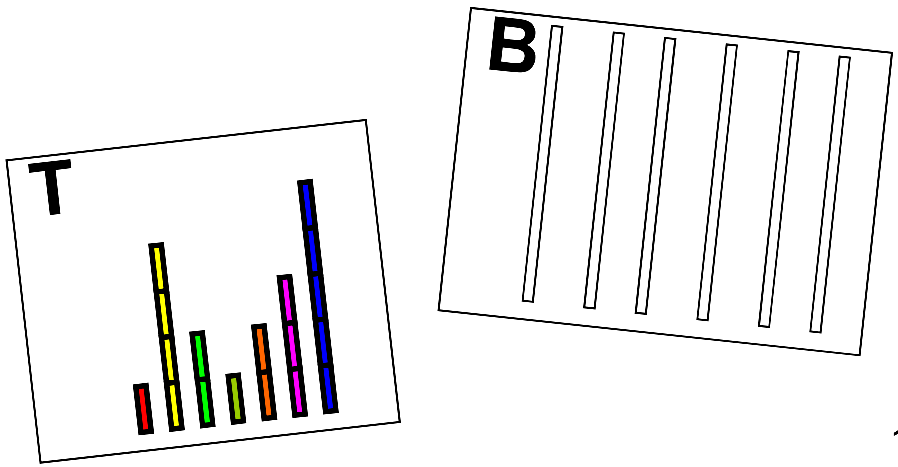
Discrete optimization methods

- **Exact** methods:
  - Exhaustive search
  - Backtracking with branch-and-bound
  - Divide-and-conquer
  - Dynamic programming – *will be shown independently*
- **Heuristic** methods:
  - Greedy approach
  - Local search
  - Tabu search
  - Genetic algorithms
  - Simulated annealing – *will be shown independently*

Example: a bin-packing problem

- We have a collection of items $T=\{t_1, t_2, t_3,\ldots,t_n\}$.
- Every item t_k has an integer size s_k.
- There is a set of bins B, each with a fixed integer size b.
  - Can hold items if the sum of their sizes is b or less.
- GOAL: Pack all items, using a minimum number of bins.
- The problem is NP-hard.



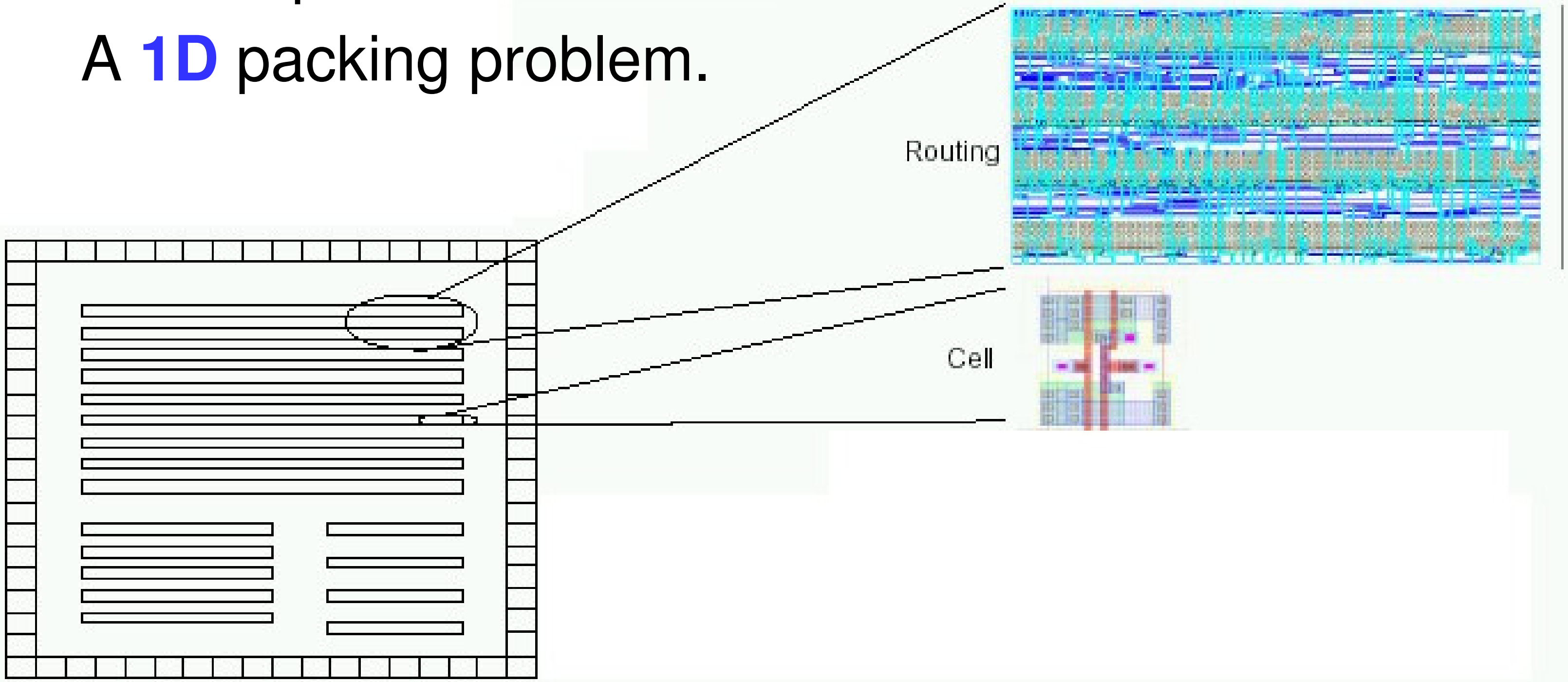


Motivation - standard cells

Cell based design:

How to pack standard cells in rows ?

A **1D** packing problem.

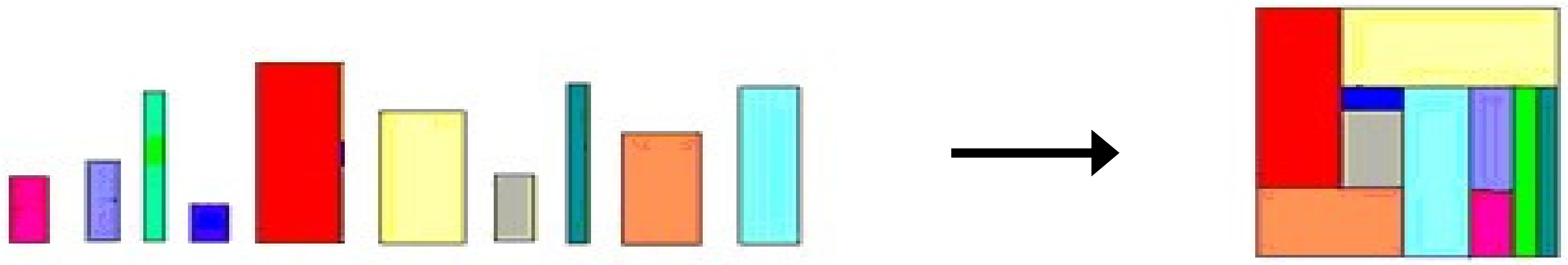


Motivation - floorplanning

Floorplanning is a :

2D packing problem !

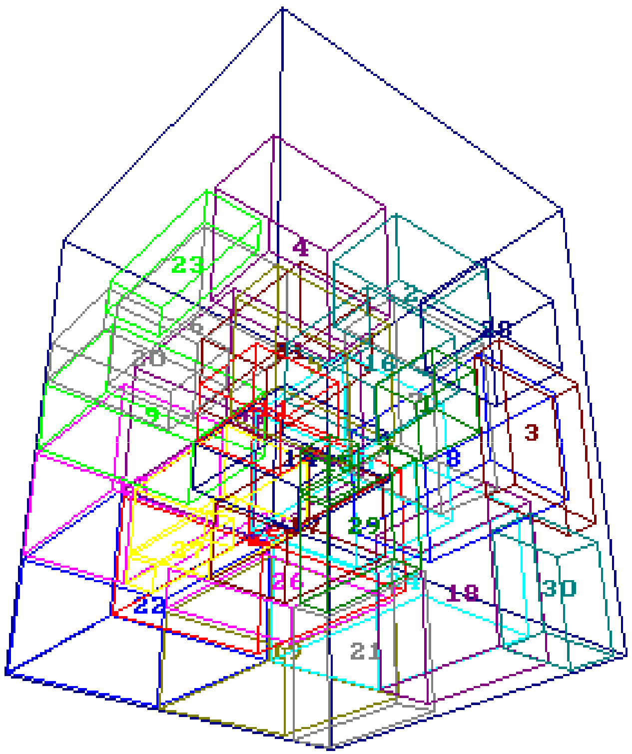
Functional units: **Compact layout:**



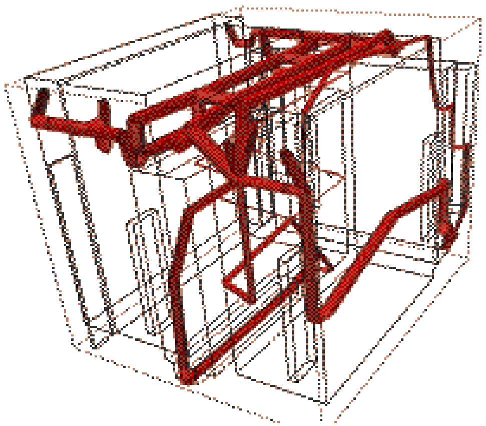
Motivation – more

There are more interesting variations :

3D packing problem !

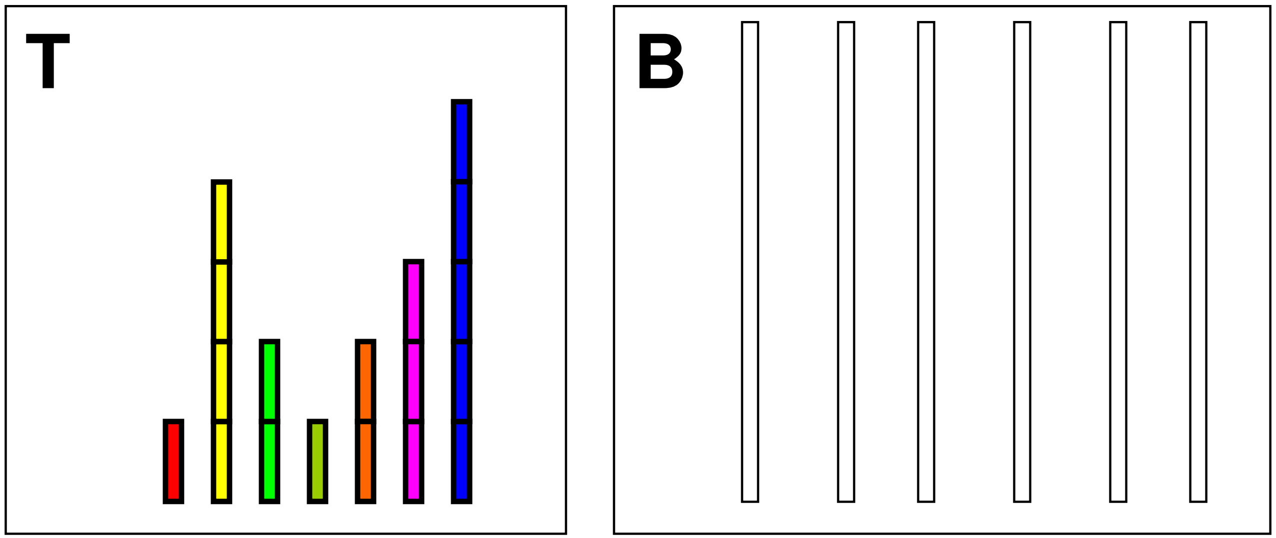


- Component fitting for high density.
- Multi-layer routing.
- Resource allocation.



Bin Packing problem

- We have bins with maximum size of 6 units.
- We have items with sizes: { 1, 4, 2, 1, 2, 3, 5 }
- The cost function is simply the number of bins - B.
- We will demonstrate several optimization methods with this simple problem.



Exhaustive search

- Generate all possible combinations, decide which ones are feasible, find one with minimum cost
- Optimal but slow - traverses the whole search-tree (<u>exponential</u>)

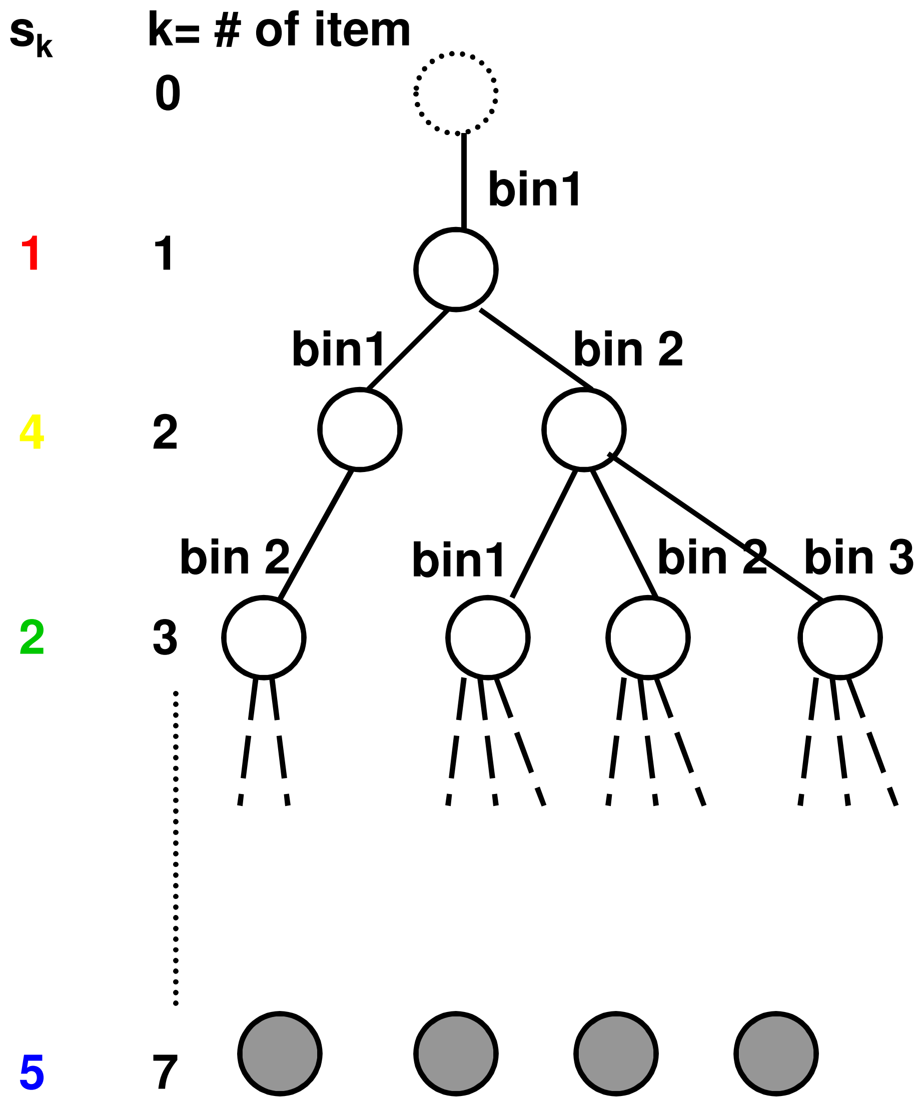
Exhaustive search by backtracking

- Systematic method of traversing a graph recursively.

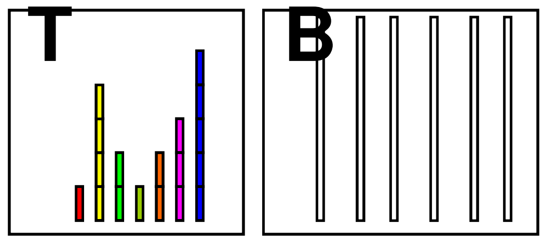
```
f(x){
  f(x+1);
}
```

- **Backtracking**:
  - A <u>*recursive*</u> way for doing an exhaustive search.
  - Start with a partial solution with as many as possible <u>unspecified</u> variables.
  - Systematically assign values to variables.
    - Try all allowed values for variable k+1, given a choice of the first k variables.
  - Until a feasible solution is found or until a “dead-end”.
  - Go back to an earlier partial solution and keep on trying.

Backtracking on our example



- The items: {1, 4, 2, 1, 2, 3, 5}
- Item $1_{(1)}$ put in <u>bin 1</u>
- Item $2_{(4)}$ has 2 options, <u>bin 1</u> or <u>bin 2</u>
- If items $1_{(1)}$ and $2_{(4)}$ are in same bin, then item $3_{(2)}$ has only one option, <u>bin 2</u>
- If items $1_{(1)}$ and $2_{(4)}$ are in separate bins, then item $3_{(2)}$ has <u>3 options</u>
- and so on …
- **This creates only feasible solutions**

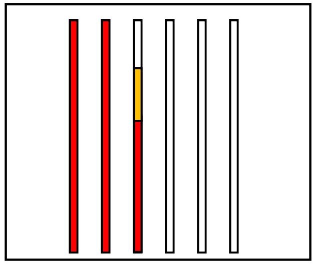


Back-tracking pseudo code

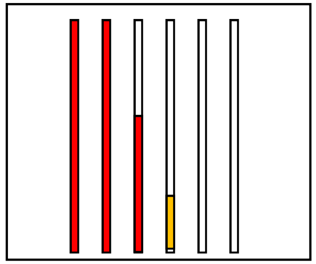
```
float best_cost;
solution_element val[n], best_solution[n];

backtrack (int k) {
        float new_cost;
        if  (k == n) {
                new_cost := cost(val);
                if (new_cost < best_cost) {
                        best_cost := new_cost;
                        best_solution := copy(val);
                }
        } else {
             foreach (el in allowed(val,k)) {
                val[k] := el;
                backtrack(k+1);
             }
        }
}
```

```
main () {
                best_cost := infinity;
                backtrack(0);
                report(best_solution);
}
```

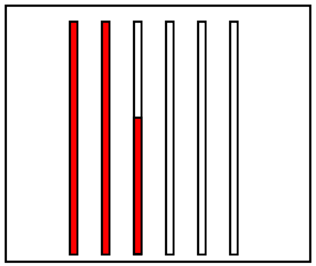
el_1



el_2



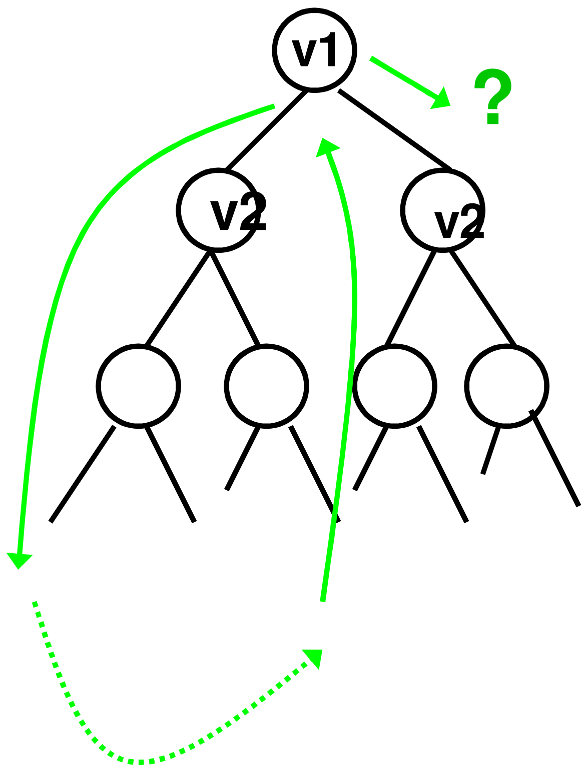
val:



Branch and Bound

- Branch & bound is refinement of backtracking
- To avoid complete enumeration of all possible solutions, we want to *prune* the tree (cut out some parts)
- For each branch, lets compute a *lower bound* for all solutions in the sub-tree that grows from it
- If that bound is higher cost than the best solution we found so far, we can skip (prune) the sub-tree!
  - Algorithm is still exponential, but can be much less on average
  - Branching selection heuristics:

    try to visit the most promising solutions early!
    It would facilitate pruning and save time
    (but no effect on exactness of solution)



Branch & bound pseudo code

```
float best_cost;
solution_element val[n], best_solution[n];

b&b (int k) {
        float new_cost;
        if  (k == n) {
                new_cost := cost(val);
                if (new_cost < best_cost) {
                        best_cost := new_cost;
                        best_solution := copy(val);
                }
        } else if (lower_bound_cost(val,k) >= best_cost
             return
        } else {
             foreach (el in allowed(val,k)) {
                val[k] := el;
                backtrack(k+1);
             }
        }
}
```

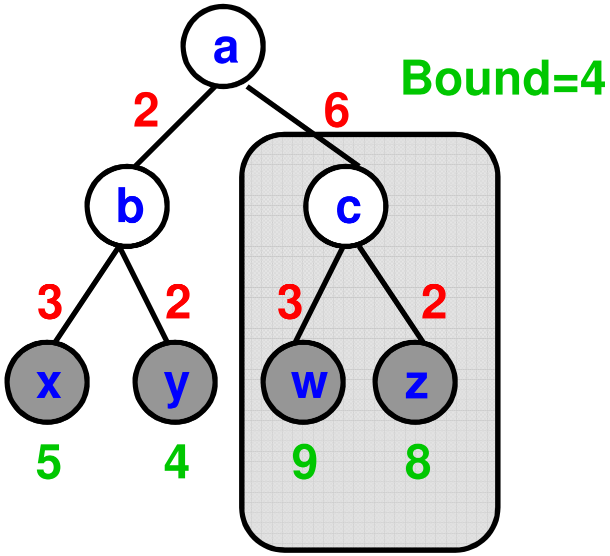
```
main () {
                best_cost := infinity;
                b&b(0);
                report(best_solution);
}
```

Bounding function

- Illustration:
- A toy problem with four possible solutions
- The cost of a leaf is the sum of the arcs on the path to it.
- Assume the bound is set to the lowest cost found.

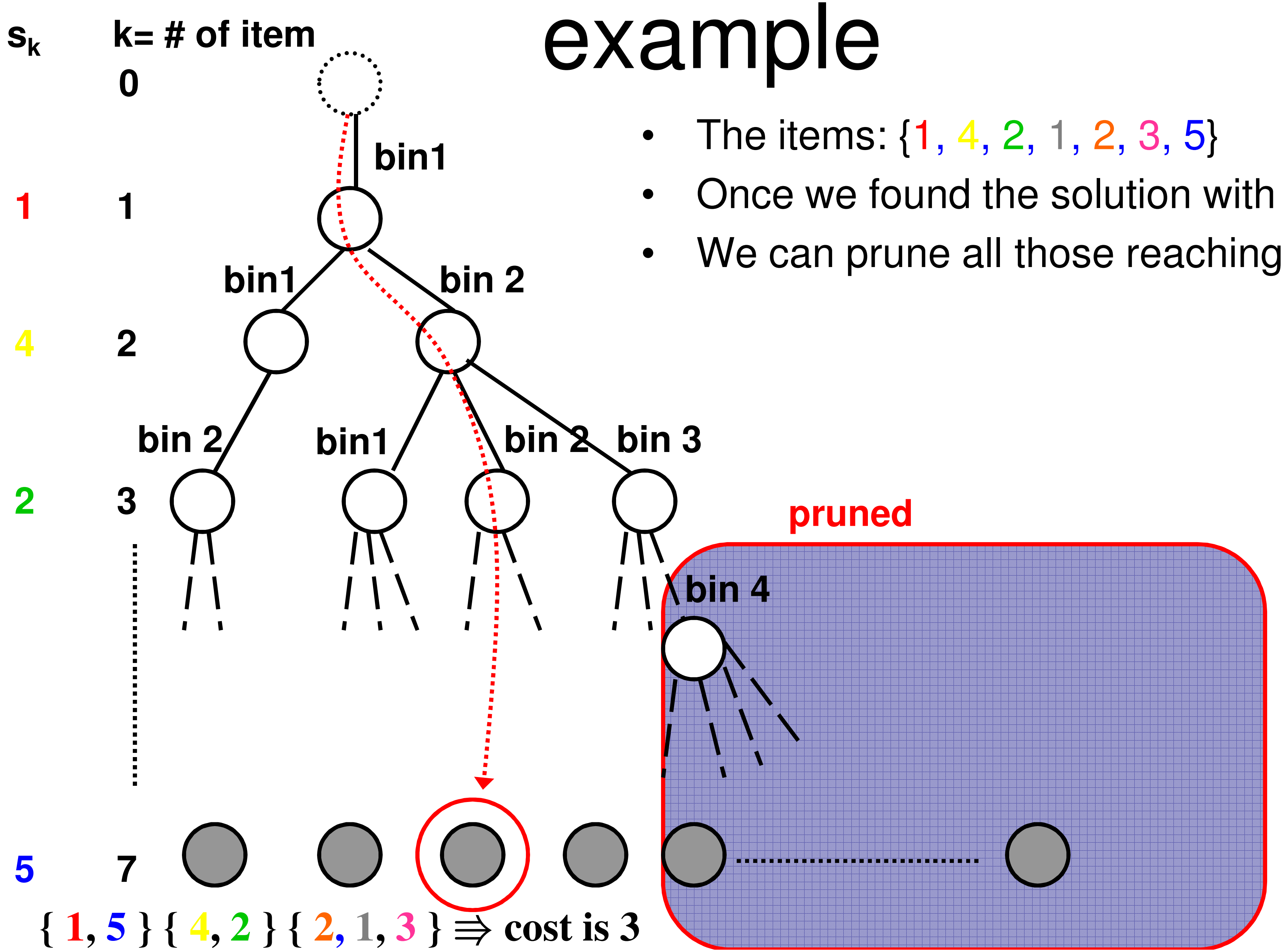
  (initialized to infinity).
- After visiting the left subtree, discovering a solution of cost 4, we can prune the whole subtree on the right (c),

  Since we know that all its solutions would cost at least 6 !
- A “sharp” /”tight” bounding function can save time
  - It should also be quick to compute.
  - What about an inaccurate, incorrect bound?



Branch and Bound on our example

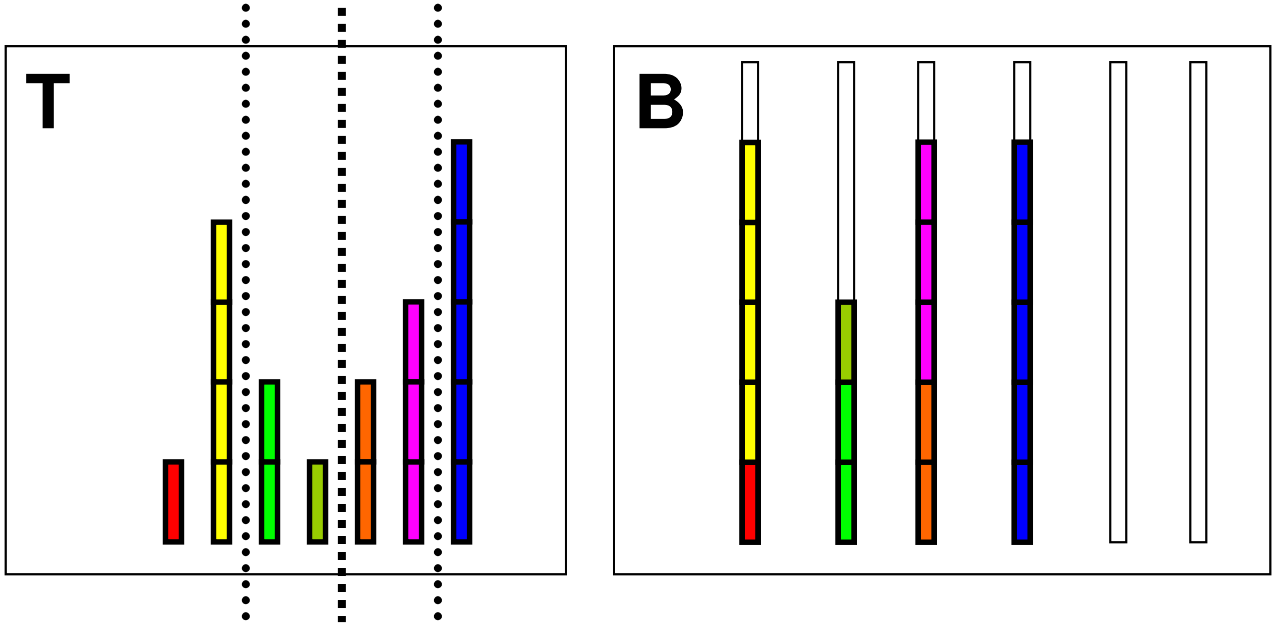
- The items: {1, 4, 2, 1, 2, 3, 5}
- Once we found the solution with 3 bins:
- We can prune all those reaching 4 !



{ 1, 5 } { 4, 2 } { 2, 1, 3 } ⇛ cost is 3

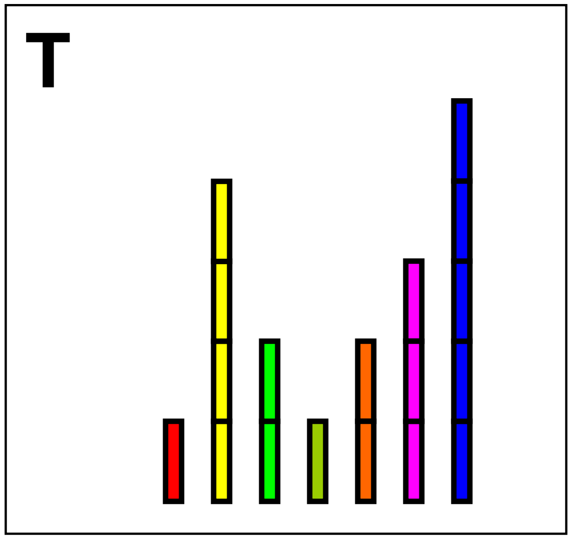
Divide-and-conquer approach

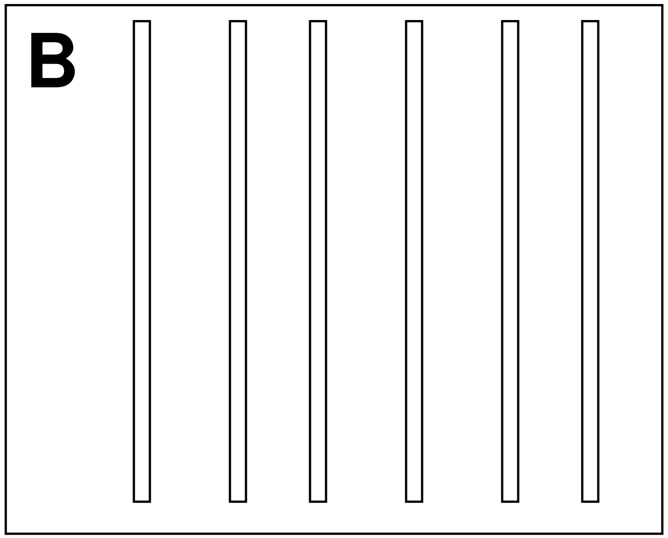
- **Divide** the problem into smaller (simpler) sub problems.
- **Conquer** the problems by solving them recursively.
  - Keep partitioning until sub problems are easy enough.
- **Combine** the solutions of the sub problems into the solution for the original problem
- In our example:
  - Let's interpret "easy enough" = sub problem fits in 1 bin.
- Partition: {1, 4} { 2, 1} { 2, 3} { 5}



Greedy approach (heuristic)

- **A greedy algorithm** always makes the choice that looks best at the moment.
- This is a simple and fast heuristic
- Usually doesn't provide global minimum
- In our case: let's try **first fit algorithm -** placing each item into the first bin in which it will fit

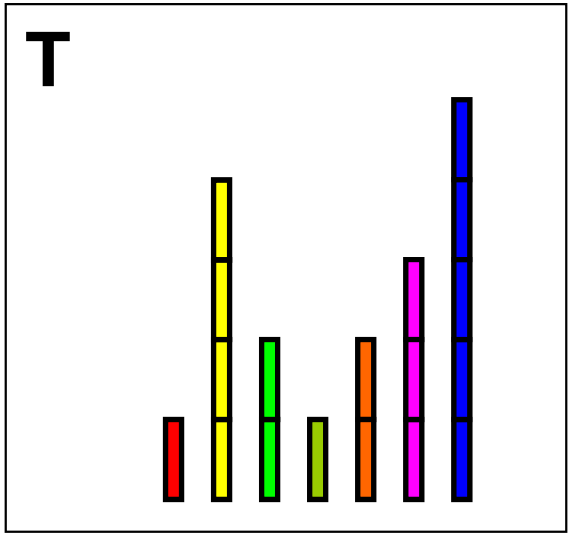


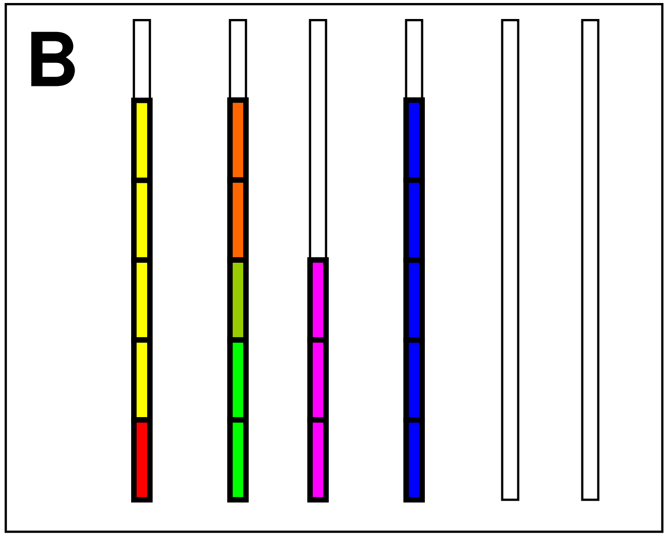


Greedy approach (heuristic)

- **A greedy algorithm** always makes the choice that looks best at the moment.
- This is a simple and fast heuristic
- Usually doesn't provide global minimum
- In our case: let's try **first fit heuristic -** placing each item into the first bin in which it will fit

{ 1, 4 } { 2, 1, 2 } { 3 } { 5 }

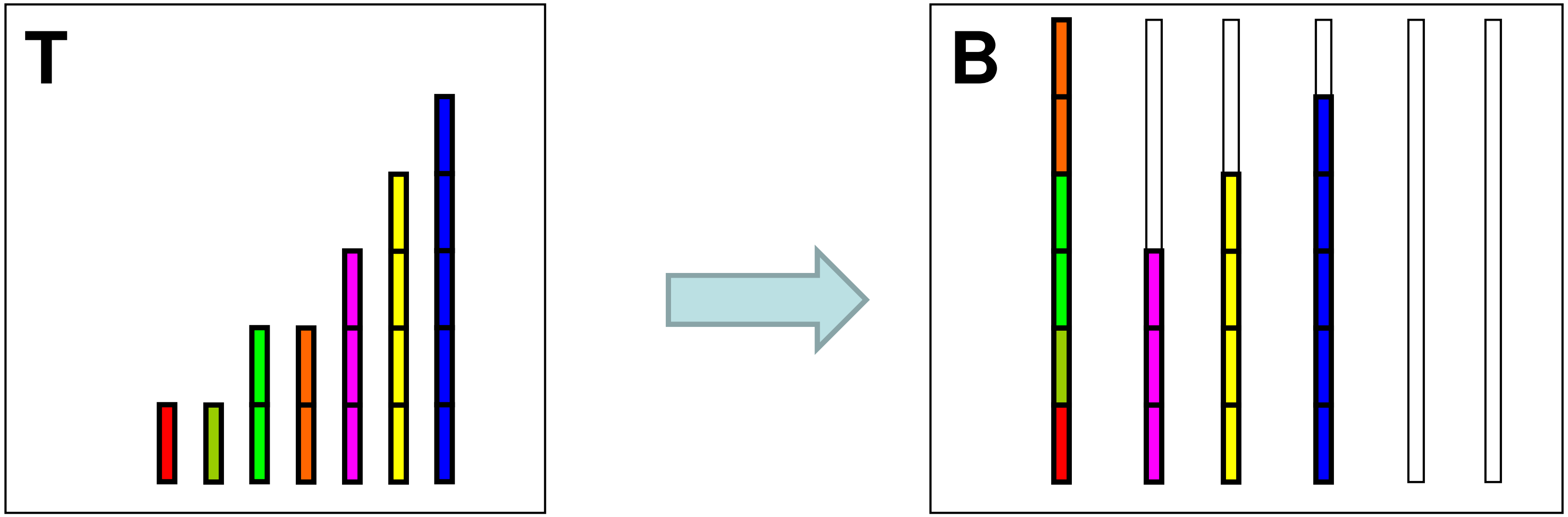




Greedy approach (heuristic)

- Now try **first fit increasing heuristic:** first sort the list of elements into increasing order and then place each item into the first bin in which it will fit

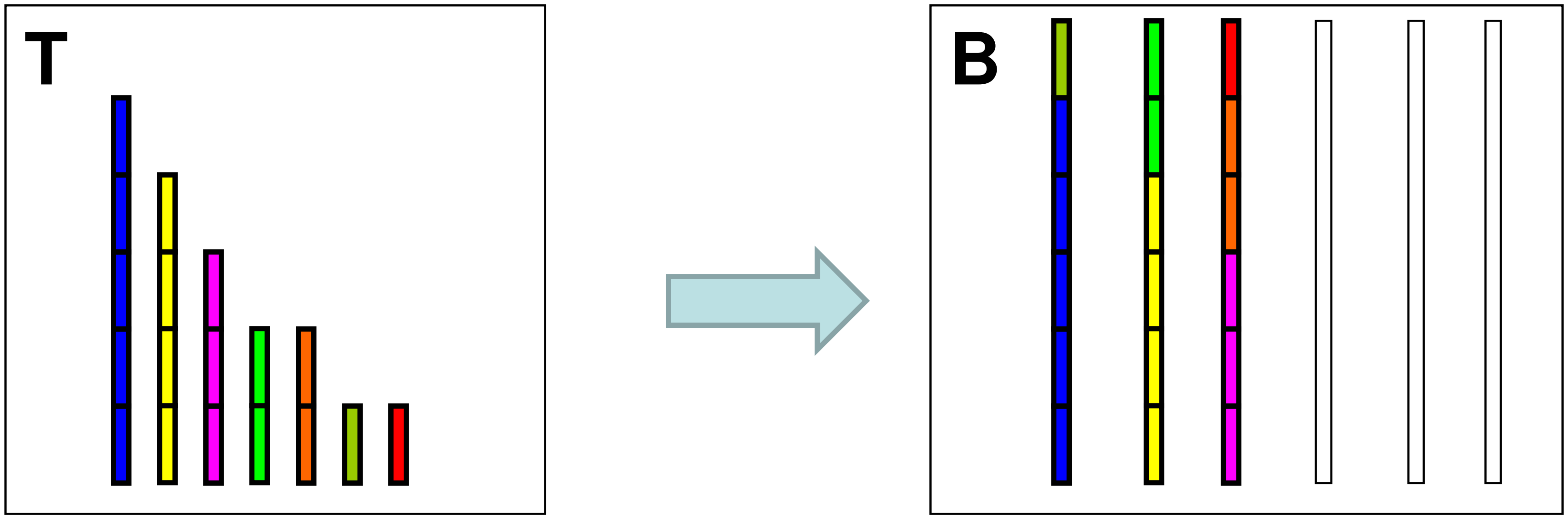
{ 1, 1, 2, 2 } { 3 } { 4 } { 5 }



Greedy approach (heuristic)

- Finally try **best fit decreasing heuristic:** first sort the list of elements into decreasing order and then try to find bin with minimum free space that still can include given item

{ 5, 1 } { 4, 2 } { 3, 2, 1 } – optimal solution!

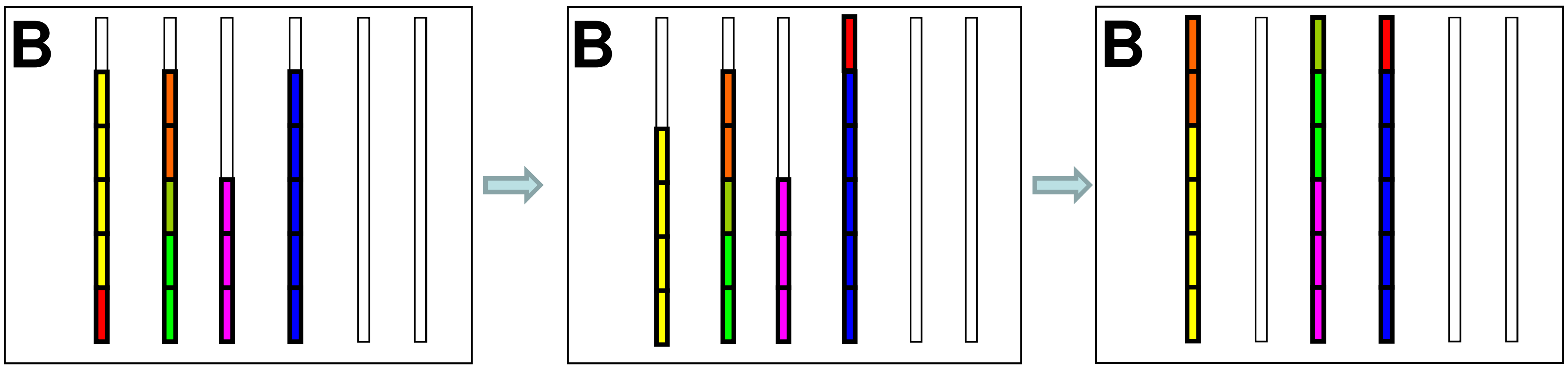


Local search

- **Local search** is the method that starts from feasible solution f and looks for better solution in a *neighborhood* N(f)
  (1) Start from feasible solution
  (2) Generate neighborhood of the solution
  (3) Choose the solution from the neighborhood with:
    better cost - *first improvement* strategy, or
    the best cost – *steepest descent* strategy
  (4) go to 2
- Non-intelligent generation of neighborhood can lead to its huge size
  – Usually will use some heuristic to do this
- Main disadvantage: can stuck in a local minimum
- Solutions:
  – repeat local search from some initial points
  – adapt the size of neighborhood during local search

Local search

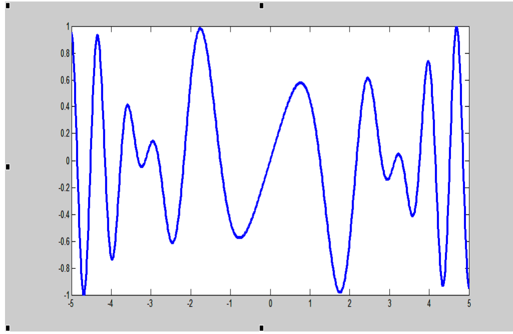
- In our case:
  - generate neighborhood of given solution by splitting the bin with minimum occupation (whenever possible), and then apply best fit heuristic



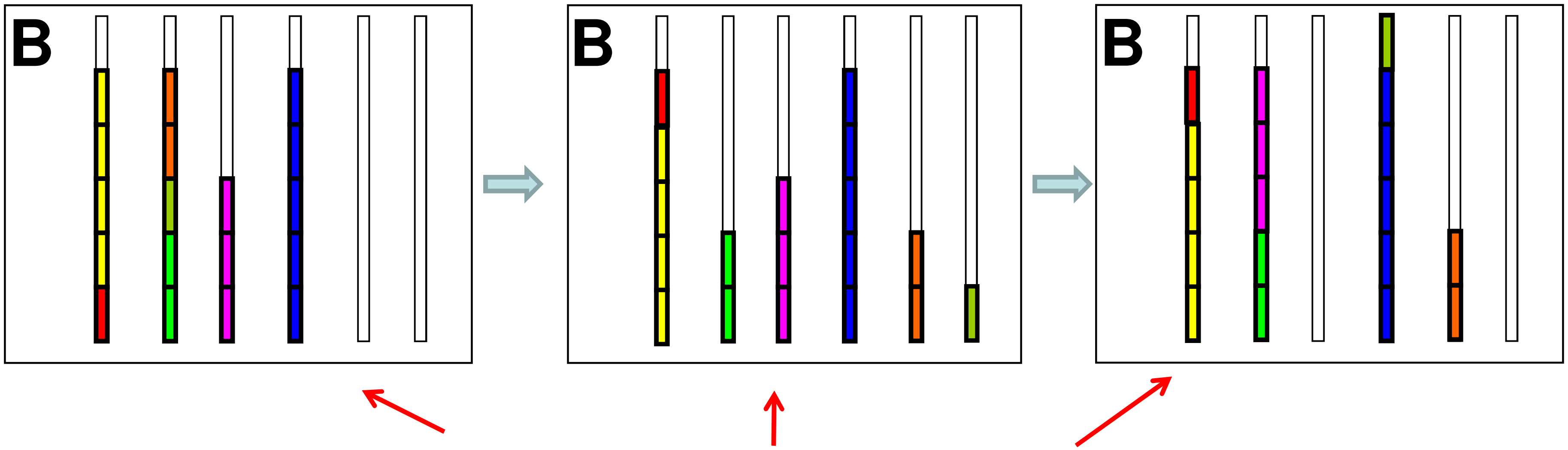
Tabu search

- Tabu search is improved version of local search
- Allows movement to the feasible solution with worse cost
  - (1) Start from feasible solution
  - (2) Generate neighborhood of the solution
  - (3) Choose the solution from the neighborhood, which is not in *tabu list*
  - (4) If its cost is better than known so far, save it
  - (5) Save the solution in *tabu list*
  - (6) go to 2
- Tabu list is the list of k last visited feasible solutions (which are taboo), to avoid cycles of length ≤ k

Tabu search



- In our case:
  - can generate solutions with the cost (i.e. number of occupied bins) worse than known so far
  - For example, totally split one of bins, then merge some of bins



These solutions are added to tabu list

Genetic Algorithms

- Instead of repetitively transforming a single current solution into a next one by the application of a move, the algorithm simultaneously keeps track of a set of feasible solutions, called the *population*
- Start with several feasible solutions (init)
  - S1 = { 1, 4 } { 2, 1 } { 2 } { 3 } { 5 }
  - S2 = { 1, 2 } { 4, 1 } { 2, 3 } { 5 }
- Obtain new solutions (*children*) by *mutations* of *parents* e.g.
  - From S1 obtain S3 = { 1, 4 } { 2, 1 } { 2, 3 } { 5 }
  - From S2 obtain S4 = { 1 } { 2 } { 4, 1 } { 2, 3 } { 5 }
- Obtain new solutions by *crossovers* of existing solutions.

  e.g. take some members of S1 and some of S2 such that the generated solutions have required characteristics:
  - From S1 and S2 obtain S5 = { 1, 2, 1 } { 4 } { 2, 3 } { 5 }
  - Eliminate some solutions to reduce population (survival of the fittest)
  - e.g. select subset of solutions - the ones requiring fewest bins (S2, S3, S5).
- Repeat procedure till a good solution is found.

Summary

- In this lesson we studied:
  - what is optimization problem
  - kinds of optimization problems, especially in CAD of VLSI
  - methods for solving optimization problems:
    - exact: backtracking, branch and bound, divide and conquer
    - heuristic: greedy, local search, tabu search, genetic
- Two more methods to come:
  - Dynamic programming (exact)
  - Simulated annealing (heuristic)