Quality of Pareto set approximations

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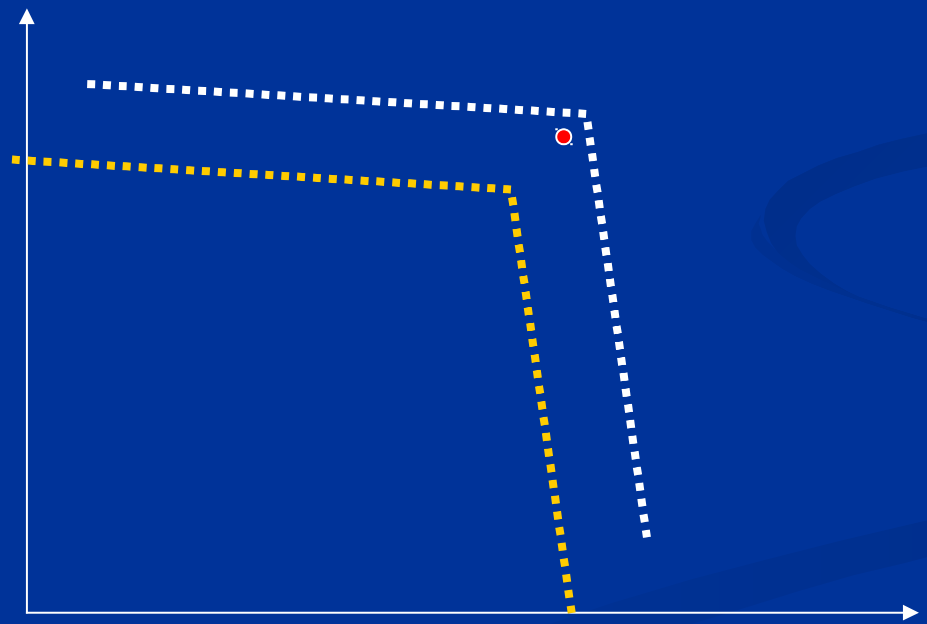
4.5 EMO, 4.5 non-EMO

Lessons learned

- This chapter is a must
- Obvious things are not obvious



Outline

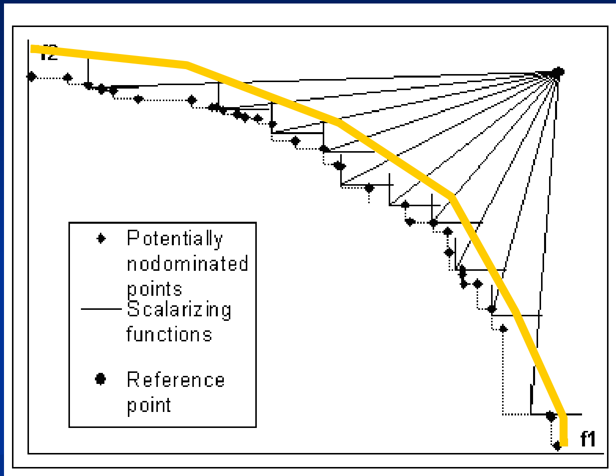
- Motivation
- Use cases
  - Testing (true Pareto front is known)
  - Practice (true Pareto front is not known)
- Comparing approximations
  - Unary measures
  - Binary measures
- Discussion and conclusions

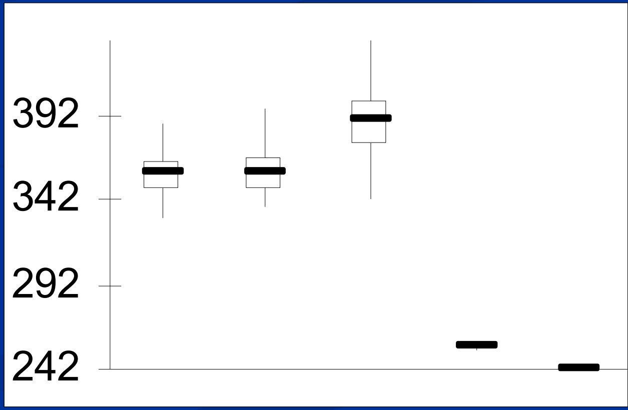
Motivation

- Comparison of algorithms
  - Pictures are valuable but not sufficient
- Design of algorithms for vector optimization
  - To guide the search
  - Stopping criteria
- Learning

Use cases

- Testing
  - True Pareto front is known
- Practice
  - True Pareto front not known in general
  - Additional useful information:
    - Lower bounds through relaxation
    - Upper bounds through random search or other methods
    - …
  - Pairwise comparison



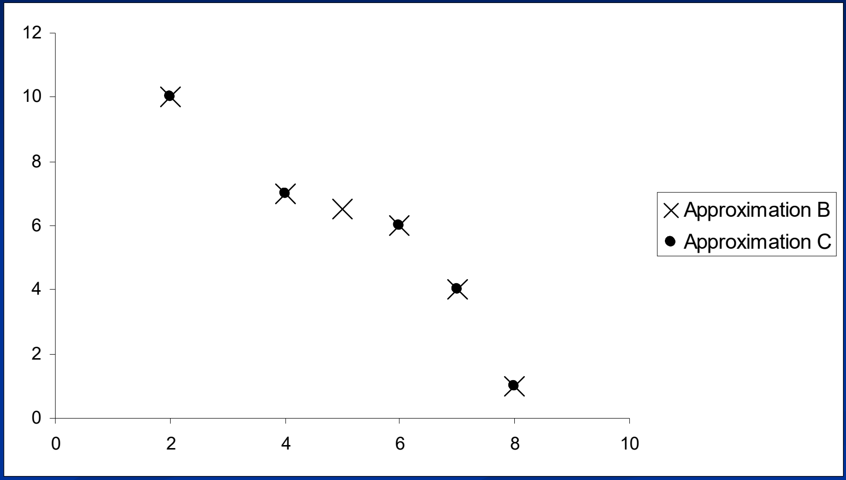


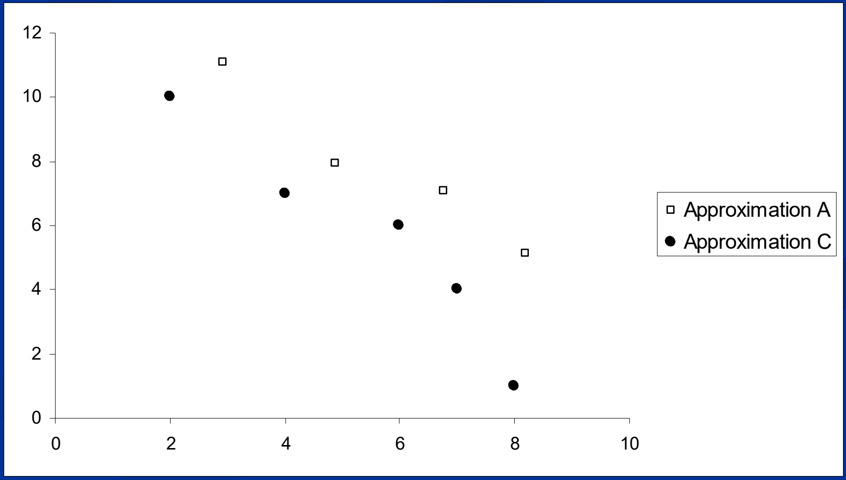
Comparing approximations

- Unary measures
  - Evaluation of a single approximation
- Binary relations
  - Purely ordinal
- Binary measures
  - Evaluation of difference between two approximations

Unary measures

- Assign a real number to a Pareto set approximation
- Relevant properties
  - Monotonicity (strict or not)
  - Uniqueness of optimum
  - Scale invariance
  - Computational requirements
  - Required information, e.g. reference set, bounds





Examples of unary measures

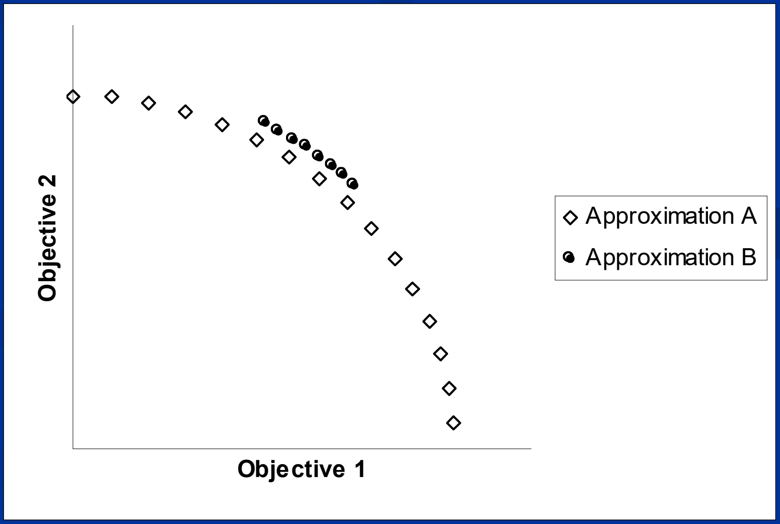
- Outer diameter
- Proportion of Pareto-optimal points found
- Cardinality
- Hypervolume
- ε-dominance
- D-measures
  - D1 - mean value of the Chenycheff distance from the Pareto front
  - D2 – asymmetric Hausdorff distance with Chebycheff metric
- R-measure - expected value of Chebycheff scalarizing function
- Uniformity measures
- Probability of improvement through random search
- Combinations of the above measures

Binary measures

- Relevant properties
  - Monotonicity (strict or not)
  - Scale invariance
  - Computational requirements
  - Required information, e.g. bounds
- Transformation of measures into relations
  - Partial orders stronger than dominance may be constructed
  - Implicit introduction of preferences

Examples of binary measures

- Difference of two unary measure values
- Pareto dominance of sets
- Binary ε-dominance
- Hypervolume of difference between two Edgeworth-Pareto hulls
- Coverage



Discussion and conclusions

- Evaluation of algorithms – not discussed here
- Taking into account preference information - not discussed here
- Monotonicity is important in theoretical sense
- In practice it *may* be relaxed
  - Learning
  - Practical optimization algorithm may benefit from this relaxation, like constraint relaxation in numerical optimization
- Computational complexity also influences the choice of performance indicator(s)