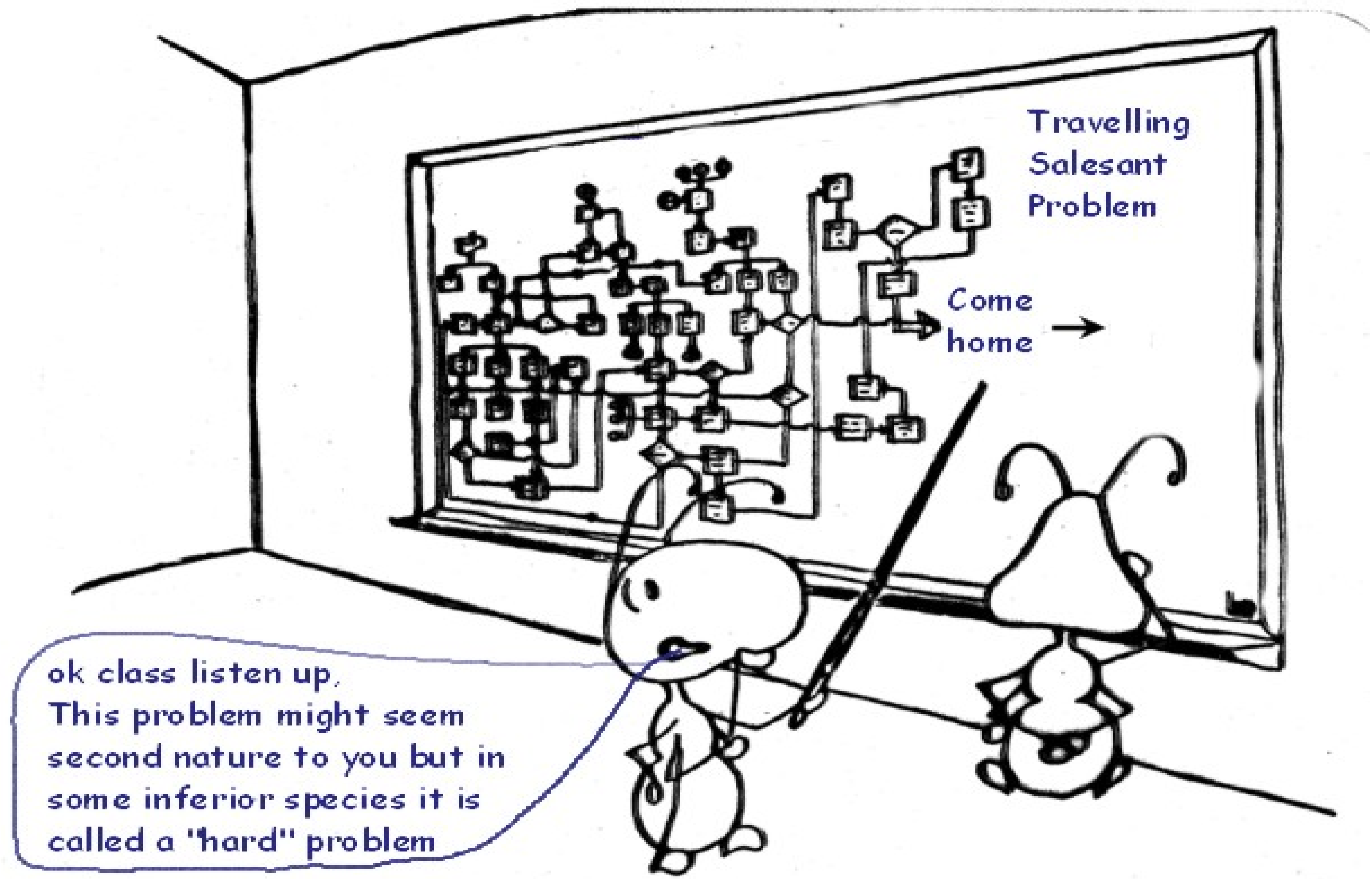
Ant Colony Optimization



Prepared by:
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Presentation Outline

- <u>Section I (Introduction)</u>
- Historical Background
- Ant System
- Modified algorithms

Danny

- <u>Section II (Applications)</u>
- TSP
- QAP

Fadi

- <u>Section III (Applications +Conclusions)</u>
- NRP
- VRP
- Conclusions, limitations and

Ahmad

Section 1

- Introduction (Swarm intelligence)
- Natural behavior of ants
- First Algorithm: Ant System
- Improvements to Ant System
- Applications

Swarm intelligence

- Collective system capable of accomplishing difficult tasks in dynamic and varied environments without any external guidance or control and with no central coordination

- Achieving a collective performance which could not normally be achieved by an individual acting alone

- Constituting a natural model particularly suited to distributed problem solving

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Inherent features

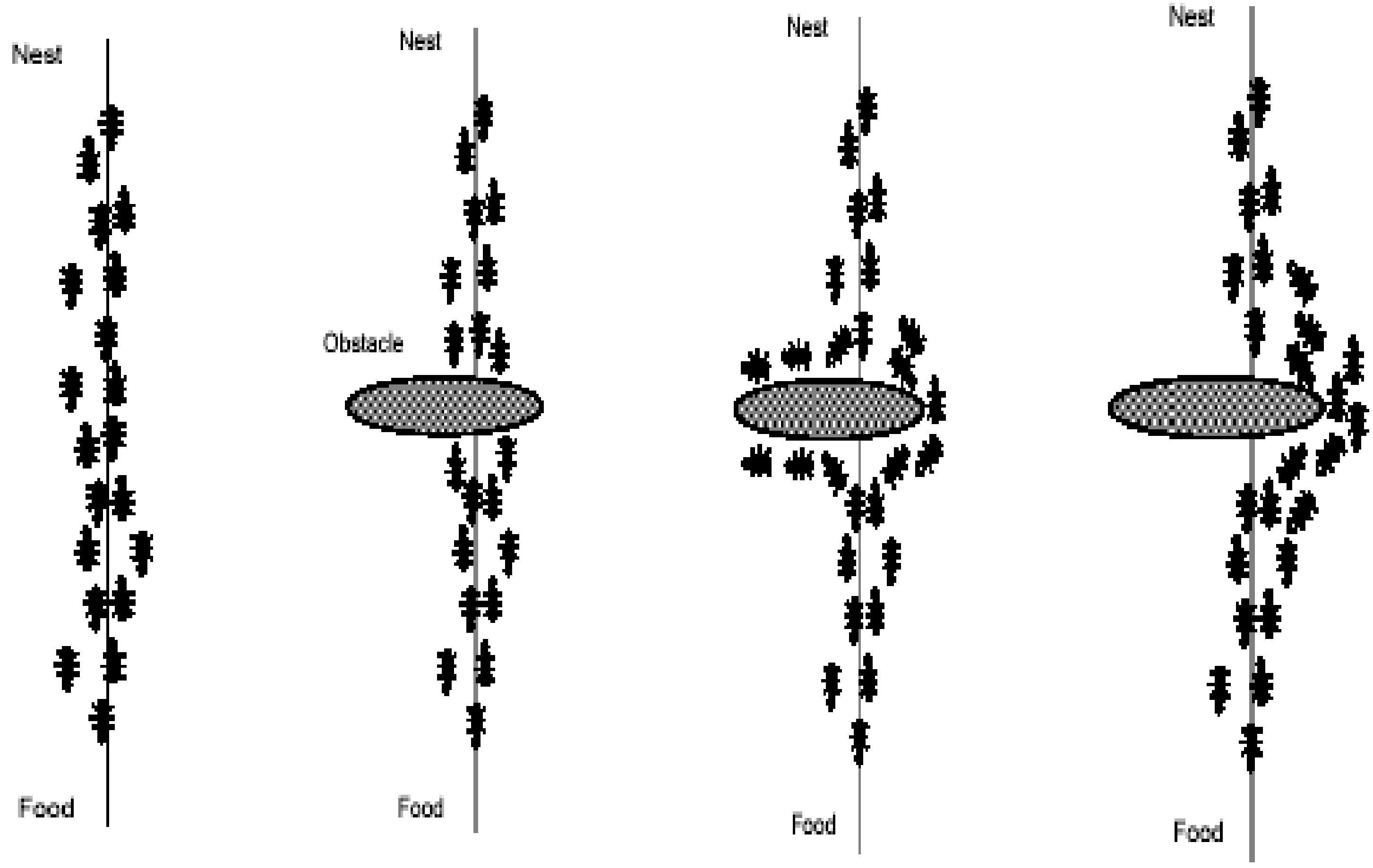
- Inherent parallelism
- Stochastic nature
- Adaptivity
- Use of positive feedback
- Autocatalytic in nature

Natural behavior of an ant

Foraging modes

- Wander mode
- Search mode
- Return mode
- Attracted mode
- Trace mode
- Carry mode

Natural behavior of ant



Work to date

Problem name	Authors	Algorithm name	Year
Traveling salesman	Dorigo, Maniezzo & Colorni	AS	1991
	Gamberdella & Dorigo	Ant-Q	1995
	Dorigo & Gamberdella	ACS &ACS 3 opt	1996
	Stutzle & Hoos	MMAS	1997
	Bullnheimer, Hartl & Strauss	AS_{rank}	1997
	Cordon, et al.	BWAS	2000
Quadratic assignment	Maniezzo, Colorni & Dorigo	AS-QAP	1994
	Gamberdella, Taillard & Dorigo	HAS-QAP	1997
	Stutzle & Hoos	MMAS-QAP	1998
	Maniezzo	ANTS-QAP	1999
	Maniezzo & Colorni	AS-QAP	1994
Scheduling problems	Colorni, Dorigo & Maniezzo	AS-JSP	1997
	Stutzle	AS-SMTTP	1999
	Barker et al	ACS-SMTTP	1999
	den Besten, Stutzle & Dorigo	ACS-SMTWTP	2000
	Merkle, Middenderf & Schmeck	ACO-RCPS	1997
Vehicle routing	Bullnheimer, Hartl & Strauss	AS-VRP	1999
	Gamberdella, Taillard & Agazzi	HAS-VRP	1999

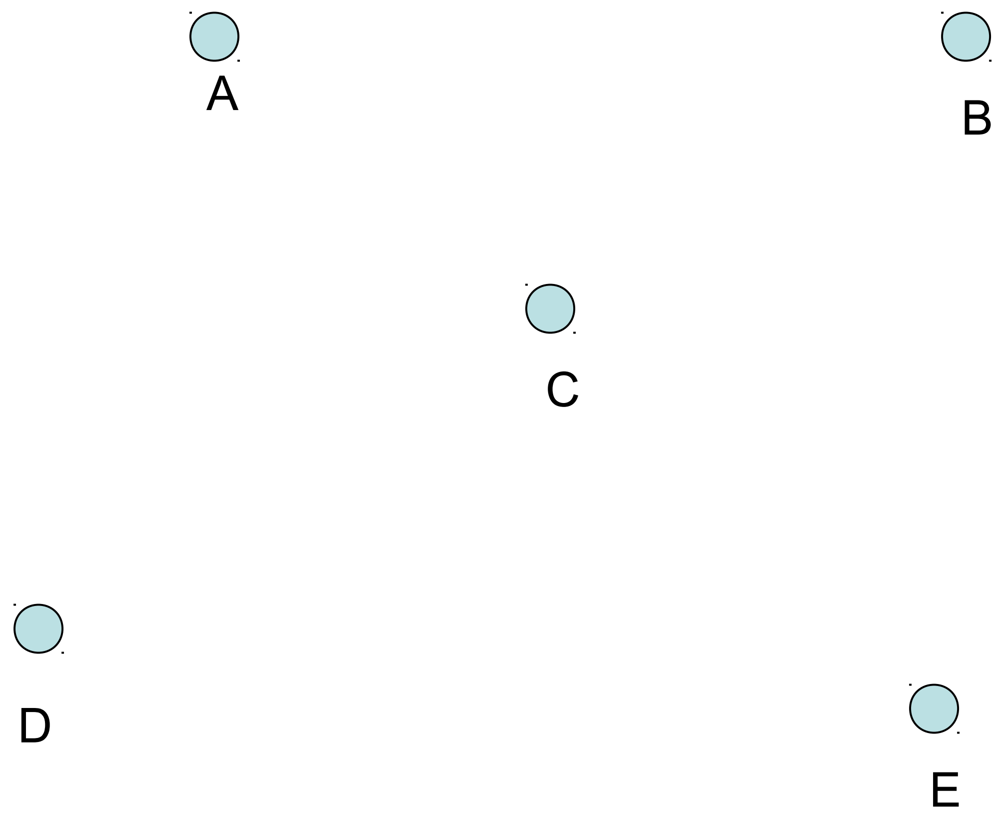
Work to date

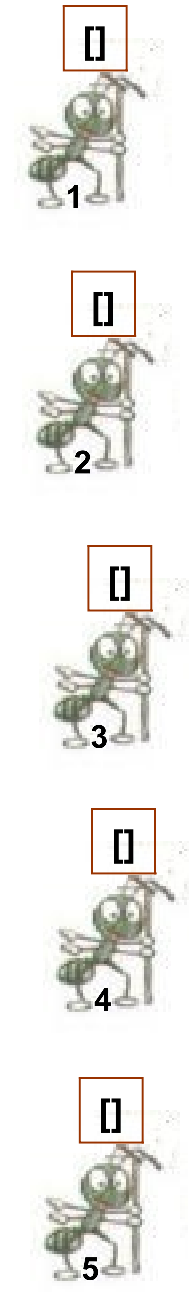
Problem name	Authors	Algorithm name	Year
Connection-oriented	Schoonderwood et al.	ABC	1996
network routing	White, Pagurek & Oppacher	ASGA	1998
	Di Caro & Dorigo	AntNet-FS	1998
	Bonabeau et al.	ABC-smart ants	1998
Connection-less	Di Caro & Dorigo	AntNet & AntNet-FA	1997
network routing	Subramanian, Druschel & Chen	Regular ants	1997
	Heusse et al.	CAF	1998
	van der Put & Rethkrantz	ABC-backward	1998
Sequential ordering	Gamberdella& Dorigo	HAS-SOP	1997
Graph coloring	Costa & Hertz	ANTCOL	1997
Shortest common supersequence	Michel & Middendorf	AS_SCS	1998
Frequency assignment	Maniezzo & Carbonaro	ANTS-FAP	1998
Generalized assignment	Ramalhinho Lourenco & Serra	MMAS-GAP	1998
Multiple knapsack	Leguizamon & Michalewicz	AS-MKP	1999
Optical networks routing	Navarro Varela & Sinclair	ACO-VWP	1999
Redundancy allocation	Liang & Smith	ACO-RAP	1999
Constraint satisfaction	Solnon	Ant-P-solver	2000

How to implement in a program

- Ants: Simple computer agents
- Move ant: Pick next component in the const. solution
- Pheromone: $\Delta\tau_{i,j}^{k}$
- Memory: M_K or $Tabu_K$
- Next move: Use probability to move ant

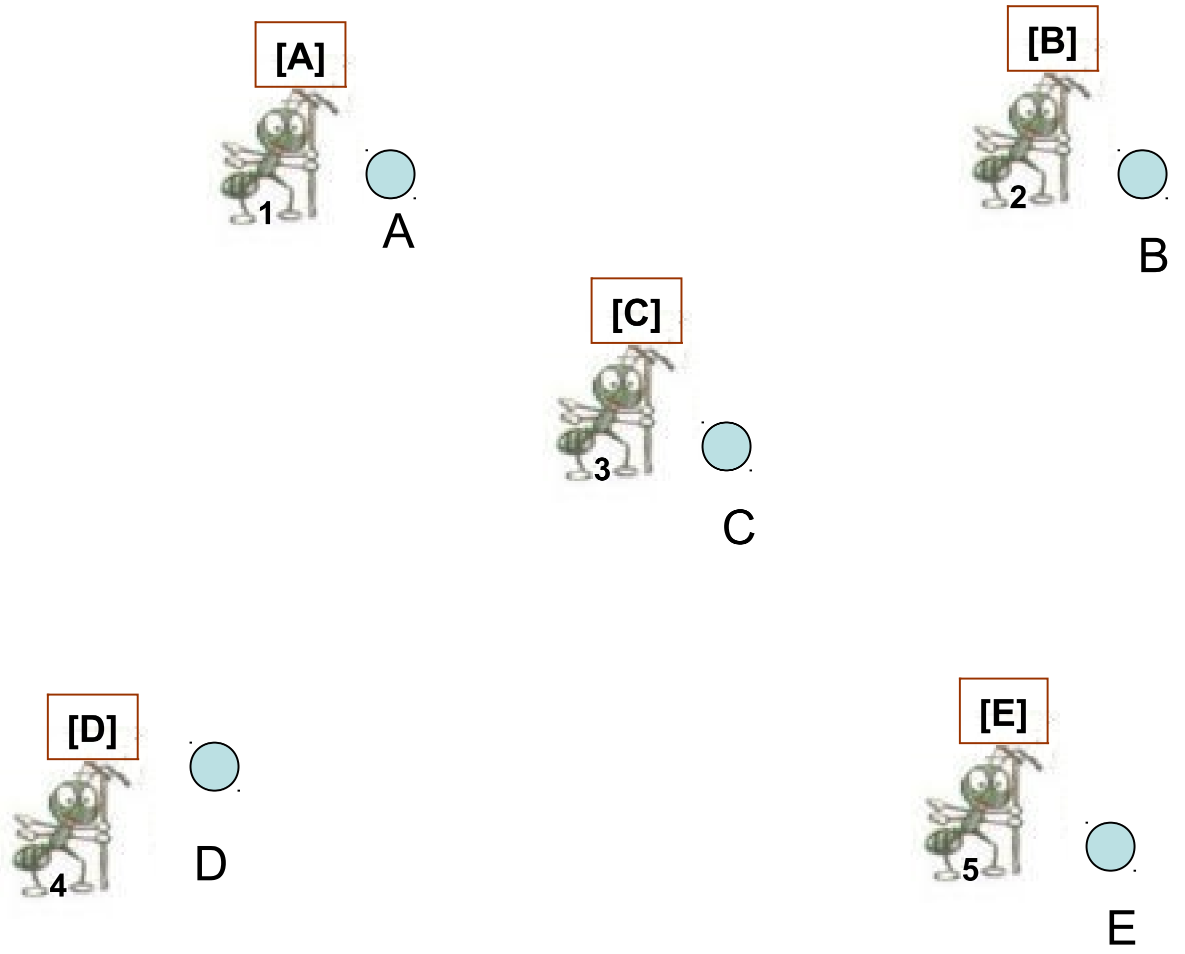
A simple TSP example



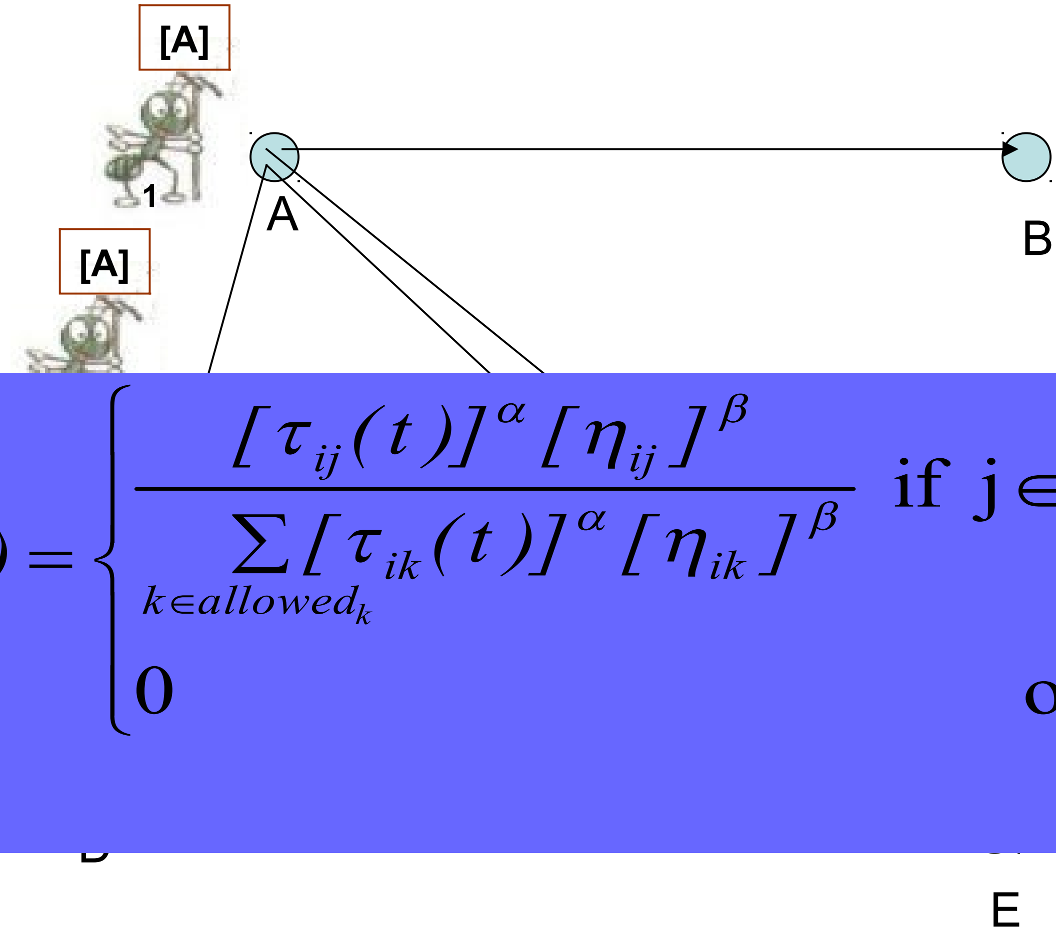


$d_{AB}=100; d_{BC}=60 \ldots; d_{DE}=150$

Iteration 1

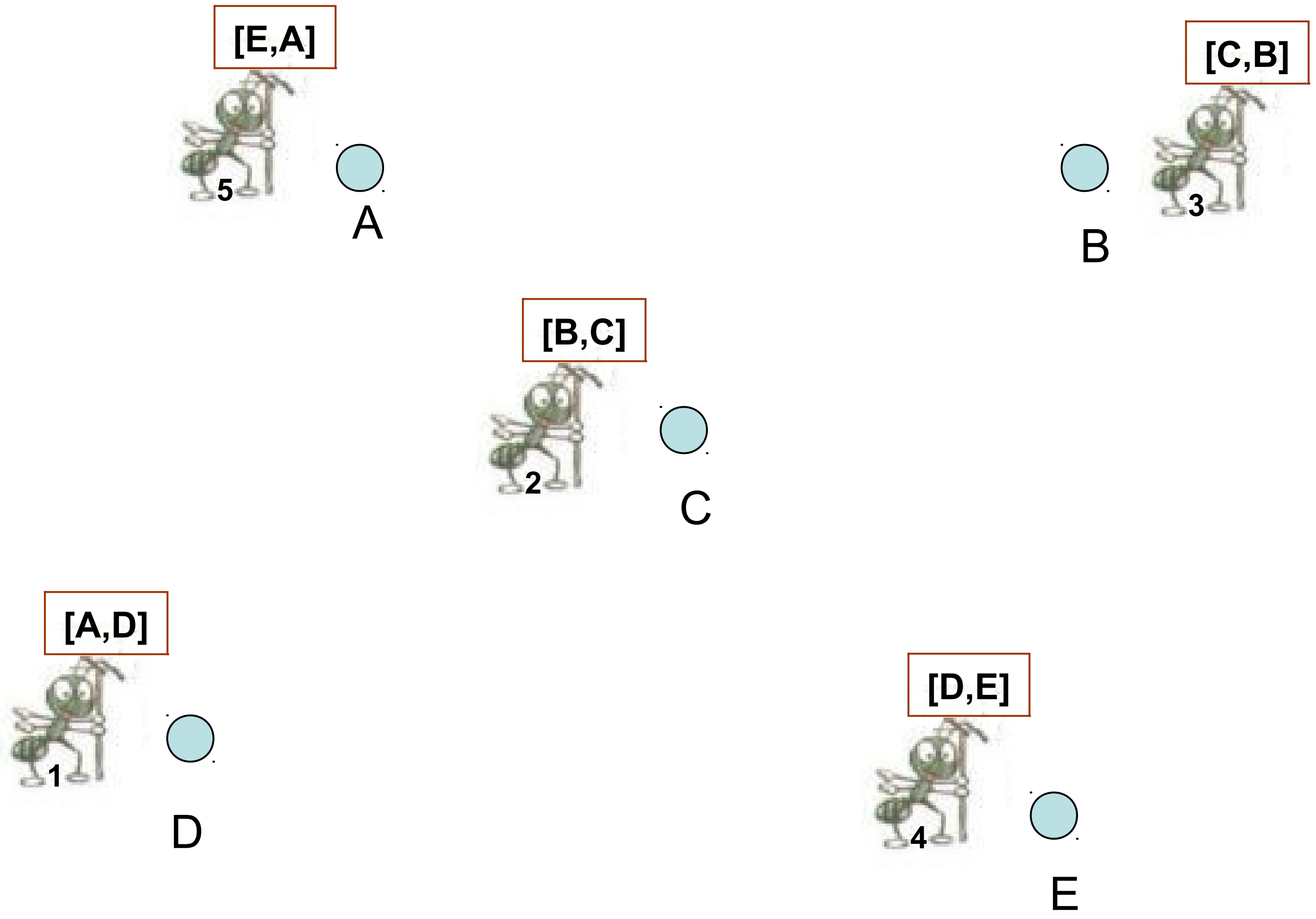


How to build next sub-solution?

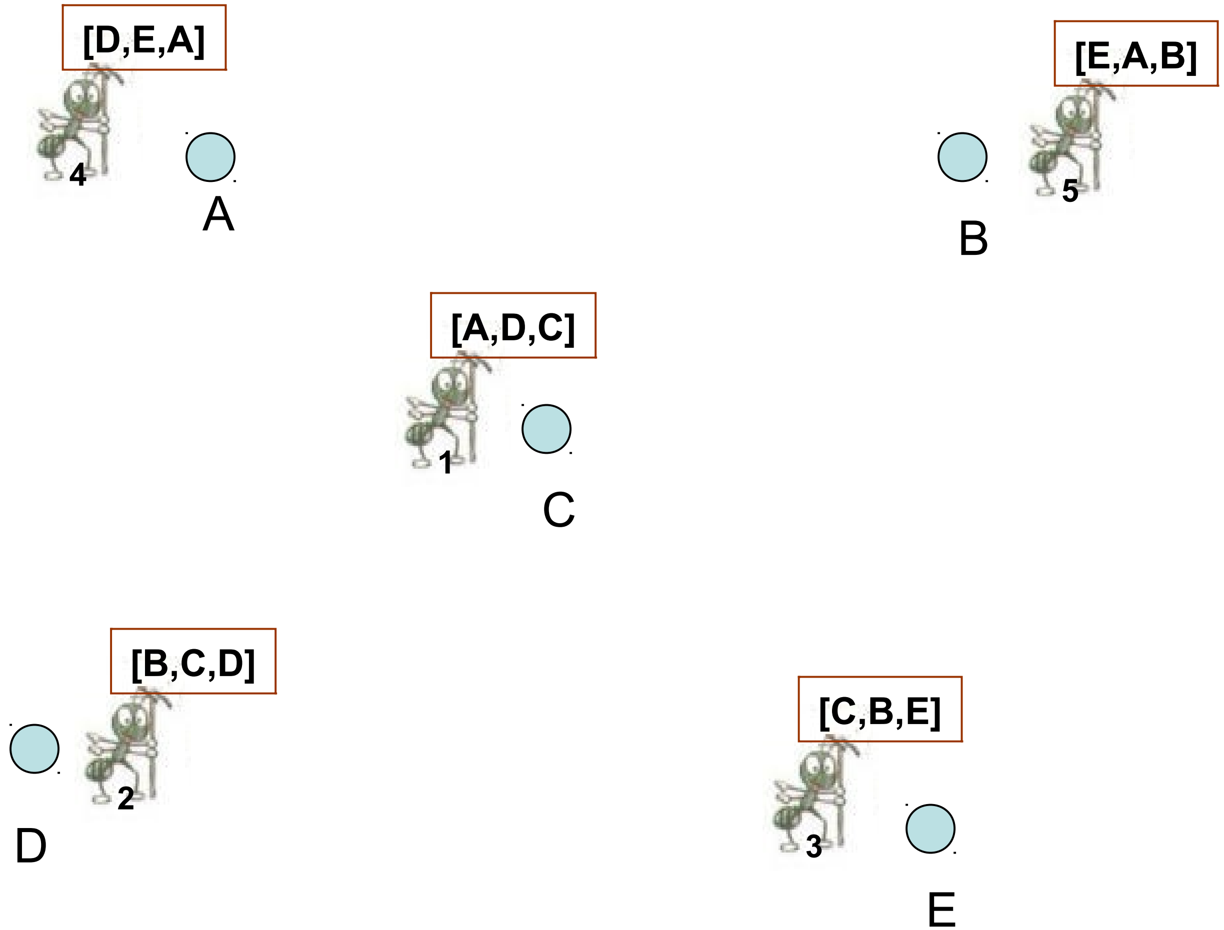


$$p_{ij}^{k}(t)=\begin{cases}\dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum\limits_{k\in allowed_k}[\tau_{ik}(t)]^{\alpha}[\eta_{ik}]^{\beta}} & \text{if } j\in \text{allowed}_{k}\\ 0 & \text{otherwise}\end{cases}$$

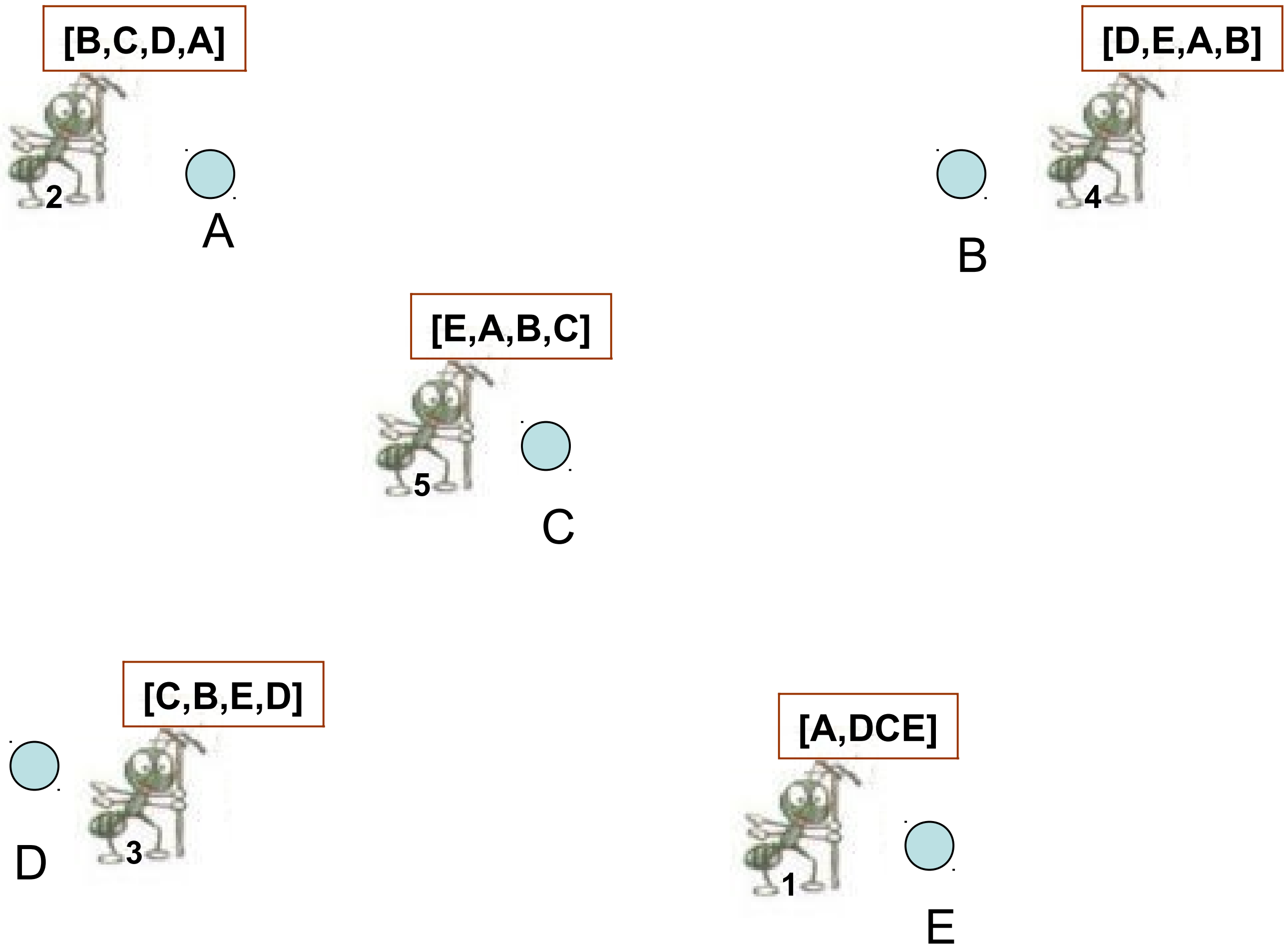
Iteration 2



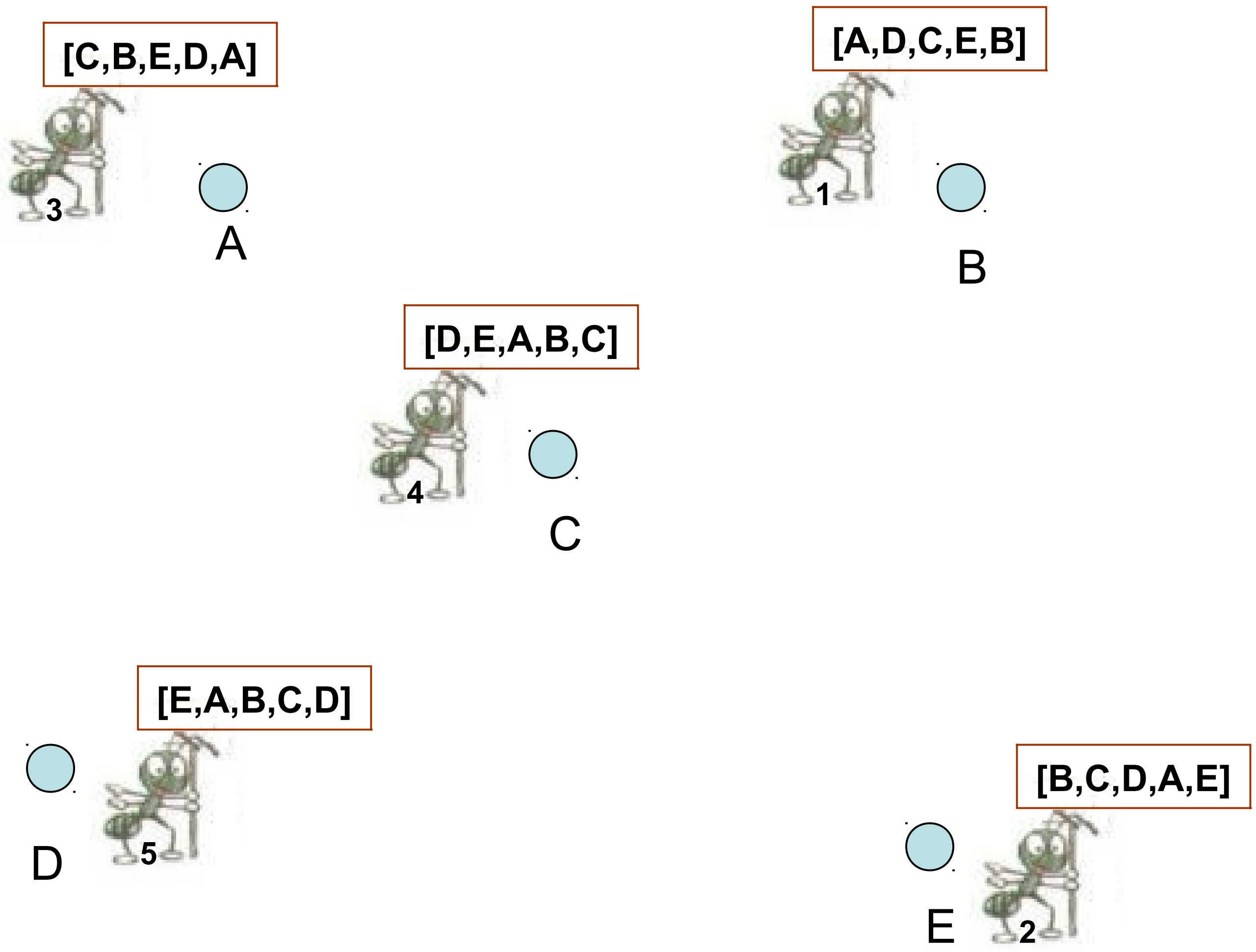
Iteration 3



Iteration 4



Iteration 5



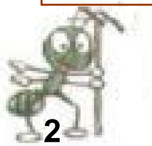
Path and Pheromone Evaluation

[A,D,C,E,B]

L_1 =300

$$\Delta\tau_{i,j}^{k} = \begin{cases} \dfrac{Q}{L_k} & if\,(i,j) \in \text{tour} \\ 0 & \text{otherwise} \end{cases}$$

[B,C,D,A,E]



L_2 =450

[C,B,E,D,A]

$$\Delta\tau_{A,B}^{total} = \Delta\tau_{A,B}^{1} + \Delta\tau_{A,B}^{2} + \Delta\tau_{A,B}^{3} + \Delta\tau_{A,B}^{4} + \Delta\tau_{A,B}^{5}$$

[D,E,A,B,C]

L_4 =280

[E,A,B,C,D]

L_5 =420

End of First Run

Save Best Tour (Sequence and length)

All ants die

New ants are born

Ant System (Ant Cycle) Dorigo [1] 1991

t = 0; NC = 0; $\tau_{ij}(t)$=c for $\Delta\tau_{ij}$=0
Place the m ants on the n nodes

Initialize

$$p_{ij}^{k}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum\limits_{k \in allowed_k}[\tau_{ik}(t)]^{\alpha}[\eta_{ik}]^{\beta}} & \text{if } j \in \text{allowed}_k \\ 0 & \text{otherwise} \end{cases}$$

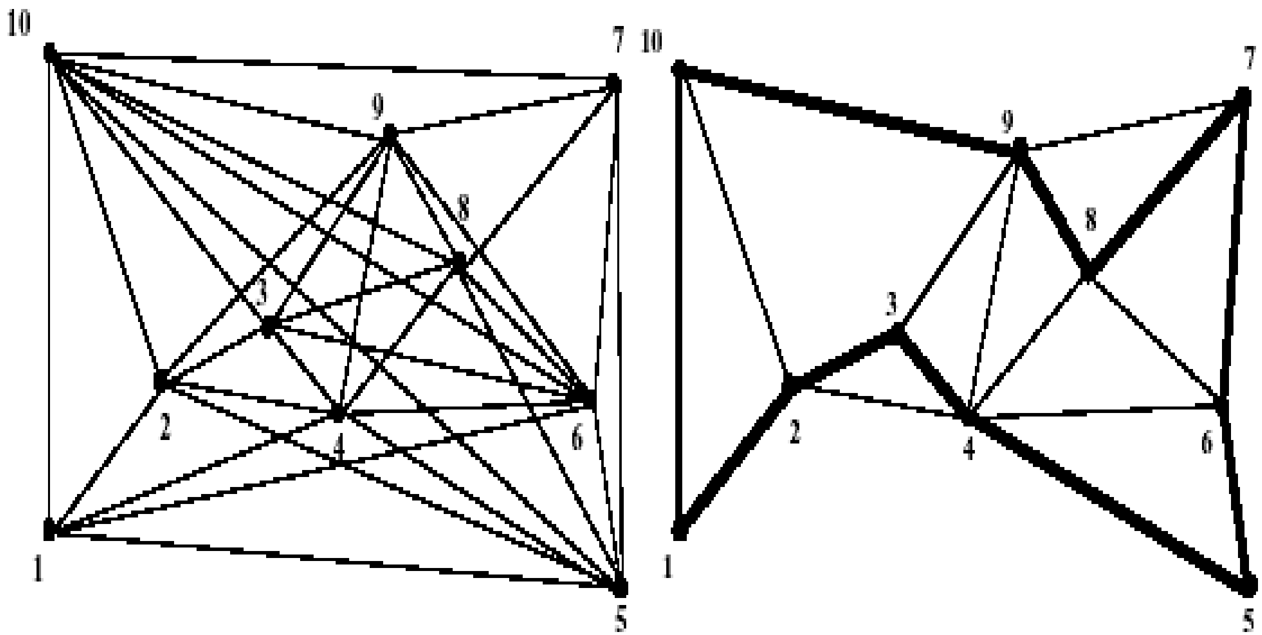
$$\tau_{ij}(t+n) = \rho\tau_{ij}(t) + \Delta\tau_{ij}$$

$$\Delta\tau_{i,j}^{k} = \begin{cases} \dfrac{Q}{L_k} \; if\,(i,j) \in \text{tour described by tabu}_k \\ 0 \qquad \text{otherwise} \end{cases}$$

End

Stopping Criteria

- Stagnation
- Max Iterations



General ACO

- A stochastic construction procedure
- Probabilistically build a solution
- Iteratively adding solution components to partial solutions
    - Heuristic information
    - Pheromone trail
- Reinforcement Learning reminiscence
- Modify the problem representation at each iteration

General ACO

- Ants work concurrently and independently
- Collective interaction via indirect communication leads to good solutions

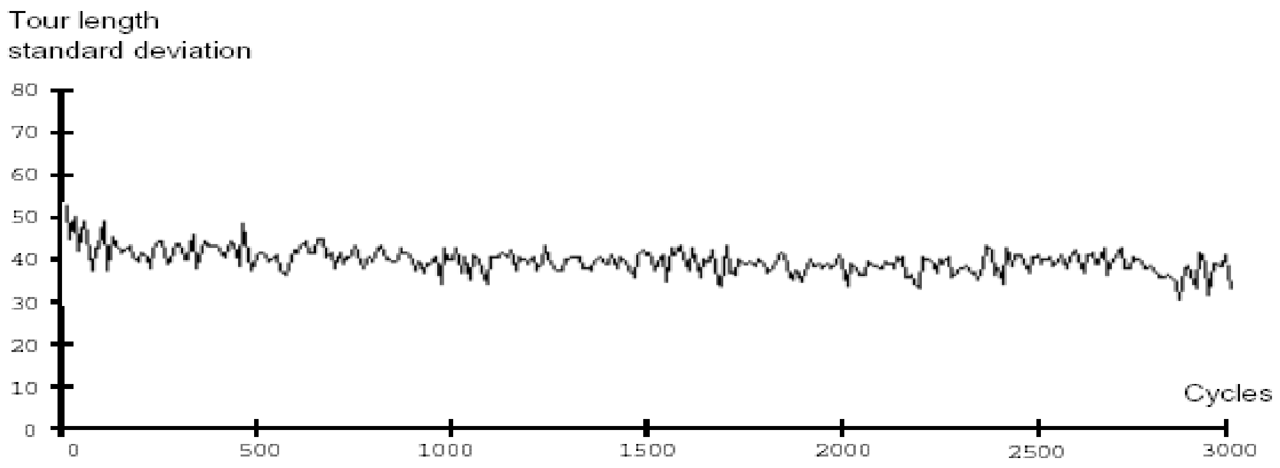
Variations of Ant System

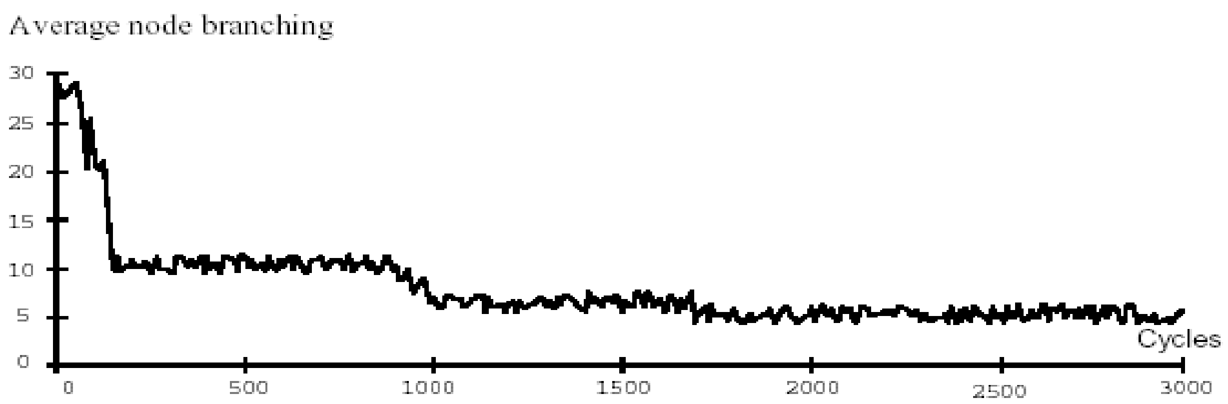
- Ant Cycle ($O(NC.n^3)$)
- Ant Density (Quantity Q)
- Ant Quantity (Quantity Q/d_{ij})

	Best parameter set	Average result	Best result
ant-density	α=1, β=5, ρ=0.99	426.740	424.635
ant-quantity	α=1, β=5, ρ=0.99	427.315	426.255
ant-cycle	α=1, β=5, ρ=0.5	424.250	423.741

Taken from Dorigo [1]

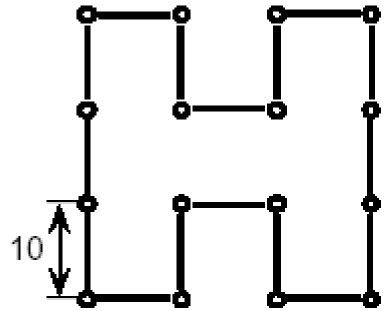
Basic Analysis

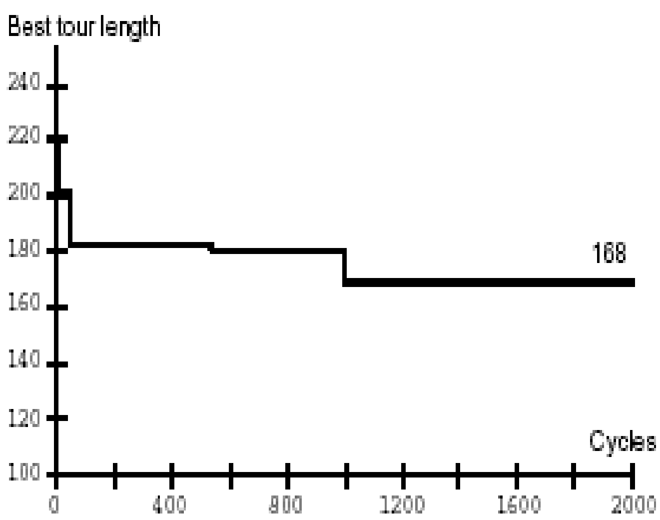




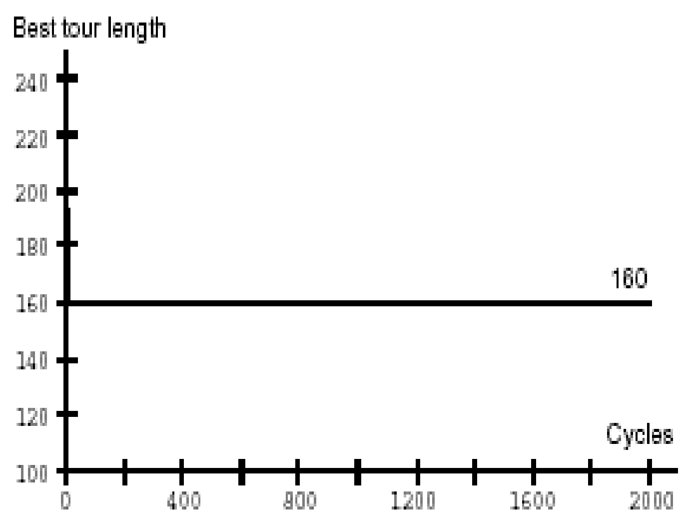
Taken from Dorigo [1]

Basic Analysis





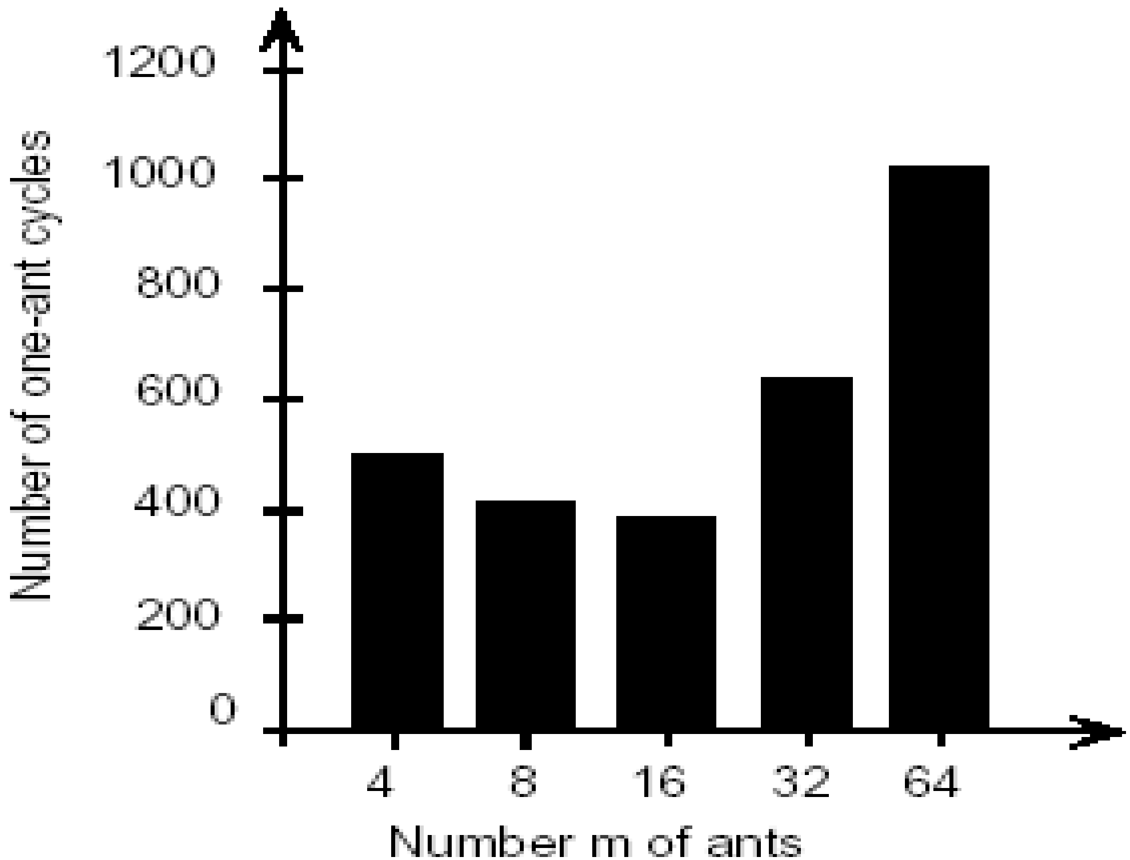
a) α=0



b) α=1

Taken from Dorigo [1]

Optimal number of ants for AS



Taken from Dorigo [1]

Versatility

- Application to ATSP is straightforward
- No modification of the basic algorithm

Some inherent advantages

- Positive Feedback accounts for rapid discovery of good solutions
- Distributed computation avoids premature convergence
- The greedy heuristic helps find acceptable solution in the early solution in the early stages of the search process.
- The collective interaction of a population of agents.

Disadvantages in Ant Systems

- Slower convergence than other Heuristics
- Performed poorly for TSP problems larger than 75 cities.
- No centralized processor to guide the AS towards good solutions

Improvements to AS

- Daemon actions are used to apply centralized actions
  - Local optimization procedure
  - Bias the search process from global information

Improvements to AS

- Elitist strategy

$$\Delta\tau_{ij}^{gb}(t) = \begin{cases} e / L^{gb}(t) & \text{if arc(i, j)} \in T^{gb} \\ 0 & \text{otherwise} \end{cases}$$

- AS_{rank}

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{r=1}^{w-1}(w-r)\Delta\tau_{ij}^{r}(t) + w\Delta\tau_{ij}^{gb}(t)$$

Improvements to AS

- ACS
  - Strong elitist strategy
  - Pseudo-random proportional rule

With Probability q_0:

$$j = arg\ max_{j \in N_i^k} \left\{ \tau_{ij}(t) \eta_{ij}^{\beta} \right\}$$

With Probability (1- q_0):

$$p_{ij}^k(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha} [\eta_{ij}]^{\beta}}{\sum\limits_{k \in allowed_k} [\tau_{ik}(t)]^{\alpha} [\eta_{ik}]^{\beta}} & \text{if } j \in \text{allowed}_k \\ 0 & \text{otherwise} \end{cases}$$

Improvements to AS

- ACS (Pheromone update)

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \rho\Delta\tau_{ij}^{best}(t)$$

  - Update pheromone trail while building the solution
  - Ants eat pheromone on the trail
  - Local search added before pheromone update

Improvements to AS

- MMAS

$$\tau_{min} \leq \tau_{ij} \leq \tau_{max}$$

  - High exploration at the beginning
  - Only best ant can add pheromone
  - Sometimes uses local search to improve its performance

Dynamic Optimization Problems

- ABC (circuit switched networks)

- AntNet (routing in packet-switched networks)

Applications

- Traveling Salesman Problem
- Quadratic Assignment Problem
- Network Model Problem
- Vehicle routing

Section II

- Traveling Salesman Problem
- Quadrature Assignment Problem

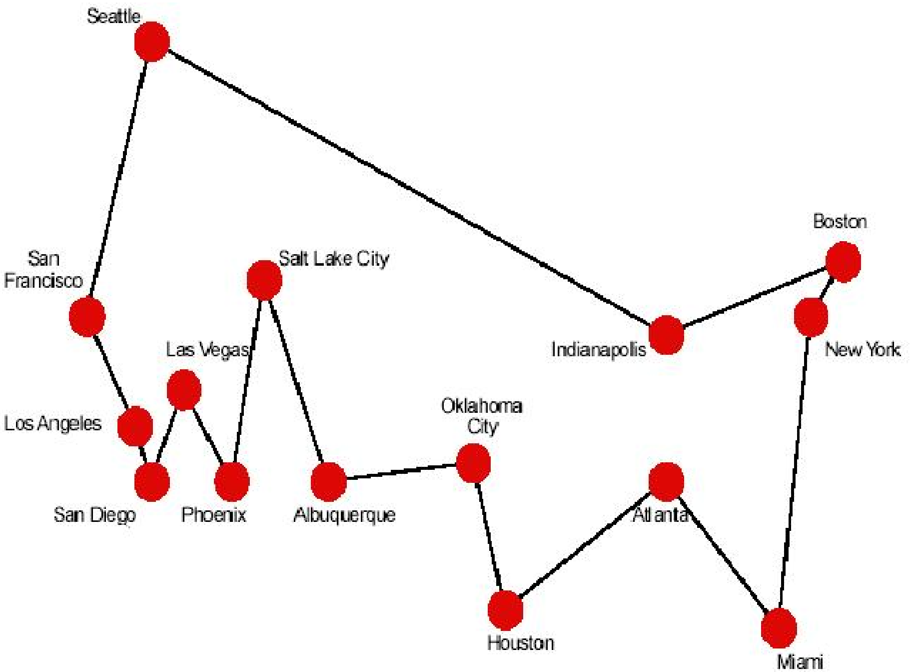
Mr. Fadi Elmasri

Travelling Salesman Problem (TSP)

TSP PROBLEM : Given N cities, and a distance function d between cities, find a tour that:

1. Goes through every city once and only once
2. Minimizes the total distance.

- Problem is NP-hard
- Classical combinatorial optimization problem to test.



ACO for the Traveling Salesman Problem

The TSP is a very important problem in the context of Ant Colony Optimization because it is the problem to which the original AS was first applied, and it has later often been used as a benchmark to test a new idea and algorithmic variants.

The TSP was chosen for many reasons:

- It is a problem to which the ant colony metaphor
- It is one of the most studied NP-hard problems in the combinatorial optimization
- it is very easily to explain. So that the algorithm behavior is not obscured by too many technicalities.

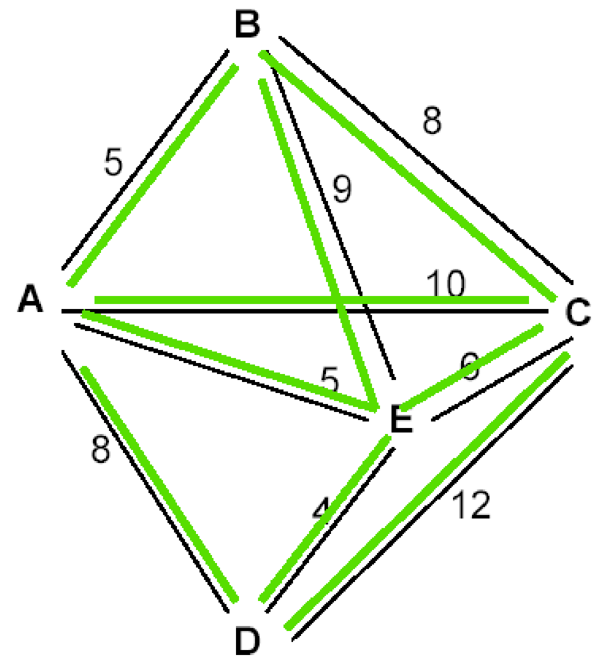
Search Space

Discrete Graph

To each edge is associated a *static value* returned by an heuristic function η (*r*,*s*) based on the edge-cost

Each edge of the graph is augmented with a pheromone trail τ (*r*,*s*) deposited by ants.

***Pheromone is dynamic and* it is learned at run-ime**



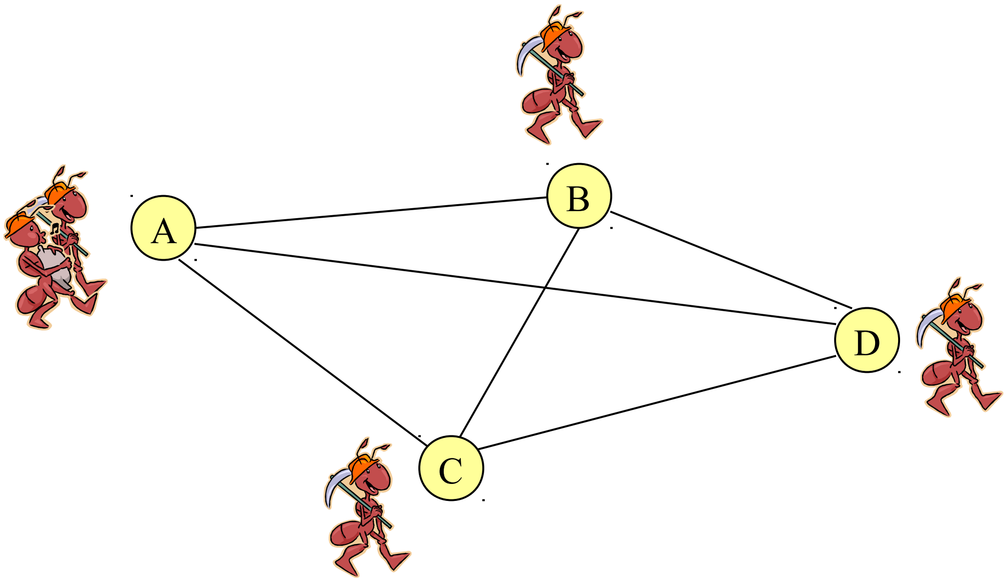
Ant Systems (AS)

Ant Systems for TSP

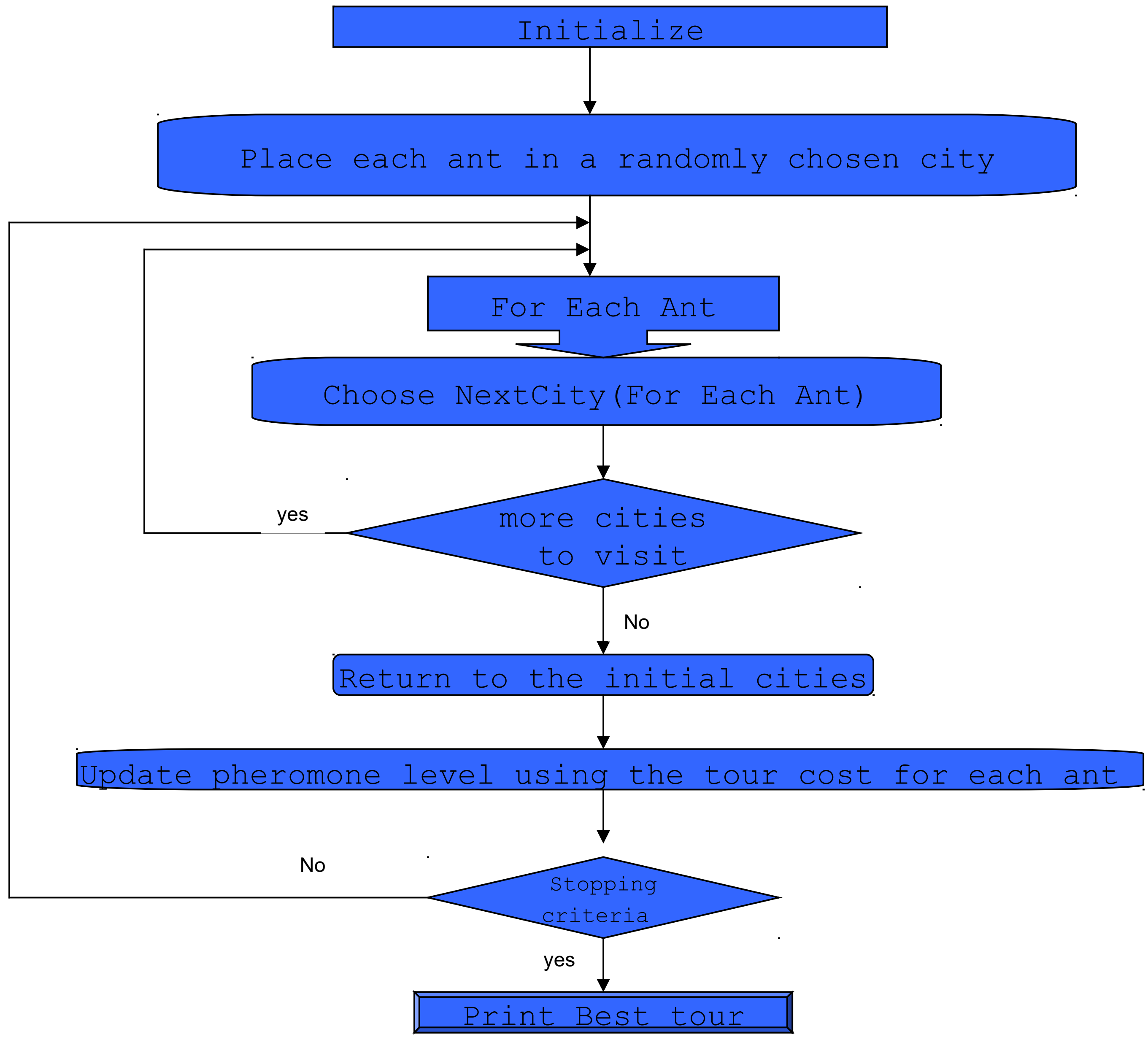
Graph (N,E): where N = cities/nodes, E = edges

d_{ij} = *the tour cost from city i to city j (edge weight)*

Ant move from one city i to the next j with some transition probability.



Ant Systems Algorithm for TSP



Rules for Transition Probability

1. Whether or not a city has been visited
   Use of a **memory**(tabu list): J_i^k : set of all cities that are to be visited

2. N_{ij} = $1/d_{ij}$ **visibility**:Heuristic desirability of choosing city j when in city i.

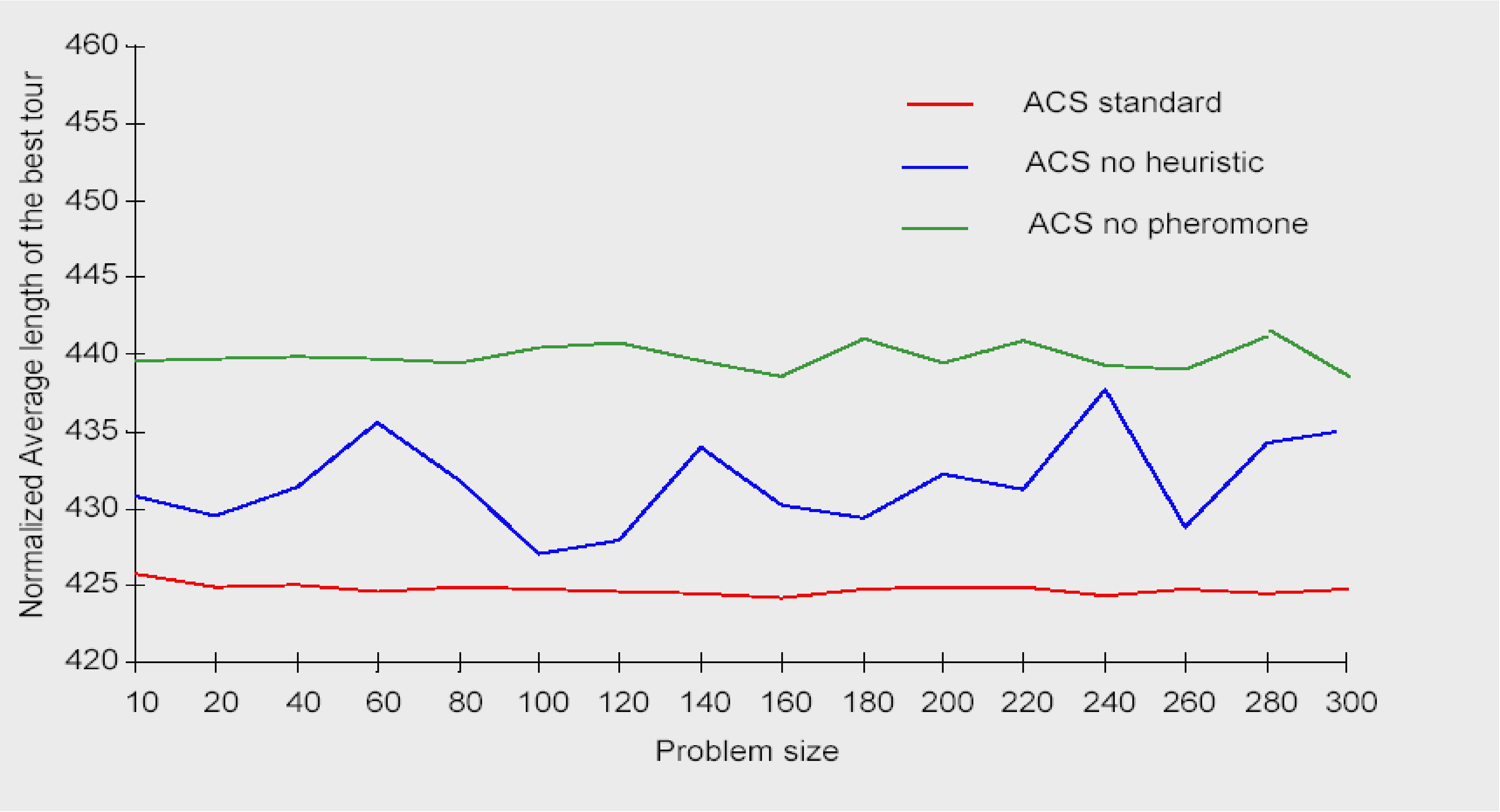
3.**Pheromone trail**: $T_{ij}(t)$ This is a global type of information

Transition probability for ant k to go from city i to city j while building its route.

$$P_{ij}^{k}(t) = \frac{[T_{ij}(t)]^{a} \cdot [n_{ij}]^{\beta}}{\sum_{l \in J_i^k} [T_{lk}(t)]^{a} \cdot [n_{lk}]^{\beta}}$$

a = 0: closest cities are selected

Pheromone trail and heuristic function: are they useful?



Comparison between ACS standard, ACS with no heuristic (i.e., we set B=0), and ACS in which ants neither sense nor deposit pheromone. Problem: Oliver30. Averaged over 30 trials, 10,000/*m* iterations per trial.

Trail pheromone in AS

After the completion of a tour, each ant lays some pheromone

$\Delta \mathrm{T}^{K}{}_{ij}(t)$

for each edge that it has used. depends on how well the ant has performed.

$$\Delta T_{ij}^{k}(t) = \begin{cases} Q / l^{k}(t) & \text{if}(i,j) \in T^{k}(t) \\ 0 & \text{if}(i,j) \notin T^{k}(t) \end{cases}$$

Trail pheromone decay = $T_{ij}(t) \leftarrow (1-p)T_{ij}(t-1) + \Delta T_{ij}(t)$

Ant Colony Optimization (ACO)

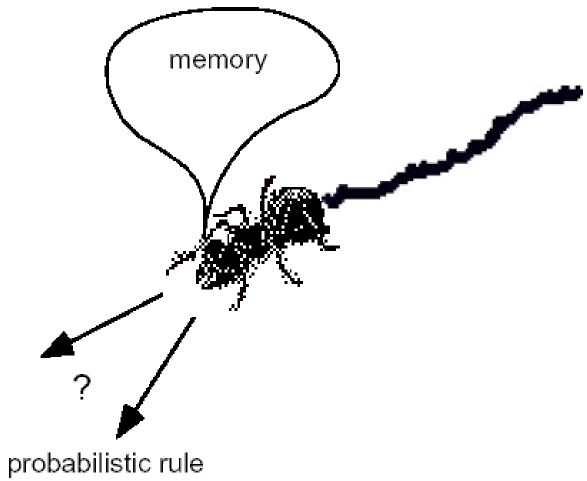
Dorigo & Gambardella introduced four modifications in AS :

1. a different transition rule,
2. Local/global pheromone trail updates,
3. use of local updates of pheromone trail to favor exploration
4. a candidate list to restrict the choice of the next city to visit.

ACS : Ant Colony System for TSP

```
Loop
   Randomly position m artificial ants on n cities
   For city=1 to n
      For ant=1 to m
         {Each ant builds a solution by adding one city after
         the other}
         Select probabilistically the next city according to
                        exploration and exploitation mechanism
         Apply the local trail updating rule
      End for
   End for
   Apply the global trail updating rule using the best ant
Until End_condition
```

ACO State Transition Rule



Next city is chosen between the **not visited** cities according to a *probabilistic* rule

Exploitation: the best edge is chosen

Exploration: each of the edges in proportion to its value

ACS State Transition Rule : Formulae

$$s = \begin{cases} \arg\max\limits_{u \in J_k(r)} \left\{ [\tau(r,u)] \cdot [\eta(r,u)]^{\beta} \right\} & \text{if } q \le q_0 \quad \text{(Exploitation)} \\ S & \text{otherwise} \quad \text{(Exploration)} \end{cases}$$

where

- **S** is a stochastic variable distributed as follows:

$$p_k(r,s) = \begin{cases} \dfrac{[\tau(r,s)] \cdot [\eta(r,s)]^{\beta}}{\sum\limits_{u \in J_k(r)} [\tau(r,u)] \cdot [\eta(r,u)]^{\beta}} & \text{if } s \in J_k(r) \\ 0 & \text{otherwise} \end{cases}$$

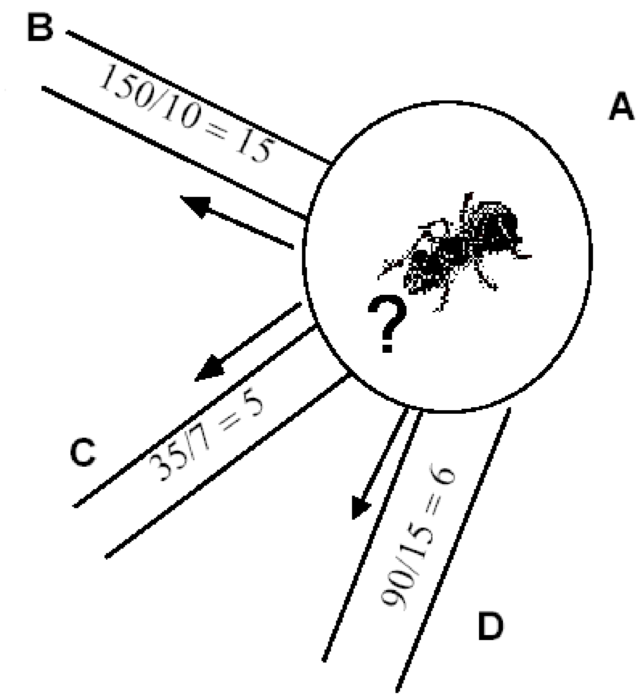
- τ is the trail
- η is the inverse of the distance
- $\mathbf{J_k(r)}$ is the set of cities still to be visited by ant **k** positioned on city **r**
- β and $\mathbf{q_0}$ are parameters

ACS State Transition Rule : example

$$\tau(A,B) = 150 \quad \eta(A,B) = 1/10$$

$$\tau(A,B) = 35 \quad \eta(A,B) = 1/7$$

$$\tau(A,B) = 90 \quad \eta(A,B) = 1/15$$



•with probability q_0 **exploitation**
(Edge AB = 15)

•with probability (1- q_0) **exploration**

AB with probability 15/26
AC with probability 5/26
AD with probability 6/26

ACS Local Trail Updating

… similar to evaporation

If an edge (r,s) is visited by an ant

$$\tau(r,s) = (1-\rho) \cdot \tau(r,s) + \rho \, \Delta\tau(r,s)$$

with $\Delta\tau(r,s)$ = τ_0

ACS Global Trail Updating

At the end of each iteration the best ant is allowed to reinforce its tour by depositing additional pheromone inversely proportional to the length of the tour

$$\tau(r,s) \leftarrow (1-\alpha)\cdot \tau(r,s) + \alpha \cdot \Delta\tau(r,s)_{Globa}$$

where

$$\Delta\tau(r,s)_{Global} = \frac{1}{\mathrm{L}_{\mathrm{best}}}$$

Effect of the Local Rule

- Local rule: learnt desirability of edges changes dynamically
- Local update rule makes the edge pheromone level diminish.
- Visited edges are less & less attractive as they are visited by the various ants.
- Favors exploration of not yet visited edges.
  This helps in shuffling the
  cities so that cities visited early in one ants tours are being
  visited later in another ants tour.

ACO vs AS

Pheromone trail update

Deposit pheromone after completing a tour in AS

Here in ACO only the ant that generated the best tour from the beginning of the trial is allowed to globally update the concentrations of pheromone on the branches (ants search at the vicinity of the best tour so far)

In AS pheromone trail update applied to all edges

Here in ACO the global pheromone trail update is applied only to the **best tour** since trial began.

ACO : Candidate List

Use of a candidate list

A list of preferred cities to visit: instead of
examining all cities, unvisited cities are examined first.

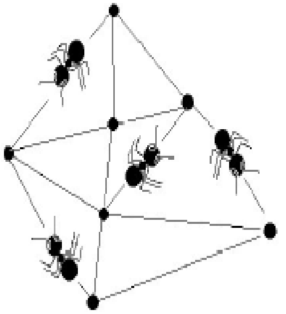
Cities are ordered by increasing distance & list is scanned sequentially.

- Choice of next city from those in the candidate list.
- Other cities only if all the cities in the list have been visited.

Performance

• Algorithm found best solutions on small problems (75 city)
• On larger problems converged to good solutions – but not the best
• On “static” problems like TSP hard to beat specialist algorithms
• Ants are “dynamic” optimizers – should we even expect good performance on static problems
• Coupling ant with local optimizers gave world class results….

Quadratic Assignment Problem(QAP)



Problem is:

- Assign n activities to n locations (campus and mall layout).

- D= $\left[d_{i,j}\right]_{n,n}$, $d_{i,j}$, distance from location i to location j

- F= $\left[f_{h,k}\right]_{n,n}$, $f_{i,j}$,flow from activity h to activity k

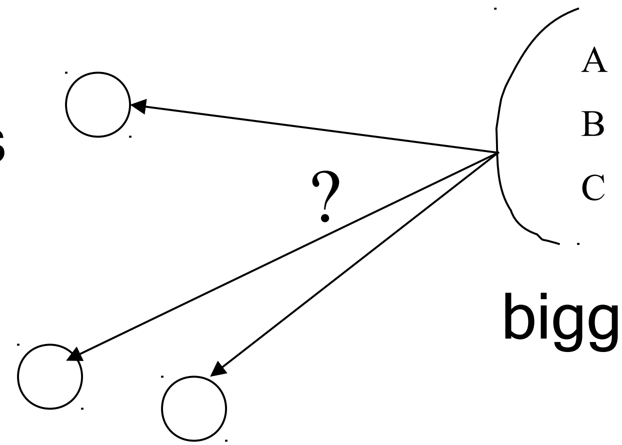
- Assignment is permutatio Π
- Minimize:

$$C(\pi) = \sum_{i,j=1}^{n} d_{ij} f_{\pi(i)\pi(j)}$$

- It's NP hard

QAP Example

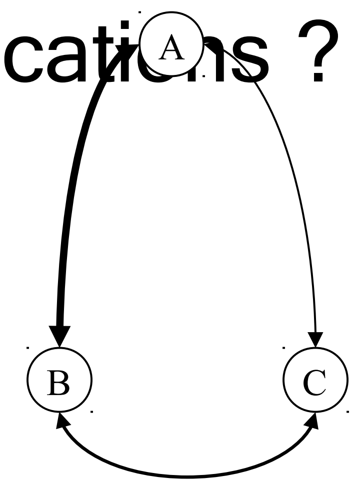
Locations



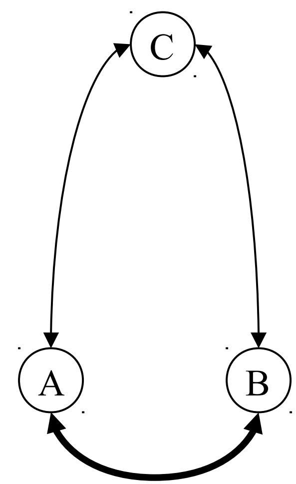
Facilities

biggest flow: A - B

How to assign facilities to locations ?



Higher cost



Lower cost

SIMPLIFIED CRAFT (QAP)

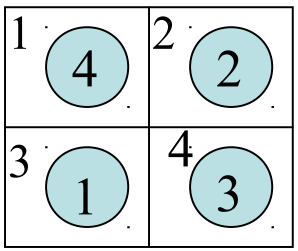
Simplification Assume all departments have equal size

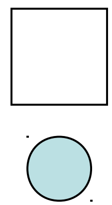
Notation $d_{i,j}$ distance between **locations** i and j

$f_{k,h}$ travel frequency between **departments** k and h

$X_{i,k} \begin{cases} 1 \text{ if department k is assigned to location i} \\ 0 \text{ otherwise} \end{cases}$

Example





Location

Department („Facility“)

Distance* $d_{i,j}$

	1	2	3	4
1	-	1	1	2
2	1	-	2	1
3	1	2	-	1
4	2	1	1	-

Frequency* $f_{k,h}$

	1	2	3	4
1	-	1	3	2
2	2	-	0	1
3	1	4	-	0
4	3	1	1	-

Ant System (AS-QAP)

Constructive method:

step 1: choose a facility j

step 2: assign it to a location i

Characteristics:

– each ant leaves trace (pheromone) on the chosen couplings (i,j)

– assignment depends on the probability (function of pheromone trail and a heuristic information)

– already coupled locations and facilities are inhibited (Tabu list)

AS-QAP Heuristic information

Distance and Flow Potentials

$$D_{ij} = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 4 & 5 \\ 2 & 4 & 0 & 6 \\ 3 & 5 & 6 & 0 \end{bmatrix} \Rightarrow D_i = \begin{bmatrix} 6 \\ 10 \\ 12 \\ 14 \end{bmatrix} \qquad F_{ij} = \begin{bmatrix} 0 & 60 & 50 & 10 \\ 60 & 0 & 30 & 20 \\ 50 & 30 & 0 & 50 \\ 10 & 20 & 50 & 0 \end{bmatrix} \Rightarrow F_i = \begin{bmatrix} 120 \\ 110 \\ 130 \\ 80 \end{bmatrix}$$

The coupling Matrix:

$$S = \begin{bmatrix} 720 & 1200 & 1440 & 1680 \\ 660 & 1100 & 1320 & 1540 \\ 780 & 1300 & 1560 & 1820 \\ 480 & 800 & 960 & 1120 \end{bmatrix}$$

$$s_{11} = f_1 \bullet d_1 = 720$$
$$s_{34} = f_3 \bullet d_4 = 960$$

Ants choose the location according to the heuristic desirability "Potential goodness"

$$\zeta_{ij} = \frac{1}{s_{ij}}$$

AS-QAP Constructing the Solution

- The facilities are ranked in decreasing order of the flow potentials
- Ant k assigns the facility i to location j with the probability given by:

$$p_{ij}^{k}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i^k} [\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}} & if \quad j \in N_i^k \end{cases}$$

where N_i^k is the feasible Neighborhood of node i

- When Ant k choose to assign facility j to location i it leave a substance, called trace “pheromone” on the coupling (i,j)

- Repeated until the entire assignment is found

AS-QAP Pheromone Update

- Pheromone trail update to all couplings:

$$\tau_{ij}(t+1) = \rho.\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

$\Delta\tau_{ij}^{k}$ is the amount of pheromone ant k puts on the coupling (i,j)

$$\Delta_{ij}^{k} = \begin{cases} \dfrac{Q}{J_{\psi}^{k}} & \mathit{if} \;\; \text{facility i is assigned to location j in the solution of ant k} \\ 0 & \text{otherwise} \end{cases}$$

- J_{ψ}^{k} …the objective function value
- Q...the amount of pheromone deposited by ant k

Hybrid Ant System For The QAP

- Constructive algorithms often result in a poor solution quality compared to local search algorithms.

- Repeating local searches from randomly generated initial solution results for most problems in a considerable gap to optimal soultion

- Hybrid algorithms combining solution constructed by (artificial) ant "probabilistic constructive" with local search algorithms yield significantly improved solution.

Hybrid Ant System For The QAP (HAS-QAP)

- HAS-QAP uses of the pheromone trails in a non-standard way.

  used to modify an existing solution,

- improve the ant’s solution using the local search algorithm.

- Intensification and diversification mechanisms.

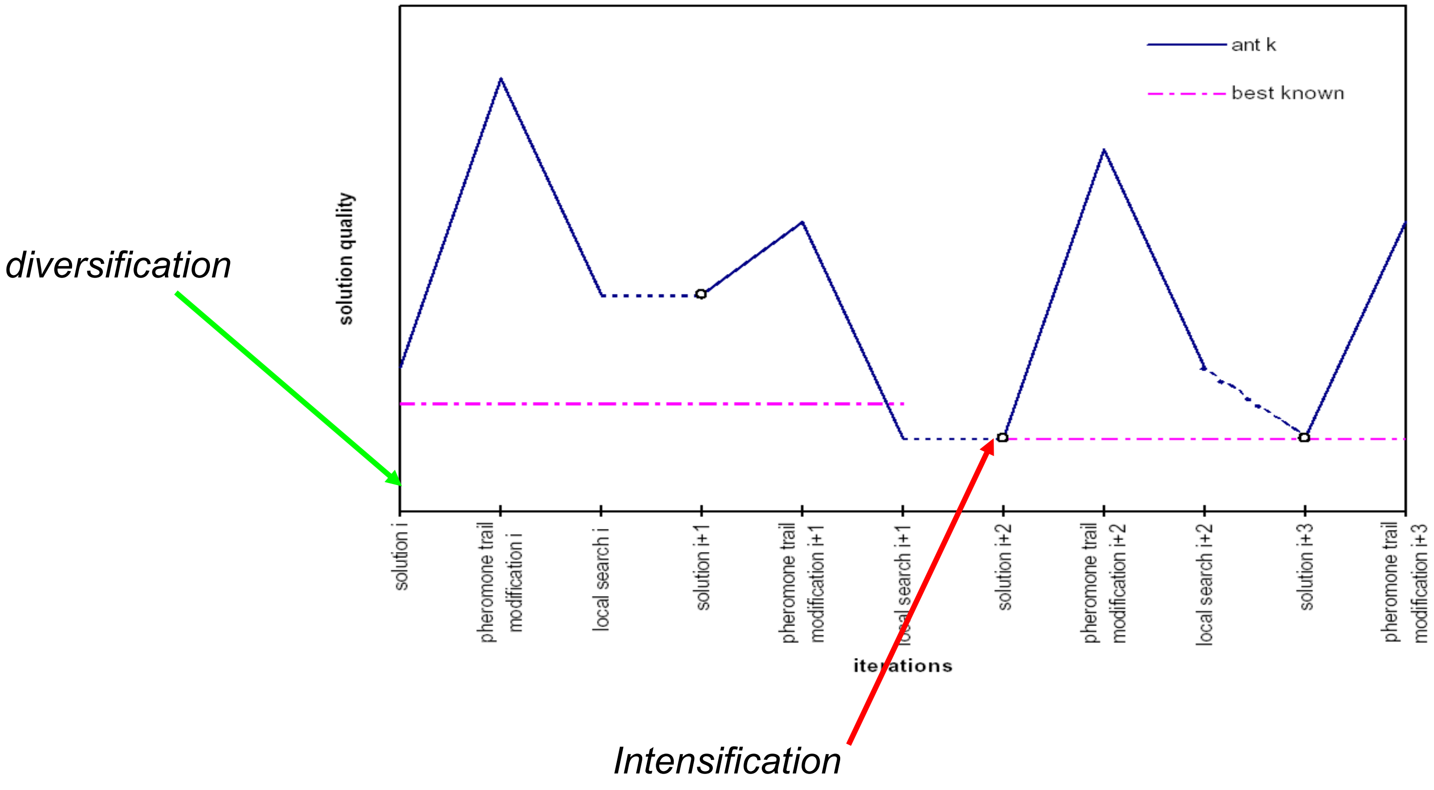
Hybrid Ant System For The QAP (HAS-QAP)

```
Generate m initial solutions, each one associated to one ant
   Initialise the pheromone trail
   For Imax iterations repeat
     For each ant k = 1,..., m do
       Modify ant k;s solution using the pheromone trail
       Apply a local search to the modified solution
       new starting solution to ant k using an intensification mechanism
     End For
     Update the pheromone trail
     Apply a diversification mechanism
End For
```

HAS-QAP Intensification& diversification mechanisms

- The intensification mechanism is activated when the best solution produced by the search so far has been improved.

- The diversification mechanism is activated if during the last *S* iterations no improvement to the best generated solution is detected.



HAS-QAP algorithms Performance

- Comparisons with some of the best heuristics for the QAP have shown that HAS-QAP is among the best as far as real world, and structured problems are concerned.

- The only competitor was shown to genetic-hybrid algorithm.

- On random, and unstructured problems the performance of HAS-QAP was less competitive and tabu searches are still the best methods.

- So far, the most interesting applications of ant colony optimization were limited to travelling salesman problems and quadratic assignment problems..

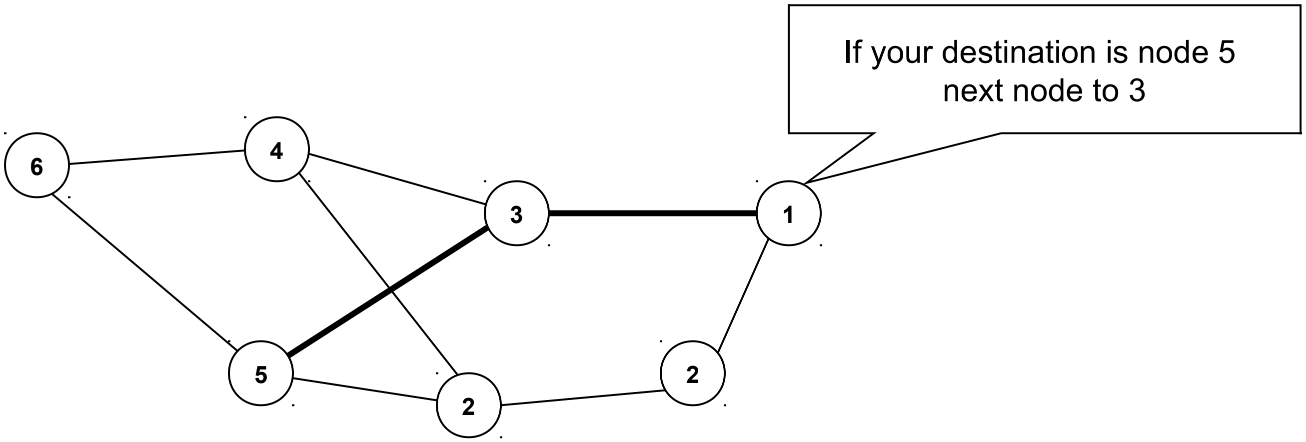
Section III

- Network Routing
- Vehicle Routing
- Conclusions

Mr. Ahmad Elshamli

ROUTING IN COMM. NETWORKS

Routing task is performed by Routers.
Routers use “Routing Tables” to direct the data.



ROUTING IN COMM. NETWORKS

Problem statement

- **Dynamic Routing**

  At any moment the pathway of a message must be as small as possible. (Traffic conditions and the structure of the network are constantly changing)

- **Load balancing**

  Distribute the changing load over the system and minimize lost calls.

ROUTING IN COMM. NETWORKS

Objective:

Minimize: Lost calls by avoiding congestion,

Minimize: Pathway

Dynamic Optimization Problem

+

Multi-Objectives Optimization Problem

ROUTING IN COMM. NETWORKS

Traditional way:

“Central Controllers”

Disadvantage:

- Communication overhead.
- Fault tolerance ~ Controller Failure.
- Scalability
- Dynamic ~ Uncertainty
- Authority.

Algorithm I

Ant-based load balancing in telecommunication networks
(Schoonderwoerd, R. -1996)

- Network has n nodes.
- Each node has its Routing Table (pheromone table) $\{R_i[n-1][k]\}$
- Initialize: equilibrium Routing table (all nodes have the same value or normalized random values)
- Each node lunches {n-1} ants (agents) each to different destination.
- Each ant select its next hop node proportionally to goodness of each neighbor node
- routing table of the node that just the ant arrived to is updated as follows:

Reference: Schoonderwoerd, R. (1996) “Ant-based load balancing in telecommunication networks” Adapt. Behav. 5, 169-207

Algorithm I (cont.)

Increase the probability of the visited link by:

$$\rho = \frac{\rho_{old} + \Delta\rho}{1 + \Delta\rho}$$

Decrease the probability of the others by :

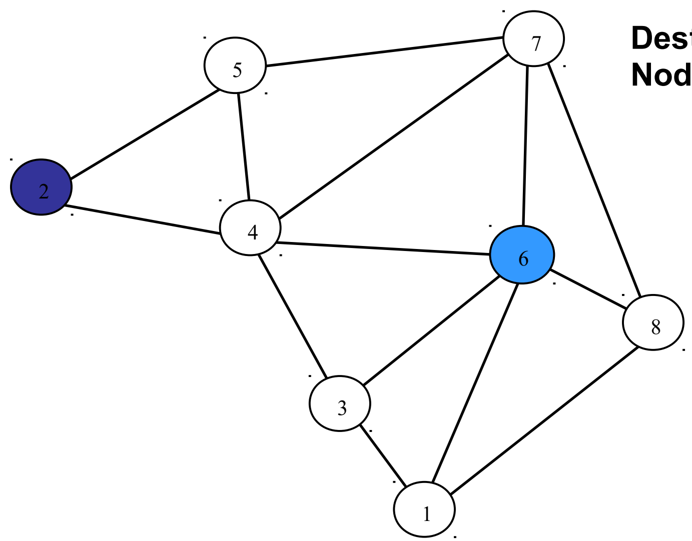
$$\rho = \frac{\rho_{old}}{1 + \Delta\rho}$$

Where $$\Delta\rho = f\left(\frac{1}{age}\right)$$

Reference: Schoonderwoerd, R. (1996) "Ant-based load balancing in telecommunication networks" Adapt. Behav. 5, 169-207

Algorithm I (cont.)

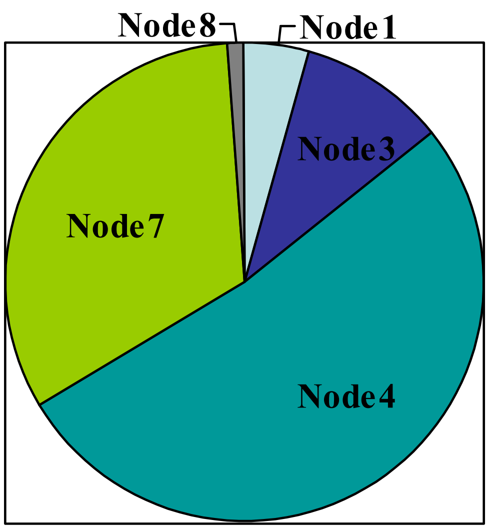
Example:



Destination Nodes

Pheromone Table @ NODE 6

	Next node 1	3	4	7	8
1	0.850	0.100	0.009	0.001	0.090
2	0.045	0.100	0.520	0.325	0.010
3	0.020	0.925	0.045	0.008	0.002
4	0.004	0.100	0.800	0.090	0.006
5	0.010	0.095	0.470	0.410	0.015
7	0.005	0.003	0.020	0.948	0.024
8	0.015	0.005	0.002	0.023	0.955



Reference: Schoonderwoerd, R. (1996) “Ant-based load balancing in telecommunication networks” Adapt. Behav. 5, 169-207

Algorithm I (cont.)

Example:

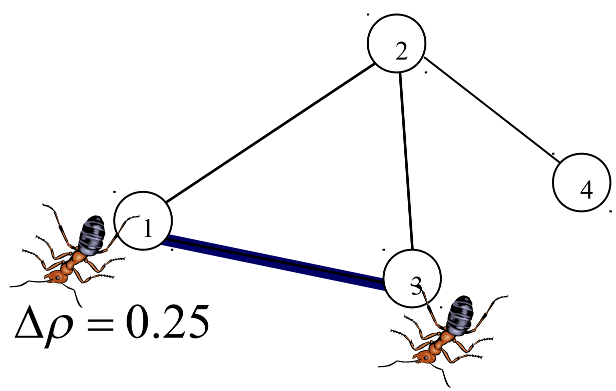
$$\rho = \frac{\rho_{old} + \Delta\rho}{1 + \Delta\rho}$$

Routing table @ node 1

Destination node	Next node 2	Next node 3
3	0.50	0.50
2	0.50	0.50
4	0.50	0.50

Destination node	Next node 2	Next node 3
3	0.40	0.60
2	0.50	0.50
4	0.50	0.50

$$\rho = \frac{\rho_{old}}{1 + \Delta\rho}$$



$\Delta\rho = 0.25$

Reference: Schoonderwoerd, R. (1996) “Ant-based load balancing in telecommunication networks” Adapt. Behav. 5, 169-207

Algorithm I (cont.)

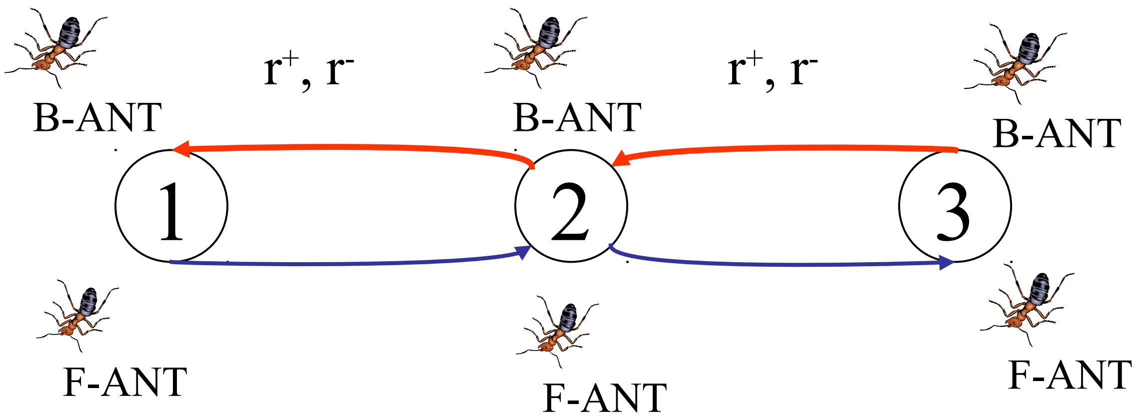
	Mean	Std. Dev.
Without load balancing	12.53%	2.04%
mobile agent	4.41%	0.85%
Ants	2.72%	1.24%

The mean percentages (ten experiments each) and standard deviations of call failures for changed call probabilities

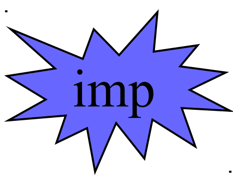
Reference: Schoonderwoerd, R. (1996) "Ant-based load balancing in telecommunication networks" Adapt. Behav. 5, 169-207

Algorithm II

AntNet
(Di Caro & Dorigo - 1997)



F-Ants also measure the quality of the trip (# nodes, Node Statistics)



VERY GOOD RESULTS, But it Generates bigger consumption of the network resources.

Reference: Schoonderwoerd, R. (1996) “Ant-based load balancing in telecommunication networks” Adapt. Behav. 5, 169-207

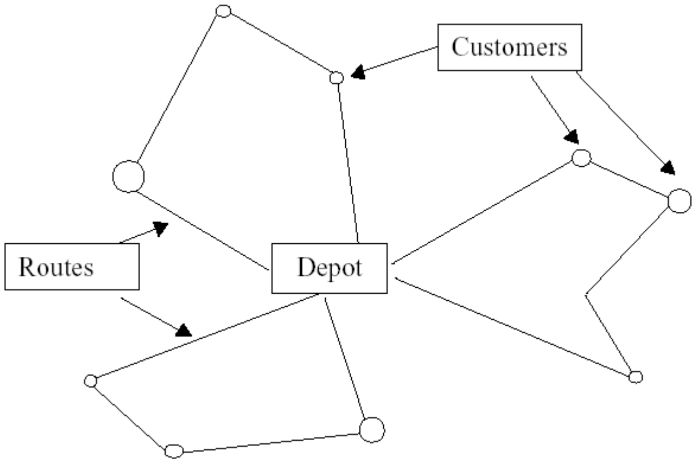
Vehicle Routing Problem with Time Windows (VRPTW)

N customers are to be visited by K vehicles

<u>Given</u>

- *Depots* (number, location)
- *Vehicles* (capacity, costs, time to leave, time on road..)
- *Customers* (demands,time windows, priority,...)
- *Route Information* (maximum route time or distance, cost on the route)

Vehicle Routing Problem with Time Windows (VRPTW)



Objective Functions to Minimize

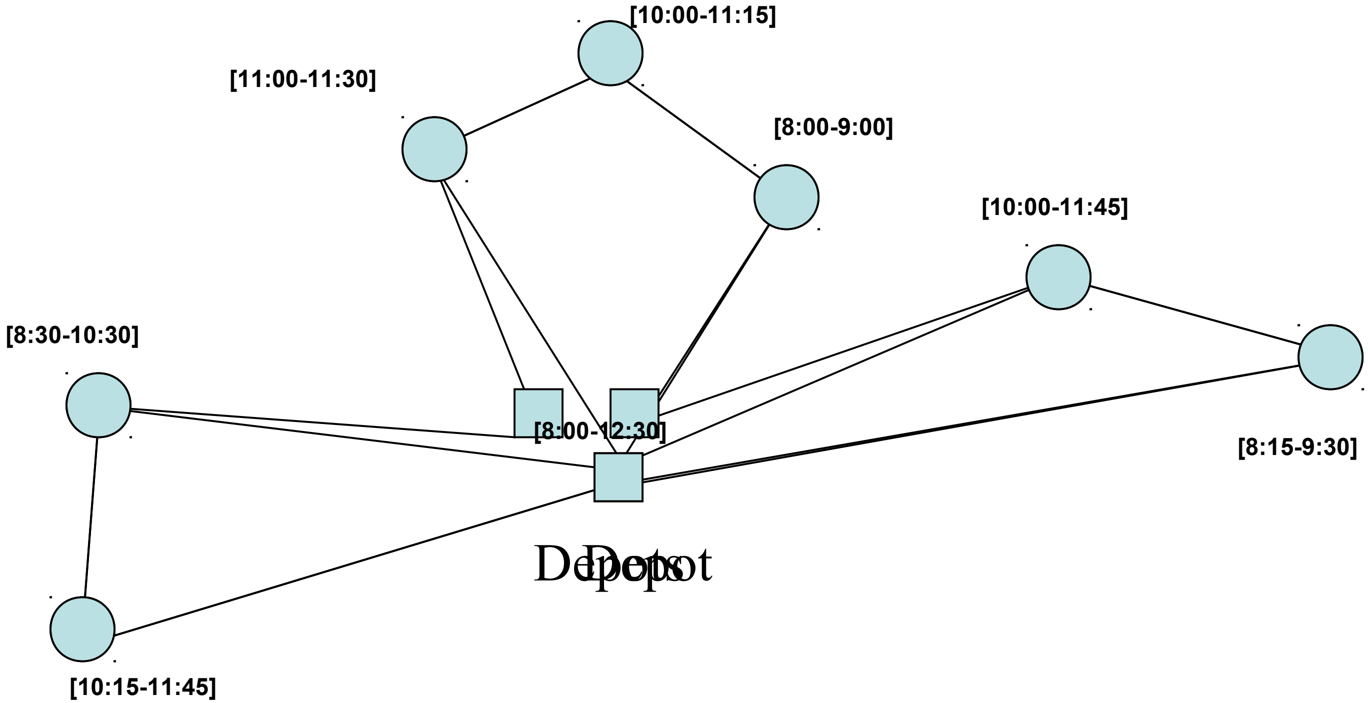
- Total travel distance
- Total travel time
- Number of vehicles

Subject to:

- Vehicles (# ,Capacity,time on road,trip length)
- Depots (Numbers)
- Customers (Demands,time windows)

Vehicle Routing Problem with Time Windows (VRPTW)

Relation with TSP?!



VRP "Simple Algorithm"

- Place ants on depots (Depots # = Vehicle #).

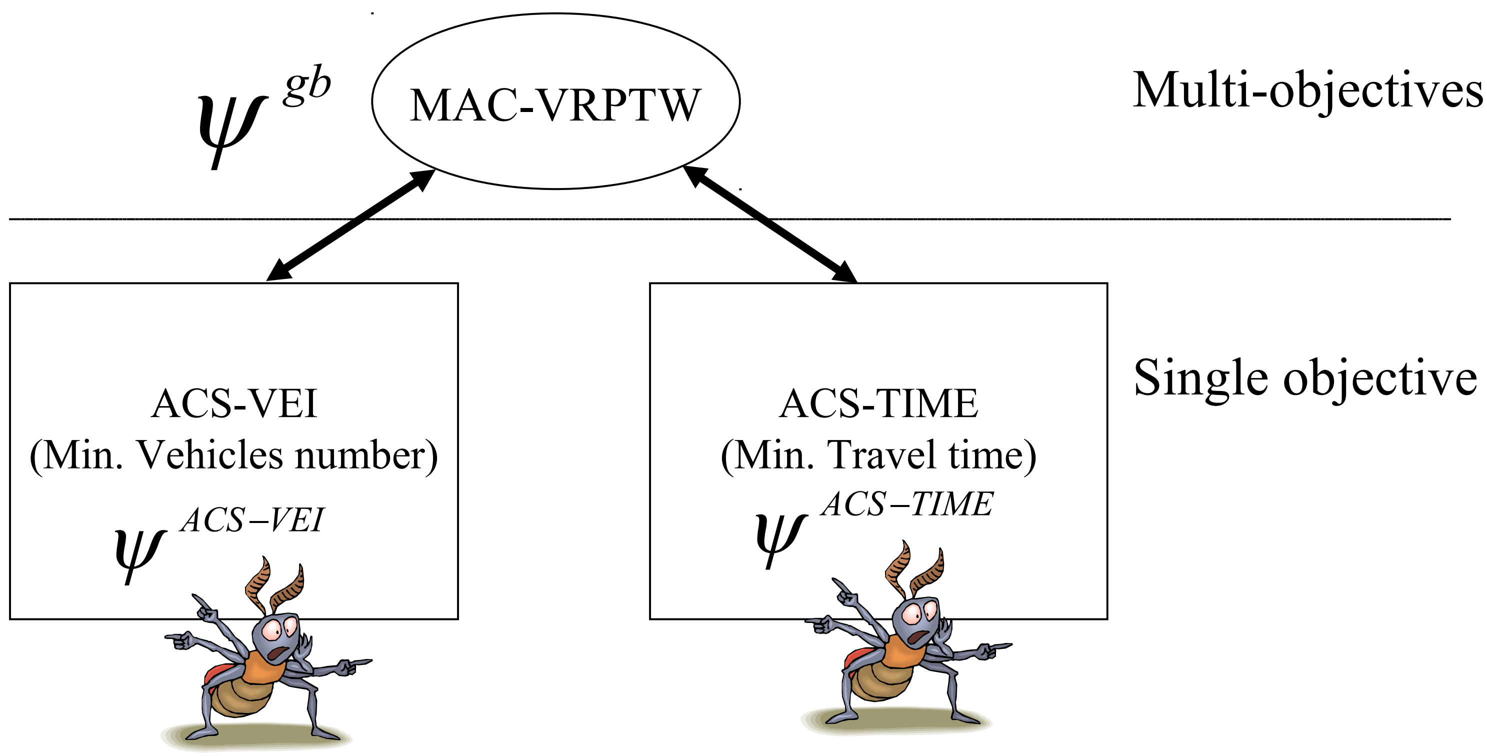
- Probabilistic choice
  - ~ (1/distance, d_i, Q)
  - ~ amount of pheromone

- If all unvisited customer lead to a unfeasible solution:
  Select depot as your next customer.

- Improve by local search.

- Only best ants update pheromone trial.

Multiple ACS For VRPTW



Gambardella L.M., Taillard 12. (1999), “Multiple ant colony system for VRPTW”

Parallel implementation

- Parallelism at the level of ants.
  - Ants works in parallel to find a solution.
- Parallelism at the level of data.
  - Ants working for sub-problems
- Functional Parallelism.
  - Ant_generation_activity()
  - Pheromone_evaporation()
  - Daemons_actions()

Good choice

Similarities with other Opt. Technique

• Populations,Elitism	~	GA
• Probabilistic,RANDOM	~	GRASP
• Constructive	~	GRASP
• Heuristic info, Memory	~	TS

Design Choices

- Number of ants.
- Balance of exploration and exploitation
- Combination with other heuristics techniques
- When are pheromones updated?
- Which ants should update the pheromone.?
- Termination Criteria

Ongoing Projects

- ***DYVO**: ACO for vehicle routing*
- ***MOSCA**: Dynamic and time dependent VRP*
- ***Ant@ptima:** Research applications*

Conclusions

- ACO is a recently proposed metaheuristic approach for solving hard combinatorial optimization problems.
- Artificial ants implement a randomized construction heuristic which makes probabilistic decisions.
- The a cumulated search experience is taken into account by the adaptation of the pheromone trail.
- ACO Shows great performance with the “ill-structured” problems like network routing.
- In ACO Local search is extremely important to obtain good results.

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Thank you