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1) Aspiration Level Approach to Interactive Multi-objective Programming
2) DEA (GDEA) & Generation of Pareto Frontier
3) Support Vector Machines based on MOP/GP
4) Approximate (Multi-objective) Optimization using SVM(SVR)

# Aspiration Level Methods in Interactive Multi-objective Programming and their Engineering Applications

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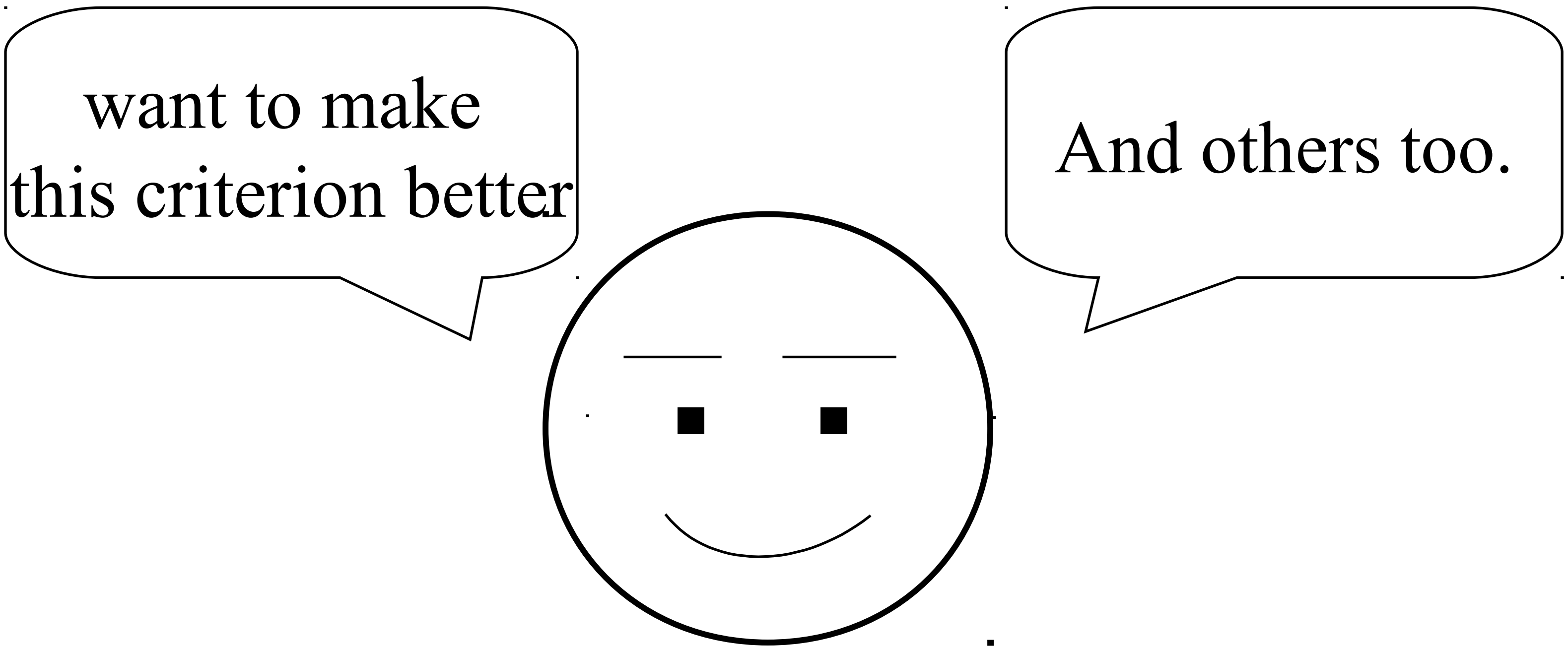
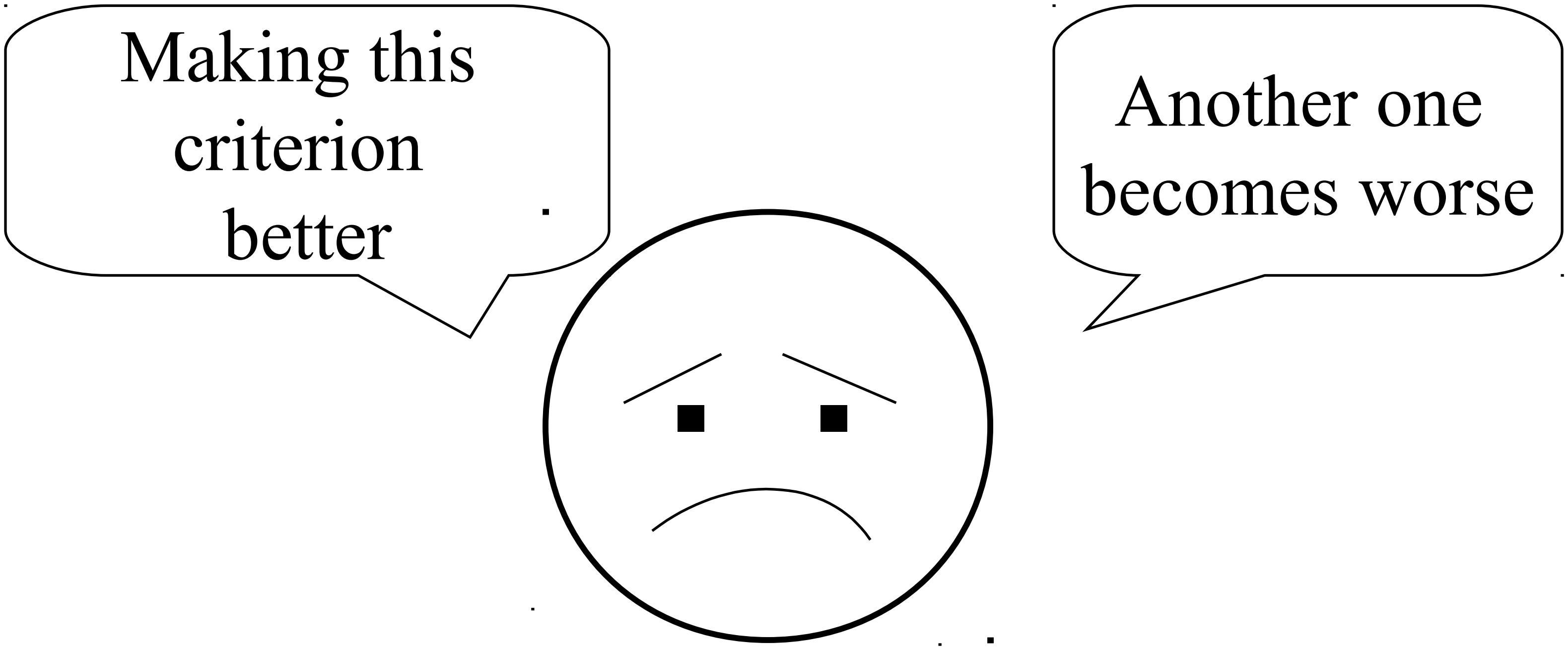
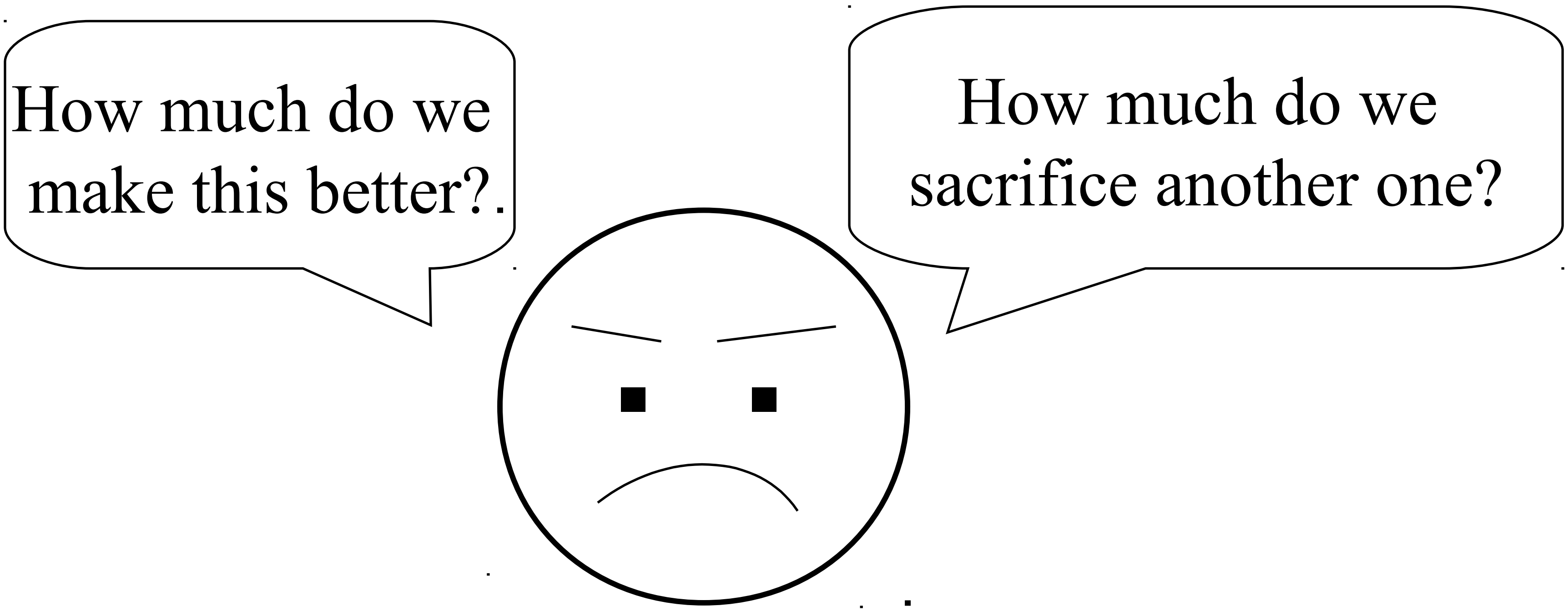
# Multiple Criteria Decision Making



# Multiple Criteria Decision Making



# Multiple Criteria Decision Making




**Trade-off Analysis**

Trade-off Analysis

↓

***Value Judgment***

# Difficulties in Value Judgment

- Multiplicity

  *Balancing among many ob*

- Inconsistency

  *DM said somethi yesterd*
  *today.*

君子豹変

Prof. Sawaragi

Sawaragi, Nakayama, Tanino: Theory of Multi-objective Optimization, *Academic Press* (1985)

*This situation is usual (reasonable)*
*because information available changes over time.*

How should we incorporate value judgment into DSS?

Main theme of MCDM

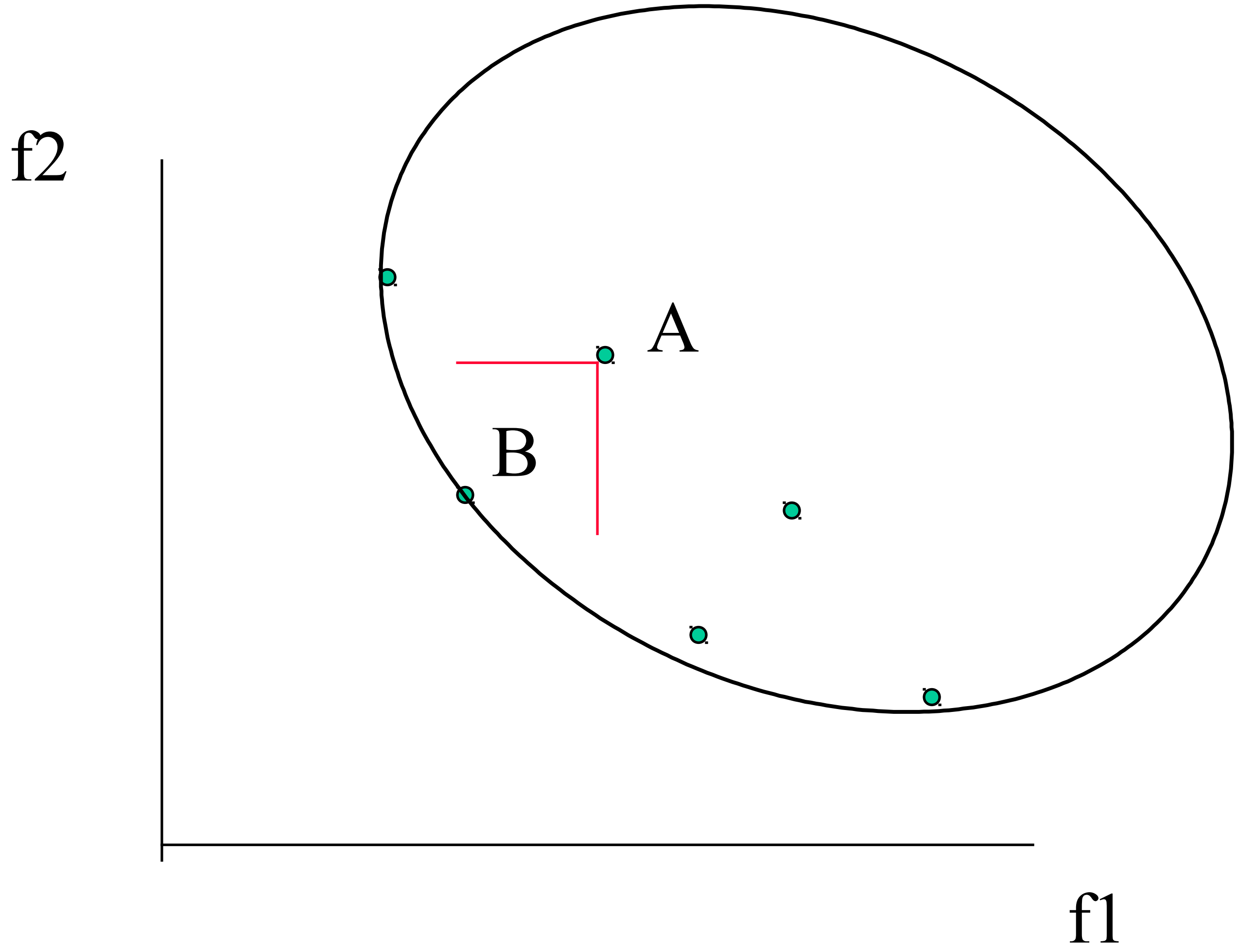
# Multi-objective Programming

$$f_1(x) \to Min$$
$$f_2(x) \to Min$$
$$\vdots$$
$$f_r(x) \to Min$$

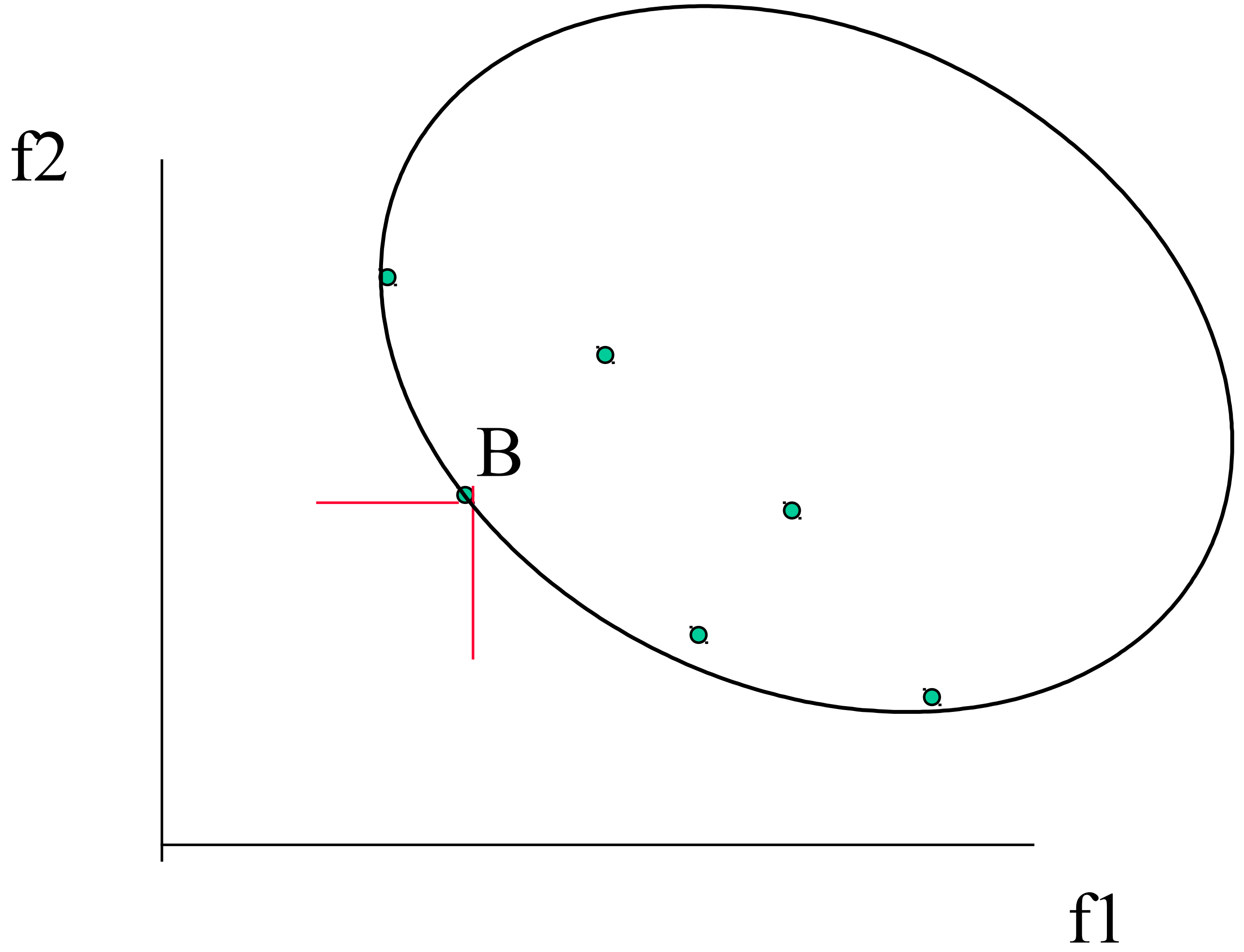
subj. to

$$x \in X$$

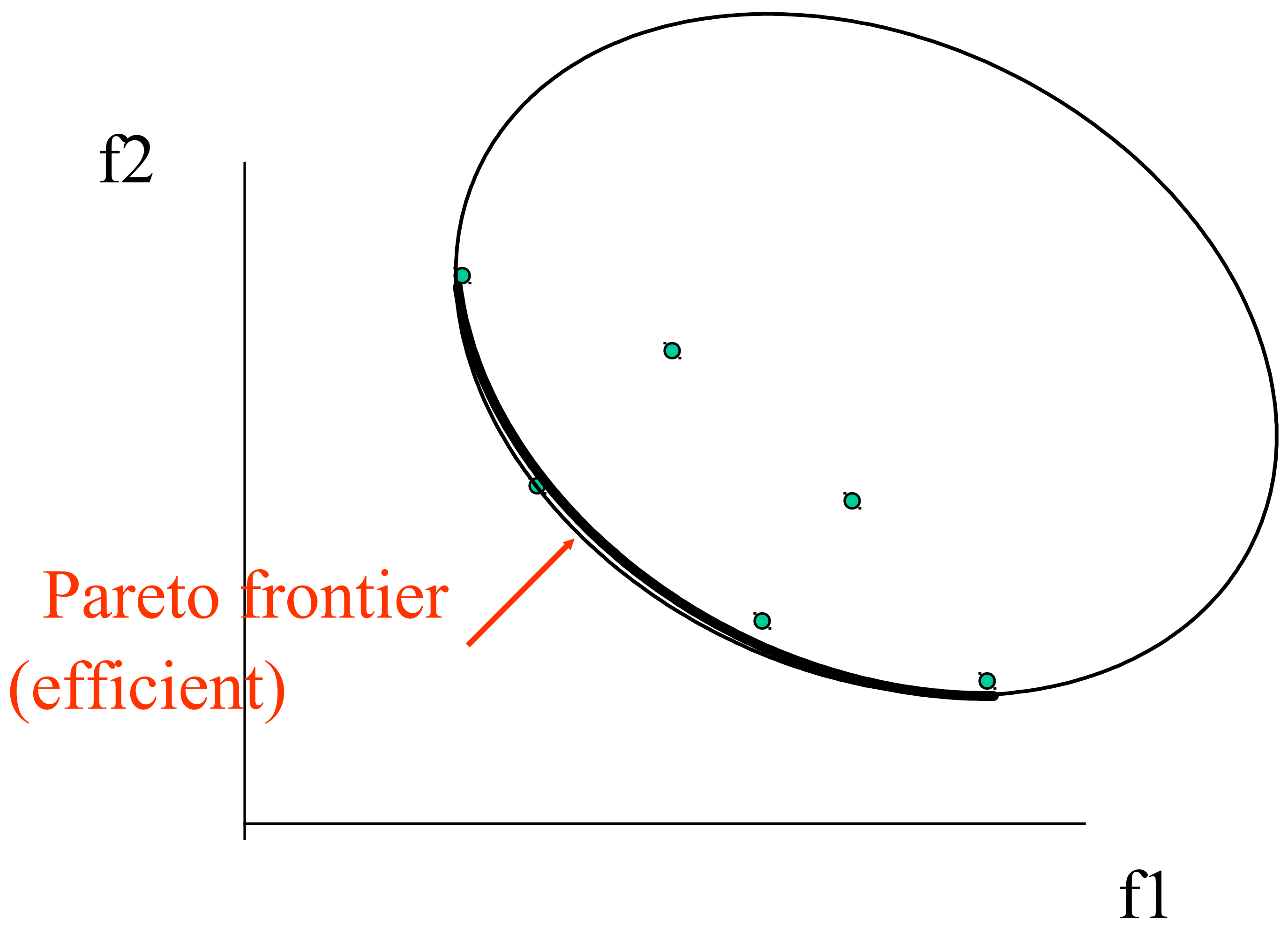
# Pareto Solution and Efficient Frontier



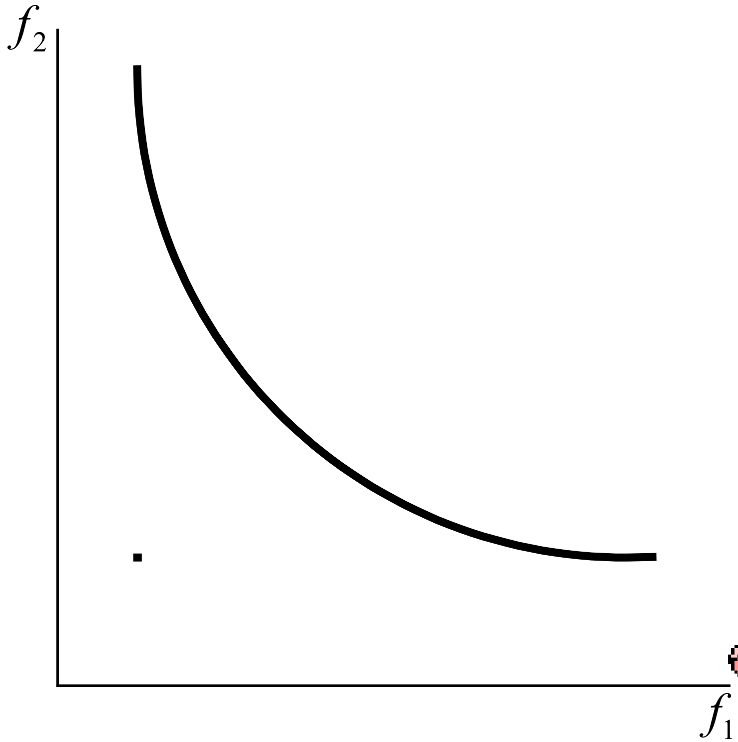
# Pareto Solution and Efficient Frontier



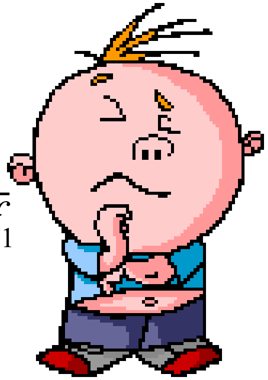
# Pareto Solution and Efficient Frontier



# Trade-off Analysis based on Pareto Frontier




- ..\..\..\..\..\MATLAB(BEAM)\beam1.fig



2-dimensional case

3-dimensional case

# Finding Pareto Solutions

- Scalarization
- Constraint transformation

Edgeworth : *Mathematical Psychics*, 1881

# Scalarization function

*Theorem* Let $y$ be a vector of the objective space. Suppose that $F$ is order preserving

i.e., $$y^1 \leq y^2 \Rightarrow F(y^1) < F(y^2)$$

then the solution minimizing $F$ is a Pareto solution.

How about the converse?

# Examples of Scalarization Function

- linearly weighted sum

$$F = w_1 f_1 + w_2 f_2 + \cdots + w_r f_r$$

- Tchebyshev type (>- monotonous)

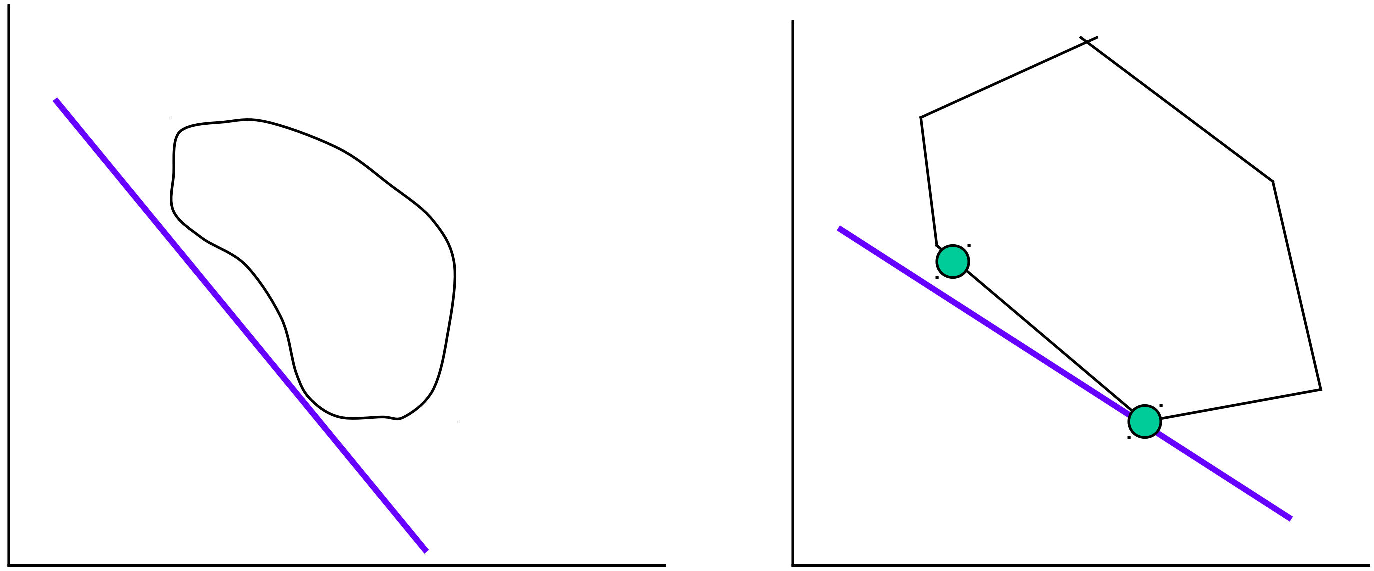
$$F = Max\{w_1 f_1, \cdots, w_r f_r\}$$

- augmented Tchebyshev type

$$F = Max\{w_1 f_1, \cdots, w_r f_r\} + \alpha \Sigma w_i f_i$$

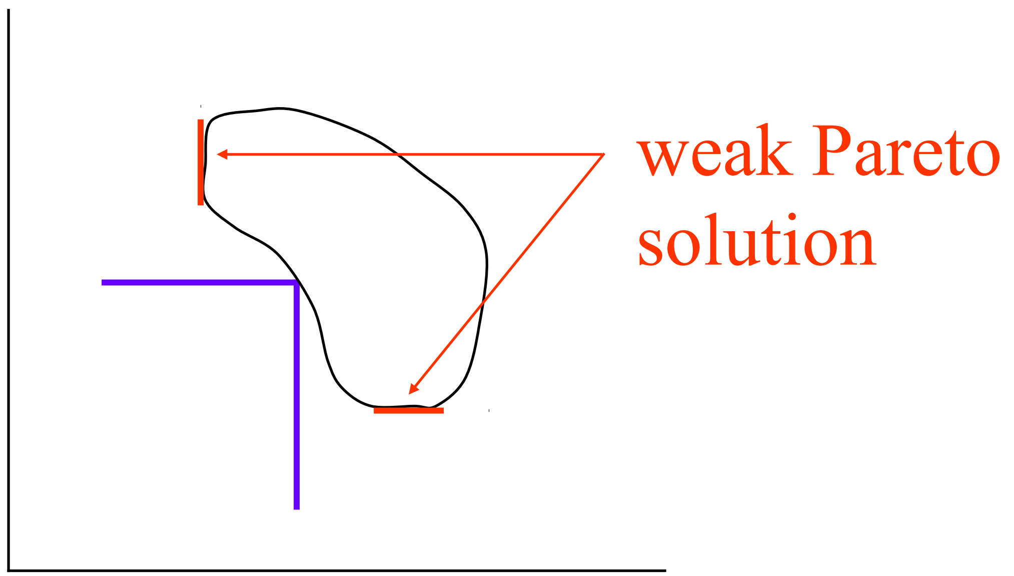
# Linearly Weighted Sum

$$F = w_1 f_1 + w_2 f_2 + \cdots + w_r f_r$$



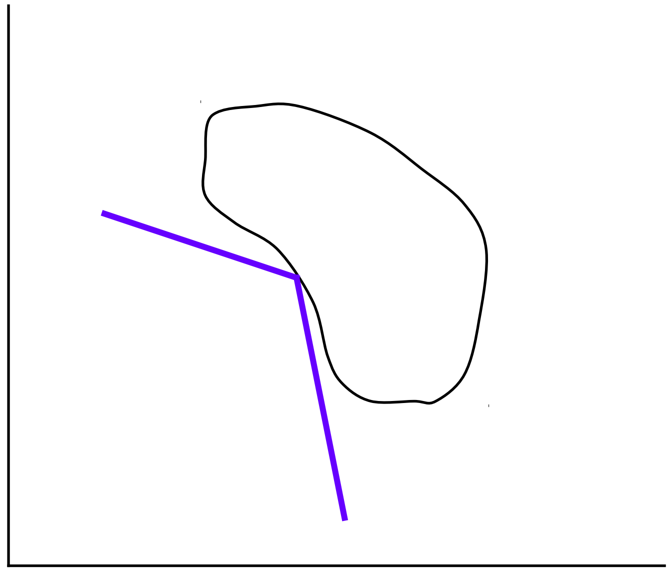
# Tchebyshev type

$$F = Max\{w_1 f_1, \cdots, w_r f_r\}$$



# Augmented Tchebyshev type

$$F = Max\{w_1 f_1, \cdots, w_r f_r\} + \alpha \Sigma w_i f_i$$



# Augmented Tchebyshev Scalarization Function

- Theorem (Nakayama-Tanino 1994)

For any $w > 0$ and $\alpha > 0$ , the solution minimizing the augmented Tchebyshev scalarinzation function is a proper Pareto solution.

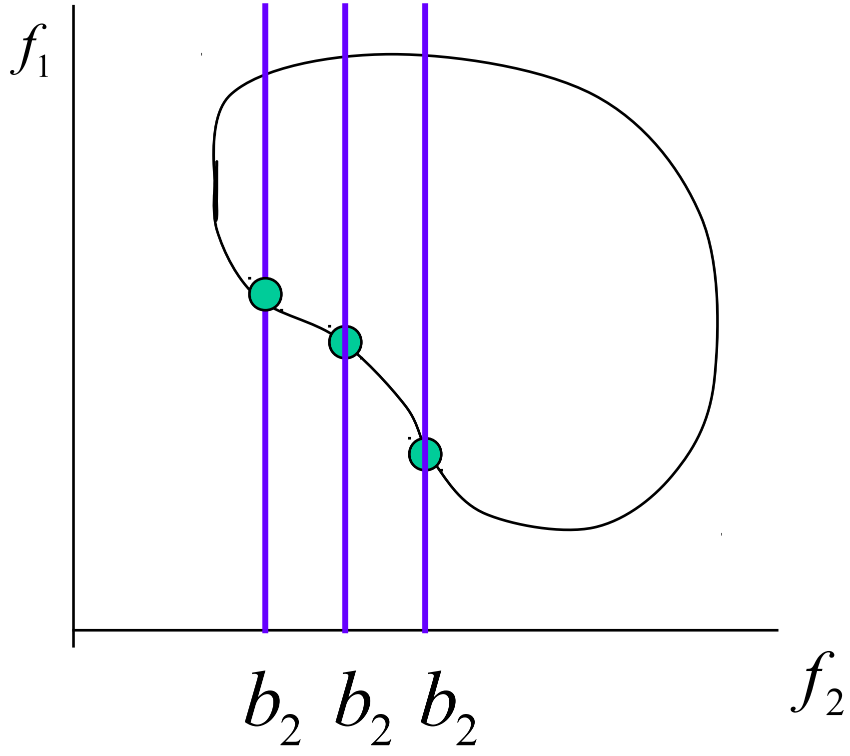
Conversely, any proper Pareto solution can be obtained by minimizing the augmented Tchebyshev scalarization function with some appropriately chosen $w > 0$, $\alpha > 0$ and an aspiration level. .

# Constraint transformation

$$f_1(x) \to Min$$

subj. to

$$f_2(x) \le b_2$$
$$\vdots$$
$$f_r(x) \le b_r$$



# In cases with more than three objective functions

Interactive Programming Method

*Eliciting preference information of DM, the solution is searched.*

⮡ local trade-off

*What should be used as information reflecting DM's preference?*

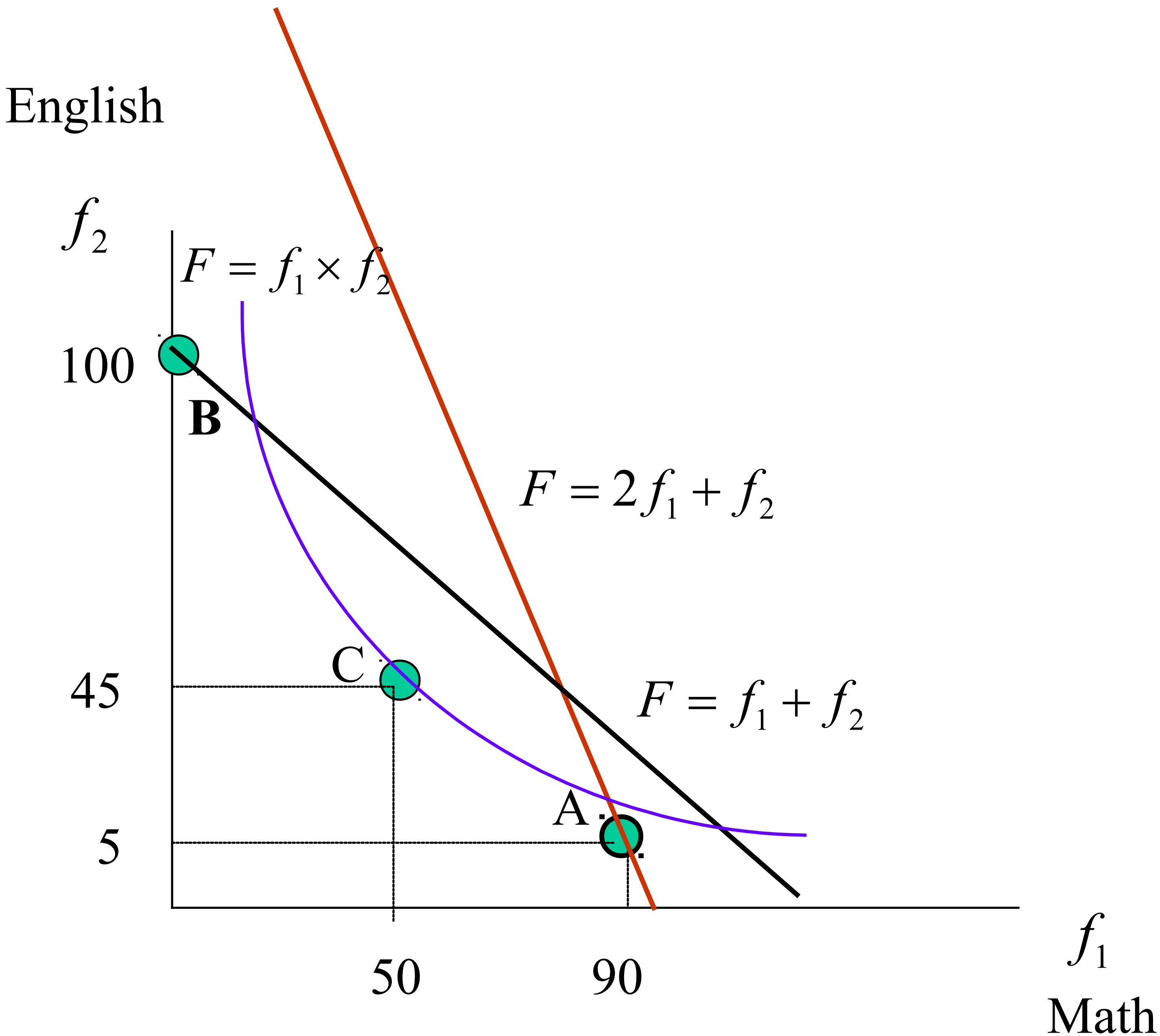
*Many people say*

**Weighting method does work.**

**We can obtain a desirable solution by adjusting the weights.**

..........?

***No!***


English
$f_2$
$F = f_1 \times f_2$
100
B
$F = 2f_1 + f_2$
C
45
$F = f_1 + f_2$
A
5
50
90
$f_1$
Math

# Example

$$\{y_1, y_2, y_3\} \rightarrow Min$$
$$\text{s.t.} \quad (y_1-1)^2+(y_2-1)^2+(y_3-1)^2 \le 1$$

$$w_1 = 1, w_2 = 1, w_3 = 1 \Rightarrow y_1 = 1-1/\sqrt{3}, \quad y_2 = 1-1/\sqrt{3}, \quad y_3 = 1-1/\sqrt{3}$$

want to improve much more

want to improve a little

$$w_1' = 10, w_2' = 2, w_3' = 1 \Rightarrow y_1 = 1-10/\sqrt{105}, \ y_2 = 1-2/\sqrt{105}, \ y_3 = 1-1/\sqrt{105}$$

worse than before

## No normalization of weights?

$$w_1 = 1, w_2 = 1, w_3 = 1 \Rightarrow w_1 = 1/3, w_2 = 1/3, w_3 = 1/3$$

$$w_1' = 10, w_2' = 2, w_3' = 1 \Rightarrow w_1' = 10/13, w_2' = 2/13, w_3' = 1/13$$

$$w_1'' = 10, w_2'' = 7, w_3'' = 1 \Rightarrow w_1'' = 10/18, w_2'' = 7/18, w_3'' = 1/18$$

$$y_1 = 1-10/\sqrt{150}, \ y_2 = 1-7/\sqrt{150}, \ y_3 = 1-1/\sqrt{150}$$

still worse than before!

# Why does not the weighting method work so well?

No positive correlation between weights in linear scalarization function and resulting solution.

Weights are not appropriate as "information" reflecting DM's preference.

→ aspiration level approach

# **Satisficing Trade-off Method**

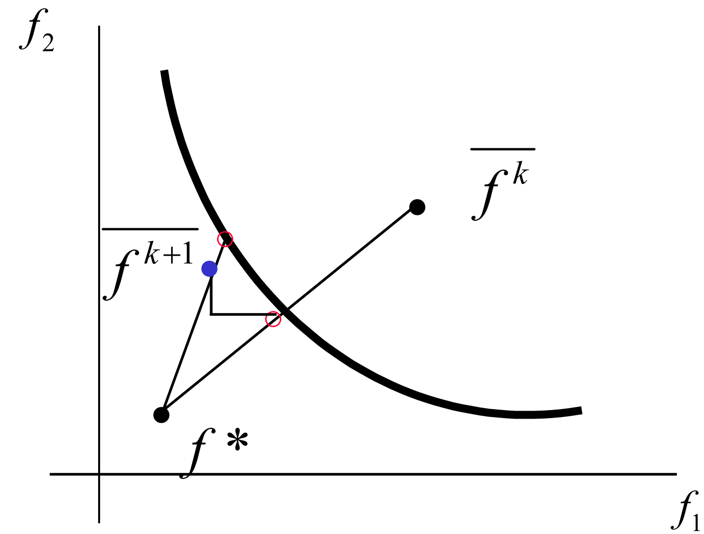
(Nakayama 1984)

$$\bar{f}^{k+1} = T \circ P(\bar{f}^{k})$$

***Operator P*** : the nearest Pareto solution to the given aspiration level $\bar{f}^{k}$

***Operator T*** : trade-off analysis （How much can we agree to relax other criteria in order to improve some criteria)

# Satisficing trade-off method




1. Set the ideal pt. $f^*$
2. Set the aspiration level $\overline{f}^{k}$
3. Show the nearest Pareto solution to the given aspiration level $\overline{f}^{k}$ by solving the following auxiliary problem:

$$\text{minimize} \quad z + \alpha \sum_{i=1}^{r} w_i (f_i(x) - \overline{f}_i^{k}) \qquad w_i = \frac{1}{\overline{f}_i - f_i^*}$$

$$\text{subj. to} \quad w_i (f_i(x) - \overline{f}_i^{k}) \leq z \quad (i = 1, \ldots, r)$$

$$x \in X$$

4. Agree with the shown Pareto solution $\Longrightarrow$ stop.

   Not agree $\Longrightarrow$ trade-off analysis

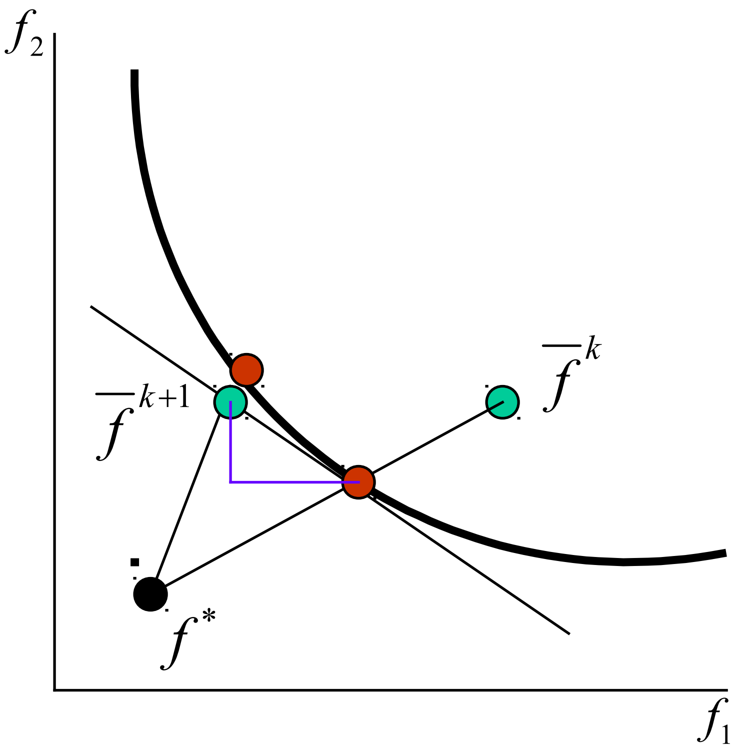
# Easy trade-off analysis (1)

## Automatic trade-off

$\overline{f}_i^{\,k+1}$ : input by DM $\quad (i \in I_I)$

$\overline{f}_s^{\,k+1} = f_s^{\,k} \quad (i \in I_A)$

$$\Delta \overline{f}_j^{\,k+1} = -\frac{\Sigma_{i \in I_I} w_i \Delta \overline{f}_i^{\,k+1}}{N \lambda_j w_j} (i \in I_R)$$

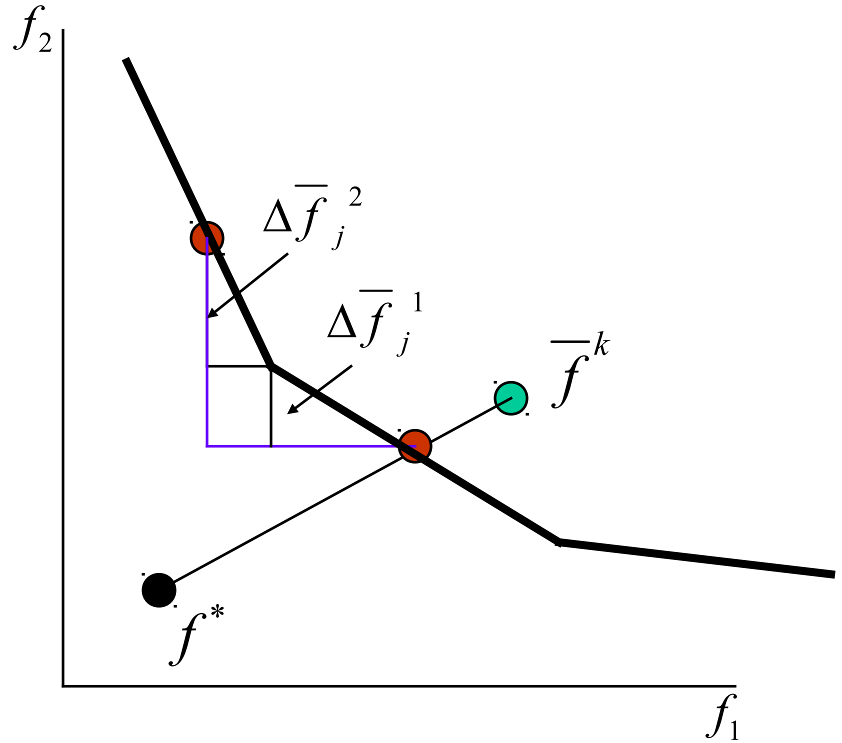


# Easy Trade-off Analysis (2)

## Exact Trade-off

$$\overline{f}_i^{k+1} : \text{input by DM} \quad (i \in I_I)$$

$$\Delta \overline{f}_j^{k+1} = \Delta \overline{f}_j^{1} + \Delta \overline{f}_j^{2} \quad (j \in I_R)$$



# Exchange of Objectives and Constraints

*(Korhonen 1987)*

$$\text{minimize} \quad z + \alpha \sum_{i=1}^{r} w_i (f_i(x) - \overline{f_i^k})$$

$$\text{subj. to} \quad w_i (f_i(x) - \overline{f_i^k}) \le \beta\, z \quad (i = 1, \ldots, r)$$

$$x \in X$$

$\beta = 1 \Rightarrow f_i :$ objective

$\beta = 0 \Rightarrow f_i :$ constraint

## Satisficing trade-off method

**Ex.**

$$\{y_1, y_2, y_3\} \rightarrow Min$$

$$\text{s.t.} \quad (y_1 - 1)^2 + (y_2 - 1)^2 + (y_3 - 1)^2 \leq 1$$

$$f^* = (0, 0, 0),\ f_* = (1, 1, 1) \Rightarrow \ w_1 = 1,\ w_2 = 1,\ w_3 = 1$$

1) $\overline{f} = (0.4,\ 0.4,\ 0.4) \Rightarrow y_1 = 0.423,\ y_2 = 0.423,\ \ y_3 = 0.423$

want to improve much more    want to improve a little

2) $\overline{f} = (0.35,\ 0.4,\ 0.5) \Rightarrow y_1 = 0.359,\ y_2 = 0.409,\ \ y_3 = 0.509$

Much better than before    A little better than before

Automatic trade-off

$$\overline{f} = (0.35,\ 0.4,\ 0.52) \Rightarrow y_1 = 0.354,\ y_2 = 0.404,\ \ y_3 = 0.524$$

# Engineering Applications by MOP Methods

- W. Stadler (ed.)
  *Multicriteria Optimization in Engineering and in the Sciences*, Plenum 1988

- M. T. Tabucanon:
  *Multiple Criteria Decision Making in Industry*, Elsevier 1988

- H. Eschenauer, J. Koski and A. Osyczka (eds.)
  *Multicriteria Design Optimization*, Springer 1990

- R. B. Statnikov:
  *Multicriteria Design*, Kluwer 1999

# Applications of Satisficing Trade-off Method

- construction accuracy control of cable stayed-bridges
- feed formulation of live stocks
- bond portfolio
- blending raw materials in cement production
- scheduling of string selection in steel manufacturing
- medical irradiation planning
- water supply planning in local governments
- lens design

# Decision Process ＝ Learning Process

- simple （簡単）
- easy （容易）
- fast （速い）

MCDM

needs easily to obtain a solution as DM desires.

→ How to incorporate the value judgment of DM into the decision support system

Aspiration level rather than weights!

# Satisficing Trade-off Method

## Optimal satisficing (aspiration level approach)

**human beings**: global judgment (aspiration level) ← ***satisficing***

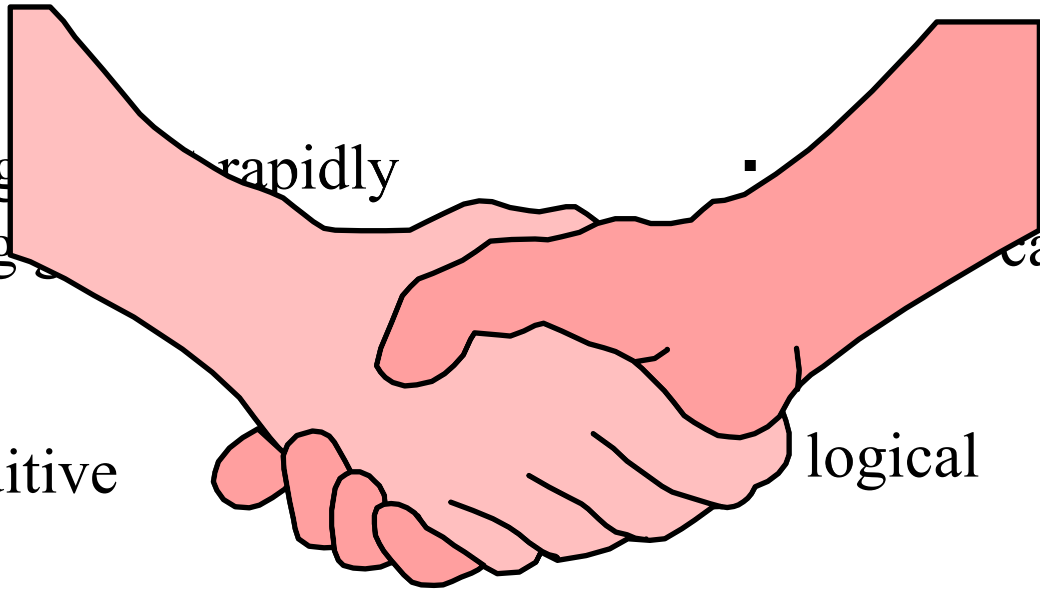
**computer:** ***optimization*** based on the augmented Tchebyshev functions

***Sharing roles***
***among human beings and computer***

# DSS for MCDM

**human beings**

**computers**



- roug rapidly making
- intuitive

- making a large calculation
- logical

Making full use of strong points
of human beings and computers respectively

# Generating Pareto Frontiers

- K. Deb:
  *Multi-objective Optimization using Evolutionary Algorithms,* Wiley 2001

- C.A. Coello Coello, D.A.Van Veldhuizen, G.B. Lamont :
  *Evolutionary Algorithms for Solving Multi-objective Problems,* Kluwer 2002

# Evolutionary Algorithms

- VEGA (Vector Evaluated Genetic Algorithm)  Schaffer (1984)
- MOGA (Multiple Objective Genetic Algorithm) Fonseca-Fleming (1993)
- NSGA (Non-Dominated Sorting Genetic Algorithm) Srinivas-Deb (1994)
- NPGA (Niched Pareto Genetic Algorithm) Horn-Nafploitis-Goldberg (1994)
- SPEA (Strength Pareto Evolutionary Algorithm) Zitzler-Thiele (1998)

# Fitness of individuals

- Convergence

  *How close is each individual to Pareto frontier?*

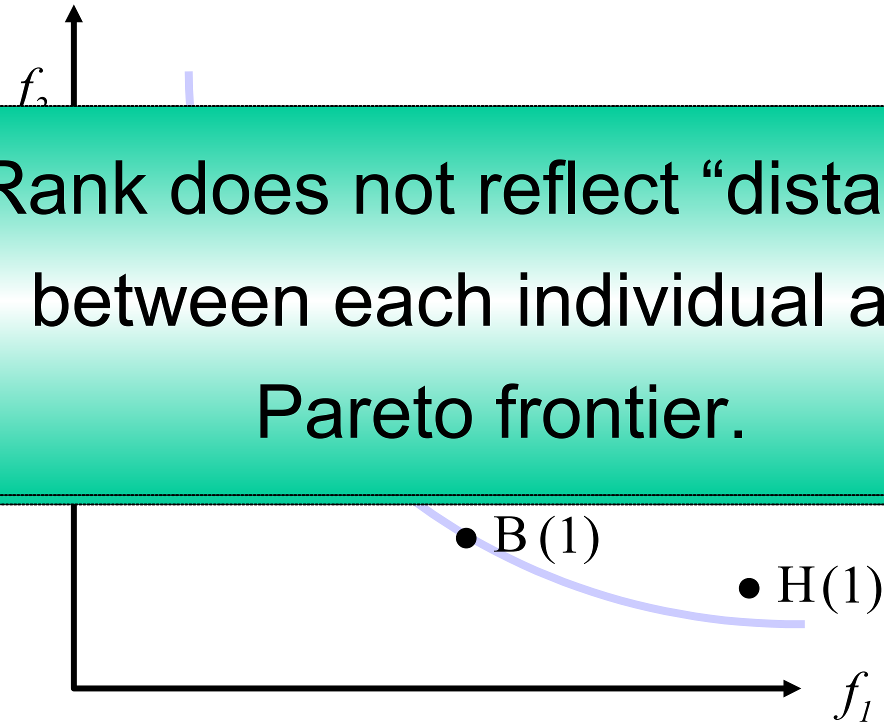
  ranking method, …

- Diversity

  *How much does the population spread over the whole Pareto frontier?*

  sharing function, …

# Convergence by Ranking

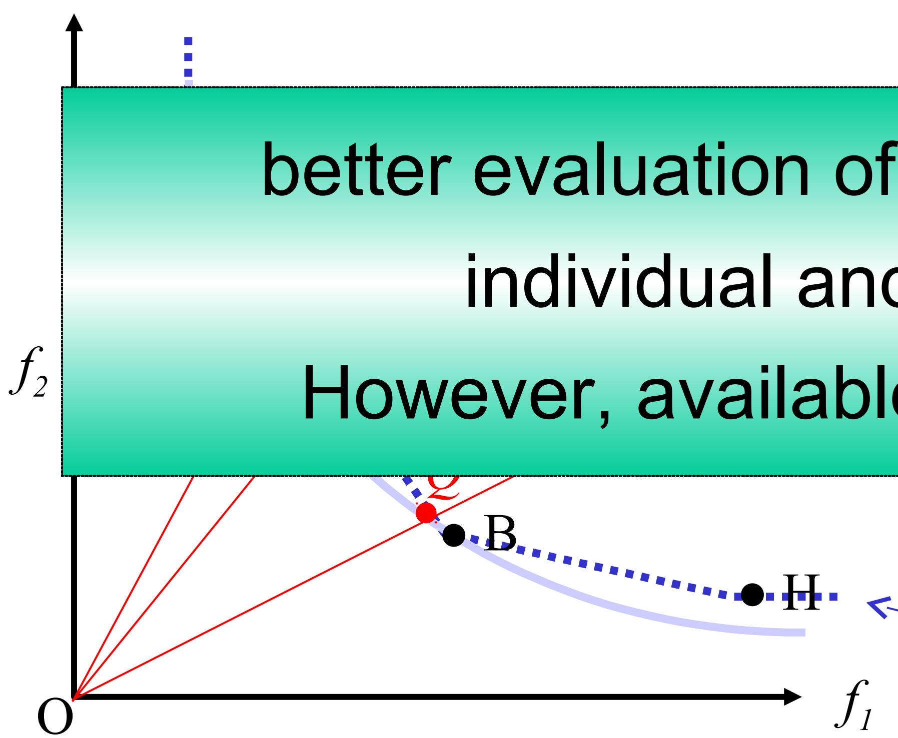



Rank does not reflect "distance" between each individual and Pareto frontier.

# Using Data Envelopment Analysis

Arakawa, Nakayama, Hagiwara, Yamakawa

（Multiobjective optimization using adaptive range genetic algorithms with data envelopment analysis, AIAA, 1998）




better evaluation of “distance” between each individual and Pareto frontier.

However, available only for convex cases.

$\{\boldsymbol{x}^1, \dots, \boldsymbol{x}^p\}$ : population

# Using Generalized DEA

Yun, Nakayama, Tanino, Arakawa

Generation of efficient frontiers in multi-objective optimization problems by generalized data envelopment analysis, EJOR, Vol.129, No. 3, pp. 586-595 (2001)

(GDEA) max $\Delta$

Available for non-convex cases.

Envelopment is based on convex cones instead of hyperplane.

$$F_i(\boldsymbol{x}) = f_i(\boldsymbol{x}) + \sum_{j=1}^{l} b_j \left[ P\left( g_j(\boldsymbol{x}) \right) \right]^a$$

$b_j$ : a penalty coefficiency

$a$ : a penalty exponent

$P(y) = \max\{y, 0\}$

# Engineering Applications

- Function forms of many criteria are not given explicitly in terms of design variables.
- …..  are evaluated via analyses (structural analysis, fluid mechanical analysis, thermo-dynamical analysis, etc.) and/or real samples.

*time consuming and expensive*

- The number of function call is important.

How to decrease the number of function calls?

# Using GDEA

– GDEA measure is the “distance” between each individual and the dotted line.

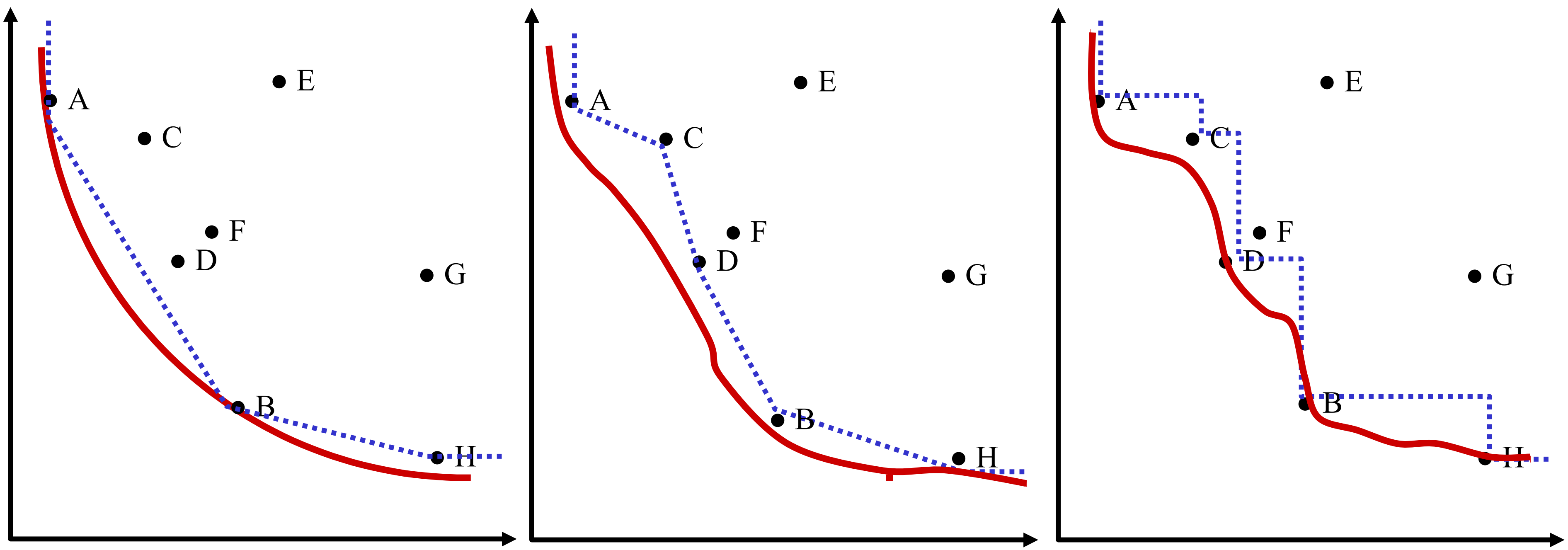



Fig. 1 sufficiently large $\alpha$ Fig. 2 . Fig. 3. Sufficiently small $\alpha$

# Support Vector Machine (Yun et. al., 2004)

- nu(ν)-SVM with 1-class
  - separating the data from the origin with maximal margin
  - for training data : $\boldsymbol{x}_1,\ldots,\boldsymbol{x}_\ell$ and given parameter ν,

separating hyperplane $h(\boldsymbol{x}) := \boldsymbol{w}^T\Phi(\boldsymbol{x}) - \rho = 0$

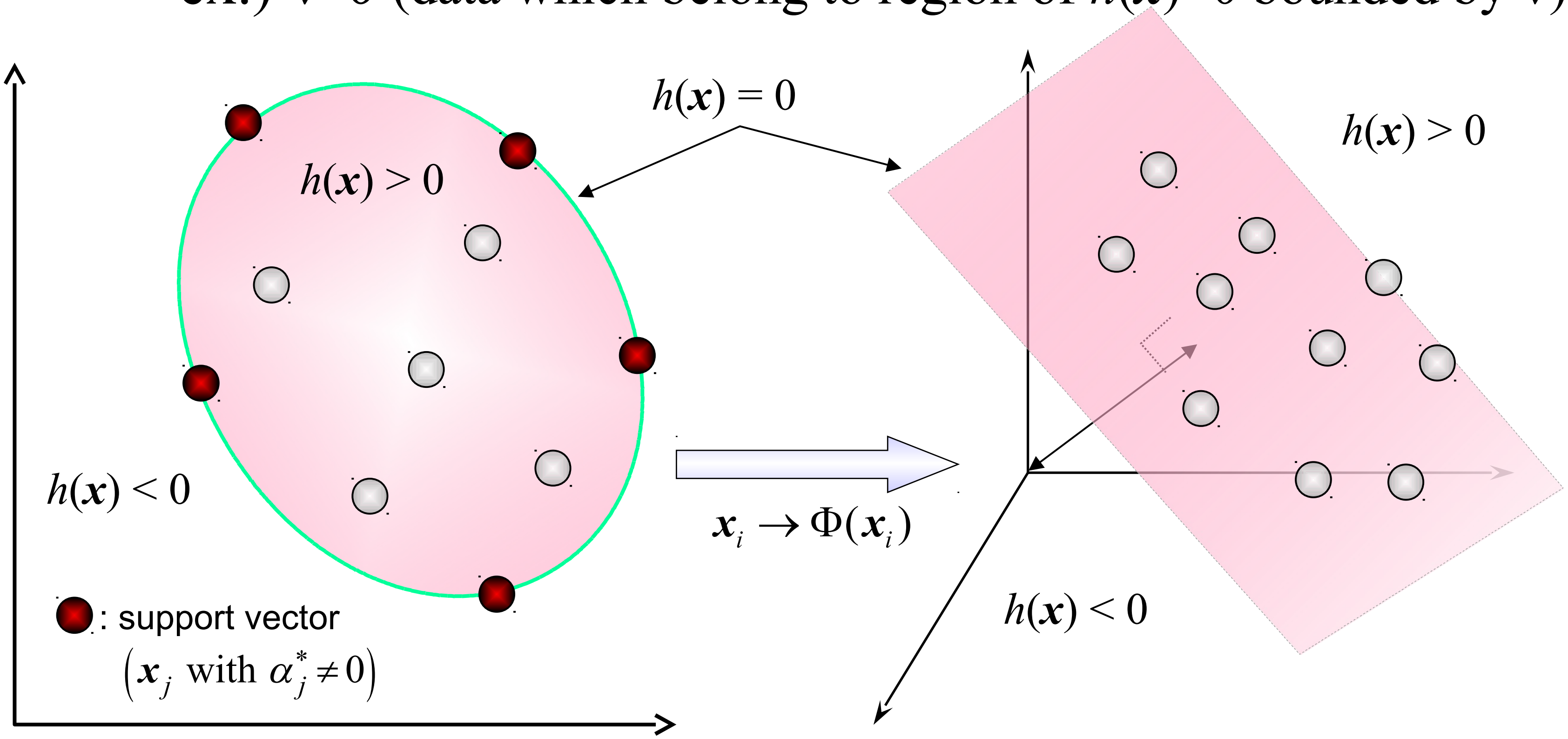
where $\boldsymbol{w}$ and $\rho$ are solved by the following problem:

$$
\begin{aligned}
&\underset{\boldsymbol{w},\rho,\xi}{\text{minimize}} && \frac{1}{2}\boldsymbol{w}^T\boldsymbol{w} + \frac{1}{\nu\ell}\sum_{i=1}^{\ell}\xi_i - \rho \\
&\text{subject to} && \boldsymbol{w}^T\Phi(\boldsymbol{x}_i) \geq \rho - \xi_i \\
& && \xi_i \geq 0,\ i = 1,\ldots,\ell \\
& && \nu \in (0,1)
\end{aligned}
$$

Lagrange ⟺ dual problem

$$
\begin{aligned}
&\underset{\boldsymbol{\alpha}}{\text{maximize}} && \frac{1}{2}\sum_{i,j=1}^{\ell}\alpha_i\alpha_j K(\boldsymbol{x}_i,\boldsymbol{x}_j) \\
&\text{subject to} && \sum_{i=1}^{\ell}\alpha_i = 1 \\
& && 0 \leq \alpha_i \leq \frac{1}{\nu\ell},\ i = 1,\ldots,\ell \\
& && \nu \in (0,1)
\end{aligned}
$$

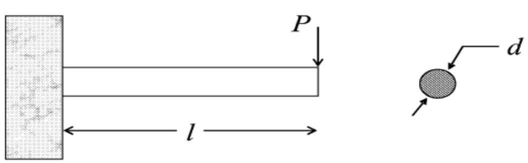
- nu(ν)-SVM with 1-class
  - ex.) $\nu \fallingdotseq 0$ (data which belong to region of $h(\boldsymbol{x})<0$ bounded by ν)




<in the input space> <in the feature space>

# Cantilever Beam Problem

- design variables
  - diameter $d$ (mm)
  - length $l$ (mm)
- objective functions
  - weight (kg)
  - end deflection (mm)
- constraints
  - maximum stress

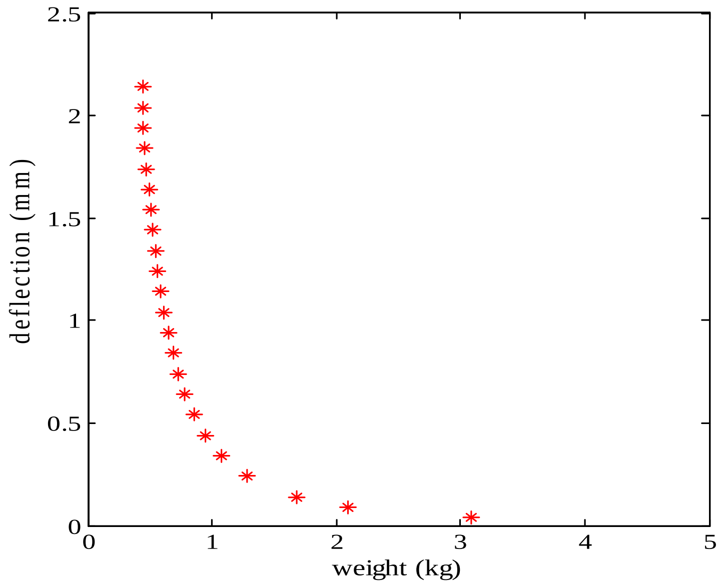



$$
\begin{aligned}
&\text{min} && f_1(d,l) := \rho \frac{\pi d^2}{4} l \\
&\text{min} && f_2(d,l) := \delta = \frac{64Pl^3}{3E\pi d^4} \\
&\text{s.t.} && \sigma_{\max} \le S_y \\
& && \delta \le \delta_{\max} \\
& && 10 \le d \le 50,\ \ 200 \le l \le 1000 \\
&\text{where}
\end{aligned}
$$

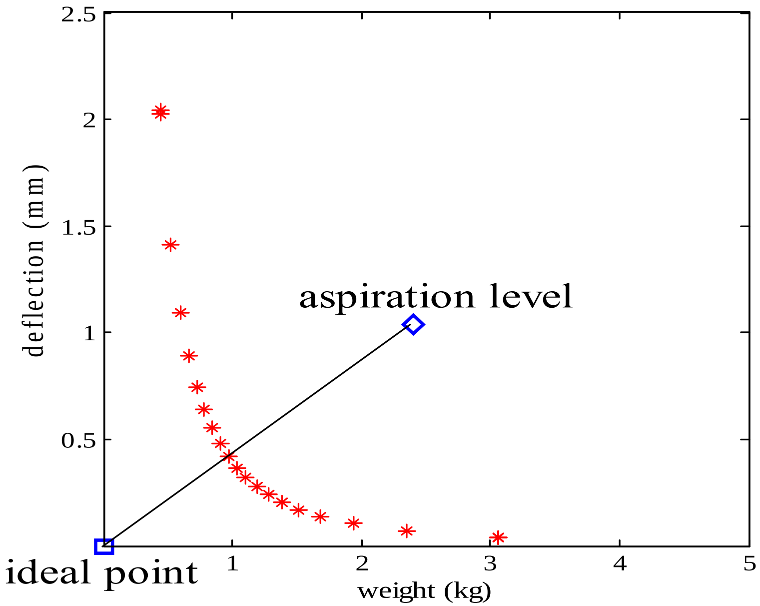
$$
\begin{cases}
\sigma_{\max} = \dfrac{32pl}{\pi d^3},\ \rho = 7800kg/m^3,\ P = 1kN \\
E = 207GPa,\ S_y = 300MPa,\ \delta_{\max} = 5mm
\end{cases}
$$

* This problem is cited from K. Deb, 2000

Constraint transformation method

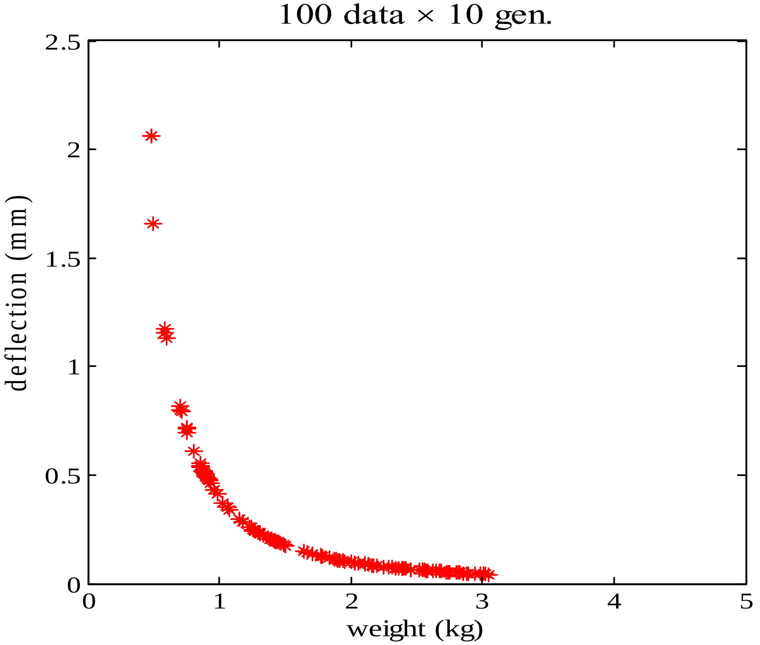



Satisficing trade-off method

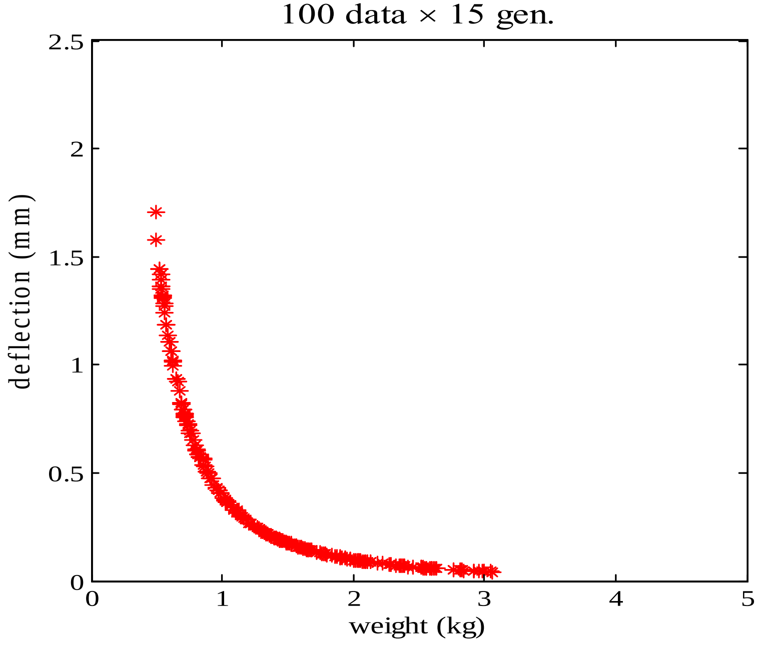



It takes about 50 function calls to obtain each Pareto point.

# GDEA



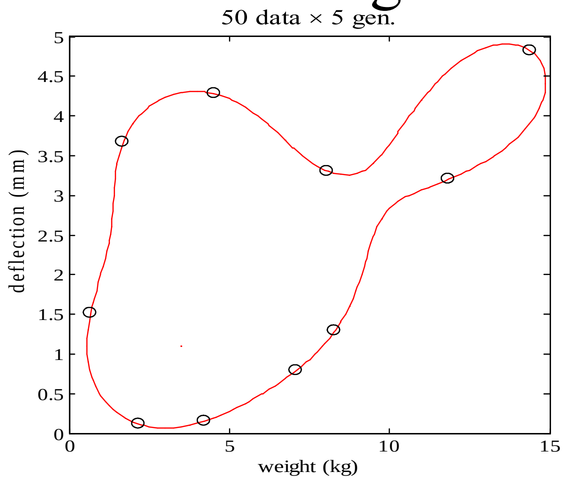
100 data × 10 gen.
deflection (mm)
weight (kg)
0
0.5
1
1.5
2
2.5
0
1
2
3
4
5

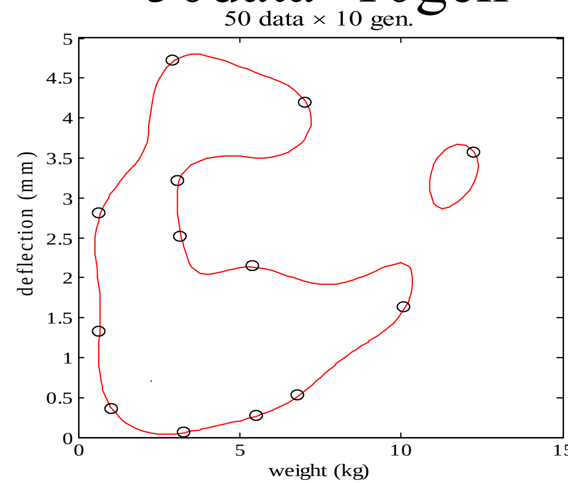
100 data × 15 gen.
deflection (mm)
weight (kg)
0
0.5
1
1.5
2
2.5
0
1
2
3
4
5

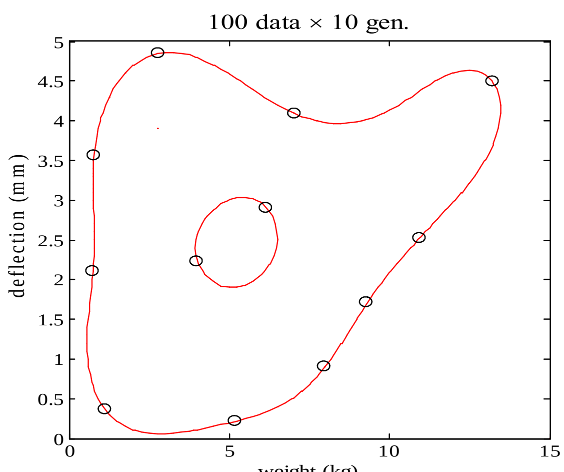
# SVM

50data×5gen

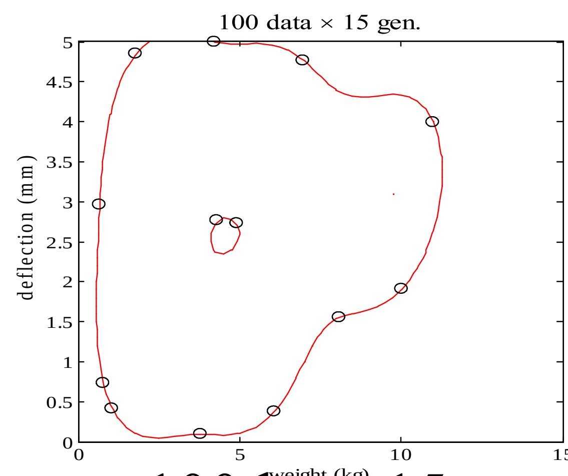



50data×10gen




100data×10gen




100data×15gen

# ZDT4

minimize $f_1(\boldsymbol{x}) = x_1$

$$f_2(\boldsymbol{x}) = g(\boldsymbol{x}) \times \left(1 - \sqrt{\frac{f_1(\boldsymbol{x})}{g(\boldsymbol{x})}}\right)$$

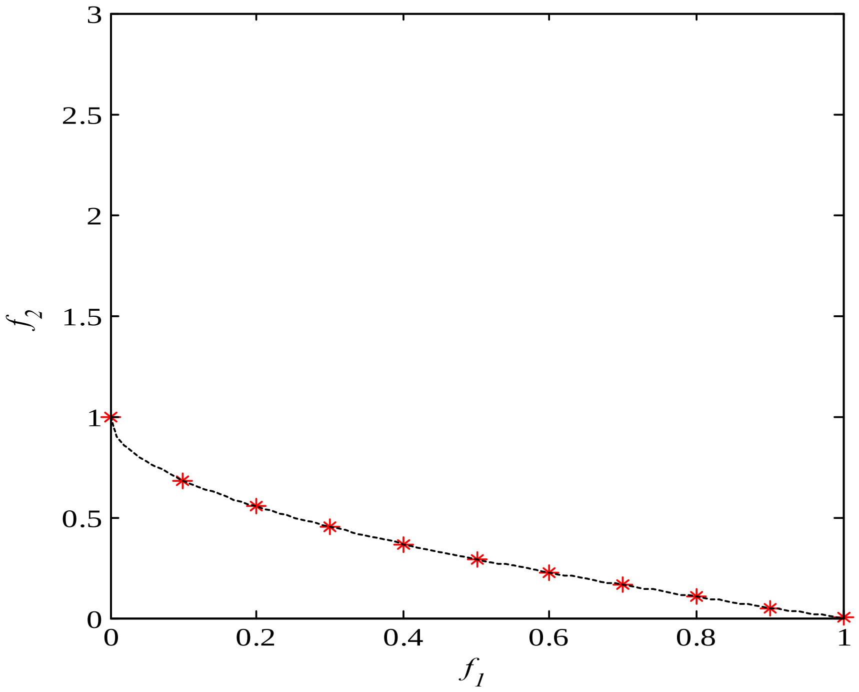
subject to $g(\boldsymbol{x}) = 1 + 10(N-1) + \sum_{i=2}^{N}\left(x_i^2 - 10\cos(4\pi x_i)\right)$

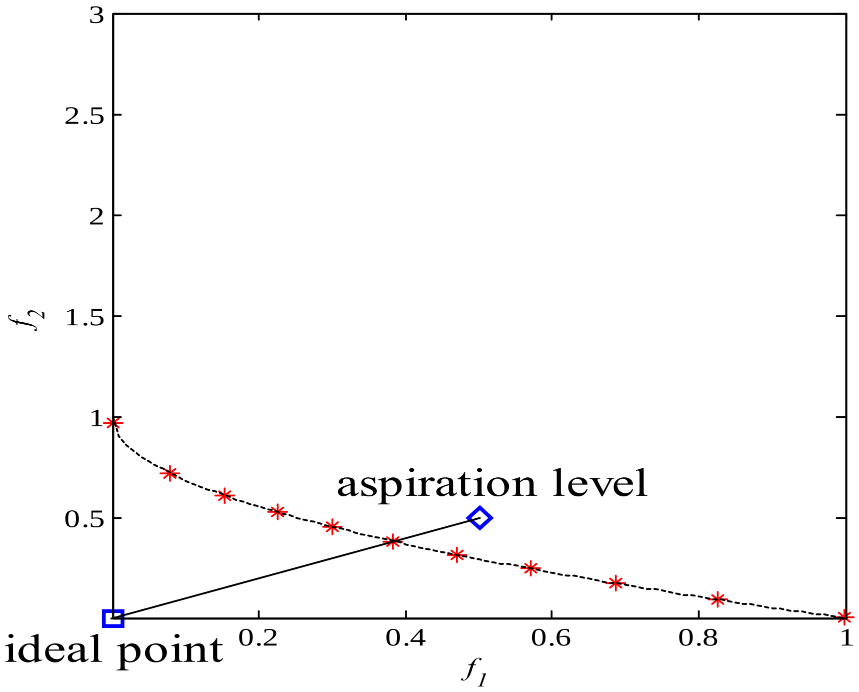
$$x_1 \in [0,1],\ x_i \in [-5,5],\ i = 1,2,\ldots,N \ \ (N=10)$$

Pareto surface satisfies $g=1$ $(x_2=x_3=\ldots=x_{10}=0)$.

Constraint transformation method

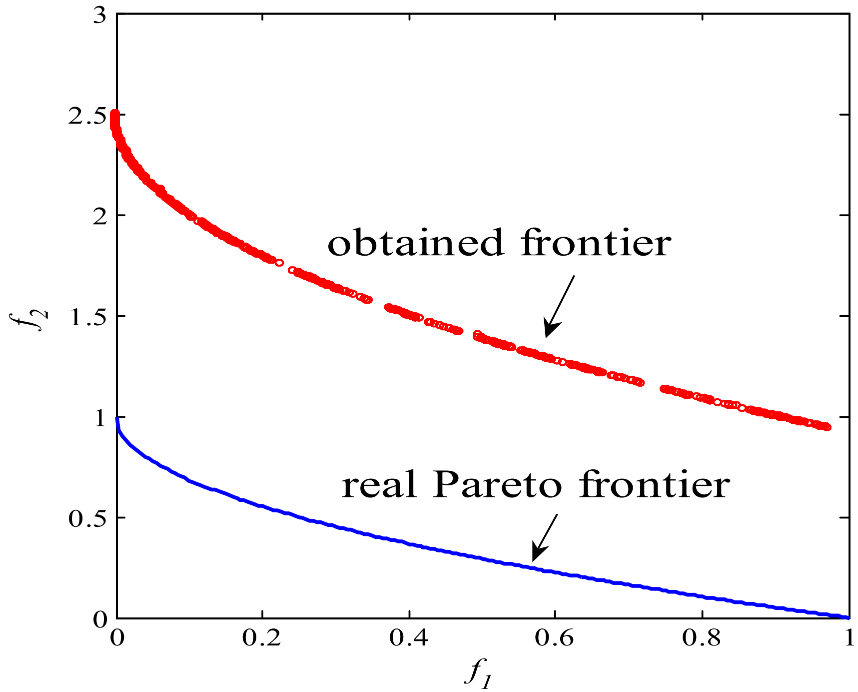
Aspiration level method




It takes about 50 function calls to obtain each Pareto point.

# MOGA & GDEA (100 data×250 gen.)



# Concluding Remarks

- Aspiration level methods are suitable for cases wit
h many objective functions in which the auxiliary
optimization problem is easily solved.

gray zone

- Generation methods of Pareto frontier are suitable
for cases with a few objective functions in which t
he auxiliary optimization problem is not so easily s
olved (e.g., combinatorial, nonsmooth).

*Usually, a large number of function calls are needed.*

# Future Subjects

- Parallel computation

- Virtual evaluation (Karakasis-Giannakoglou 2004)
  (Nakayama-Yun 2005)

  *function approximation by computational intelligence*

- Combining evolutionary methods and interactive methods

# Combining aspiration level methods and generation methods of Pareto frontier

- Y. Yun, H. Nakayama, M. Arakawa

  Multiple Criteria Decision Making with Generalize
  d DEA and an Aspiration Level Method

  *European Journal of Operational Research* ,　Vo
  l.158 ,　pp. 697-706 (2004)

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