Varieties of functions

Suppose we have a function f (from the source X to the target Y). Then it is called

Total function	$\rightarrow$	-->	if $dom(f) = X,\ ran(f) \subseteq Y$
Partial function	⇸	+->	if $dom(f) \subseteq X,\ ran(f) \subseteq Y$
Total injection	$\rightarrowtail$	>->	if $dom(f) = X,\ ran(f) \subseteq Y$ and one-to-one function
Partial injection	⤔	>+>	if $dom(f) \subseteq X,\ ran(f) \subseteq Y$ and one-to-one function
Total surjection	$\twoheadrightarrow$	->>	if $dom(f) = X,\ ran(f) = Y$
Partial surjection	⤀	+->>	if $dom(f) \subseteq X,\ ran(f) = Y$
(Total) Bijection	⤖	>->>	if $dom(f) = X,\ ran(f) = Y$ and one-to-one function

Varieties of functions: examples

- Injection: a function with 1-to-1 relationship between the source and target sets (e.g., an array without repeating elements)
- Example: $VU\_id \in PERSON \rightarrowtail\!\!\!\!\!\!\!\!\rightarrow ID$

  It is a partial injection: not all persons have a VU identification number, however, id is unique for each person
- Advantage: a reverse relation for an injection is also a function!
- Example: $VU\_id^{\sim} \in ID \rightarrowtail PERSON$

  A total injection this time
- Other examples:
  $Capital \in \text{COUNTRY} \rightarrowtail CITY$, and
  $Capital\_of \in CITY \rightarrowtail\!\!\!\!\!\!\!\!\rightarrow COUNTRY$ (where $Capital\_of = Capital^{\sim}$)

Varieties of functions: examples

- Surjection: a function that covers all the target set
- Example:
  $married \in WIFE \twoheadrightarrow HUSBAND$

  It is a total surjection
- Another example:
  $Capital\_of \in CITY \nrightarrow COUNTRY$

  A partial surjection this time

Varieties of functions: examples

- Bijection: a total function that is both injection and surjection
- Example:
  $married \in WIFE \rightarrowtail\!\!\!\!\!\rightarrow HUSBAND$

  It is a bijection (in many countries)
- Bijections relate sets with the same power (length)
- Another example:
  $VU\_account \in ID \rightarrowtail\!\!\!\!\!\rightarrow VU\_ACCOUNT$

  A bijection: both sets are of the same length and one-to-one relationships in both directions