Basic operations with relations

- Initialising (for $R \in S \leftrightarrow T$):
  $R := \{s_1 \mapsto t_1,\ s_2 \mapsto t_2,\ \dots\}$ or $R := S \times \{t_0\}$
  where $s_i \mapsto t_i$ are constructed pairs from $S \leftrightarrow T$
- Adding, merging elements (using set operations):
  $R := R \cup \{s \mapsto t\}$ or $R := R \cup Q$
  where Q is a relation of the same type, i.e., $Q \in S \leftrightarrow T$
- Checking membership or inclusion:
  $s \mapsto t \in R$ or $R \subseteq Q$
- Removing elements (filtering on the relation domain):
  $R := \{s_1, s_2, \dots\} \lhd R$ or $R := \{s_1, s_2, \dots\} \unlhd R$
- Removing elements (filtering on the relation range):
  $R := R \rhd \{t_1, t_2, \dots\}$ or $R := R \unrhd \{t_1, t_2, \dots\}$

More operations on relations

- Inverse relation R^{-1} (ascii R~).
  Includes all such $(x \mapsto y)$ that $(y \mapsto x) \in R$

- Example: for the relation
  $$owns\_camera = \{Jonas \mapsto Canon, Vaidas \mapsto Nikon,$$
  $$Vaiva \mapsto Sony, Jonas \mapsto Sony, Sandra \mapsto Pentax\}$$
  its inverse is
  $$owns\_camera^{-1} = \{Canon \mapsto Jonas, Nikon \mapsto Vaidas,$$
  $$Sony \mapsto Vaiva, Sony \mapsto Jonas, Pentax \mapsto Sandra\}$$

- Relational image $R[S]$. Returns a subset of the relation range related to any element from the given set S

- Example: for the relation *owns_camera* defined above
  $owns\_camera[\{Jonas, Vaiva\}] = \{Canon, Sony\}$

More operations on relations (cont.)

- Relational composition $R_1 ; R_2$.
  Composition of relations $R_1 \in S \leftrightarrow T$ and $R_2 \in T \leftrightarrow U$ is relation $\{(s \mapsto u) \mid \exists t.\ (s \mapsto t) \in R_1 \wedge (t \mapsto u) \in R_2\}$

- Example: Suppose we are given a relation between camera models and their brands $cmodels \in CMODEL \leftrightarrow CAMERA$,
  for instance,
  $cmodels \ = \ \{350 \mapsto Canon, 60D \mapsto Canon, D5200 \mapsto Nikon, \ldots\}$

  Then the previously defined $owns\_camera$ can be built as a result of a composition of the relations $owns\_cmodel \in PERSON \leftrightarrow CMODEL$ and $cmodels \in CMODEL \leftrightarrow CAMERA$, i.e.,

  $$owns\_camera \ = \ owns\_cmodel \,;\, cmodels$$

More operations on relations (cont.)

- Relational overriding $R_1 \Leftarrow\!\!+ R_2$ (ascii R_1 <+ R_2).

  Updating the relation R_1 by R_2:
  $R_1 \Leftarrow\!\!+ R_2 \;\; = \;\; (dom(R_2) \lhd\!\!- R_1) \cup R_2$

- Example:
  $owns\_camera \Leftarrow\!\!+ \{Jonas \mapsto Nikon\}$

  Two pairs $Jonas \mapsto Canon$ and $Jonas \mapsto Sony$ are removed, one pair $Jonas \mapsto Nikon$ is added

More operations on relations (cont.)

- Identity relation: $id \in S \leftrightarrow S$

  The relation contains pairs $(s \mapsto s)$ for all $s \in S$
  (each element is only related to itself)

- Example: suppose we have the relation

  $$connected \in CITY \leftrightarrow CITY$$

  containing all the road connections between cities
  (encoding a graph representation of roads between cities)

  Then the requirements "Roads can only be between different cities" can be formulated as

  $$connected \cap id = \varnothing$$