

CHAPTER ONE

Introduction

In this book, gravity waves on water and their interaction with oscillating systems (having zero forward speed) are approached from a somewhat interdisciplinary point of view. Before the matter is explored in depth, a comparison is briefly made between different types of waves, including acoustic waves and electromagnetic waves, drawing the reader's attention to some analogies and dissimilarities. Oscillating systems for generating or absorbing waves on water are analogous to loudspeakers or microphones in acoustics, respectively. In electromagnetics, the analogues are transmitting or receiving antennae in radio engineering, and light-emitting or light-absorbing atoms in optics.

The discussion of waves in this book is almost exclusively limited to waves of sufficiently low amplitudes for linear analysis to be applicable. Several other books (see, e.g., the monographs by Mei et al. [1], Faltinsen [2], Sarpkaya [3] or Chakrabarti [4]) treat the subject of large ocean waves and extreme wave loads—which are important for determining the survivability of ships, harbours and other ocean structures, including devices for conversion of ocean-wave energy. In contrast, the purpose of the present book is to convey a thorough understanding of the interaction between waves and oscillations when the amplitudes are low, which is true most of the time. For example, for a wave-power plant, the income is determined by the annual energy production, which is essentially accrued during most times of the year, when amplitudes are low—that is, when linear interaction is applicable. On the other hand, as with many other types of ocean installations, wave-power plants have their expenses, to a large extent, determined by the extreme-load design.

There are several steps in the process of converting wave energy to some useful form, such as electrical energy, by means of an immersed oscillating wave-energy-conversion (WEC) unit. In the first conversion step, energy may be converted to some form of kinetic and/or potential energy, to possibly be stored in flywheels and or gas accumulators of a hydraulic machinery. Such energy storage may, for certain applications, be desirable in order to even out fluctuations of the incident wave energy on a minute-to-minute scale [5]. In a

second conversion step, mechanical energy may be delivered, for example, through the shaft of a turbine or some other type of hydraulic motor, to, for example, an electrical generator, which may serve a tertiary conversion to electric energy. The second conversion step may possibly be circumvented by the application of linear electric generators. The present book deals with the first conversion step, the *primary wave-energy conversion*—that is, how to reduce the energy in an incident wave while, correspondingly, increasing the energy input to an oscillating WEC system. The technological/engineering aspects related to conversion and useful application of wave energy are not covered in the present book. Readers interested in such subjects are referred to other literature [5–11].

The content of the subsequent chapters is outlined in the following paragraphs. At the end of each chapter, except the first, there is a collection of problems.

Chapter 2 gives a mathematical description of free and forced oscillations in the time domain as well as in the frequency domain. An important purpose is to introduce students to the very useful mathematical tool of the complex representation of sinusoidal oscillations. The mathematical connection between complex amplitudes and Fourier transforms is treated. Linear systems are discussed in a rather general way, and for a causal linear system in particular, the Kramers–Kronig relations are derived. A simple mechanical oscillating system is analysed to some extent. Then the concept of mechanical impedance is introduced. Moreover, a discussion of energy accounting in the system is included to serve as a tool for physical explanation, in subsequent chapters, of the so-called hydrodynamic added mass and other analogue quantities.

In Chapter 3, a brief comparison is made of waves on water with other types of waves, in particular with acoustic waves. The concepts of wave dispersion, phase velocity and group velocity are introduced. In addition, the transport of energy associated with propagating waves is considered, and the radiated power from a radiation source (wave generator) is mathematically expressed in terms of a phenomenologically defined radiation resistance. The radiation impedance, which is a complex parameter, is introduced in a phenomenological way. For mechanical waves (such as acoustic waves and waves on water), its imaginary part may be represented by an added mass. Finally, in Chapter 3, an analysis is given of the absorption of energy from a mechanical wave by means of a mechanical oscillation system of the simple type considered in Chapter 2. The optimum parameters of this system for maximising the absorbed energy are discussed. The maximum is, for such a simple oscillating system, obtained at resonance. Chapter 3 is rather elementary and intended as a general introduction for readers not familiar with hydrodynamics. It may be skipped by readers already familiar with hydrodynamics.

From Chapter 4 and onward, a deeper hydrodynamic discussion of water waves is the main subject. With the assumption of inviscid and incompressible fluid and irrotational fluid motion, the hydrodynamic potential theory is

developed. With the linearisation of fluid equations and boundary conditions, the basic equations for low-amplitude waves are derived. In most of the following discussions, either infinite water depth or finite, but constant, water depth is assumed. Dispersion and wave-propagation velocities are studied, and plane and circular waves are discussed in some detail. Also, non-propagating, evanescent, plane waves are considered. Another studied subject is wave-transported energy and momentum. The spectrum of real sea waves is treated only briefly in the present book. The rather theoretical Sections 4.7 and 4.8, which make extensive use of Green's theorem, may be omitted at the first reading, and then be referred to as needed during the study of the remaining chapters of the book. Whereas most of Chapter 4 is concerned with discussions in the frequency domain, the last section contains discussions in the time domain. As 'Linear Interactions' is part of the present book's title, nonlinear wave energy converters, such as the Norwegian 'Tapchan' or the Danish 'Wave Dragon', are outside the scope of the present book, except for a simple exercise, though, related to Chapter 4 (Problem 4.11).

The subject of Chapter 5 is the interactions between waves and oscillating bodies, including wave generation by oscillating bodies as well as forces induced by waves on the bodies. Initially, six-dimensional generalised vectors are introduced which correspond to the six degrees of freedom for the motion of an immersed (three-dimensional) rigid body. The radiation impedance, known from the phenomenological introduction in Chapter 3, is now defined in a hydrodynamic formulation, and, for a three-dimensional body, extended to a 6×6 matrix. In a later part of the chapter, the radiation impedance matrix is extended to the case of a finite number of interacting, radiating, immersed bodies. For this case, the generalised excitation force vector is decomposed into two parts, the Froude–Krylov part and the diffraction part, which are particularly discussed in the 'small-body' (or 'long-wavelength') approximation. From Green's theorem (as mentioned in the summary of Chapter 4), several useful reciprocity theorems are derived, which relate excitation force and radiation resistance to each other or to 'far-field coefficients' (or 'Kochin functions'). Subsequently, these theorems are applied to oscillating systems consisting of concentric axisymmetric bodies or of two-dimensional bodies. The occurrence of singular radiation-resistance matrices is discussed in this connection. While most of Chapter 5 is concerned with discussions in the frequency domain, two sections, Sections 5.3 and 5.9, contain discussions in the time domain. In the latter section, motion response is the main subject. In the former section, two hydrodynamic impulse-response functions are considered, where one of them is causal and, hence, has to obey the Kramers–Kronig relations.

The extraction of wave energy by means of an oscillating immersed WEC body is the subject of Chapter 6, which starts by explaining wave absorption as a wave-interference phenomenon: to absorb a wave, the oscillating WEC body needs to radiate a wave which interferes destructively with the incident wave. Thus, energy is removed from the incident wave and converted to

some form of useful energy. The WEC body thus needs to displace water, push water during half part of a wave period and suck water during the remaining part. Unfortunately, some proposed devices, claimed to be wave energy absorbers, seem to essentially just follow the water motion of the incident wave.

Moreover, in Chapter 6, WEC units are classified according to their horizontal extension relative to the wavelength. A formula is derived for the power that is absorbed from an incident plane wave by means of a WEC body, if it is oscillating in only one degree of freedom. This absorbed power is positive, provided the body's oscillation has an amplitude and a phase within a certain region relative to the amplitude and phase of the incident wave. This region may be illustrated by the wave-power 'island', where the highest point corresponds to the maximum power which may be absorbed from the incident wave. Moreover, a section of Chapter 6 is devoted to the subject of optimum control aiming at maximising absorbed wave power. The Budal upper bound (BUB), as well as the Keulegan–Carpenter number, are additional important matter in a section of Chapter 6. Here, in contrast to the previous sections, the WEC body's hull volume size and shape are matters of concern. The final section of Chapter 6 concerns wave-power absorption by means of a WEC body oscillating in more than one mode.

Toward the end of the chapter (Section 6.5), a study is made of the absorption of wave energy by means of an immersed body oscillating in several (up to six) degrees of freedom. This discussion provides a physical explanation of the quite frequently encountered cases of singular radiation-resistance matrices, as mentioned earlier (also see Sections 5.7 and 5.8). However, the central part of Chapter 6 is concerned with wave-energy conversion which utilises only a single body oscillating in just one degree of freedom. With the assumption that an external force is applied to the oscillating system, for the purpose of power takeoff and optimum control of the oscillation, this discussion has a different starting point than that given in the last part of Chapter 3. The conditions for maximising the converted power are also studied for the case in which the body oscillation has to be restricted as a result of its designed amplitude limit or because of the installed capacity of the energy-conversion machinery.

Oscillating water columns (OWCs) are mentioned briefly in Chapter 4 and considered in greater detail in Chapter 7, where their interaction with incident waves and radiated waves is the main subject of study. Two kinds of interaction are considered: the radiation problem and the excitation problem. The radiation problem concerns the radiation of waves resulting from an oscillating dynamic air pressure above the internal air–water interfaces of the OWCs. The excitation problem concerns the oscillation which is due to an incident wave when the dynamic air pressure is zero within the OWC's air chamber. Comparisons are made with corresponding wave–body interactions. Also, wave-energy extraction by an OWC WEC is discussed. The last part of Chapter 7 concerns WECs which oscillate in unconventional modes of motion—that is, modes other than the

six rigid-body modes considered in Chapter 5. The analysis of such WECs is facilitated by the use of generalised modes, which are described by shape functions.

The main subject of Chapter 8, the last chapter of the book, is related to possible WEC systems for a more distant future, arrays of WEC bodies and WEC arrays based on oscillating bodies as well as OWCs. Also, two-dimensional WEC devices, verbally described in Section 6.1, are considered in some mathematical detail in Chapter 8.