2.1 Wave theory

2.1.1 Introduction

A classification of different types of waves is given by Kinsman (1965), based on the amount of energy as a function of the frequency. He considered tidal waves, gravity waves, capillary waves, etc. In the context of this master thesis, only gravity waves are considered, which have a period between 1 s and 30 s and contain the largest amount of energy. This wave energy is an indirect form of solar energy, since gravity waves are produced by wind action and wind arises through conversion of heat energy supplied by the sun. However, wave energy is more spatially concentrated than wind or solar energy (Falcão, 2010).

The wave characteristics of a progressive wave are illustrated in Figure 2-1. Such a wave travels in a particular direction and transfers energy. Its wave motion can be described by the Airy theory. This is a linear wave theory, which assumes waves with a small wave steepness s (see Eq.(2.1)) and a small relative wave height (see Eq.(2.2)). The elevations relative to the water surface at rest are also small (*the small amplitude wave theory*) and forces due to surface tension are assumed to be negligible.

$$s = {}^{H}/_{L} << 1 \tag{2.1}$$

$${}^{H}/_{d} << 1 \tag{2.2}$$

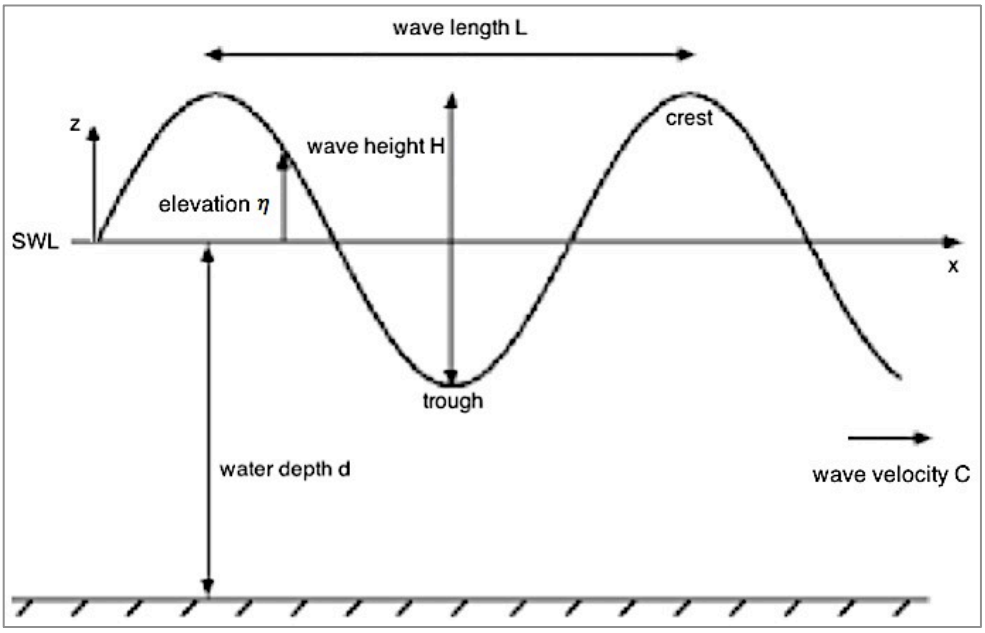

Figure 2-1: Wave characteristics used in the Airy theory (not drawn to scale).

The Airy theory is only an approximation of the real wave mechanism, but it has proven to be sufficient for a wide range of engineering applications. A short background of the linear potential theory, which forms the base of this wave theory, is given (Troch, 2007) to provide insight in the different formulas.

2.1.2 Linear potential theory

2.1.2.1 Boundary conditions problem for a regular wave

The motion of a fluid is described by the equation of continuity (conservation of mass) and the Navier-Stokes equations (conservation of momentum). When an incompressible, inviscid fluid and an irrotational, two dimensional flow are assumed, those equations can be written in a simplified linearized form, respectively the Laplace equation (Eq.(2.3)) and Bernoulli's equation (Eq.(2.4)).

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \tag{2.3}$$

$$-\frac{\partial \varphi}{\partial t} + \frac{p}{\rho} + gz = 0 \tag{2.4}$$

φ is the velocity potential of the incident regular progressive wave, p is the pressure, g the gravitational acceleration and ρ is the water density. The coordinates x and z are indicated in Figure 2-1, while the parameter t represents time. To derive the velocity potential φ, several boundary conditions need to be required. The dynamic boundary condition on the free surface states that the pressure is equal to the atmospheric pressure. The kinematic boundary expresses that there is no transport of fluid through the free surface by demanding a vertical velocity component of a fluid particle equal to the vertical velocity of the free surface. Transport of fluid is also not allowed through the seabed, resulting in a kinematic boundary condition of a normal velocity component of a fluid particle equal to zero.

2.1.2.2 Velocity potential of the incident wave

A solution for the velocity potential and the corresponding boundary conditions can be found based on the method of separation of variables and is given by following formula:

$$\varphi(x,z,t) = -\frac{ag}{\omega}\frac{\cosh\big(k_w(d+z)\big)}{\cosh(k_w d)}\sin\,(k_w x - \omega t) \tag{2.5}$$

The wave amplitude a equals $H/2$, the wave number k_w can be written as $2\pi/L$ and the pulsation ω as $2\pi/T$. T is the wave period and equals the inverse of the wave frequency f, while d is the water depth.

2.1.2.3 Wave motion

The equation of the wave motion is given by the elevation η on the free surface relative to the still water level (SWL) and equals:

$$\eta(x,t) = \frac{H}{2}\cos\,(k_w x - \omega t) \tag{2.6}$$

It is a two dimensional sinusoidal wave, periodic in time and space. The velocity C is expressed as:

$$C = \frac{L}{T} = \frac{gT}{2\pi}\tanh k_w d \tag{2.7}$$

This is the velocity of an individual wave. Another parameter is the group velocity, which differs most often from the individual velocity. It is the velocity of a group of waves, which consists of the superposition of a finite number of regular waves. It is given by following formula:

$$C_g = n \cdot C = \frac{1}{2}\left(1 + \frac{2k_w d}{\sinh\,(2k_w d)}\right) \cdot C \tag{2.8}$$

2.1.2.4 Pressure

The total pressure in a particular point in the water consists of the hydrostatic pressure and the hydrodynamic pressure, represented respectively by the first and second term in Eq.(2.9). The hydrostatic pressure is caused by the water column above the considered point, while the hydrodynamic pressure is associated with the acceleration of the fluid particles.

$$p(z) = \rho g(-z) + \rho g \frac{H}{2} \frac{\cosh\big(k_w(d+z)\big)}{\cosh(k_w d)} \cos\,(k_w x - \omega t) \tag{2.9}$$

2.1.2.5 Energy

The total energy consists of two components: the kinetic energy E_k, due to the velocity of the fluid particles, and the potential energy E_p, due to the elevation of the free water surface. The kinetic component can be calculated based on the horizontal and vertical velocity component of a fluid particle, respectively u and w. The amount per unit of crest length is given by Eq.(2.10).

$$E_k = \frac{1}{2}\rho \int_0^L \int_{-d}^{\eta} (u^2 + w^2)\; dz\; dx = \frac{1}{16}\rho g H^2 L \tag{2.10}$$

The elevation η relative to SWL is considered to determine the potential energy per unit of crest length:

$$E_p = \frac{1}{2}\rho g \int_0^L \eta^2\; dx = \frac{1}{16}\rho g H^2 L \tag{2.11}$$

The total energy is the sum of both components and is given in Eq.(2.12) per unit of water surface.

$$E = \frac{E_k + E_p}{L} = \frac{1}{8}\rho g H^2 \tag{2.12}$$

2.1.2.6 Power

The power is the amount of energy transported per time unit by the wave. It is calculated by the product of a force and a velocity through a surface perpendicular on the wave propagation (De Rouck, 2011). The force is equal to the dynamic pressure multiplied by the section and the velocity equals the horizontal velocity of the fluid particles.

$$P = \frac{1}{T}\int_0^T \int_{-d}^{\eta} (p + \rho g z)\; u\; dz\; dt = E C_g \tag{2.13}$$

2.1.2.7 Irregular waves

Thanks to the linear theory, the elevation η relative to SWL of irregular waves in nature at a time t on a certain position can be described by the superposition of a finite number N of regular sinusoidal waves (De Rouck, 2011):

$$\eta(t) = \sum_{i=1}^{N} a_i \cos(\omega_i t - \phi_i) \tag{2.14}$$

The parameters a_i, ω_i and ϕ_i are respectively the wave amplitude, the wave pulsation and the phase shift of the compounding wave i.

The wave motion is described by a characteristic wave height and wave period. In the time domain, the significant wave height H_s equals the arithmetical mean of the third highest wave heights registered during a certain time interval of wave observation. Different characteristic wave periods are used, like the mean wave period T_m of all observed waves or the significant wave period T_s. The latter is the mean period of the same waves used to calculate H_s. In the frequency domain, the significant wave height H_{m0} is related to the area below the wave spectrum. This wave spectrum gives the amount of energy as a function of the frequency within the irregular wave train. The characteristic wave period is often the peak period T_p, which corresponds with the frequency that has the largest amount of energy in the wave spectrum.

2.1.2.8 Wave-body interactions

When a body is placed in the waves, wave-body interactions will occur (De Backer, 2009). Besides the velocity potential of the incident waves, there are two additional problems that need to be considered: the radiation problem and the diffraction problem.

The radiation problem is related to the forced harmonic motion of the body in initially still water. During this motion, waves are radiated and the corresponding flow is described by the radiation potential. The diffraction problem is noticed when the body is kept fixed in a regular wave field. The incident waves are diffracted in its shadow zone and those diffracted waves are described by the diffraction potential. The total velocity potential is the linear sum of the incident, radiated and diffracted potential.

The extra potentials need to satisfy two additional boundary conditions. The first condition is on the body: a fluid particle on the submerged surface of a moving body is not able to go through or come out of the body boundary. This means that the normal velocity component of a fluid particle equals the normal velocity component of the body at its surface. The second boundary condition expresses the conservation of energy at infinity.

2.2 Point absorber

2.2.1 Introduction

A WEC has to remove energy from the incident waves. Three conversion steps can be considered (Falnes, 2007). The primary conversion includes the energy transfer from the sea to the WEC. Mechanical energy in the form of kinetic and/or potential energy captured in the WEC is then converted to more useful energy by means of conversion machinery (e.g. turbine, motor). An electric generator completes the power train by a tertiary conversion step into electric energy.

An oscillating device, like a point absorber, generates waves and when those waves interfere destructively with the incident waves, the latter ones are reduced or – in optimum conditions – eliminated. Falnes (1995) summarizes this in a paradoxical statement that "*to destroy a wave means to create a wave*". He illustrates the principle for an infinite line of oscillating small bodies, evenly interspaced a short distance as shown in Figure 2-2. The undisturbed incident wave field is given by curve a. When the bodies oscillate in heave, a symmetric wave is generated (curve b). Asymmetric wave generation is given in curve c. If those three waves occur simultaneously, the resultant wave is given by the superposition represented in curve d, which illustrates 100 % absorption of wave energy. This maximum energy capture is only possible when the devices oscillate with an optimal amplitude and phase. For the specific case mentioned there, the generated waves in b and c need to have an amplitude of half the incident wave amplitude from a and the same phase, which is in turn correct with respect to the phase of the incident wave.

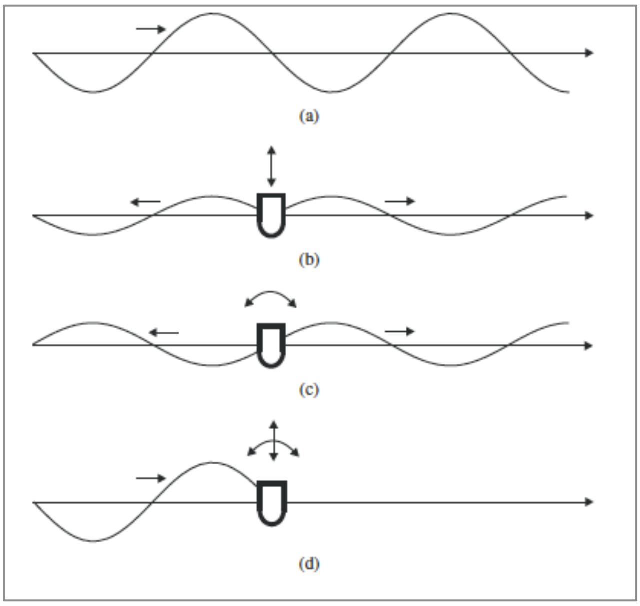

Figure 2-2: Wave energy absorption by an infinite line (perpendicular to the figure) of oscillating small floating bodies, evenly interspaced a short distance (Falnes, 1995).

The situation in the experimental research of this master thesis consists of one point absorber, which is a WEC-type characterized by small horizontal dimensions compared to the incident wave length. Generally, a point absorber can oscillate in one ore more degrees of freedom. The movement here is restricted to oscillation in heave: as a response to incident waves, the device moves up and down. Circular waves are radiated away from the immersed surface by this motion and they interfere with the incident plane wave. The wave pattern is given in Figure 2-3 (Falnes, 1995). In addition, incident waves are also reflected in front of the buoy and diffracted behind the buoy. The resultant wave climate is determined by the superposition of all these components. Damping the buoy motion by a PTO-system absorbs its energy, which can then be converted into electricity by a generator.

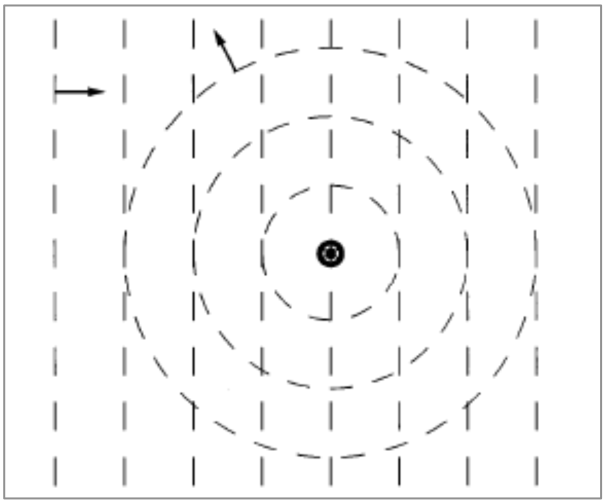

Figure 2-3: Wave pattern from the incident plane wave and the radiated circular wave (Falnes, 1995).

An oscillating system is considered as an efficient absorber when its natural frequency coincides with the frequency of the incident waves, since operating at near-resonance conditions ensures the largest power absorption. Then, the body velocity is in phase with the excitation force from the waves. For a heaving buoy, this force is approximately in phase with the water surface elevation of the incident wave (Falnes, 1995). Two important issues have to be considered when applying this requirement to a real situation (Falcão, 2010). First of all, the oscillation frequency of the device is very often too high compared with typical ocean-wave frequencies. Secondly, real waves are not single-frequency, but they are irregular waves, which cover a range of frequencies and wave heights characterized by a peak frequency and a significant wave height.

For frequencies within the resonance bandwidth, optimum phase condition is also satisfied approximately (Falnes, 1995). Since point absorbers have small horizontal dimensions, they have a rather narrow resonance bandwidth, so phase control is important. There are several principles to phase control the oscillating system in order to absorb a maximum amount of energy. A first possibility contains tuning of the system by adding a supplementary mass in order to adjust the natural frequency of the system to the incident wave frequency. In irregular waves, each individual wave is characterized by its own frequency, making it impossible to tune the device to all components. Therefore, a constant optimal value for the supplementary mass is used, experimentally derived depending on the applied wave class. A second method is latching phase control (Falnes, 1995), which can be used when the wave frequencies are lower than the natural frequency of the device. Optimum phase can be approximately achieved by locking the device at the largest deflection where the velocity becomes zero. A clamping mechanism is therefore used, which releases the device a certain time (about one quarter of the natural period) before the next peak of the wave exciting force occurs. The principle is illustrated in Figure 2-4. Curve a gives the elevation of the water surface due to the incident wave. Curve b gives the vertical position of a device operating at resonance, meaning its natural frequency equals the incident wave frequency and optimum power absorption occurs. Curve c gives the vertical position of a device with a higher frequency that is phase controlled by keeping it in a fixed vertical position during certain time intervals. In practice, the wave force needs to be known a certain time into the future to enable latching phase control. Irregular waves have a stochastic distribution of the time interval between crests and troughs, so input signals from sensors measuring the waves and forces are necessary.

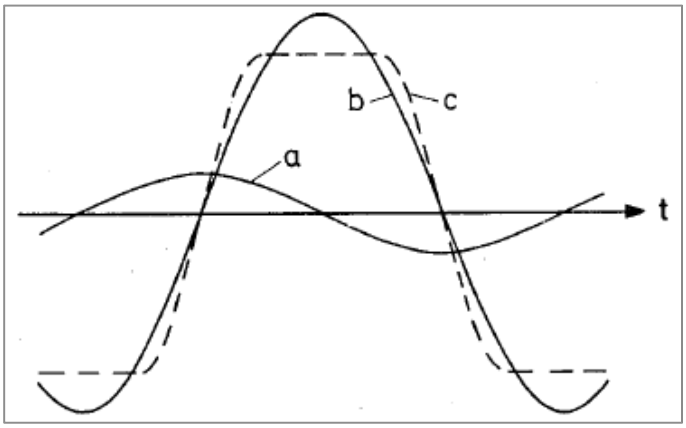

Figure 2-4: Principle sketch of phase control by latching (Falnes, 1995).

Other possible methods to phase control a point absorber are an additional spring term with negative spring coefficient and freewheeling (Nolan *et al.*, 2005). In the latter principle, the device moves undamped between its extrema building up its velocity, until a limit value of the velocity is exceeded and damping is applied.

None of the methods shortly described before is used in the concept of the WEC in this project. The aim is namely not on maximum power absorption, but on examining the wake effects of a WEC-farm. The design needs to simulate the real impact of a WEC on the wave climate by a simple concept of energy extraction. Adding phase control to each of the 20 buoys would be too expensive. However, it should be noticed that pursuing optimum power absorption would realise a more realistic wake.

2.2.2 Equation of motion

The equation of motion of a point absorber is given by Newton's second law:

$$m \cdot \ddot{\vec{x}} = \vec{F} \tag{2.15}$$

Generally, a freely floating body in ocean waves has six degrees of freedom: 3 translational modes (surge, sway and heave) and 3 rotational modes (roll, pitch and yaw) as illustrated in Figure 2-5. The xy-plane of the right-handed coordinate system is the horizontal plane, parallel to the still water surface. The z-axis is positive in the upward direction and coincides with the axis of symmetry of the device, while the origin point corresponds to the centre of gravity.

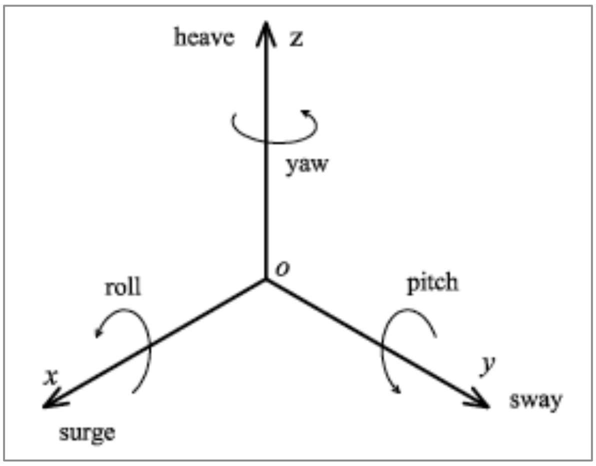

Figure 2-5: Six degrees of freedom of a freely floating structure in ocean waves (De Backer, 2009).

The point absorber considered here is restricted to heave motion only. The vectorial Eq.(2.15) can be reduced to a one-dimensional equation for the vertical translation along the z-axis.

$$m \cdot \ddot{z} = F_z \tag{2.16}$$

The device with mass m has a position z from its equilibrium position during its heaving motion. Simultaneously, it encounters a total force F_z in the direction of the degree of freedom, which is composed of different components. These components are given in Eq.(2.17) (De Backer, 2009).

$$m \frac{d^2 z(t)}{dt^2} = F_{ex} + F_{rad} + F_{res} + F_{damp} + F_{tun} \tag{2.17}$$

To describe the different components, a linear theory is applied which is allowed when the amplitude of the vertical motion of the buoy is small.

F_{ex} is the exciting wave force due to incident waves, which equals the sum of the Froude-Krylov force and the diffraction force. The first component is the force experienced by the body as if there was no disturbance of the incident wave field by the body. The second component is linked to the diffraction problem: when the floater is held fixed in a wave, it causes diffraction of the incident waves in its shadow zone. The resultant of the hydrodynamic pressure field represents the vertical wave action on the device.

F_{rad} is the radiation force due to the buoy motion. It is associated with the radiation problem, concerning the hydrodynamic pressure field that arises when a floater is forced to move in a harmonic oscillation in initially still water. It equals the component according to the z-direction of the resultant pressure field on the buoy. It can be decomposed in two terms with the aid of the linear theory: a linear added mass term and a linear hydrodynamic damping term:

$$F_{rad} = -m_a(\omega) \frac{d^2 z(t)}{dt^2} - b_{hyd}(\omega) \frac{dz(t)}{dt} \tag{2.18}$$

The first term is proportional to the acceleration of the buoy with the hydrodynamic coefficient of added mass $m_a(\omega)$ as proportionality factor, which corresponds to a volume of water that is moved when the buoy is oscillating. This term thus accounts for the inertia of the water surrounding the buoy and is a hydrodynamic inertia force. The second term is a hydrodynamic damping force, which accounts for the radiation damping. The energy dissipated due to this damping equals the energy transferred to the waves radiated away. It is proportional with the velocity according the hydrodynamic damping coefficient $b_{hyd}(\omega)$.

F_{res} is the hydrostatic restoring force and is equal to the resultant of the Archimedes force F_{arch} and the gravity force F_g. The force is connected to the geometry of the buoy and its instantaneous vertical displacement from its equilibrium position according following formula:

$$F_{res} = F_{arch} - F_g = \rho_w V(t)g - mg = -kz(\mathrm{t}) \tag{2.19}$$

ρ_w is the density of water and $V(t)$ is the instantaneous submerged buoy volume. The linear spring constant or hydrostatic restoring coefficient k is expressed as follows, in which A_w is the waterline area:

$$k = \rho_w g A_w \tag{2.20}$$

The energy absorption by the PTO-system is simulated by an external damping force F_{damp}. In practical applications, this force is typically non-linear, but for simplicity, it is often considered as proportional to the velocity. The proportionality factor is the linear external damping coefficient b_{ext}.

$$F_{damp} = -b_{ext}\frac{dz(t)}{dt} \tag{2.21}$$

F_{tun} is the tuning force to phase control the buoy by adding a supplementary mass m_{sup}, which corresponds to an additional inertia force.

$$F_{tun} = -m_{sup}\frac{d^2z(t)}{dt^2} \tag{2.22}$$

As mentioned before, phase control by tuning the natural frequency of the device to the frequency of the incident waves is not considered within this research.

Compared to Eqs.(1.54 – 1.55) from De Backer (2009), a minus sign is added in Eqs.(2.21 – 2.22) in order to achieve Eq.(2.23) by substituting the aforementioned equations into Eq.(2.17). Eq.(2.23) corresponds then to Eq.(1.56) from De Backer (2009).

$$\left(m + m_a(\omega) + m_{sup}\right)\frac{d^2z(t)}{dt^2} + \left(b_{hyd}(\omega) + b_{ext}\right)\frac{dz(t)}{dt} + kz(t) = F_{ex}(\omega, t) \tag{2.23}$$

All parameters are dependent on the buoy geometry, while m_a, b_{hyd} and F_{ex} also depend on the frequency ω of the incident wave. F_{ex} is in addition linked to the wave amplitude.

The total response $z\ (t)$ is the sum of the free response and the forced oscillation or steady-sate oscillation (Loccufier, 2011):

$$z\ (t) = z_{free}(t) + z_{forced}(t) \tag{2.24}$$

The free oscillation is the response on the initial conditions and it forms the homogeneous solution of Eq.(2.23). It can be mathematically expressed by Eq.(2.25) for an underdamped system, which is generally the case for a heaving point absorber.

$$z_{free}(t) = z_{Af} \cdot \exp\left(-\zeta_d \omega_n t\right) \cdot \sin\left(\omega_d t + \beta_f\right) \tag{2.25}$$

In this equation, z_{free} gives the buoy position relative to its equilibrium position. z_{Af} is the amplitude of the vertical motion, β_f the phase angle, ζ_d the damping factor and ω_d the damped natural angular frequency, which is related to the natural angular frequency ω_n by following equation:

$$\omega_d = \sqrt{1 - \zeta_d^2} \cdot \omega_n \tag{2.26}$$

The amplitude z_{Af} and phase angle β_f can be calculated with the aid of the angular frequencies $\omega_n$and ω_d, the damping factor ζ_d and the initial conditions of the system, namely the initial position q_0 and the initial velocity $\dot{q}_0$.

$$z_{Af} = \sqrt{q_0^2 + (\dot{q}_0 + \zeta_d \omega_n q_0)^2 \cdot \frac{1}{\omega_d^2}} \tag{2.27}$$

$$\beta_f = \arctan \frac{q_0}{\frac{1}{\omega_d} \cdot (\dot{q}_0 + \zeta_d \omega_n q_0)} \tag{2.28}$$

Based on the damping in the system, the oscillations fade exponentially and eventually disappear after a certain time. The envelope can be mathematically described as:

$$z_{envelope}(t) = z_{Af} \cdot \exp(-\zeta_d . \omega_n . t) \tag{2.29}$$

Both the motion curve and the envelope are illustrated in Figure 2-6.

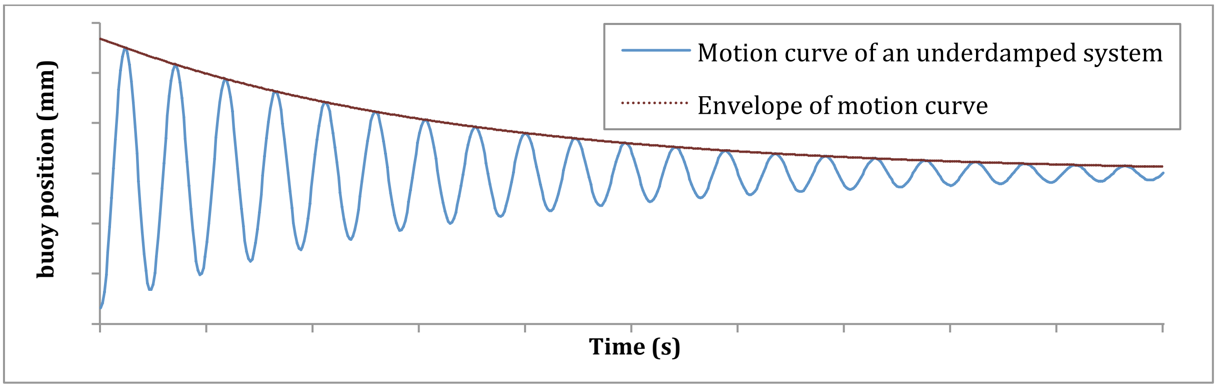

Figure 2-6: Exponential extinguishing decay curve of an underdamped system.

The steady-state oscillation of the device is the response on the exciting force when it has no initial divergence from its equilibrium position and no initial velocity ($z_{free} = 0$). This particular solution of the differential equation (Eq.(2.23)) is a sinusoidal function:

$$z_{forced}(t) = z_{As} \cdot \sin(\omega t + \beta_{mot}) \tag{2.30}$$

z_{As} is the amplitude of the steady-state oscillation and β_{mot} is the phase angle. They can be expressed by following formulas:

$$z_{As}(\omega) = \frac{F_{ex,A}(\omega)}{\sqrt{\left[k - \left(m + m_a(\omega) + m_{sup}\right) \cdot \omega^2\right]^2 + \left[\left(b_{hyd}(\omega) + b_{ext}\right) \cdot \omega\right]^2}} \tag{2.31}$$

$$\beta_{mot} = \beta_{Fex} - \arctan\left(\frac{\left(b_{hyd}(\omega) + b_{ext}\right) \cdot \omega}{k - \left(m + m_a(\omega) + m_{sup}\right) \cdot \omega^2}\right) \tag{2.32}$$

In the decay test in Chapter 5 and Chapter 6, the buoy position is described by the free oscillation formulas. There is no exciting force present, meaning z_{As} equals zero and thus z_{forced} too. During the other tests, the buoy is brought in motion by the incident waves. Its initial position q_0 and velocity $\dot{q}_0$ are equal to zero since the buoy starts from its equilibrium position at rest. Therefore, z_{Af} equals zero and thus z_{free} too. Its motion is then described by the steady-state oscillation.

The behaviour of a heaving point absorber shows good agreement with a mass-spring-damper system with one degree of freedom subjected to an external force in the direction of the degree of freedom. The formulas mentioned before have the same form. The natural frequency ω_n of the system can also be calculated as for a mass-spring-damper system:

$$\omega_n = \sqrt{\frac{k}{m_{tot}}} = \sqrt{\frac{k}{m + m_a(\omega) + m_{sup}}} \tag{2.33}$$

2.2.3 Power absorption

2.2.3.1 Definition

The average absorbed power over a period T of a point absorber equals the difference between the average excited power and the average radiated power (De Backer, 2009).

$$P_{abs,av} = P_{ex,av} - P_{rad,av} \tag{2.34}$$

The power can be calculated as the product of the velocity and the force. A point absorber subjected to a sinusoidal force with amplitude $F_{ex,A}$ undergoes a harmonic oscillation with a sinusoidal velocity with amplitude v_A. The average exciting power is given by Eq.(2.35) with γ the phase shift between the force and the velocity.

$$P_{ex,av} = \frac{1}{2} F_{ex,A} v_A \cos\gamma \tag{2.35}$$

The average radiated power depends on the hydrodynamic radiation damping coefficient b_{hydr}:

$$P_{rad,av} = \frac{1}{2} b_{hydr} v_A^2 \tag{2.36}$$

The average power absorption can also be expressed as the power absorbed by the PTO-system:

$$P_{abs,av} = \frac{1}{2} b_{ext} v_A^2 \tag{2.37}$$

2.2.3.2 Efficiency

The absorption width λ_p is defined as the width of the wave front that contains the same available power as the power absorbed by the device in the wave field. It is equal to the ratio of the absorbed power to the average available power per unit width of the wave front.

$$\begin{aligned} \lambda_p &= \frac{P_{abs}}{P_{avail}} \\ &= \frac{2 \cdot L}{\pi} \cdot \frac{b_{hyd}(\omega) \cdot b_{ext} \cdot \omega^2}{\left[k - \left(m + m_a(\omega) + m_{sup}\right) \cdot \omega^2\right]^2 + \left[\left(b_{hyd}(\omega) + b_{ext}\right) \cdot \omega\right]^2} \end{aligned} \tag{2.38}$$

The capture width includes the power losses due to friction and other dissipative effects. It represents the useful or produced power, which equals the absorbed power minus the losses, and is thus smaller than the absorption width.

The absorption efficiency is calculated as the ratio of the absorption width to the diameter of the point absorber. This efficiency can exceed the value of 100 %, meaning that the absorbed power can be larger than the available power over the device diameter. This phenomenon is called the *point absorber effect*.

Different authors, among them Budal and Falnes (1975), have proven on the base of a linear theory that the maximum energy that can be absorbed by a heaving point absorber in a regular wave with length L is equal to the available wave energy in the incident wave front of width L divided by 2π.

$$\lambda_{p,max} = \frac{L}{2 \cdot \pi} \tag{2.39}$$

This is only a theoretical optimum, since second order effects become important for large buoy velocities. The linear theory is then no longer valid.