

**FROM FORMAL SPECIFICATION TO FULL PROOF:  
A STEPWISE METHOD**

*by*

Lavinia Burski



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DEPARTMENT OF COMPUTER SCIENCE  
SCHOOL OF MATHEMATICAL AND COMPUTER SCIENCES  
HERIOT-WATT UNIVERSITY

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# Abstract

Proving formal specifications in order to find logical errors is often a difficult and labour-intensive task. This thesis introduces a new and stepwise toolkit to assist in the translation of formal specifications into theorem provers, using a number of simple steps based on the MathLang Framework. By following these steps, the translation path between a Z specification and a formal proof in Isabelle could be carried out even by one who is not proficient in theorem proving.

# Acknowledgements

I dedicate this thesis to my loving and supportive boyfriend, Jeff.

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# Acronyms

**ASM** Abstract state machine.

**CGa** Core Grammatical aspect.

**DRa** Document Rhetorical aspect.

**GPSa** General Proof Skeleton aspect.

**Gpsa** General Proof Skeleton aspect.

**GpsaOL** General Proof Skeleton ordered list.

**Hol-Z** Hol-Z.

**IEC** International Electrotechnical Commission.

**MathLang** MathLang framework for mathematics.

**PPZed** Proof Power Z.

**SIL** Safety Integrity Levels.

**SMT** Satisfiability Modulo Theories.

**TSa** Text and Symbol aspect.

**UML** Unified Modeling Language.

**UTP** Unifying theories of programming.

**ZCGa** Z Core Grammatical aspect.



**ZDRa** Z Document Rhetorical aspect.

**ZMathLang** MathLang framework for Z specifications.

# Glossary

**computerisation** The process of putting a document in a computer format.

**formal methods** Mathematically rigorous techniques and tools for the specification, design and verification of software and hardware systems.

**formalisation** The process of extracting the essence of the knowledge contained in a document and providing it in a complete, correct and unambiguous format.

**halfbaked proof** The automatically filled in skeleton also known as the Half-Baked Proof.

**partial correctness** A total correctness specification  $[P] \ C \ [Q]$  is true if and only if, whenever  $C$  is executed in a state satisfying  $P$  and if the execution of  $C$  terminates, then the state in which  $C$ 's execution terminates satisfies  $Q$ .

**semi-formal specification** A specification which is partially formal, meaning it has a mix of natural language and formal parts.

**total correctness** A total correctness specification  $[P] \ C \ [Q]$  is true if and only if, whenever  $C$  is executed in a state satisfying  $P$ , then the execution of  $C$  terminates, after  $C$  terminates  $Q$  holds.

# Chapter 1

## Background

Introduction stating formal methods are a type of mathematics. Explanation of where formal languages came from in mathematics etc

### 1.1 History of mathematics

Intro....

#### 1.1.1 Right from the beginning

- logic and aristotle
- Frege Grundgesetze + Cantor + Russel discovered paradoxes
- Russel inventing type theory
- Zermelo added axiomisation
- Fraenkel + skolem extended to ZF set theory (which Z is based on)

#### 1.1.2 Computerisation of Maths and Proof Systems

- typesetting systems like L<sup>A</sup>T<sub>E</sub>X
- proof assistants and automated theorem provers
- Semantical oriented document representations like OpenMath and OMDoc

### 1.1.3 Conclusion

In summary....

## 1.2 MathLang for mathematics

Intro....

### 1.2.1 Overview and Goals

### 1.2.2 Detailed information on CGa

- Reference Zenglers quote
- Weak type theory into CGa

### 1.2.3 Detailed information on DRa

- relations
- instances

### 1.2.4 Detailed information on skeletons

### 1.2.5 information on TSa

### 1.2.6 A full worked examples in mathlang

### 1.2.7 MathLang for Z

- [2] states what to do to make formal methods industrial strength
- [1] stating in future work mathlang should be developed to cope with more mathematics (formal spec is a type of mathematics)
- diagram of math text to theorem prover using mathlang + diagram of specification to theorem prover using mathlang

### 1.2.8 Conclusion

## 1.3 History of formal methods and formal languages

I don't know if history of formal methods should be a separate section or keep it as a subsection under 'history of mathematics'.

- definitions of 'formal language', 'formal method' and 'formal specification'
- the first formal language is thought to be used by Frege in his Begriffsschrift (1879), Begriffsschrift meaning 'concept of writing' described as 'formal language of pure thought'
- broad history of formal methods
  - 1940's, Alan Turing annotated the properties of program states to simplify the logical analysis of sequential programs
  - 1960's Floyd, Hoare and Naur recommended using axiomatic techniques to prove programs meet their specification.
  - 1970's Dijkstra used formal calculus to aid development of non-deterministic programs
- Formal methods today
- why use formal methods in industry (design errors like Therac-25 1985, NASA's Checkout Launch and Control System (CLCS) cancelled 9/2002, added level of rigor)
- types of formal methods (Z, B method, ABS)
- Success of formal methods (B27 Traffic Control System, SHOLIS project, Data Acquisition, Monitoring and Commanding of Space Equipment)
- Weakness of formal methods (Low-level ontologies, Limited Scope, Cost, Poor tool feedback)

- What needs to be done to make formal methods industrial strength?
  - Bridge gap between real world and mathematics
  - Mapping from formal specifications to code (preferably automated)
  - Patterns identified
  - Level of abstraction should be supported
  - Tools needed to hide complexity of formalism
  - Provide visualization of specifications
  - Certain activities not yet formulizable methods
  - No one model has been identified which should be used for software)

ZMathLang covers items 1, 3, 5, 6, 7(semi formal spec)

### **1.3.1 Conclusion**

## **1.4 Z Syntax and semantics**

## **1.5 Proving systems for Z**

### **1.5.1 Levels of Rigor**

- Level 1 represents the use of mathematical logic to specify the system.
- Level 2 uses pencil-and-paper proofs.
- Level 3 is the most rigorous application of formal methods.

## **1.6 Conclusion**

# Bibliography

- [1] R. Lamar. *A Partial Translation Path from MathLang to Isabelle*. PhD thesis, Heriot-Watt University, 2011.
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