$[NAME]$

$[SURNAME]$

$[TELEPHONE]$

$$
\begin{array}{|l}
\_\_\ TelephoneDirectory _____ \\
\ persons : NAME \rightarrow SURNAME \\
\ phoneNumbers : NAME \nrightarrow TELEPHONE \\
\hline
\ \operatorname{dom} phoneNumbers = \operatorname{dom} persons \\
\end{array}
$$

$$
\begin{array}{|l}
\_\_\ InitTelephoneDirectory _____ \\
\ TelephoneDirectory' \\
\hline
\ phoneNumbers' = \{\} \\
\ persons' = \{\} \\
\end{array}
$$

$$
\begin{array}{|l}
\_\_\ AddPerson _____ \\
\ \Delta TelephoneDirectory \\
\ name? : NAME \\
\ surname? : SURNAME \\
\ phone? : TELEPHONE \\
\hline
\ name? \mapsto surname? \notin persons \\
\ persons' = persons \cup \{name? \mapsto surname?\} \\
\end{array}
$$

$$
\begin{array}{|l}
\_\_\ AddNumber _____ \\
\ \Delta TelephoneDirectory \\
\ n? : NAME \\
\ s? : SURNAME \\
\ p? : TELEPHONE \\
\hline
\ n? \mapsto s? \in persons \\
\ p? \notin \operatorname{ran} phoneNumbers \\
\ phoneNumbers' = phoneNumbers \cup \{name? \mapsto phone?\} \\
\end{array}
$$

$$
\begin{array}{|l}
\_\_\ RemovePerson _____ \\
\ \Delta TelephoneDirectory \\
\ n? : NAME \\
\ s? : SURNAME \\
\ p? : TELEPHONE \\
\hline
\ n? \mapsto s? \in persons \\
\ n? \mapsto p? \notin phoneNumbers \\
\ persons' = persons \setminus \{n? \mapsto s?\} \\
\end{array}
$$

```
┌─ RemoveNumber ─────────────────────────────────
│ Δ TelephoneDirectory
│ p? : TELEPHONE
├────────────────────────────────────────────────
│ p? ∈ ran phoneNumbers
│ ∃ m : dom persons •
│ m ↦ p? ∈ phoneNumbers ∧
│ phoneNumbers' = phoneNumbers \ {m ↦ p?}
└────────────────────────────────────────────────
```

$OUTPUT ::= success \mid alreadyInDirectory \mid nameNotInDirectory \mid numberInUse \mid numberDoesntExist$

```
┌─ Success ──────────────────────────────────────
│ message! : OUTPUT
├────────────────────────────────────────────────
│ message! = success
└────────────────────────────────────────────────
```

```
┌─ AlreadyInDirectory ───────────────────────────
│ Ξ TelephoneDirectory
│ message! : OUTPUT
│ n? : NAME
│ s? : SURNAME
├────────────────────────────────────────────────
│ n? ↦ s? ∈ persons
│ message! = alreadyInDirectory
└────────────────────────────────────────────────
```

```
┌─ NameNotInDirectory ───────────────────────────
│ Ξ TelephoneDirectory
│ message! : OUTPUT
│ n? : NAME
│ s? : SURNAME
├────────────────────────────────────────────────
│ n? ↦ s? ∉ persons
│ message! = nameNotInDirectory
└────────────────────────────────────────────────
```

```
┌─ NumberInUse ──────────────────────────────────
│ message! : OUTPUT
│ p? : TELEPHONE
├────────────────────────────────────────────────
│ p? ∈ ran phoneNumbers
│ message! = numberInUse
└────────────────────────────────────────────────
```

```
┌─ NumberDoesntExist ─────────────────────────────────
│ message! : OUTPUT
│ p? : TELEPHONE
├──────────────────
│ p? ∉ ran phoneNumbers
│ message! = numberDoesntExist
└─────────────────────────────────────────────────────
```

$TotalAddPerson \mathrel{\widehat{=}} (AddPerson \wedge Success) \vee AlreadyInDirectory$

$TotalRemovePerson \mathrel{\widehat{=}} (RemovePerson \wedge Success)$
$\vee NameNotInDirectory$

$TotalAddNumber \mathrel{\widehat{=}} (AddNumber \wedge Success)$
$\vee NameNotInDirectory \vee NumberInUse$

$TotalRemoveNumber \mathrel{\widehat{=}} (RemoveNumber \wedge Success)$
$\vee NumberDoesntExist \vee NameNotInDirectory$

$TotalPhone \mathrel{\widehat{=}} TotalAddPerson \vee TotalRemovePerson$
$\vee TotalAddNumber \vee TotalRemoveNumber$

$TotalPhoneAndTotalAddPerson \mathrel{\widehat{=}} TotalPhone \vee TotalAddPerson$