

**FROM FORMAL SPECIFICATION TO FULL PROOF:
A STEPWISE METHOD**

by

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Abstract

Proving formal specifications in order to find logical errors is often a difficult and labour-intensive task. This thesis introduces a new and stepwise toolkit to assist in the translation of formal specifications into theorem provers, using a number of simple steps based on the MathLang Framework. By following these steps, the translation path between a Z specification and a formal proof in Isabelle could be carried out even by one who is not proficient in theorem proving.

Acknowledgements

I dedicate this thesis to my loving and supportive boyfriend, Jeff.

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Acronyms

ASM Abstract state machine.

CGa Core Grammatical aspect.

DRa Document Rhetorical aspect.

GPSa General Proof Skeleton aspect.

Gpsa General Proof Skeleton aspect.

GpsaOL General Proof Skeleton ordered list.

Hol-Z Hol-Z.

IEC International Electrotechnical Commission.

MathLang MathLang framework for mathematics.

PPZed Proof Power Z.

SIL Safety Integrity Levels.

SMT Satisfiability Modulo Theories.

TSa Text and Symbol aspect.

UML Unified Modeling Language.

UTP Unifying theories of programming.

ZCGa Z Core Grammatical aspect.

ZDRa Z Document Rhetorical aspect.

ZMathLang MathLang framework for Z specifications.

Glossary

computerisation The process of putting a document in a computer format.

formal methods Mathematically rigorous techniques and tools for the specification, design and verification of software and hardware systems.

formalisation The process of extracting the essence of the knowledge contained in a document and providing it in a complete, correct and unambiguous format.

halfbaked proof The automatically filled in skeleton also known as the Half-Baked Proof.

partial correctness A total correctness specification $[P] C [Q]$ is true if and only if, whenever C is executed in a state satisfying P and if the execution of C terminates, then the state in which C 's execution terminates satisfies Q .

semi-formal specification A specification which is partially formal, meaning it has a mix of natural language and formal parts.

total correctness A total correctness specification $[P] C [Q]$ is true if and only if, whenever C is executed in a state satisfying P , then the execution of C terminates, after C terminates Q holds.

Chapter 1

Evaluation and Discussion

In this chapter we go through a few case studies and discuss the difference between the specification translations if any. Table 1.1 shows the specifications we have translated into Isabelle using ZMathLang. We have classified these examples to show the different types of specifications which can be translated using the ZMathLang toolkit. In this chapter we take one example from each class and describe in more detail how the translation was done.

Examples using only terms	Examples using sets and terms
Vending Machine	Birthday Book
SteamBoiler	ClubState
Incomplete translations	Clubstate2
Autopilot	GenDB
A specification which fails ZCGa	ModuleReg
A specification which fails ZDRa	ProjectAlloc
	Timetable
	Videoshop
	TelephoneDirectory
	ZCGa

Table 1.1: A table showing the specifications we have translated into Isabelle using ZMathLang

We have categorised the specification into three groups; specifications which

only use terms, specifications which use both terms and set and specifications which the translation is incomplete for a variety of reasons. All the specifications we have translated are ‘state based specifications’, which means they operate within a state and to change the state their may become precondition and postconditions within the state. Some specifications are described differently such as functional specifications, however those type of specifications are out of the scope of this thesis.

1.1 Complexity of specifications

This section we analyse the complexity of the specifications we have translate using ZMathLang. First we check the complexity of the raw \LaTeX specification file, without any annoatation. Then we discuss the complexity of the ZCGa annotated specifications and ZDRa annotated specifications and how this affects the translation into Isabelle.

1.1.1 Raw Latex Count

Table 1 how long each specification is by amount of lines of code and environments uses. We have listed the specifications in decreasing complexity of how many lines of \LaTeX the raw specification has.

Specification	Environment				Lines of \LaTeX
	Zed	Schema	Axdef	Total	
Steamboiler	10	34	3	47	507
ProjectAlloc	4	17	0	21	213
VideoShop	3	15	0	18	166
TelephoneDirectory	6	11	0	17	133
ClubState	4	11	1	16	129
ZCGa	2	9	0	11	128
GenDB	2	7	0	9	114
Timetable	1	6	1	8	92
BirthdayBook	3	7	0	10	83
AutoPilot	2	3	0	5	83
ClubState2	1	6	1	8	80
Vending Machine	4	7	0	11	68
ModuleReg	1	3	0	4	43

Table 1.2: How many zed, schema and axdef environments and lines of \LaTeX code makes up each specification

We list information about how many different environments and lines of \LaTeX make up each specification in table 1.2. The environment numbers count how many different types of environments exist within the specification. That is how many ‘ $\backslash\text{begin}\{\text{schema}\}\dots\backslash\text{end}\{\text{schema}\}$ ’ or ‘ $\backslash\text{begin}\{\text{zed}\}\dots$ ’ etc. We add up the total amount of environments in the specification. From the table we can see that for most of the specifications the more lines of \LaTeX there is then the total amount of environments increase. However, there are three exceptions to this trend. The ‘*BirthdayBook*’ specification, ‘*ClubState2*’ specification and ‘*Vending Machine*’ specification. Specifications for systems can always be written in a variety of ways and still have the same meaning. Even formal specifications can be written different ways. For example one may have the following declarations:

$$t : \mathbb{N}$$

$$l : \mathbb{N}$$

However, this declaration can also be written as the following:

$$t, l :: \mathbb{N}$$

Thus removing a line. Formal specifications can also include comments written in natural language which are not part of the formal script. These extra comments about the specification may have also added to the line count in table 1.2.

1.1.2 ZCGa Count

In this section, we evaluate the ZCGa annotations on the specifications. We describe how many of each ZCGa annotations occurs for each specification we have translated.







Specification	ZCGa WeakTypes					
						
Steamboiler	297	26	282	595	4	0
ProjectAlloc	98	43	113	154	165	0
VideoShop	87	31	75	119	95	0
TelephoneDirectory	78	26	53	72	50	0
ClubState	75	17	51	55	51	0
ZCGa	73	27	67	35	133	0
GenDB	45	24	71	117	121	1
Timetable	35	15	53	48	114	0
BirthdayBook	26	11	24	28	19	0
AutoPilot	16	9	19	31	2	0
ClubState2	34	7	37	22	72	0
Vending Machine	16	7	21	37	0	0
ModuleReg	20	6	18	13	31	0

Table 1.3: How many of each grammatical category exists in each specification.

The amount of times a ZCGa weak type occurs in each specification is shown in table 1.3. We remind the reader the colours corresponding to each grammatical type are: `schematext` , `declaration` , `expression` , `term` , `set` and `definition` . In this instance we don't use `specification` as we assume each document contains a single specification.

In our sample set we only have one specification (GenDB) with a 'definition' annotation. This `definition` is locally defined within the specification. The '*Vending Machine*' specification only uses `terms` and therefore there are no ZCGa `term` annotations. However the '*SteamBoiler*' specification also only uses `term` yet there are 4 `set` ZCGa annotations. This is because some of the `terms` used in the specification have to be introduced by a `set`. For example in the *SteamBoiler* specification we have the following annotation:

```
\begin{zed}
\set{State} ::= \term{init} | \term{norm} |
\term{broken} | \term{stop}
\end{zed}
```

Although the set `State` is annotated as a set, it is not used in any of the schema's in the rest of the specification. It is only defined to present the terms `init`, `norm`, `broken` and `stop` which are used in the specification.

We expect there to be more `schemaText`'s then `declarations` and `expressions` combined as `schemaText` contains all `declarations`, `expressions` and `SchemaNames` however, from the table we can see that this is not always the case. For example in the *ProjectAlloc* example, there are 98 `schemaText`, 43 `declarations` and 113 `expressions`. The reason for this could be because a single `expression` can in itself contain many `expressions`. For example the following `schemaText` has been taken from the *ProjectAlloc* specification:

```
\text{\expression{\forall
\declaration{\term{lec}: \expression{\dom maxPlaces}}\
@ \expression{\term{\# (\set{\set{allocation}
\rres \set{\{\term{lec}\}})}} \leq \term{\set{maxPlaces}~\term{lec}}}}
```

In this example we can see that there contains 1 annotated `schemaText` but 3 `expressions`. Another reason why there may be more `expressions` than `schemaText` is because when annotating a specification with ZCGa, `declarations` also contain `expressions`. If we have the following example, again taken from the ProjectAlloc specification:

```
\text{\declaration{\set{studInterests}, \set{lecInterests}:
\expression{PERSON \pfun\iseq TOPIC}}}
```

The ZCGa text contains 1 annotation of `SchemaText`, 1 annotation of a `declaration`, 2 annotations of `sets` and 1 annotation of an `expression`. Since this is the case we expect to see more expressions than declarations in every specification, which is true according to table 1.3.

1.1.3 ZDRa Count

In this section we analyse the amount of ZDRa instances and relations are labeled for each of the specifications we translated. We give details of the amount of instances in table 1.4 and give details of the amount of relations in each specification in table 1.5.

Specification	ZDRa Instances									
	A	SS	IS	CS	OS	TS	PRE	PO	O	SI
Steamboiler	6	2	2	21	6	6	21	23	12	1
ProjectAlloc	0	1	1	5	11	0	11	6	22	1
VideoShop	0	1	1	3	10	0	13	4	20	1
TelephoneDirectory	0	1	1	4	5	5	8	5	10	1
ClubState	1	1	1	4	6	4	9	6	11	0
ZCGa	0	1	1	6	1	0	6	7	2	1
GenDB	0	1	1	4	2	0	6	5	4	1
Timetable	1	1	1	4	0	0	4	5	0	1
BirthdayBook	0	1	1	1	4	2	4	2	8	1
AutoPilot	0	2	0	1	1	0	1	1	2	0
ClubState2	1	2	1	3	0	0	3	4	0	2
Vending Machine	0	1	0	3	0	3	3	2	0	0
ModuleReg	0	1	0	2	0	0	2	2	0	1

Table 1.4: How many of each ZDRa instances exists in each specification.

From table 1.4 we can see that all specifications have either 1 or 2 statesSchema's. For state base specification it should be the case that then specification has at least 1 state. Most state based specifications have stateInvariants that must be conformed to through all the changes of the specification. However this is not a must and some specification (even from our sample) do not have any stateInvariants.

All precondition must have a corresponding postcondition or output, therefore we can say:

Lemma 1.1.1. $precondition \longrightarrow postcondition \vee output$

The table supports this informatio as there are more combined postconditions and outputs then there are precondition. However not all postconditions and outputs need to have a precondition, they can be executed without one. Therefore the number of preconditions does not need to equal the total number of postcondition and outputs.

Specification	ZDRa Relations				
	initiaOf	requires	allows	totalises	uses
Steamboiler	2	28	21	24	92
ProjectAlloc	1	16	11	0	16
VideoShop	0	15	13	0	142
TelephoneDirectory	1	11	8	14	8
ClubState	1	12	9	14	12
ZCGa	1	9	6	0	7
GenDB	1	8	6	0	6
Timetable	1	6	4	0	6
BirthdayBook	1	7	4	6	5
AutoPilot	0	2	1	0	2
ClubState2	1	6	3	0	6
Vending Machine	0	2	0	2	8
ModuleReg	0	3	2	0	2

Table 1.5: How many of each ZDRa relations exists in each specification.

We can cross reference the table showing the amount of instances (table 1.4) with the table showing the relations (table 1.5). For example, the relation *initialOf* can only occur if the specification has an *initialSchema*. Not all specifications have an *initialSchema* and therefore do not have an *initialOf* relation.

There is also an equal amount of *allows* relations as there is *preconditions*. As was written previously, all preconditions must have a corresponding output or postcondition, therefore the relation ‘*allows*’ links each precondition to its corresponding postcondition or output. However, the vendingMachine specification is an exception to this as the preconditions are written as entire schema’s. For example we have the following instance in the vending machine specification:

```
\draschema{PRE3}{
\begin{schema}{some\_stock}
stock: \nat
```

```
\where
stock > 0
\end{schema}}
```

This chunk of specification describes an entire schema as a precondition. The totalising schema then joints the precondition to their corresponding output or postcondition. The specification is written in this way as it is a personal choice of writing the specification formally. All other specifications in our sample set are written in the style where the precondition and corresponding output or postcondition are written inside the same schema environment.

Obviously, the relation ‘*totalises*’ only occurs in specifications where `totaliseSchema`’s are present. Therefore the ‘*totalise*’ relation is not necessary in all specifications.

VideoShop specification is one of the largest specifications (in terms of lines of `LATEX`) in our sample set however it has quite a small amount of relations

Complete Evaluation and discussion chapter

1.2 Case Studies

This section describes a few specification case studies in which we have used the ZMathLang tool kit to translate and prove formal specifications into the Isabelle automated theorem prover. The first case study present a formal specification only using terms, the second is a formal specification where both sets and terms are used and therefore the syntax used in Isabelle is more complex. The final case study we present is a partial translation of a specification which is not fully formalised but on it’s way to becoming fully formal.

1.2.1 Case Study 1: A specification using only terms.

The following case study is based on the *Steamboiler* [8] specification which has been translated and proved in Isabelle using the ZMathLang framework. This case study only uses variables which are terms. The steamboiler specification is the

larges from our examples. It is made up of 507 lines of \LaTeX code, 10 zed environments, 34 schemes and 3 axiom definitions. When annotating with ZCGa there were 297 schematext, 26 declarations, 282 expressions, 595 terms and 4 sets. When annotated with ZDRa there were 6 axioms, 2 stateSchema's, 2 initialSchema's, 21 changeSchema's, 6 outputSchema's, 6 totaliseSchema's, 21 preconditions, 23 post-conditions, 12 outputs and 1 set of stateInvariants.

1.2.1.1 Natural Language Specification of the Steamboiler

The steam boiler itself is a water level and steam quantity measuring device, with four pumps and four pump controllers. There is a valve for emptying the boiler.

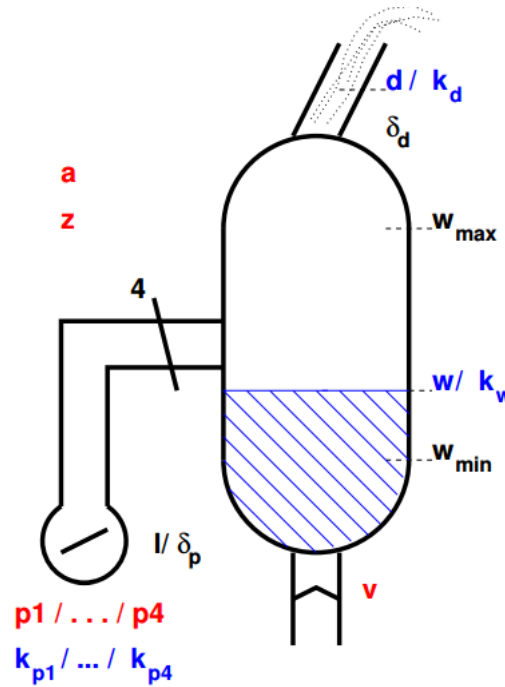


Figure 1.1: A diagram showing a theoretical Steamboiler.

An example of how the steamboiler could look is shown in figure 1.1. The variables of the steamboiler are shown in table 1.6.

find out what l does

variables	description
w_{min}	minimal water level
w_{max}	maximal water level
l	
d_{max}	maximal quantity of steam exiting the boiler
δ_p	error in the value of the pumps
δ_d	error in steam
w	water level
d	amount of steam exiting the boiler
$k_{p,i}$	pump i works/broken
k_w	water level measuring device works/broken
k_d	steam amount measuring device works/broken
p_i	pump i on/off
v	valve open/closed
a	boiler on/off
z	state init/norm/broken/stop

Table 1.6: The variables of the steamboiler and their descriptions.

The full formal specification for the steamboiler is 10 pages long which can be found in [15]. Therefore we have given small examples taken from the full specification.

1.2.1.2 ZMathLang steps for the steamboiler case study.

<pre> \documentclass{article} \usepackage{zmathlang} \begin{document} \begin{zed} State ::= init norm broken stop \end{zed} \begin{zed} OnOff ::= on off \end{zed} \begin{zed} OpenClosed ::= open closed \end{zed} Physical Constants \begin{axdef} w_{min}: \nat \\\ w_{max}: \nat \\\ w_{opt}: \nat \\\ l: \nat \\\ d_{max}: \nat \\\ \delta_p: \nat \\\ \delta_d: \nat \\\ \where w_{min} < w_{max} \end{axdef} Measured values \begin{schema}{Input} w?: \nat \\\ </pre>	<pre> State ::= init norm broken stop OnOff ::= on off OpenClosed ::= open closed hysical Constants w_{min} : \N w_{max} : \N l : \N d_{max} : \N \delta_p : \N \delta_d : \N w_{min} < w_{max} Measured values Input w? : \N d? : \N ontrol values Pumps p_1, p_2, p_3, p_4 : OnOff SteamBoiler0 Pumps v : OpenClosed a : OnOff z : State uxiliary Schemata PumpsOff Pumps' p'_1 = off \wedge p'_2 = off \wedge p'_3 = off \wedge p'_4 = off PumpsOn Pumps' p'_1 = on \wedge p'_2 = on \wedge p'_3 = on \wedge p'_4 = on </pre>
--	---

Figure 1.2: The formal specification
L^AT_EX code for the steamboiler sys-
tem.

We show the L^AT_EX code for part of the raw steamboiler specification in figure 1.2 and it's pdflatex counterpart in figure 1.3.

We then annotate the specification using ZCGa and ZDRa labels.

Figure 1.3: The formal specification
for the steamboiler system.

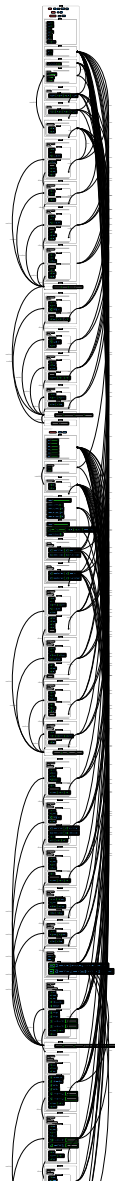


Figure 1.4: An example of the original steamboiler specification annotated in ZCGa and ZDRa.

Since we only have a warning and no errors when checking the steamboiler specification we can now generate a goto graph and dependency graphs for it.

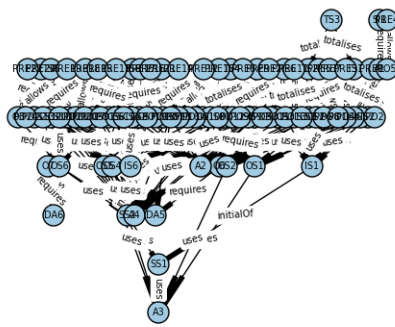
```

Messages
Spec Grammatically Correct
Messages
Warning! Specification not correctly
totalised
Specification is Rhetorically Correct

```

Figure 1.5: The outputting result when checking the steamboiler specification with the ZCGa and ZDRa checkers.

Dependency Graph of T1



GoTo graph of T1

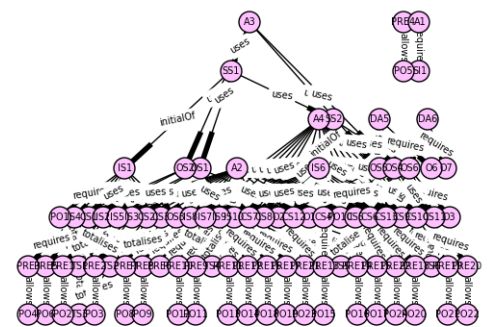


Figure 1.6: The dependency graph produced for the steamboiler specification.

Figure 1.7: The goto graph produced for the steamboiler specification.

The dependency and goto graphs are shown in figures ?? and 1.7 respectively. Since there are a lot of ZDRa instances and therefore a lot of nodes, both the dependency graph and goto graph are cluttered. We will discuss this as a limitation in the next section.

From the goto graph the ZMathLang tool kit automatically generates a general proof skeleton, which uses the order from the goto graph to order the instances in how they should appear in any theorem prover. Part of the skeleton for the steamboiler specification is shown in figure 1.8.


```
axiom A1
stateInvariants SI1
axiom A2
axiom A3
stateSchema SS1
initialSchema IS1
postcondition P01
changeSchema CS7
precondition PRE8
postcondition P09
changeSchema CS2
precondition PRE2
```

Figure 1.8: Gpsa for the steamboiler specification.

We can now translate the Gpsa into Isabelle syntax using the ZMathLang toolkit.

```

theory steamboilerSkelton
imports
Main

begin
  (*DATATYPES*)

  record SS1 =
    (*DECLARATIONS*)

  locale ln2 =
    fixes (*GLOBAL DECLARATIONS*)
    assumes SI1
    begin

    definition IS1 ::
      "(*IS1_TYPES*) => bool"
    where
      "IS1 (*IS1_VARIABLES*) == (P01)"

    definition CS7 ::
      "(*CS7_TYPES*) => bool"
    where

    definition TS3 ::
      "(*TS3_TYPES*) => bool"
    where
      "TS3 (*TS3_VARIABLES*) == (*TS3_EXPRESSION*)"

    end

    record SS2 = SS1 +

    definition IS2 ::
      "(*IS2_TYPES*) => bool"
    where
      "IS2 (*IS2_VARIABLES*) == (P010)"

    definition OS5 ::
      "(*OS5_TYPES*) => bool"
    where
      "OS5 (*OS5_VARIABLES*) == (O4)"

    definition OS4 ::
      "(*OS4_TYPES*) => bool"
    where
      "OS4 (*OS4_VARIABLES*) == (O3)"

    lemma CS7_L1:
      "( $\exists$  (*CS7_VARIABLESANDTYPES*) .
      (PRE8)
       $\wedge$  (P09)
       $\longrightarrow$  ((SI1)
       $\wedge$  (SI1')))"
    sorry

    lemma CS2_L2:
      "( $\exists$  (*CS2_VARIABLESANDTYPES*) .
      (PRE8)
       $\wedge$  (P09)
       $\longrightarrow$  ((SI1)
       $\wedge$  (SI1')))"
    sorry

    lemma CS5_L3:
      "( $\exists$  (*CS5_VARIABLESANDTYPES*) .
      (PRE5)
       $\wedge$  (P06)
       $\longrightarrow$  ((SI1)
       $\wedge$  (SI1')))"

```

Figure 1.9: Part of the isabelle skeleton for the steamboiler specification.

Part of the isabelle skeleton for the steamboiler specification is shown in figure 1.9. Since the steamboiler example has 2 stateSchema's the ZMathLang toolset creates 2 isabelle **records** in the theory file. The top left image shows the beginning part of the isabelle skeleton, where the first stateSchema (or record) sets the state of the theory. Midway down the theory file the first record **ends** and a new one is

added with the line `record SS2 = SS1 +.` Towards the end the Isabelle skeleton there are lemma's to check the consistency for all state changing schema's (CS) in the format described in chapter ?? section ??.

1.2.2 Case Study 2: A specification using both terms and sets.

This case study based is on the *ModuleReg* specification which uses both terms and sets. The specification has been translated into Isabelle using the ZMathLang framework. The entire ZMathLang works for the ModuleReg example is shown in chapter ??.

1.2.3 Case Study 3: A semi formal specification.

In this case study we present the *AutoPilot* specification. The specification is a semi formal specification and has been partially translated into Isabelle. The parts which have been translated are written formally and have been annotated accordingly. This gives an example of a specification which is written in natural language and is on it's way to being formalised.

We have taken the natural language specification for an autopilot system from [16] and started to formalise it.

The mode-control panel contains four buttons for selecting modes and three displays for dialing in or displaying values. The system supports the following four modes:

- attitude control wheel steering (att_cws)
- flight path angle selected (fpa_sel)
- altitude engage (alt_eng)
- calibrated air speed (cas_eng)

Only one of the first three modes can be engaged at any time. However, the cas_eng mode can be engaged at the same time as any of the other modes. The pilot engages a mode by pressing the corresponding button on the panel. One of the three modes, att_cws, fpa_sel, or alt_eng, should be engaged at all times. Engaging any of the first three modes will automatically cause the other two to be disengaged since only one of these three modes can be engaged at a time.

There are three displays on the panel: and altitude [ALT], flight path angle [FPA], and calibrated air speed [CAS]. The displays usually show the current values for the altitude, flight path angle, and air speed of the aircraft. However, the pilot can enter a new value into a display by dialing in the value using the knob next to the display. This is the target or "pre-selected" value that the pilot wishes the aircraft to attain. For example, if the pilot wishes to climb to 25,000 feet, he will dial 25,000 into the altitude display window and then press the alt_eng button to engage the altitude mode. Once the target value is achieved or the mode is disengaged, the display reverts to showing the "current" value.

If the pilot dials in an altitude that is more than 1,200 feet above the current altitude and then presses the alt_eng button, the altitude mode

The mode-control panel contains four buttons for selecting modes and three displays for dialing in or displaying values. The system supports the following four modes:

- attitude control wheel steering (att_cws)
- flight path angle selected (fpa_sel)
- altitude engage (alt_eng)
- calibrated air speed (cas_eng)

events ::= press_att_cws | press_cas_eng | press_alt_eng | press_fpa_sel

Only one of the first three modes can be engaged at any time. However, the cas_eng mode can be engaged at the same time as any of the other modes. The pilot engages a mode by pressing the corresponding button on the panel. One of the three modes, att_cws, fpa_sel, or alt_eng, should be engaged at all times. Engaging any of the first three modes will automatically cause the other two to be disengaged since only one of these three modes can be engaged at a time.

mode_status ::= off | engaged

off_eng
mode : mode_status
mode = off ∨ mode = engaged

AutoPilot

Figure 1.10: An example of the original Autopilot specification.

Figure 1.11: An example of the Autopilot specification partially formalised.

1.2.3.1 ZMathLang steps for the autopilot case study.

We give the informal specification in figure 1.10 and one which we are beginning to formalised in figure 1.11. We have highlighted in red the parts which we have formalised in figure 1.11. The formalised parts of the semi formal specification are taken from the text in the informal specification.

We then annotate the partial formal specification in ZCGa annotations and ZDRa annotations taken from chapters ?? and ?? respectively. Once annotated we can check the annotated document for ZCGa and ZDRa errors.

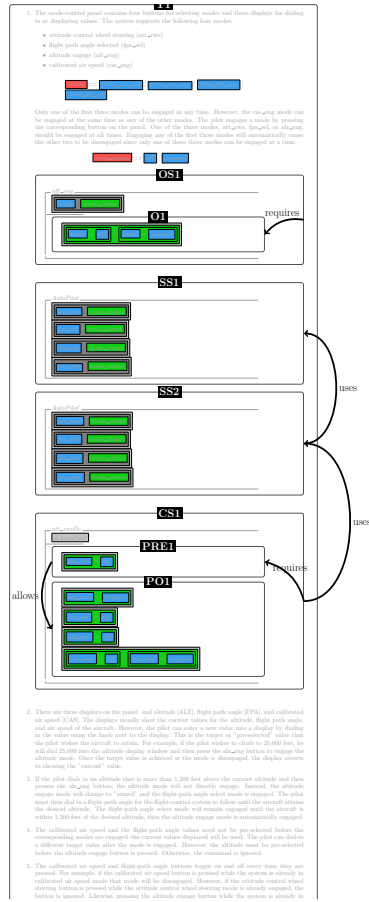


Figure 1.12: An example of the original Autopilot specification annotated in ZCGa and ZDRa.

Even though the specification is not fully formalised we can still annotate it with ZCGa and ZDRa and check for the correctness of the parts which have been annotated (shown in figures 1.12 and 1.13). When checking with ZDRa we have a warning message telling the user that the specification is not correctly totalised. That is there is a precondition outstanding with not postcondition counter part. This does not matter for now as we can still carry on with the translation.

When checking the specification for ZDRa, ZMathLang has also produced a dependency graph and goto graphs (shown in figures 1.14 and 1.15):

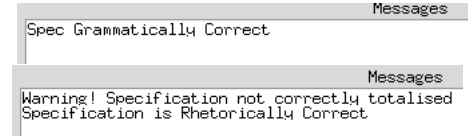


Figure 1.13: The outputting result when checking the autopilot specification with the ZCGa and ZDRa checkers.

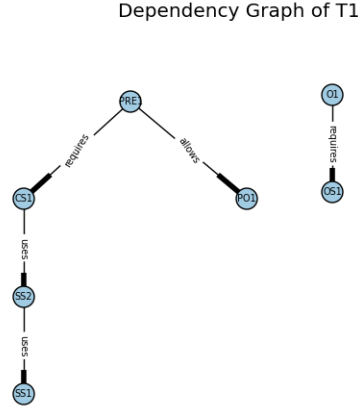


Figure 1.14: The dependency graph produced for the autopilot specification.

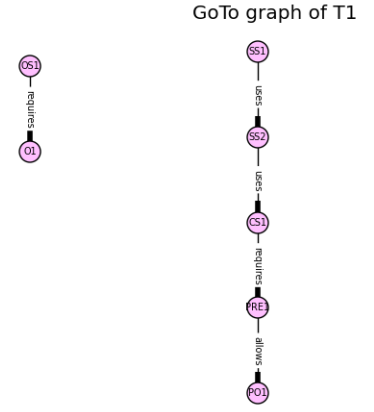


Figure 1.15: The goto graph produced for the autopilot specification.

With the dependency graph (figure 1.14) we can say that *SS2* uses *SS1*, *CS1* uses *SS2*, *PRE1* requires *CS1* and allows *PO1*. Which makes up the main tree dependencies. *OS1* and *O1* are separate as they do not have any relations with any parts of the main tree, the only dependency they have is on each other where *O1* requires *OS1*.

We can say that the dependency graph *describes* the relation between instances and the goto graph (figure 1.15) *orders* the instances in a way as to parse through a theorem prover.

We can now generate a general proof skeleton for the Autopilot specification even though it is not fully formalised (shown in figure 1.16). We can clearly see that the arrow has changed direction for the *OS1* and *O1* relationship from the dependency graph. Again since these two instances are not dependent on any part of the main tree they are separate. However in the dependency graph described the relation that *O1* *requires* *OS1* (*O1* root and *OS1* child) the goto graph flips this relationship as in a theorem prover we would need *OS1* to appear before *O1* since *O1* requires *OS1* to exist. We can also say that *SS2* uses *SS1* therefore *SS2* needs *SS1* to exist for itself to exist. Then *CS1* uses *SS2* therefore *CS1* needs *SS2* to exist for itself to exist.

exit. We can say that *PRE1* requires *CS1* and allows *PO1*. Therefore *PO1* needs *PRE1* to exist before it is allowed to exist itself.

```
stateSchema SS1
outputSchema OS1
output O1
stateSchema SS2
changeSchema CS1
precondition PRE1
postcondition P01
```

Figure 1.16: Gpsa for the Autopilot specification.

Since the Autopilot specification has passed the ZCGa and ZDRa checks we can then generate a Gpsa for the specification using the goto graph produced in the previous stage. The way this is done is described in section ???. Note that even though there is a *changeSchema* instance, there are no *stateInvariants* in the specification (as yet). Therefore ZMathLang does not generate any lemma's to prove in this case since ZMathLang only checks for consistency across the specification and thus need state invariants to be present.

```

theory gpsaln2
imports
Main

begin
(*DATATYPES*)

record SS1 =
(*DECLARATIONS*)

locale ln2 =
fixes (*GLOBAL DECLARATIONS*)
begin

definition OS1 ::
  "(*OS1_TYPES*) => bool"
where
  "OS1 (*OS1_VARIABLES*) == (O1

definition CS1 ::
  "(*CS1_TYPES*) => bool"
where
  "CS1 (*CS1_VARIABLES*) ==
    (PRE1
    ^ (P01)"

end
end

```

Figure 1.17: The Isabelle skeleton produced for the autopilot specification.

```

theory 5
imports
Main

begin
datatype events = press_att_cws
| press_cas_eng | press_alt_eng |
  press_fpa_sel
datatype mode_status = off | engaged

record AutoPilot =
  ALT_ENG :: "mode_status"
  CAS_ENG :: "mode_status"
  ATT_CWS :: "mode_status"
  FPA_SEL :: "mode_status"

locale theautopilot =
  fixes alt_eng :: "mode_status"
  and cas_eng :: "mode_status"
  and att_cws :: "mode_status"
  and fpa_sel :: "mode_status"
begin

definition off_eng ::
  "mode_status => bool"
where
  "off_eng mode == (mode = off ∨ mode =

definition att_cwsDo ::
  "mode_status => mode_status => mode_st
  mode_status => bool"
where
  "att_cwsDo fpa_sel' cas_eng' att_cws'
  alt_eng' ==
    (att_cws = off)
    ∧ (att_cws' = engaged)
    ∧ (fpa_sel' = off)
    ∧ (alt_eng' = off)
    ∧ (cas_eng' = off ∨
      cas_eng' = engaged)"

end
end

```

Figure 1.18: The autopilot specification in Isabelle syntax.

ZMathLang can automatically translate the Gpsa into Isabelle syntax (figure 1.17), this is now an Isabelle skeleton. The Isabelle skeleton has not yet taken the ZCGa information as one can get to this step with just the ZDRa annotated document. Once the Isabelle is filled in (figure 1.18) we have the annotated specification in Isabelle form. This can now give the user an idea of how to input their specification into Isabelle syntax, without them having prior knowledge of Isabelle. It is important to note that this is as far as the ZMathLang translation goes. Since there are no state Invariants with this case study no lemma's to check for consistency have been generated. The user can add the state Invariants in their raw \LaTeX specification, or fully formalise their specification. Another way to fully prove their specification is to add other properties to the Isabelle document.

1.3 Analysing examples

1.4 Reflection and Discussion

Assumptions

-

Limitations

- If we totalise preconditions e.g. `\totalises{TS#}{PRE#}` and all preconditions have been totalised then no warning. If we totalise schemas with preconditions within them e.g. `\totalises{TS#}{CS#}` then the ZDRa checker doesn't pick up on it.
- Specification = 1 theory. Can't do more than 1 specification in 1 document.
- when viewing goto and dep graph if there are a lot of nodes they are all bundled together. Would be better if spaced out more.

1.5 Conclusion

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