Master's degree in Computer Engineering for Robotics and Smart Industry

Robotics, Vision and Control

Report on the assignments given during the 2021/2022 a.y.

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Part I
Robotics

1 Assignment 1

1.1 Implement in MATLAB 3rd-, 5th-, 7th-order polynomials for $q_i > q_f$ and $q_i < q_f$ and for $t \in [t_i, t_f]$ and $t \in [0, \Delta T]$

All polynomial trajectories can be expressed as:

$$
\begin{aligned}
q(t) &= a_7(t-t_i)^7 + a_6(t-t_i)^6 + a_5(t-t_i)^5 + a_4(t-t_i)^4 + a_3(t-t_i)^3 + a_2(t-t_i)^2 + a_1(t-t_i) + a_0 \\
\dot{q}(t) &= 7a_7(t-t_i)^6 + 6a_6(t-t_i)^5 + 5a_5(t-t_i)^4 + 4a_4(t-t_i)^3 + 3a_3(t-t_i)^2 + 2a_2(t-t_i) + a_1 \\
\ddot{q}(t) &= 42a_7(t-t_i)^5 + 30a_6(t-t_i)^4 + 20a_5(t-t_i)^3 + 12a_4(t-t_i)^2 + 6a_3(t-t_i) + 2a_2 \\
\dddot{q}(t) &= 210a_7(t-t_i)^4 + 120a_6(t-t_i)^3 + 60a_5(t-t_i)^2 + 24a_4(t-t_i) + 6a_3 \\
\ddddot{q}(t) &= 840a_7(t-t_i)^3 + 360a_6(t-t_i)^2 + 120a_5(t-t_i) + 24a_4
\end{aligned}
$$

For the 3rd-order polynomial $a_7 = a_6 = a_5 = a_4 = 0$ while for the 5th-order polynomial $a_7 = a_6 = 0$.

The problem of determining the a_i coefficients of the polynomials is solved by setting up a system of equations using initial and final conditions on velocity (3rd-order), velocity and acceleration (5th-order), velocity, acceleration and jerk (7th-order).

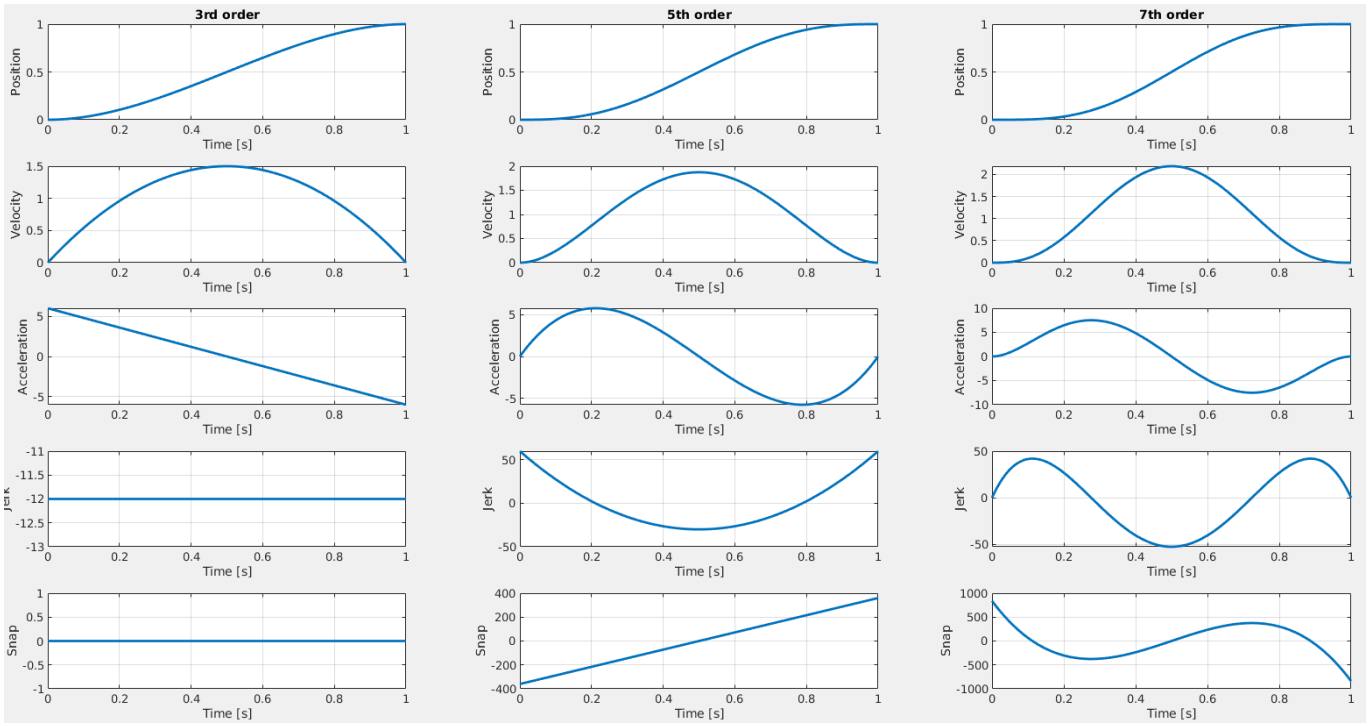

Figure 1: 3rd-, 5th-, 7th-order polynomial trajectories with $q_i < q_f$ and $t \in [0, \Delta T]$, $q_i = 0, q_f = 1, \Delta T = 1, v_i = v_f = a_i = a_f = j_i = j_f = 0$.

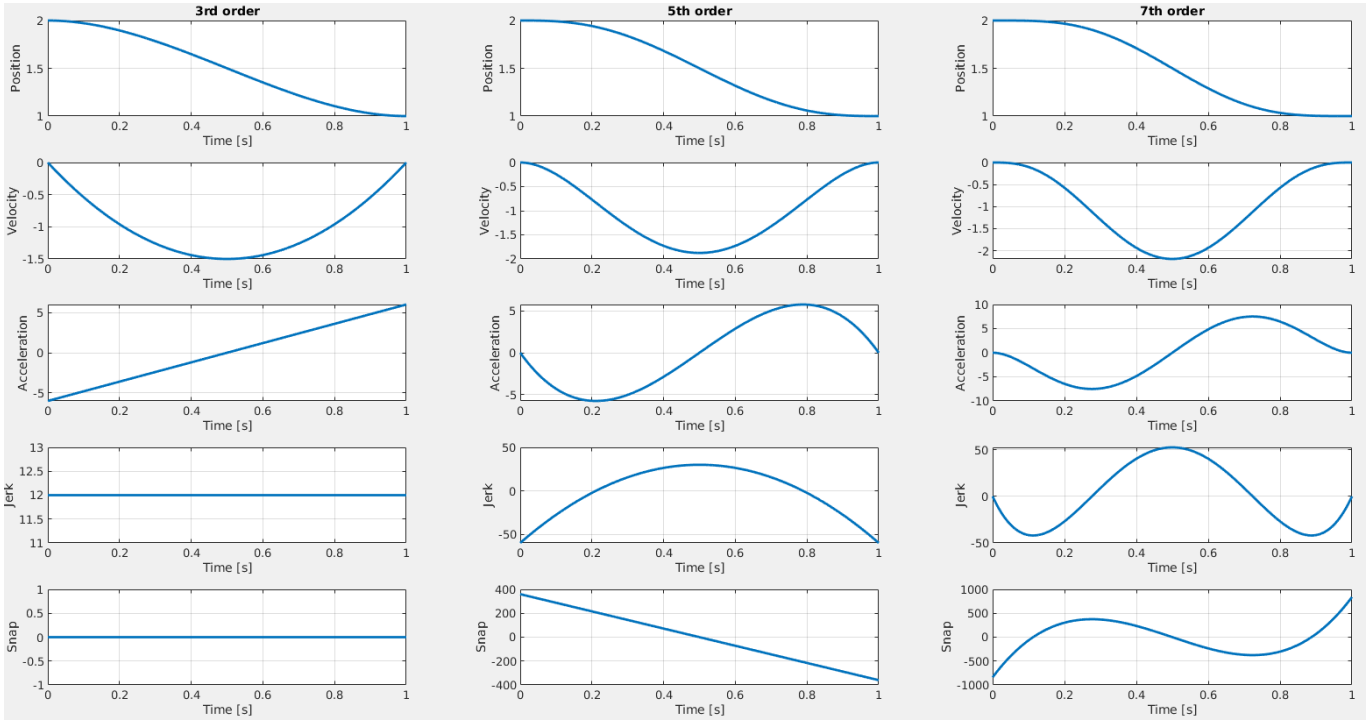

Figure 2: 3rd-, 5th-, 7th-order polynomial trajectories with $q_i > q_f$ and $t \in [0, \Delta T]$, $q_i = 2, q_f = 1, \Delta T = 1, v_i = v_f = a_i = a_f = j_i = j_f = 0$.

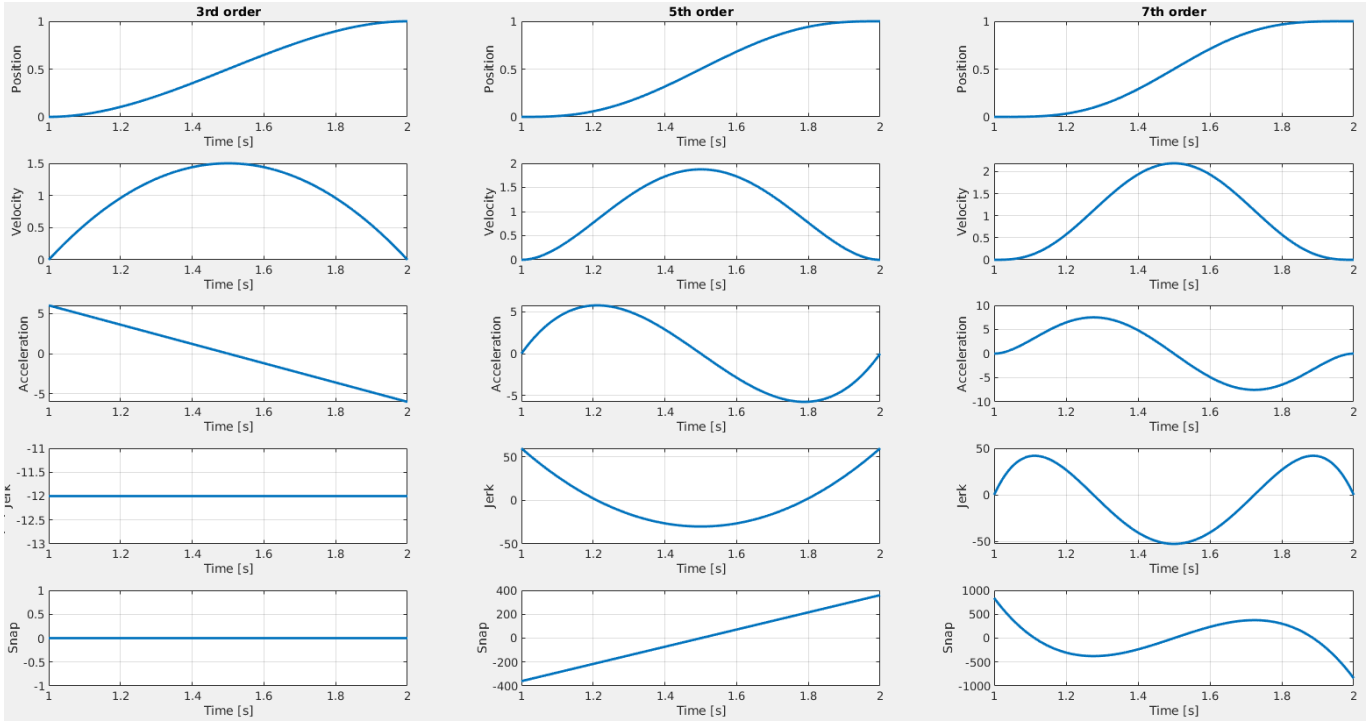

Figure 3: 3rd-, 5th-, 7th-order polynomial trajectories with $q_i < q_f$ and $t \in [t_i, t_f]$, $q_i = 0, q_f = 1, t_i = 1, t_f = 2, v_i = v_f = a_i = a_f = j_i = j_f = 0$.

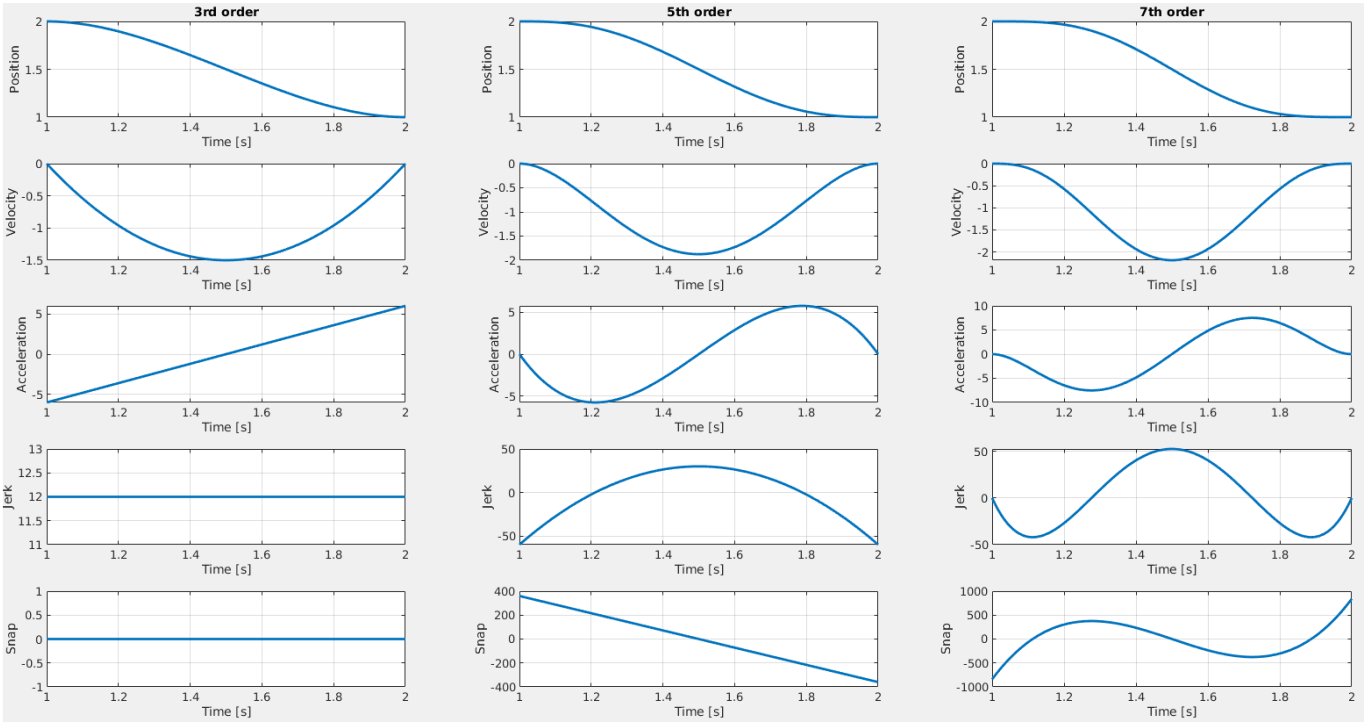

Figure 4: 3rd-, 5th-, 7th-order polynomial trajectories with $q_i > q_f$ and $t \in [t_i, t_f]$, $q_i = 2, q_f = 1, t_i = 1, t_f = 2, v_i = v_f = a_i = a_f = j_i = j_f = 0$.

2 Assignment 2

2.1 Implement in MATLAB the trapezoidal trajectory taking into account the different constraints.

The general expression for a trapezoidal velocity profile is:

$$q(t) = \begin{cases} q_i + \dot{q}_i(t - t_i) + \frac{\dot{q}_c - \dot{q}_i}{2t_a}(t - t_i)^2 & t_i \leq t \leq t_a + t_i \\ q_i + \dot{q}_i\frac{t_a}{2} + \dot{q}_c(t - t_i - \frac{t_a}{2})^2 & t_a + t_i \leq t \leq t_f - t_d \\ q_f - \dot{q}_f(t_f - t) - \frac{\dot{q}_c - \dot{q}_f}{2t_d}(t_f - t)^2 & t_f - t_d \leq t \leq t_f \end{cases}$$

If the initial and final velocities are null, then $t_a = t_d = t_c$.
If $\dot{q}_i = \dot{q}_f = 0$ then the possible constraints are:

- t_c:

$$\ddot{q}_c = \frac{q_f - q_i}{t_c t_f - t_c^2} \quad \dot{q}_c = \ddot{q}_c t_c$$

- $\ddot{q}_c$:

$$t_c = \frac{t_f}{2} - \frac{1}{2}\sqrt{\frac{t_f^2\ddot{q}_c - 4(q_f - q_i)}{\ddot{q}_c}} \quad \dot{q}_c = \ddot{q}_c t_c$$

- $\dot{q}_c$:

$$t_c = \frac{q_i - q_f + \dot{q}_c t_f}{\dot{q}_c} \quad \ddot{q}_c = \frac{\dot{q}_c^2}{q_i - q_f + \dot{q}_c t_f}$$

- $\ddot{q}_c, \dot{q}_c$:

$$t_c = \frac{\dot{q}_c}{\ddot{q}_c} \quad t_f = \frac{\dot{q}_c^2 + \ddot{q}_c(q_f - q_i)}{\dot{q}_c\ddot{q}_c}$$

In all cases the feasibility condition is that $2t_c \leq t_f - t_i$.
If $\dot{q}_i, \dot{q}_f \neq 0$ the possible constraints are:

- $\ddot{q}_{c,max}$:

$$\dot{q}_c = \frac{1}{2}(\dot{q}_i + \dot{q}_f + \ddot{q}_{c,max}\Delta T + \sqrt{\ddot{q}_{c,max}^2\Delta T^2 - 4\ddot{q}_{c,max}\Delta q + 2\ddot{q}_{c,max}(\dot{q}_i + \dot{q}_f)\Delta T - (\dot{q}_i - \dot{q}_f)^2} \quad t_a = \frac{\dot{q}_c - \dot{q}_i}{\ddot{q}_{c,max}}\ t_d = \frac{\dot{q}_c - \dot{q}_f}{\ddot{q}_{c,max}}$$

The trajectory is feasible when the argument of the square root is positive, and when the maximum acceleration satisfies:

$$\ddot{q}_{c,max}\Delta q > \frac{\left|\dot{q}_i^2 - \dot{q}_f\right|^2}{2} \quad \ddot{q}_{c,max} \geq \ddot{q}_{c,lim} = \frac{2\Delta q - (\dot{q}_i - \dot{q}_f)\Delta T + \sqrt{4\Delta q^2 - 4\Delta q(\dot{q}_i + \dot{q}_f)\Delta T + 2(\dot{q}_i^2 + \dot{q}_f^2)\Delta T^2}}{\Delta T^2}$$

when $\ddot{q}_{c,max} = \ddot{q}_{c,lim}$ there is no constant velocity phase.

- $\ddot{q}_{max}, \dot{q}_{max}$: First we compute the condition:

$$\ddot{q}_{c,max}\Delta q \gtreqless \dot{q}_{c,max}^2 - \frac{\dot{q}_i^2 + \dot{q}_f^2}{2}$$

If the above is $>$ then:

$$\dot{q}_c = \dot{q}_{c,max} \quad t_a = \frac{\dot{q}_{c,max} - \dot{q}_i}{\ddot{q}_{c,max}} \quad t_d = \frac{\dot{q}_{c,max} - \dot{q}_f}{\ddot{q}_{c,max}} \quad \Delta T = \frac{\Delta q\ddot{q}_{c,max} + \dot{q}_{c,max}^2}{\ddot{q}_{c,max}\dot{q}_{c,max}}$$

If the above is $\leq$ then:

$$\dot{q}_c = \dot{q}_{c,lim} = \sqrt{\ddot{q}_{c,max}\Delta q + \frac{\dot{q}_i^2 + \dot{q}_f^2}{2}} < \dot{q}_{c,max} \quad t_a = \frac{\dot{q}_{c,lim} - \dot{q}_i}{\ddot{q}_{c,max}} \quad t_d = \frac{\dot{q}_{c,lim} - \dot{q}_f}{\ddot{q}_{c,max}}$$

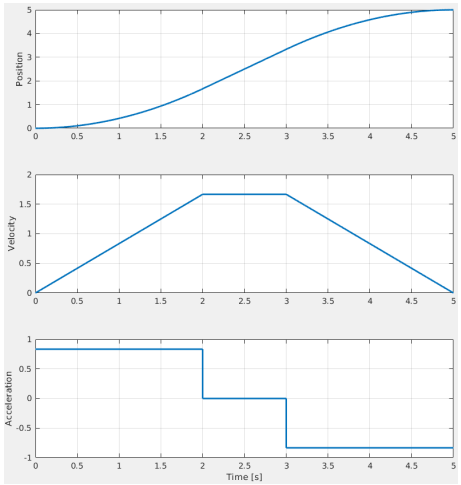

Figure 5: $t_c = 2s$

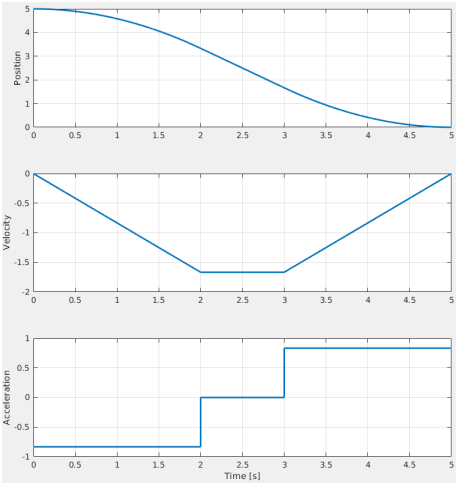

Figure 6: $t_c = 2s, q_f > q_i$

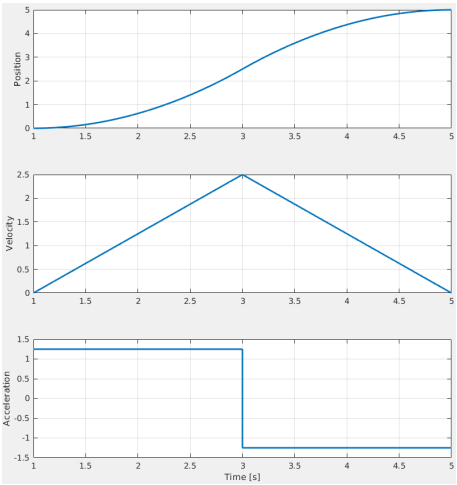

Figure 7: $t_c = 2s, t_i \neq 0$

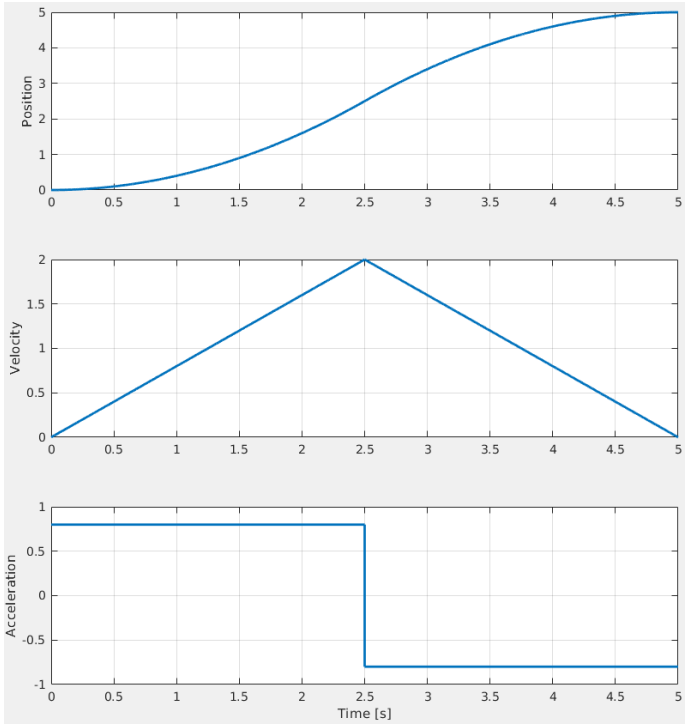

Figure 8: $v_c = v_{c,lim} = 2$

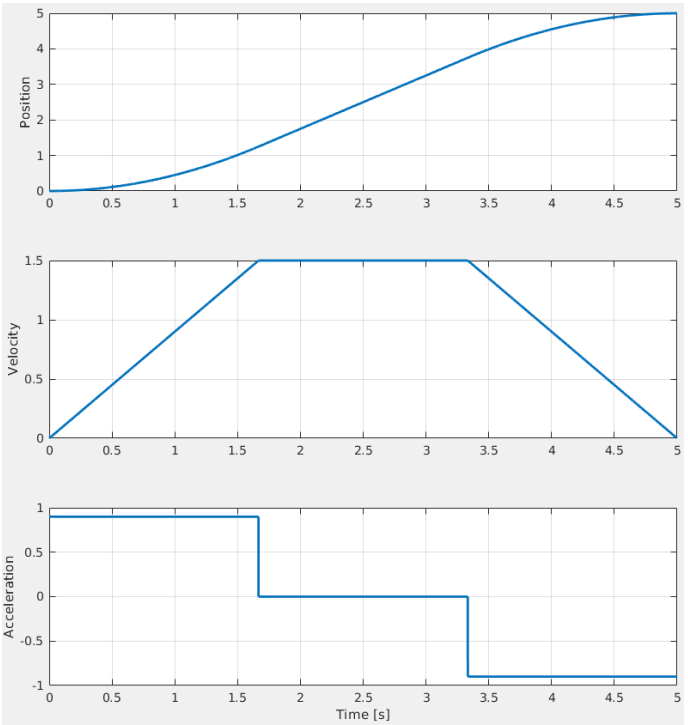

Figure 9: $v_c = 2s$

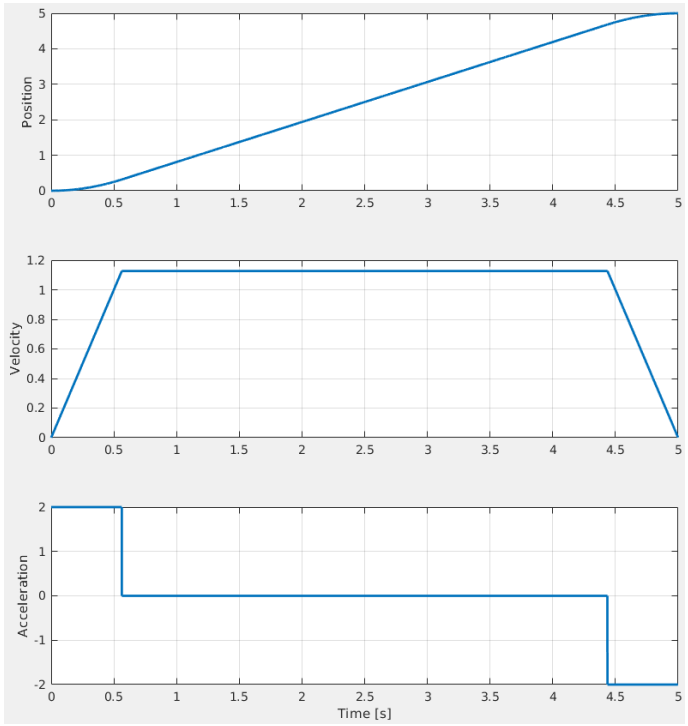

Figure 10: $a_c = 2$

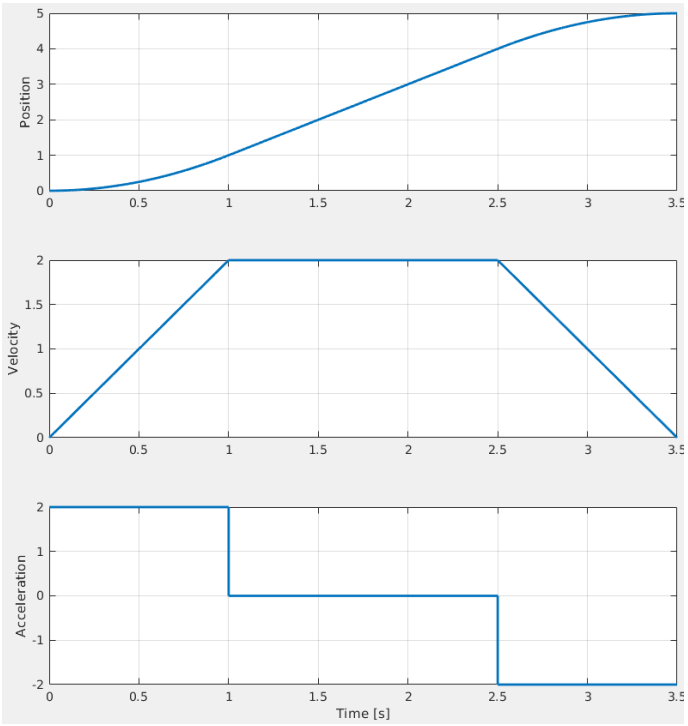

Figure 11: $a_c, v_c = 2s$

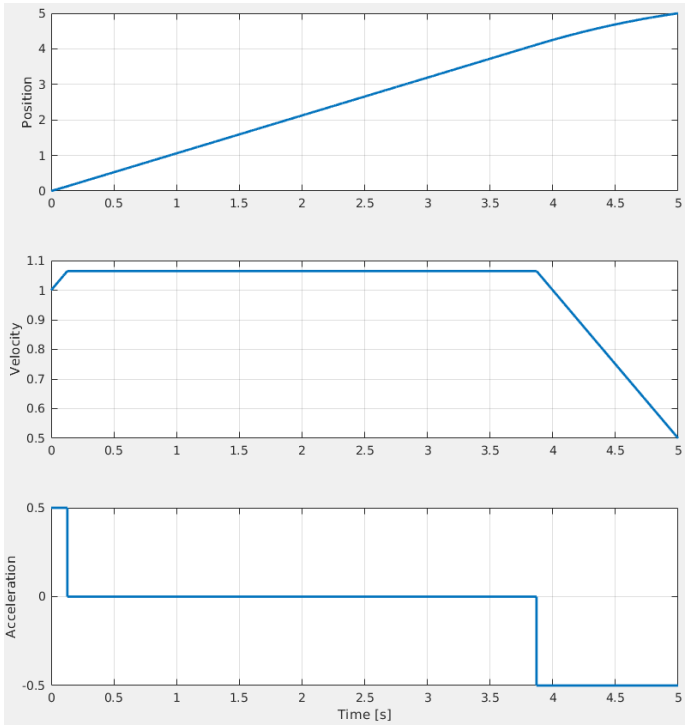

Figure 12: $\dot{q}_i = 1, \dot{q}_f = 0.5, \ddot{q}_{max} = 0.5$

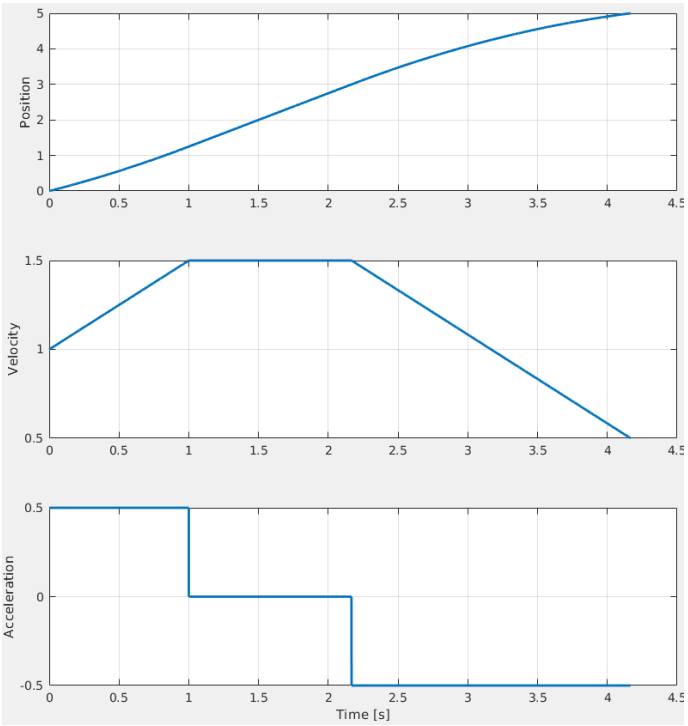

Figure 13: $\dot{q}_i = 1, \dot{q}_f = 0.5, \dot{q}_{max} = 1.5, \ddot{q}_{max} = 0.5$

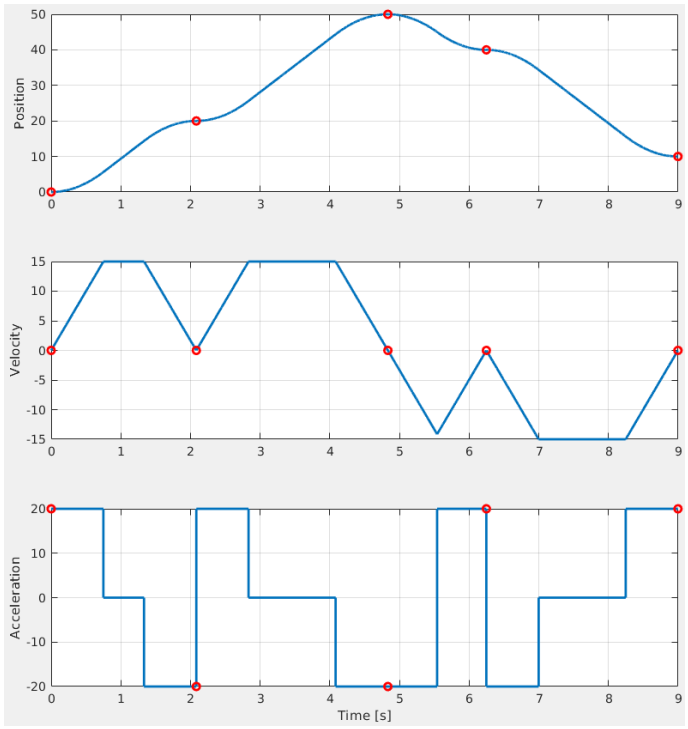

Figure 14: Multipoint trajectory without velocity heuristic

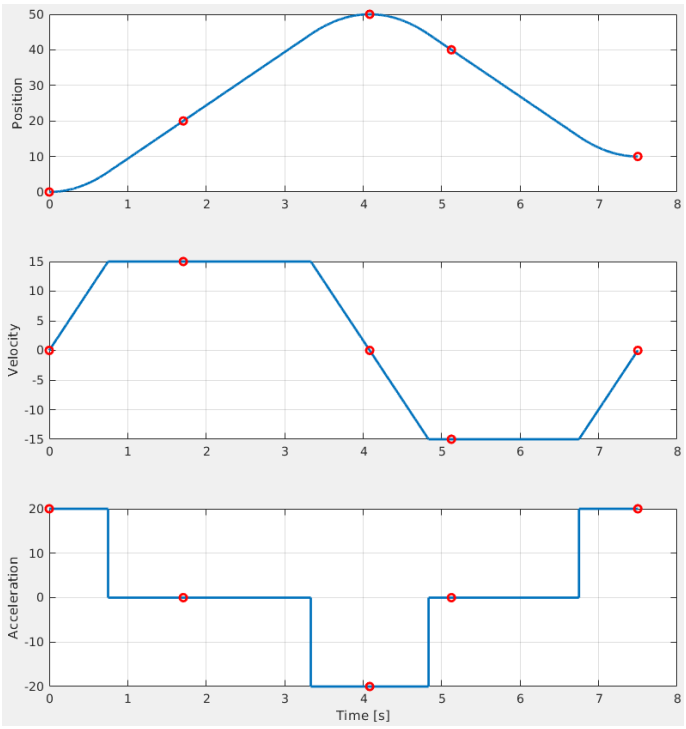

Figure 15: Multipoint trajectory with velocity heuristic

To generate a multipoint trajectory we simply repeat the computation for each consecutive point pair. To improve the resulting velocity and acceleration profiles we can apply an heuristic to assign velocities to each waypoint, so that a less jagged profile is obtained:

$$
\begin{aligned}
&\dot{q}(t_i) = \dot{q}_i \\
&\dot{q}(t_k) = \begin{cases} 0 & \text{if } sign(\Delta Q_k) \neq sign(\Delta Q_{k+1}) \\ sign(\Delta Q_k)\dot{q}_{max} & \text{if } sign(\Delta Q_k) = sign(\Delta Q_{k+1}) \end{cases} \\
&\dot{q}(t_f) = \dot{q}_f
\end{aligned}
$$

3 Assignment 3

3.1 Interpolating polynomials with computed velocities at path points and imposed velocity at initial/final points.

To implement multipoint trajectories we concatenate cubic splines. The interpolating trajectory is then:

$$q(t) \coloneqq \{\Pi_k(t), t \in [t_k, t_{k+1}], k = 0, \ldots, n-1\}$$

where:

$$\Pi(t) = a_3^k(t-t_k)^3 + a_2^k(t-t_k)^2 + a_1^k(t-t_k) + a_0^k$$

Fixing velocities on all path points yields:

$$\begin{cases} a_0^k &= q_k \\ a_1^k &= \dot{q}_k \\ a_2^k &= \frac{1}{T_k}\left(\frac{3(q_{k+1}-q_k)}{T_k} - 2\dot{q}_k - \dot{q}_{k+1}\right) \\ a_3^k &= \frac{1}{T_k^2}\left(\frac{^2(q_k-q_{k+1})}{T_k} + \dot{q}_k + \dot{q}_{k+1}\right) \end{cases}$$

where $T_k = t_{k+1} - t_k$.

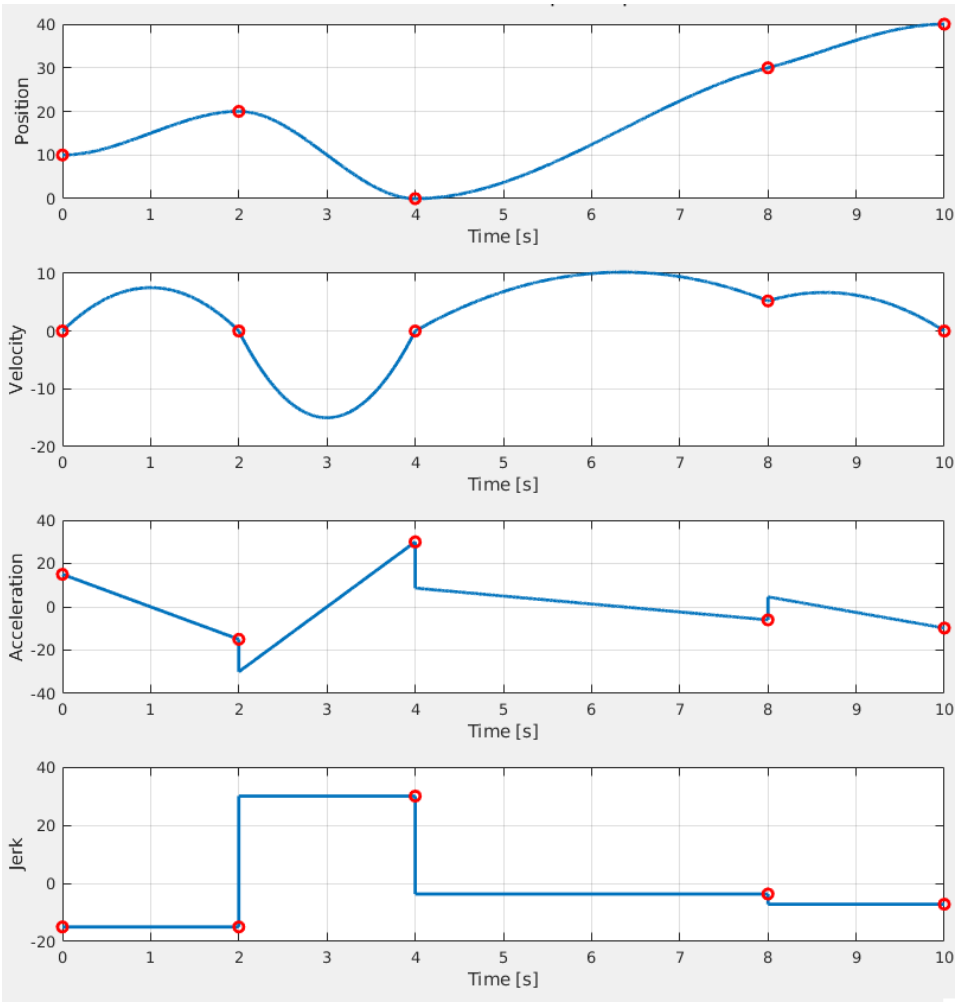

Figure 16: Trajectory through $q_k = \begin{bmatrix}10 & 20 & 0 & 30 & 40\end{bmatrix}$ at times $t_k = \begin{bmatrix}0 & 2 & 4 & 8 & 10\end{bmatrix}$ with velocities $\dot{q}_k = \begin{bmatrix}0 & 0 & 0 & 5.2 & 0\end{bmatrix}$

3.2 Interpolating polynomials with continuous accelerations at path points and imposed velocity at initial/final points (+ Thomas algorithm)

Path point velocities are not generally known. We can estimate them with Euler's approximation:

$$v_k = \frac{q_k - q_{k-1}}{t_k - t_{k-1}} \implies \begin{cases} \dot{q}(t_0) & = \dot{q}_0 \\ \dot{q}(t_k) & = \begin{cases} 0 & \text{if } sign(\Delta Q_k) \neq sign(v_{k+1}) \\ \frac{v_k + v_{k+1}}{2} & \text{if } sign(v_k) = sign(v_{k+1}) \end{cases} \\ \dot{q}(t_n) & = \dot{q}_n \end{cases}$$

Imposing acceleration continuity at path points results in the definition of a linear system $A\dot{q} = c$, where A is a tridiagonal matrix. Thanks to this property the system can be solved for $\dot{q}$ efficiently using Thomas' algorithm. Given:

$$\begin{bmatrix} b_1 & c_1 & 0 & \dots & & & \\ a_2 & b_2 & c_2 & 0 & \dots & & \\ 0 & \ddots & \ddots & \ddots & 0 & \dots & \\ & 0 & a_k & b_k & c_k & 0 & \dots \\ & & 0 & \ddots & \ddots & \ddots & 0 \\ & & & 0 & a_{n-1} & b_{n-1} & c_{n-1} \\ & & & & 0 & a_n & b_n \end{bmatrix} \begin{bmatrix} q_1 \\ \vdots \\ q_k \\ \vdots \\ q_n \end{bmatrix} = \begin{bmatrix} d_1 \\ \vdots \\ d_k \\ \vdots \\ d_n \end{bmatrix}$$

Thomas' algorithm is as follows:

Forward elimination:

for $k = 2 : 1 : n$ **do**
  $m \leftarrow \frac{a_k}{b_{k-1}}$
  $b_k \leftarrow b_k - mc_{k-1}$
  $d_k \leftarrow d_k - md_{k-1}$
end for

Backward substitution:

$x_n \leftarrow \frac{d_n}{b_n}$
for $k = 2 : 1 : n$ **do**
  $x_k \leftarrow \frac{d_k - c_k x_{k+1}}{b_k}$
end for

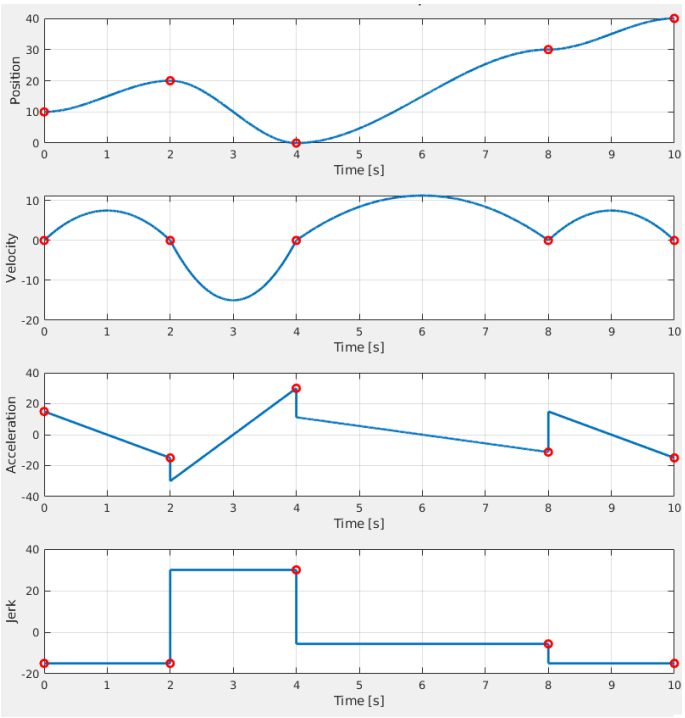

Figure 17: Trajectory interpolation with Euler's approximation.

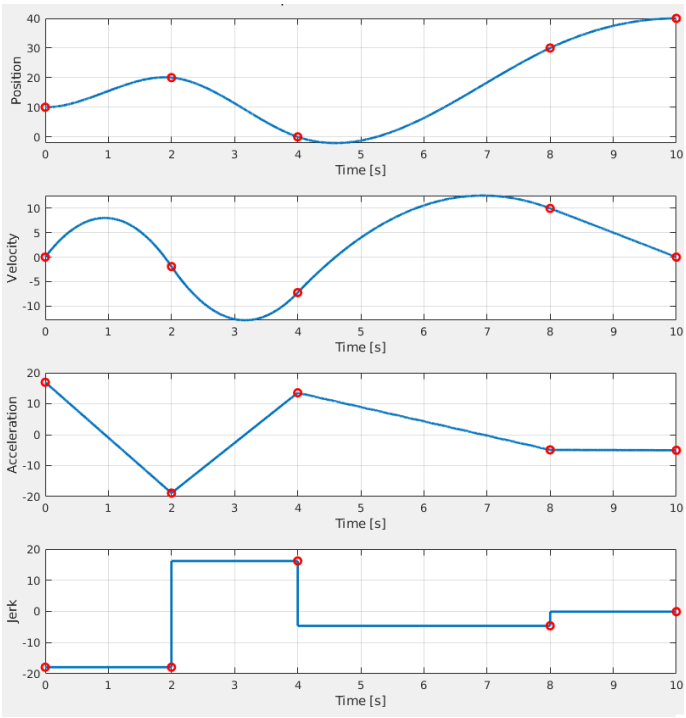

Figure 18: Trajectory interpolation with continuous accelerations.

4 Assignment 4

5 Assignment 5

6 Assignment 6

7 Assignment 7

Part II
Vision