Mobile Robotics, Localization: Observation Models

Material based on the book Probabilistic Robotics (Thrun, Burgard, Fox) [PR]; Chapter 6.3, 6.4

Part of the material is based on lectures from Cyrill Stachniss

Summary

- Introduction to probabilistic observation models
- Beam models for Range Finders [Chapter 6.3]
- Likelihood models for Range Finders [Chapter 6.4]
- Feature-based models [Chapter 6.6]

Introduction to probabilistic observation models

Observation models for state estimation

- Estimate state x of a system given observations z and commands u
- **Goal**

$$P(x_t | z_{1:t}, u_{1:t})$$

- Recursive state estimation:
  - Prediction step:

$$\overline{Bel}(x_t) = \int P(x_t | x_{t-1}, u_t) Bel(x_{t-1}) dx_{t-1}$$

  - Correction step:

$$Bel(x_t) = \eta P(z_t | x_t) \overline{Bel}(x_t)$$

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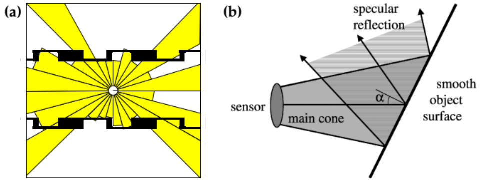
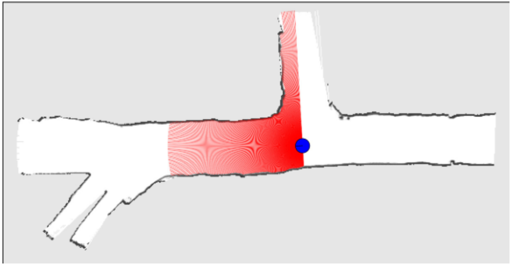
Range Sensors



Typical scan of an ultrasound sensor and possible issues [PR]



Reading for a SICK LMS sensor in a coal mine, source [PR], courtesy Dirk Hähnel

Model for Laser Scanners

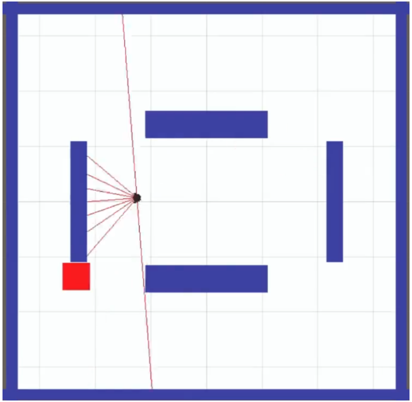
- Scan z consists of k measurements

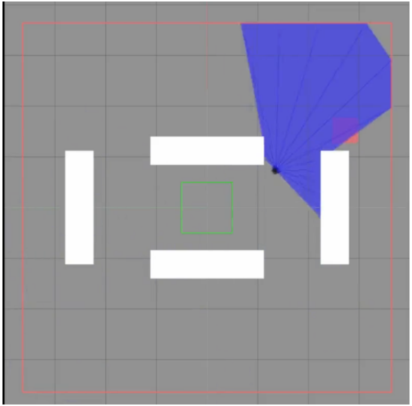
$$z_t = \{z_t^1, \ldots, z_t^k\}$$

- Individual measurements are independent **given** the sensor position

$$P(z_t | x_t, m) = \prod_{i=1}^{k} P(z_t^i | x_t, m)$$

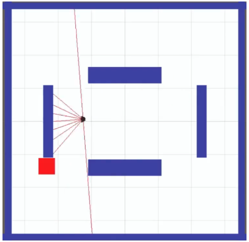
Beam-based sensor model

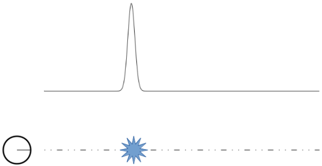




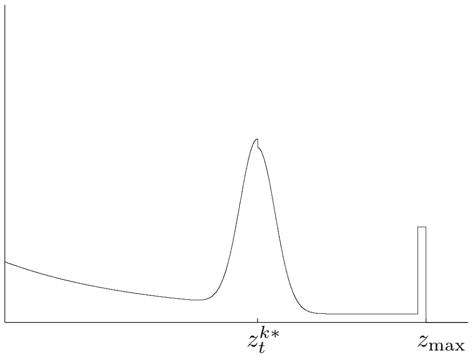
$$P(z_t|x_t, m) = \prod_{i=1}^{k} P(z_t^i|x_t, m)$$

Simple Ray-Cast Model





Advanced Ray-Cast Model

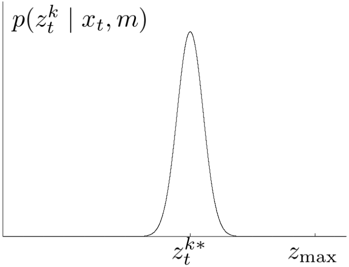


Mixture distribution typically used for $P(z_t|x_t, m)$, source [PR]

Local measurement Noise

$$P_{hit}(z_t^k | x_t, m) = \begin{cases} \eta \mathcal{N}(z_t^{k*}, \sigma_{hit}^2) & \text{if } 0 \leq z_t^k < z_{max} \\ 0 & \text{otherwise} \end{cases}$$

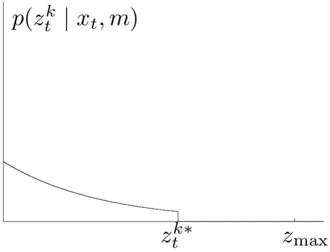
$$\mathcal{N}(z_t^{k*}, \sigma_{hit}^2) = \frac{1}{\sqrt{2\pi\sigma_{hit}^2}} e^{-\frac{1}{2}\frac{(z_t^k - z_t^{k*})^2}{\sigma_{hit}^2}}, \quad \eta = \left(\int_0^{z_{max}} \mathcal{N}(z_t^{k*}, \sigma_{hit}^2) dz_t^k \right)^{-1}$$



Gausian ditribution for local measurement noise, source [PR]

Unexpected obstacle

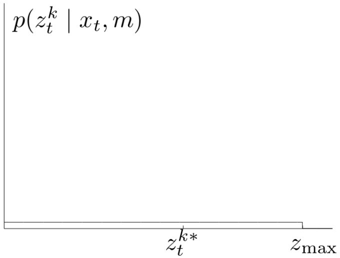
$$P_{short}(z_t^k | x_t, m) = \begin{cases} \eta \lambda_{short} e^{-\lambda_{short}} & \text{if } 0 \leq z_t^k \leq z_t^{k*} \\ 0 & \text{otherwise} \end{cases}, \quad \eta = 1 - e^{-\lambda_{short} z_t^{k*}}$$



Exponential distribution for measurement noise, source [PR]

Random error

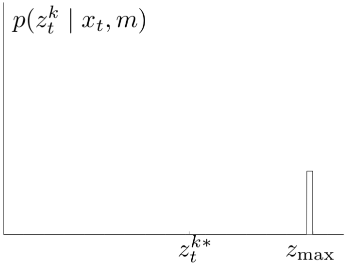
$$P_{rand}(z_t^k|x_t, m) = \begin{cases} \frac{1}{z_{max}} & \text{if } 0 \leq z_t^k < z_{max} \\ 0 & \text{otherwise} \end{cases}$$



Uniform distribution for measurement noise, source [PR]

Failure

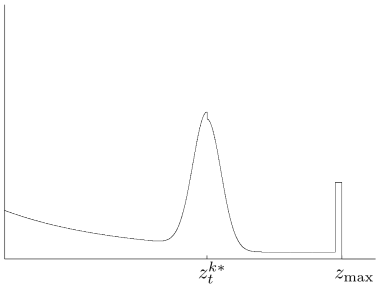
$$P_{max}(z_t^k|x_t,m) = I(z_t^k = z_{max}) \begin{cases} 1 & \text{if } z_t^k = z_{max} \\ 0 & \text{otherwise} \end{cases}$$



Uniform distribution for maximum readings, source [PR]

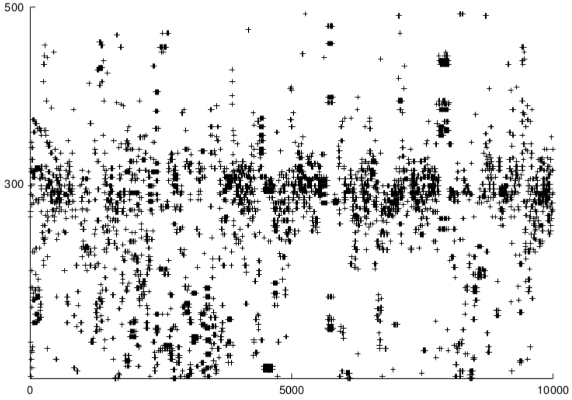
Mixture for advanced Ray-Cast Model

$$P(z_t^k|x_t,m) = \begin{pmatrix} z_{hit} \\ z_{short} \\ z_{rand} \\ z_{max} \end{pmatrix}^T \cdot \begin{pmatrix} P_{hit}(z_t^k|x_t,m) \\ P_{short}(z_t^k|x_t,m) \\ P_{rand}(z_t^k|x_t,m) \\ P_{max}(z_t^k|x_t,m) \end{pmatrix}$$



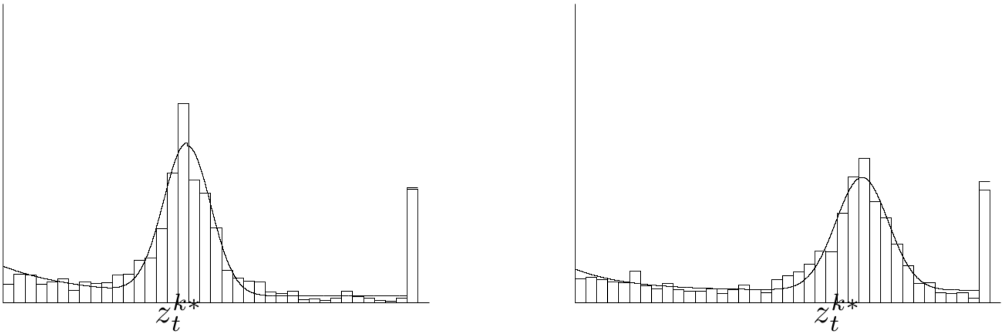
Mixture distribution typically used for $P(z_t|x_t,m)$, source [PR]

Raw data measurement, Laser



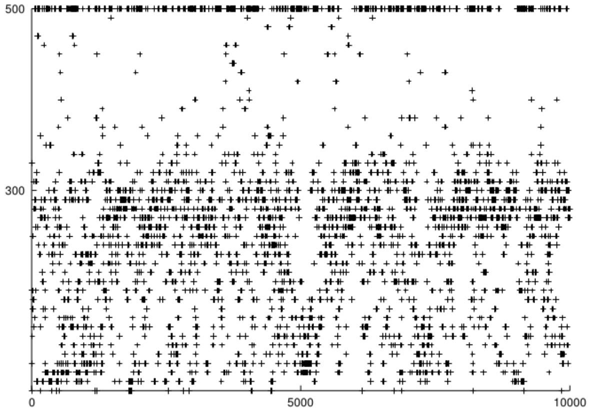
Laser data for office environments with a true range of 300 cm and a max range of 500 cm, source [PR]

Maximum Likelihood Estimation: results



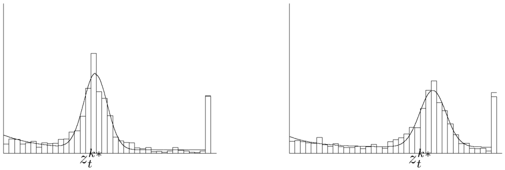
Maximum likelihood estimation of laser data, left range is 300 cm, right rage is 400 cm (different data-set), source [PR]

Raw data measurement, Sonar

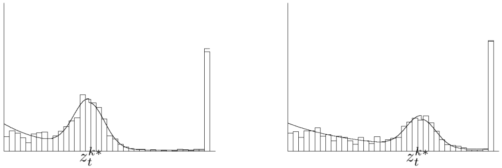


Sonar data for office environments with a **true** range of 300 cm and a max range of 500 cm, source [PR]

Maximum Likelihood Estimation: result comparison

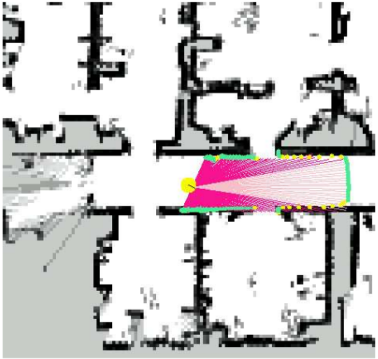


Laser, source [PR]

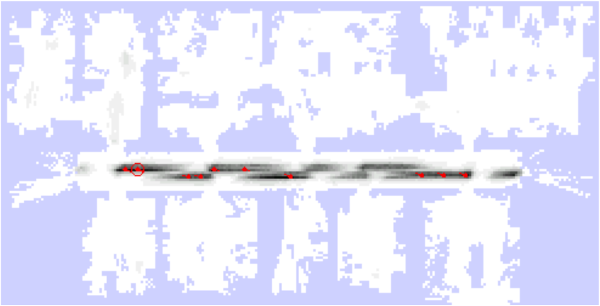


Sonar, source [PR]

Observation model in action

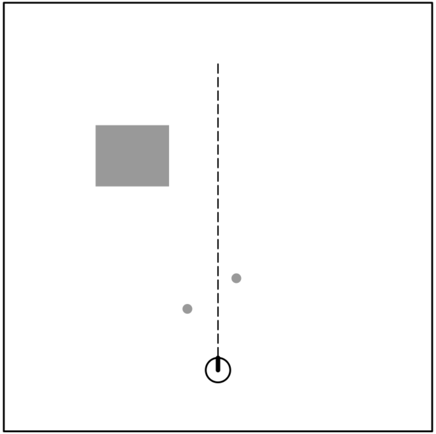


Scan reading acquired in a given position, source [PR]

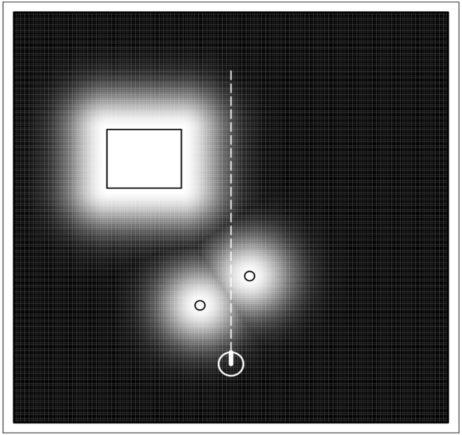


$P(z_t|x_t, m)$ evaluated for every possible x_t, source [PR]

Likelihood field model I

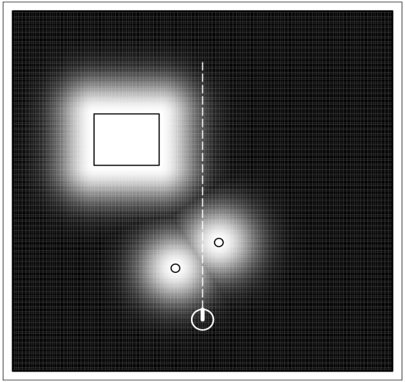


Example environment with three obstacles, source [PR]

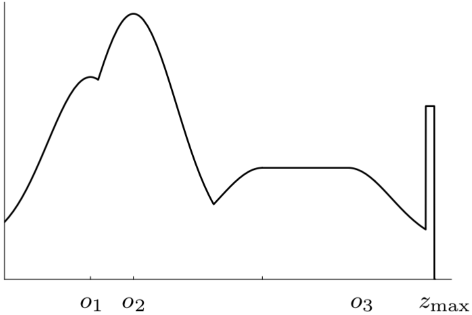


Likelihood field, source [PR]

Likelihood field model II

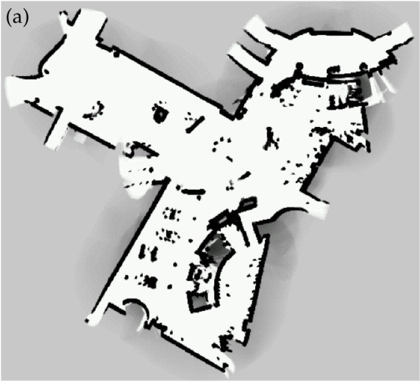


Likelihood field, source [PR]

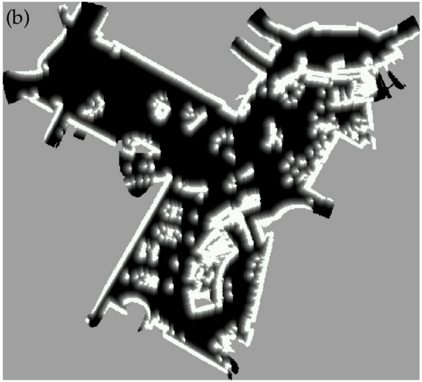


Sensor probability $P(z_t^k | x_t, m)$ for our likelihood field, source [PR]

Likelihood field model: example



Indoor Map (San Jose Tech Museum), source [PR]



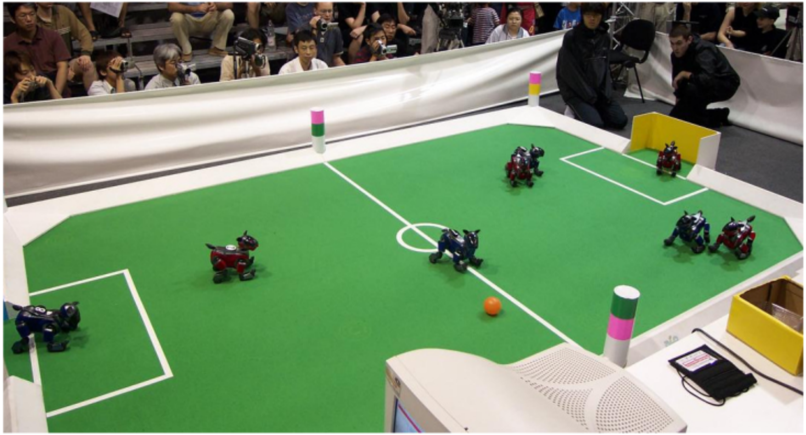
Resulting likelihood field, source [PR]

Model for Landmarks with Range-Bearing Sensors

- Range-Bearing $z_t^i = (r_t^i, \phi_t^i)^T$
- Pose $(x, y, \theta)^T$
- Observation of feature j at location $(m_{j,x}, m_{j,y})^T$

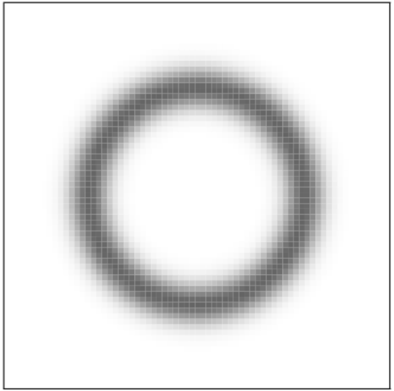
$$\begin{pmatrix} r_t^i \\ \phi_t^t \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{j,x} - x)^2 + (m_{j,y} - y)^2} \\ atan2(m_{j,y} - y, m_{j,x} - x) - \theta \end{pmatrix} + \begin{pmatrix} \epsilon_{\sigma_r^2} \\ \epsilon_{\sigma_\phi^2} \end{pmatrix}$$

Landmark-Based localization: AIBO RoboCup Soccer

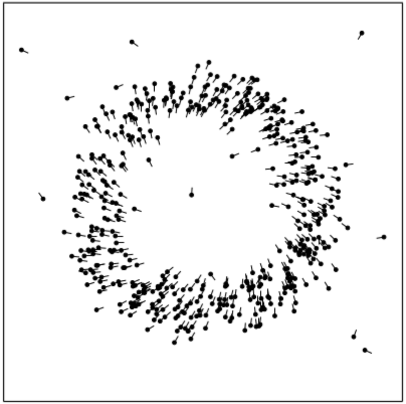


Landmarks for AIBO RoboCup soccer league, source [PR]

Landmark detection model



Posterior of robot position after detecting a landmark at distance of 5 meters and at a relative angle of 30 degrees (projected on 2D), source [PR]



Sample robot pose from the same posterior distribution, lines indicate orientation of the pose, source [PR]

Model for Landmarks with Bearing Sensors

- Bearing $z_t^i = (\phi_t^i)^T$
- Pose $(x, y, \theta)^T$
- Observation of feature j at location $(m_{j,x}, m_{j,y})^T$

$$\phi_t^t = atan2(m_{j,y} - y, m_{j,x} - x) - \theta + \epsilon_{\sigma_\phi^2}$$

Summary

- Observation model is a key component for recursive state estimation
- Focused on range sensors
- Assume measurements are independent given sensor position
- Advanced Ray-Cast model (mixture)
  - accurate, computationally demanding
- Likelihood field
  - less accurate, works well in practice, much faster
- Observation models for Landmark
  - Range-Bearing
  - Bearing only