

# 1.

---

(a)

```
1 // a[0]: three times
2 int a[] = {2, 4, 0, -1, 392, 34, 2, 3, 4, 6, 10, 3, 2, 999};
3 cout << count(a, a + sizeof(a)/sizeof(int), a[0]) << '\n'; // 3
```

(b)

```
1 using Mat = vector<vector<int> >;
2 Mat transpose(Mat &m, int Row, int Col) {
3     Mat trans(Col, vector<int>(Row));
4     for (int i=0; i<Row; ++i)
5         for (int j=0; j<Col; ++j)
6             trans[j][i] = m[i][j];
7     return trans;
8 }
```

## 2. && 3.

---

(2.)

(a)  $\sum_{i=0}^n i^2 = \frac{n(n+1)(2n+1)}{6} = f(n)$ , let  $g(n) = n^3$

$\exists C_0 = \frac{1}{6} \forall n \geq 1$  s.t.  $C_0 g(n) \leq f(n) \quad \text{--- (1)}$

$\exists C_1 = 6 \forall n \geq 1$  s.t.  $f(n) \leq C_1 g(n) \quad \text{--- (2)}$

By (1), (2)  $\frac{1}{6} n^3 \leq f(n) \leq 6n^3 \quad \forall n \geq 1$

$\therefore f(n) = \sum_{i=0}^n i^2 = \Theta(n^3)$

(b)

$f(n) = n!$ ,  $g(n) = n^n$

(1)  $0 \leq f(n) \quad \forall n \geq 1$

(2)  $\exists C = 1, \forall n \geq 1$  s.t.  $f(n) \leq C \cdot g(n)$

By (1), (2)  $0 \leq f(n) \leq n^n \quad \forall n \geq 1$

$\therefore f(n) = n! = O(n^n)$

(3.)

(a)  $\lim_{n \rightarrow \infty} \frac{n^2}{n} = \infty \Rightarrow$  there doesn't exist  $n_0 \in \mathbb{R}^+$  and  $c \in \mathbb{R}^+$  s.t.  $\forall n \geq n_0$ ,  $n^2 \leq c \cdot n$

$\therefore 10n^2 + 9 \neq O(n)$

(b)  $\lim_{n \rightarrow \infty} \frac{n^2 / \log n}{n^2} = 0 \Rightarrow$  there doesn't exist  $n_0 \in \mathbb{R}^+$  and  $c \in \mathbb{R}^+$  s.t.  $\forall n \geq n_0$ ,

$c \cdot n^2 \leq n^2 / \log n$

$\therefore \frac{n^2}{\log n} \neq \Theta(n^2)$

4.

```
1 class Complex {
2     public:
3         int real, img;
4         Complex(): real(0), img(0) {};
5         Complex(const int &r, const int &i): real(r), img(i) {};
6     };
```

5.

```

1  class Quadratic {
2      public:
3          int a, b, c;
4          Quadratic(const int &a, const int &b, const int &c) {
5              a = _a;
6              b = _b;
7              c = _c;
8          }
9          Quadratic operator+(const Quadratic &q) {
10             return Quadratic(a+q.a, b+q.b, c+q.c);
11         }
12     };

```

## 6.

```

1  #ifndef _CSTRING_
2  #define _CSTRING_
3  #include <cstring>
4  #endif
5
6  // "Bag" is a "class template"; "Bag<int>" is a "class"
7  // abstract class
8  template<typename T>
9  class Bag {
10     public:
11         Bag() {}
12         virtual ~Bag() = default;
13
14         virtual int Size() { return tail - top; }
15         virtual bool IsEmpty() { return tail == top; }
16         virtual T Element() const = 0;
17
18         virtual void Push(const T) = 0;
19         virtual void Pop() = 0;
20
21     protected:
22         T *array; // stores elements
23         int capacity; // the maximum number can be used to store elements,
without reallocation
24         int top; // array position of top element
25         int tail; // array position of tail element
26 };
27
28 template<typename T>
29 class Queue: public Bag<T> {
30     public:
31         Queue(int queueCapacity = 10) {
32             this->capacity = queueCapacity;
33             this->array = new T[queueCapacity];
34             this->top = this->tail = size = 0;
35         }
36         ~Queue() { delete[] this->array; }
37
38         int Size() { return size; }
39         bool IsEmpty() { return size == 0; }
40         int Capacity() { return this->capacity; }

```

```

41
42 // return "front" element in queue
43 T Element() const {
44     return this->array[this->top % this->capacity];
45 }
46
47 // push an element at the end of queue
48 void Push(const T var) {
49     this->top %= this->capacity;
50     this->tail %= this->capacity;
51
52     // cyclic
53     if (this->tail == this->top) {
54         // full
55         if (size == this->capacity) {
56             this->capacity = (this->capacity + 1) * 2;
57             T *new_block = new T[this->capacity];
58             memcpy(new_block, this->array + this->top, sizeof(T)*
(size));
59             delete this->array;
60
61             this->array = new_block;
62             this->tail = size;
63             this->top = 0;
64         }
65     }
66     this->array[(this->tail)++] = var;
67     ++size;
68 }
69 void Pop() {
70     if (IsEmpty()) return;
71     this->top %= this->capacity;
72     ++(this->top);
73     --size;
74 }
75 private:
76     int size; // number of elements
77 };

```

## 7.

*\*\* ABC*

## 8.

```

1 class Entry {
2     public:
3         int row, col, val;
4 };
5 class SparseMatrix {
6     public:
7         int size;
8         Entry *entries;
9         SparseMatrix(SparseMatrix &_M) {
10             entries = new Entry[_M.size];
11             for (int i=0; i<_M.size; ++i) {

```

```

12         entries[i] = _M.entries[i];
13     }
14 }
15 };

```

- Computing time (time complexity):  $O(N)$  ( $N$  is the number of **non-zero** entries.)

## 9.

```

1  #include <bits/stdc++.h>
2  using namespace std;
3
4  void insertion_sort(int arr[], int len) {
5      for (int i=1; i<len; ++i) {
6          int at = arr[i], j;
7          for (j=i-1; j>=0; --j) {
8              if (arr[j] > at)
9                  arr[j+1] = arr[j];
10             else break;
11         }
12         arr[j+1] = at;
13     }
14 }
15
16 int main() {
17     int arr[] = {3, 2, 4, 1, 5};
18     insertion_sort(arr, 5);
19     for (int i=0; i<5; ++i)
20         cout << arr[i] << ' ';
21     return 0;
22 }

```

## 10.

```

1  bool is_palin(string &s) {
2      int len = s.length();
3      for (int i=0; i<len/2; ++i) {
4          if (s[i] != s[len-i-1]) return false;
5      }
6      return true;
7  }

```