

1.

```
1  template<class T>
2  class Node {
3      public:
4          T data;
5          Node *nxt;
6
7          Node (): nxt(nullptr) {}
8          Node (const T _data): data(_data) {}
9  };
10
11  template<class T>
12  class linkedQueue {
13      public:
14          linkedQueue(): _size(0) {
15              head = new Node<T>;
16              tail = head;
17              head->nxt = tail;
18          }
19
20          void push(const T &data) {
21              Node<T> *tmp = new Node<T>(data);
22              if (tail != nullptr)
23                  tail->nxt = tmp;
24              tail = tmp;
25              ++_size;
26          }
27          void pop() {
28              if (_size == 0) return;
29              Node<T> *tmp = head->nxt;
30              delete head;
31              head = tmp;
32              --_size;
33          }
34          T front() const {
35              return head->nxt->data;
36          }
37          int size() const {
38              return this->_size;
39          }
40      private:
41          int _size;
42          Node<T> *head, *tail;
43  };
44
45  /* test
46  #include <iostream>
47  #include "linked_queue.h"
48  using namespace std;
49
50  int main() {
51      linkedQueue<int> que;
52      for (int i=0; i<5; ++i) {
53          que.push(i);
54      }
55      cout << "size: " << que.size() << '\n';
```

```

56     for (int i=0; i<5; ++i) {
57         cout << que.front() << ' ';
58         que.pop();
59     }
60     return 0;
61 }
62 */

```

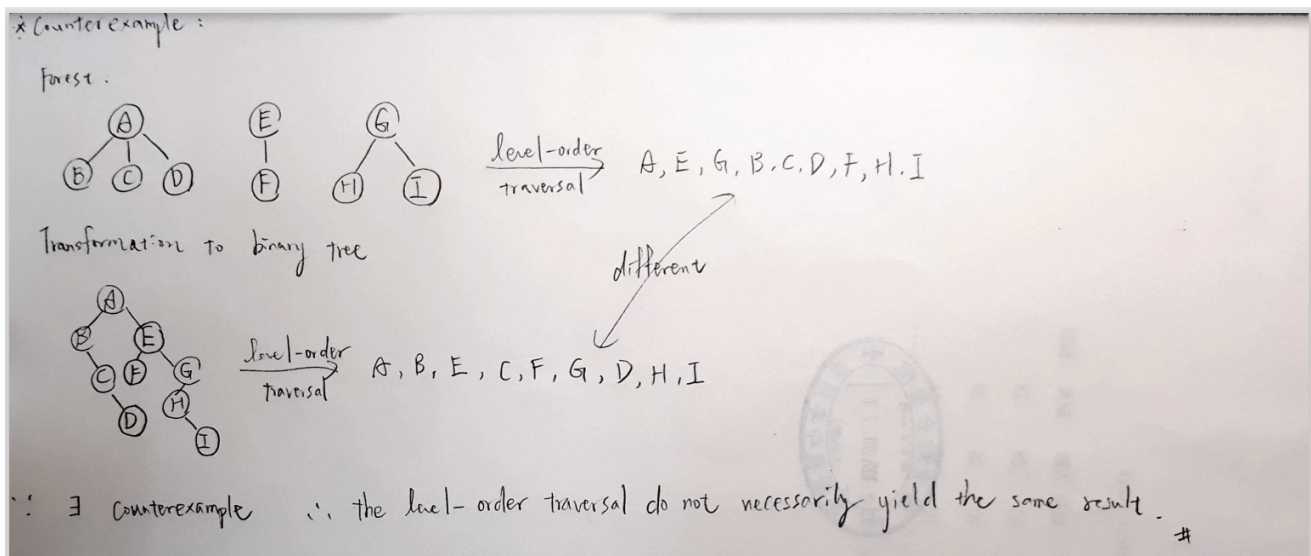
2.

```

1 void SwapTree(BST *node) {
2     if (node == nullptr) return;
3     SwapTree(node->lch);
4     SwapTree(node->rch);
5     swap(node->lch, node->rch);
6 }

```

3.



4.

Suppose binary tree  $T$  has  $n$  nodes.

Let the preorder sequence of  $T$  be  $P = P_1 P_2 \dots P_n$  and the inorder sequence of it be  $I = I_1 I_2 \dots I_n$ .

Let subsequence of  $I$  be  $I'$ .

First, by definition, the root of  $T$  is  $P_1$ , and find  $j$  such that  $I_j = P_1$ .

The left subsequence of  $I$ :  $I_l = I_1 I_2 \dots I_{j-1}$  is the left subtree of  $T$ , and  $I_r = I_{j+1} I_{j+2} \dots I_n$  is the right subtree of  $T$ . (definition of inorder)

If  $I_l$  is not empty, then  $P_2$  is the root of  $T$ 's left subtree; if it is empty, then  $P_2$  is that of  $T$ 's right subtree. (definition of preorder)

While incrementing  $i$ , recursively find  $j$  such that  $I'_j = P_i$ , and divide  $I'$  into subsequences, until  $i = n$ .

A binary tree is then constructed.

Since all procedures all carried out by definition, the constructed binary tree is unique, and it is  $T$ .

5.

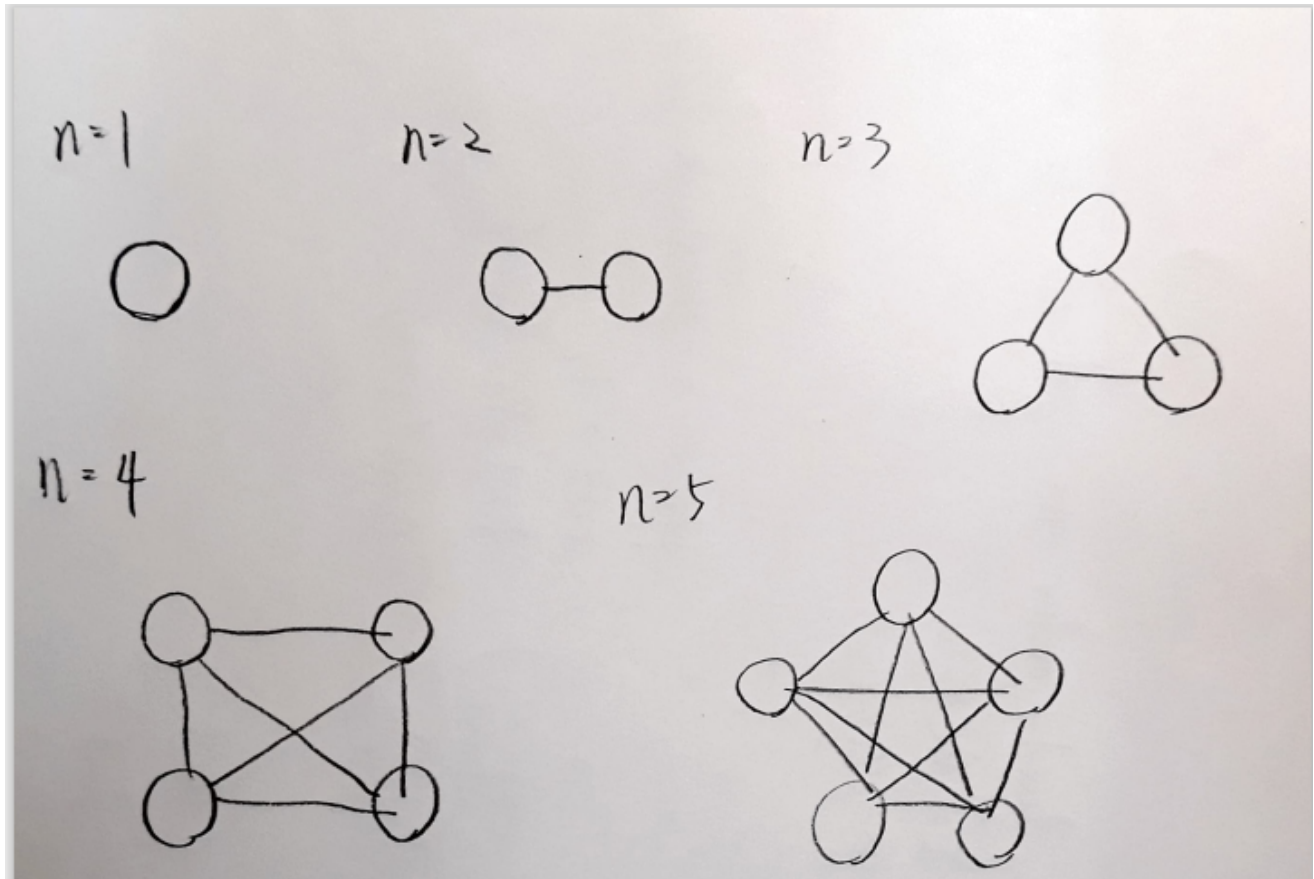
Suppose there are  $E$  edges.

Each edge is connected to exactly two vertices, which contributed to 2 degrees to the sum of degree of vertices in an undirected graph.

$\therefore$  there are  $E$  edges

$\therefore$  the sum of degree of vertices is  $2E$

6.



Suppose there are  $n$  vertices in a complete undirected graph.

$\therefore$  each pair of distinct vertices is connected by an edge

$\therefore$  the number of edges is  $\binom{n}{2} = \frac{n(n-1)}{2}$

7.

```

1  const int V = 100; // number of vertices
2  vector<int> G[V]; // adjacency list
3  bool vis[V]; // true if visited
4
5  void bfs(int st) {
6      queue<int> que;
7      que.push(st);
8      fill(vis, vis+V, 0);
9
10     while (que.size()) {
11         auto v = que.front(); que.pop();
12         vis[v] = true;
13         cout << v << ' ';
14
15         for (auto u: G[v]) {

```

```

16         if (vis[u]) continue;
17         que.push(u);
18     }
19 }
20 }

```

## 8.

Suppose there are  $n$  vertices, numbered from 1 to  $n$ , in a complete graph  $G$ .

Let  $P_i$  denotes a permutation of  $\{1, 2, \dots, n\}$ .

Since  $G$  is a complete graph, every pair of vertices are guaranteed to be connected. Hence,  $P_i$  can be interpreted as a traversing order of a spanning tree of  $G$ .

However,  $(1, 2, \dots, n-1, n) = (n, n-1, \dots, 2, 1)$  in terms of spanning tree, which means traversing the same tree from one end or from the other end.

Therefore, there are at least  $S = \frac{n!}{2}$  distinct spanning trees which can be derived from  $G$ . ( $\forall n \geq 2$ )

For  $n = 1$ , define that  $S = 1$ .

$$\because n! > 2^n \Rightarrow \frac{n!}{2} > 2^{n-1}$$

$$\therefore S = \frac{n!}{2} \geq 2^{n-1} - 1$$

## 9.

```

1  #include <queue>
2  #include <vector>
3  #define QUEUE
4  #define VECTOR
5
6  // no error detection
7  class TopoIterator {
8      public:
9          // number of vertices and adjacency list of graph
10         TopoIterator(int _v, std::vector<int> _graph[]) {
11             iterator = 0;
12             V = _v;
13             G.resize(V);
14             for (int i=0; i<V; ++i) {
15                 G[i] = _graph[i];
16             }
17             sort();
18         }
19
20         void sort() {
21             std::queue<int> que;
22             std::vector<int> indeg(V);
23
24             for (auto &vec: G) {
25                 for (auto &u: vec) {
26                     ++indeg[u];
27                 }
28             }
29
30             for (int u=0; u<V; ++u) {
31                 if (indeg[u] == 0) {
32                     que.push(u);
33                     sorted_seq.push_back(u);
34                 }
35             }
36
37             while (que.size()) {

```

```

38         int v = que.front(); que.pop();
39         for (auto &u: G[v]) {
40             if (--indeg[u] == 0) {
41                 que.push(u);
42                 sorted_seq.push_back(u);
43             }
44         }
45     }
46 }
47 // return the id of current node
48 int at() const {
49     return sorted_seq[iterator];
50 }
51 // move forward for one step (cyclic)
52 void advance() {
53     (++iterator) %= V;
54 }
55 // return the whole sorted sequence
56 std::vector<int> get_all() const {
57     return sorted_seq;
58 }
59
60 private:
61     int V, iterator;
62     std::vector<std::vector<int> > G;
63     std::vector<int> sorted_seq;
64 };

```

## 10.

1. Since there is a **negative-weighted** edge, distance from 0 to 1 is actually shorter if path  $0 \rightarrow 2 \rightarrow 1$  is taken. However, *ShortestPath* will take  $0 \rightarrow 1$  directly. Hence it will not work.
2.  $0 \rightarrow 2 \rightarrow 1 \rightarrow 3 \rightarrow 4 \rightarrow 6$