

As y is equal distribution therefore $p(y=0)$ or $p(y=1)$ equals $1/2$

The entropy $H(y_s) = -2*(1/2)*\lg(1/2) = 1$

Similarly, the entropy $H(z_s) = -2*(1/2)*\lg(1/2) = 1$

According to conditional entropy of X given Y is:

$$H(X|Y) = - \sum_j p(Y=y_i) \sum_i p(X=x_i|Y=y_i) \lg p(X=x_i|Y=y_i)$$

As for $H(y_s|x_t)$, X has three value: 0, 1, 2 and probability should be $1/4$, $1/2$ and $1/4$ respectively.

$$\text{Therefore, } H(y_s | x_t) = -\sum_t p(X=x_t) \sum_s p(Y=y_s | X=x_t) \lg p(Y=y_s | X=x_t) = 1/2$$

When X has value 0, there is only one circumstances that y and z all are 0 therefore for $X=0$, the entropy should be 0.

When X has value 2, there is only one circumstances that y and z all are 1 therefore for $X=2$, the entropy should be 0.

When X has value 1, there is only two circumstances either y or z is 1 and the other is 0, therefore for $X=1$, the entropy should be $1/2$.

$$\text{Therefore, } H(y_s | x_t) = -(1/4+1/2+1/4)*(0+0+1/2)*\lg(1/2) = 1/2$$

$$\text{Similary, } H(z_s | x_t) = - \sum_t p(X=x_t) \sum_s p(Z=z_s | X=x_t) \lg p(Z=z_s | X=x_t) = 1/2$$

$$\text{And } H(z_s | x_t) = 1/2$$

In conclusion:

$H(y_s | x_t) < H(y_s)$ and $H(z_s | x_t) < H(z_s)$ therefore information flows can happen from y and z to x.

$$H(y_s) = -2 \left(\frac{1}{2} \lg \left(\frac{1}{2} \right) \right) = \lg 2 = 1$$

$$H(y_s | x_t) = - \sum_{t=0}^2 p(X=x_t) \left[\sum_{s=0}^1 p(Y=y_s | X=x_t) \lg p(Y=y_s | X=x_t) \right]$$

$$= - [P(X=0) [P(Y=0|X=0) \lg P(Y=0|X=0) + P(Y=1|X=0) \lg P(Y=1|X=0)] +$$

$$P(X=1) [P(Y=0|X=1) \lg P(Y=0|X=1) + P(Y=1|X=1) \lg P(Y=1|X=1)] +$$

$$P(X=2) [P(Y=0|X=2) \lg P(Y=0|X=2) + P(Y=1|X=2) \lg P(Y=1|X=2)]]$$

$$= - [\frac{1}{4} [1 \lg 1 + 0 \lg 0] + \frac{1}{2} [\frac{1}{2} \lg \frac{1}{2} + \frac{1}{2} \lg \frac{1}{2}] + \frac{1}{4} [0 \lg 0 + 1 \lg 1]]$$

$$= - [0 + \frac{1}{2} \lg \frac{1}{2} + 0]$$

$$= - \frac{1}{2} \lg \frac{1}{2}$$

$$= \frac{1}{2}$$

$X := Y + Z$, information flow from y to x

