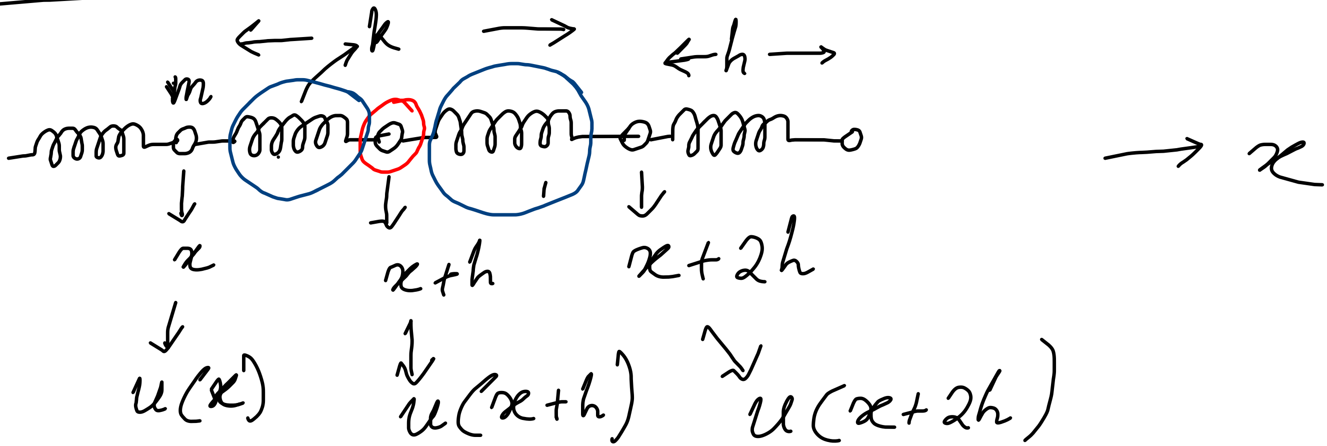
Waves



$$F_{Hook} = F_{x+2h} - F_x$$

$$= k\left[u(x+2h,t) - u(x+h,t)\right] - k\left[u(x+h,t) - u(x,t)\right]$$

$$F_{\text{Newton}} = m\,a(t) = m \frac{\partial^2}{\partial t^2} u(x+h,t)$$

$$\boxed{\frac{\partial^2}{\partial t^2} u(x+h,t) = \frac{k}{m}\Big[u(x+2h,t) - u(x+h,t) - u(x+h,t) + u(x,t)\Big]}$$

N masses $\quad L = Nh \quad K = k/N \quad M = Nm$

$$\boxed{\frac{\partial^2}{\partial t^2} u(\underline{x+h},t) = \frac{KL^2}{M} \frac{u(x+2h,t) - 2u(\underline{x+h},t) + u(x,t)}{h^2}}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2}$$

$N \to \infty$ $\qquad$ $h \to 0$

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \frac{KL^2}{M}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f''(x) = \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2}$$

$N \to \infty \qquad h \to 0$

$$\boxed{\frac{\partial^2 u(x,t)}{\partial t^2} = \underbrace{\frac{KL^2}{M}}_{c^2} \frac{\partial^2 u(x,t)}{\partial x^2}}$$

Wave equation

Solution

$$\psi(x,t) = \underline{X(x)\,T(t)}$$

$$\frac{\partial \psi}{\partial t} = X(x)\,\frac{dT(t)}{dt}$$

$$\frac{\partial^2 \psi}{\partial t^2} = X(x)\,\frac{d^2T(t)}{dt^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = T(t)\,\frac{d^2X(x)}{dx^2}$$

$\psi(x+t)$

$\sin(x+t)$

$\sin(kx+\omega t)$

$\sin kx \cos\omega t$
$\quad + \cos kx \sin\omega t$

$kx+\omega t$

$$X(x)\frac{d^2T(t)}{dt^2} = c^2\, T(t)\frac{d^2X(x)}{dx^2} \qquad \div\; X(x)T(t)$$

$$\frac{1}{T(t)}\frac{d^2T(t)}{dt^2} = c^2\frac{1}{X(x)}\frac{d^2X(x)}{dx^2} = \text{constant}$$

$$= -\omega^2$$

Separation of variables

$$\frac{d^2X(x)}{dx^2} = -\frac{\omega^2}{c^2}X(x)$$

$$\frac{d^2X(x)}{dx^2} + \frac{\omega^2}{c^2}X(x) = 0$$

$$\frac{\omega^2}{c^2} \rightarrow k^2$$

$$\frac{d^2T(t)}{dt^2} + \omega^2 T(t) = 0$$