PHY 106 **Quantum Physics**
Assignment 1 17 January 2025
Submit by 22 January 2025

1. Show that the function $\psi(x,t) = f_1(ct-x) + f_2(ct+x)$ satisfies the one-dimensional wave equation.

2. Consider the one-dimensional wave equation

$$\frac{\partial^2\psi}{\partial t^2} = c^2\frac{\partial^2\psi}{\partial x^2}. \tag{1}$$

Find the traveling wave solution satisfying the initial conditions $\psi(x,0) = g(x)$ and $\dot{\psi}(x,0) = 0$.

3. For gravity waves in a liquid, the phase velocity c depends on the wavelength λ as $c = A\sqrt{\lambda}$, A is a constant. Show that the group velocity is half the phase velocity.

4. The phase velocity c of deep water waves obeys the relationship

$$c^2 = \frac{g\lambda}{2\pi} + \frac{2\pi S}{\rho\lambda}, \tag{2}$$

where g, ρ, and S are the acceleration due to gravity, the density of water, and the surface tension of water, respectively.

   (a) Find the wavelength λ_0 at which the waves do not disperse in water.

   (b) Show that for $\lambda << \lambda_0$, $c_g = 3c/2$.

   (c) Show that for $\lambda >> \lambda_0$, $c_g = c/2$.