Wave equation

$$\frac{\partial^2 \psi}{\partial t^2} = c^2 \frac{\partial^2 \psi}{\partial x^2}$$

$$\psi(x,t) = X(x)\, T(t)$$

$$\frac{1}{T(t)} \frac{d^2 T(t)}{dt^2} = c^2 \frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} = -\omega^2$$

$$\boxed{\begin{aligned} &\frac{d^2 X}{dx^2} + k^2 \underline{X(x)} = 0 \\ &\frac{d^2 T}{dt^2} + \omega^2 T = 0 \end{aligned}}$$

$$k^2 = \frac{\omega^2}{c^2}$$

ODE

$$\frac{d^2T}{dt^2} + \omega^2 T = 0$$

$$X(x) = A_1 e^{ikx} + A_2 e^{-ikx}$$

$$T(t) = B_1 e^{i\omega t} + B_2 e^{-i\omega t}$$

$$\psi(x,t) = \left(A_1 e^{ikx} + A_2 e^{-ikx}\right)\left(B_1 e^{i\omega t} + B_2 e^{-i\omega t}\right)$$

$$= A_1 B_1 e^{i(\omega t + kx)} + A_2 B_2 e^{-i(\omega t + kx)}$$
$$+ A_1 B_2 e^{-i(\omega t - kx)} + A_2 B_1 e^{i(\omega t - kx)}$$

$$\psi(x,t) = a\sin(\omega t - kx)$$

Angular frequency

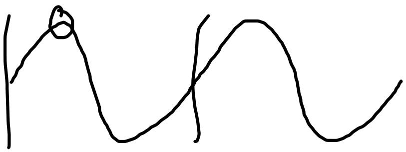
$$\nu = \frac{\omega}{2\pi}$$

Amplitude

Wavenumber

$a\cos(\omega t - kx) \rightarrow$ Forward

Phase



$a\sin(\omega t + kx) \rightarrow$ Backward

Time period $T \rightarrow$

$$\nu = \frac{1}{T}$$

Superposition of waves

$$\psi_1(x,t) = \psi_0 \sin(kx - \omega t)$$

$$\psi_2(x,t) = \psi_0 \sin(kx - \omega t + \phi)$$

$$\psi_1 + \psi_2 = \psi = \psi_0 \sin(kx - \omega t) + \psi_0 \sin(kx - \omega t + \phi)$$

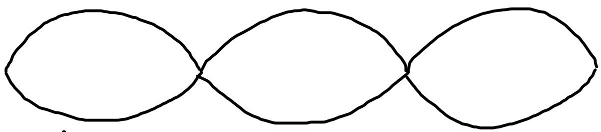
$$\boxed{\psi = 2\psi_0 \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)}$$

$\phi = 0$ Max $\qquad\qquad$ $\phi = \pi$ Min

Interference

$$\Psi = \psi_0 \sin(kx - \omega t) + \psi_0 \sin(kx + \omega t)$$

$$= 2\psi_0 \cos\omega t \sin kx$$



Standing wave

$kx = 0, \pi, 2\pi, \ldots$

$kx = n\pi$

$n = 0, \pm 1, \pm 2, \ldots$

Nodes

$$\frac{2n+1}{2}\pi$$

Different frequencies

$$\psi(x,t) = \psi_0 \sin(k_1 x - \omega_1 t) + \psi_0 \sin(k_2 x - \omega_2 t)$$

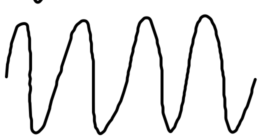
$$= 2\psi_0 \cos\left[\frac{k_1 - k_2}{2} x - \frac{\omega_1 - \omega_2}{2} t\right]$$

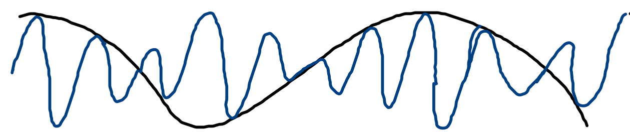
Beat frequency

$$\sin\left[\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t\right]$$

sound frequency

sine wave





$$\frac{d}{dx}\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right)$$

$$= 0+1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

Set $x=1 \longrightarrow 2.718\cdots$

$$\frac{d^2X}{dx^2} = -k^2 X \qquad X = e^{ikx}$$

$$X(x) = Ae^{ikx} \qquad X = Be^{-ikx} \qquad i=\sqrt{-1}$$

$$Ae^{ikx} + Be^{-ikx}$$

General solution

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$