Willingness to Pay and Price Optimization

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Agenda

- New Product Pricing
- Relationship between WTP and Demand
- Optimal Pricing based on WTP estimates

Importance of Monetization

- Companies spend tremendous resources for value creation
- Often little thought given to capturing value
  - Heuristics
  - Tradition.. Always done this way

Growing Need for Smarter Pricing

- Global competition, --- entry of low-cost suppliers
- Product proliferation --- shorter product lifecycles
- Deregulation and privatization of industries
- A shift from variable costs to fixed costs
- Technologies for searching/monitoring price changes and targeting

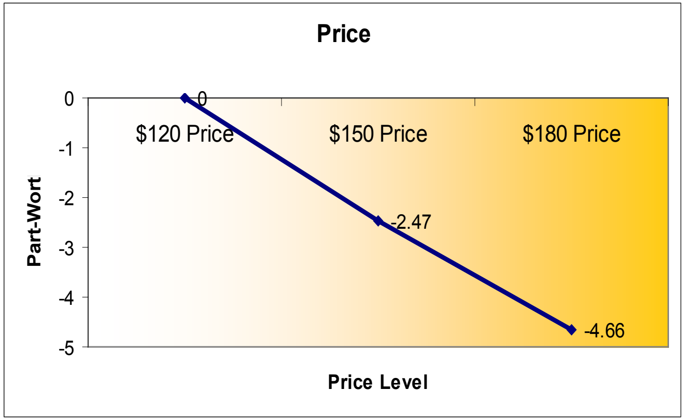
New Product Pricing

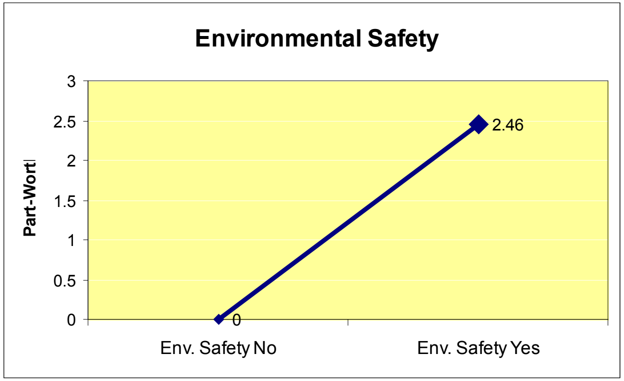
Methods

Contingent Valuation

Conjoint Analysis

Car Batteries: Value of Environmental Safety





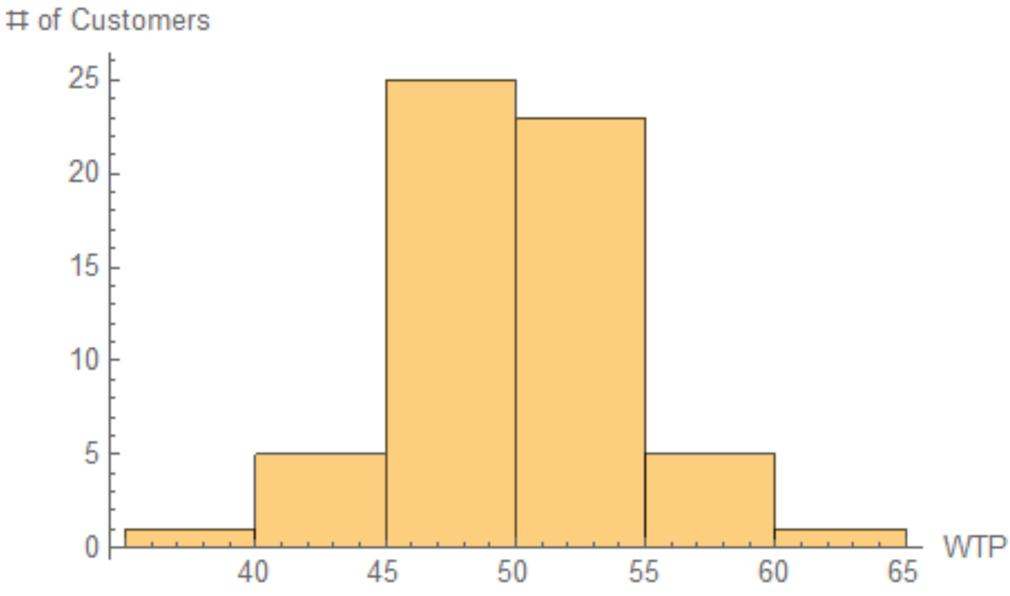
Price Optimization using WTP Measures

- Market size 10,000 customers, Marginal cost $15
- What is the optimal price and optimal profit?
- Draw the demand curve

Customer	WTP
1	47
2	44
3	49
4	30
5	49
6	30
7	32
8	52
9	31
10	48
11	47
12	48

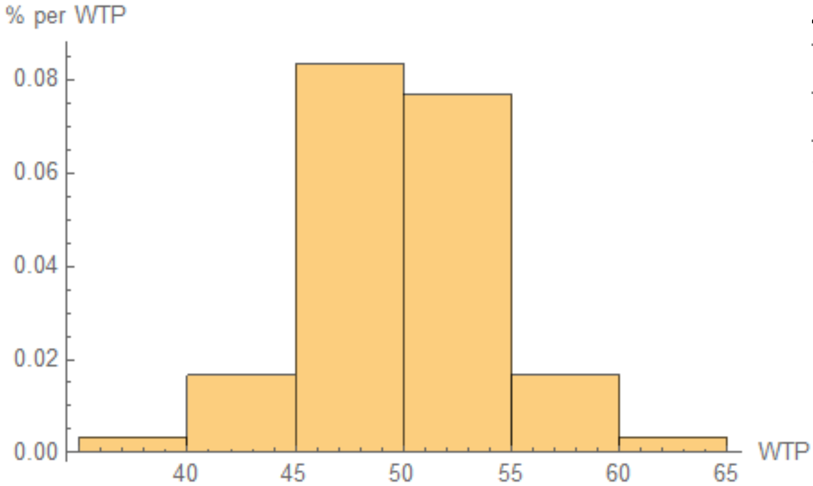
Histogram

{48, 56, 50, 42, 49, 57, 53, 49, 51, 49, 49, 50, 50, 47, 47,
50, 54, 43, 59, 54, 46, 53, 51, 43, 58, 47, 47, 46, 49, 52,
52, 59, 48, 43, 53, 45, 50, 50, 48, 46, 54, 47, 45, 49, 52,
48, 48, 46, 52, 51, 61, 49, 50, 40, 53, 51, 39, 46, 47, 50}



Density Histogram

- Density is the proportion of cases per unit



Bin	Count	Proportion	Density
35–40	1	0.017	0.0033
40–45	5	0.083	0.017
45–50	25	0.42	0.083
50–55	23	0.38	0.077
55–60	5	0.083	0.017
60–65	1	0.017	0.0033

$$\frac{5}{60} = 0.083$$

$$\frac{0.083}{5} = 0.017$$

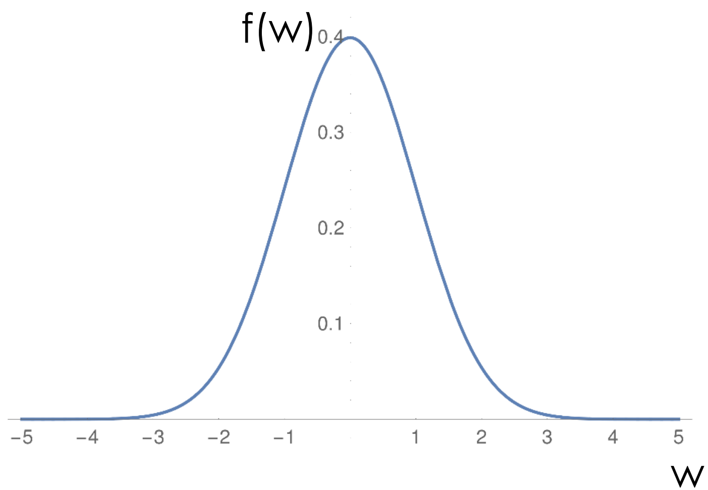
WTP distributions

- We will model WTP data using probability distributions
- Distributions can be represented by their
  - Probability density functions, i.e., p.d.f
  - Cumulative distribution function, i.e., c.d.f

Normal Distribution: PDF

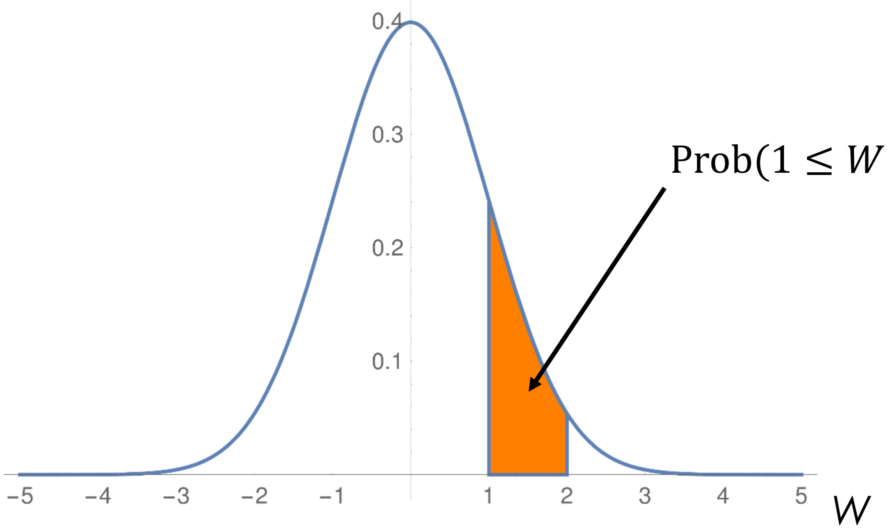
- The pdf of a normal distribution with mean μ and standard deviation σ is given by

$$f(w;\mu,\sigma)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(w-\mu)^2}{2\sigma^2}\right)$$



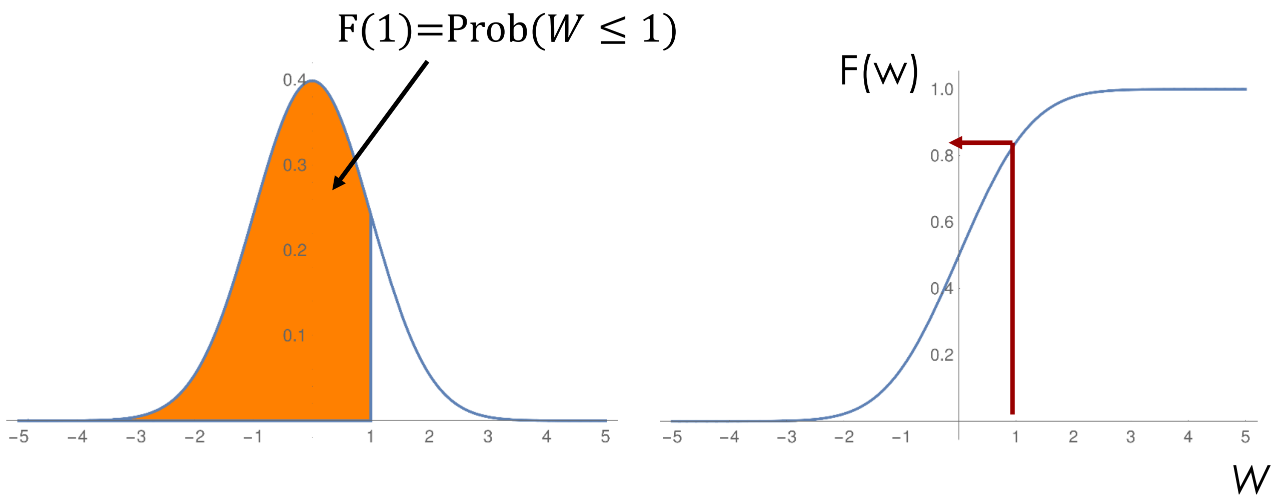
Normal Distribution: PDF

- The pdf shows the density at a point.
  - is always positive, and the area under it totals 1
- The probability that the random variable *W* falls between two values is



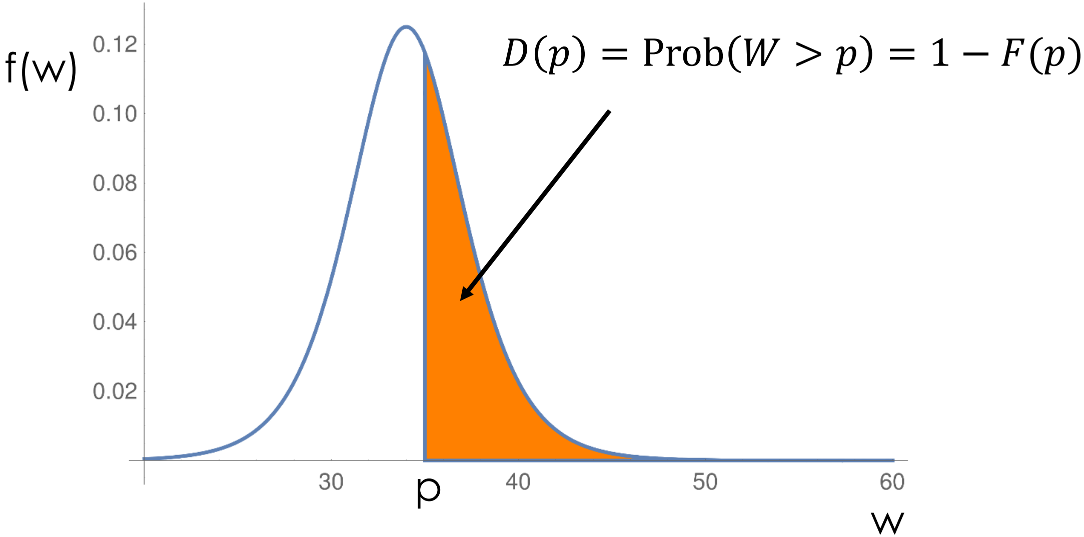
Normal Distribution: CDF

- The c.d.f, F(w) gives the probability $\text{Prob}(W \leq w)$



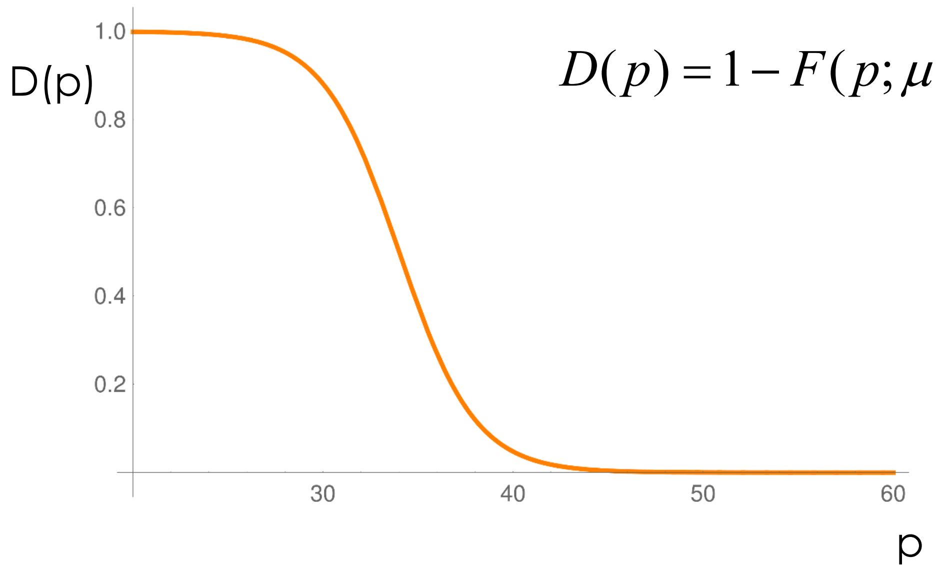
Demand

- The demand at a given price *p* is given by the orange area



Demand Curve

- The demand curve is given by

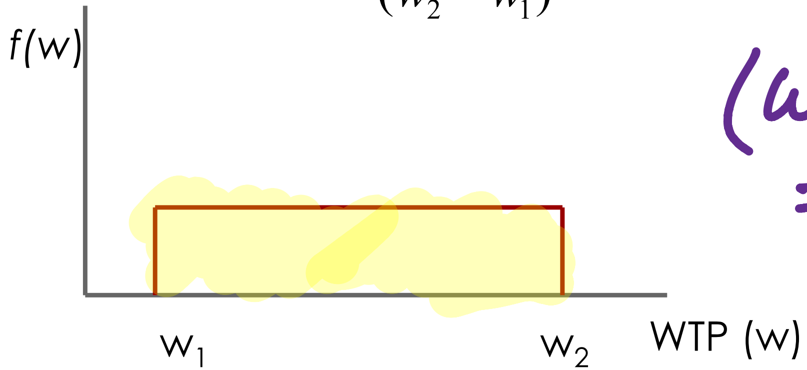


$$D(p) = 1 - F(p; \mu, \sigma)$$

Uniform WTP distribution

- Suppose the willingness to pay are distributed uniformly between w_1 and w_2
- The probability density function of the uniform distribution is given by

$$f(w) = \frac{1}{(w_2 - w_1)}$$

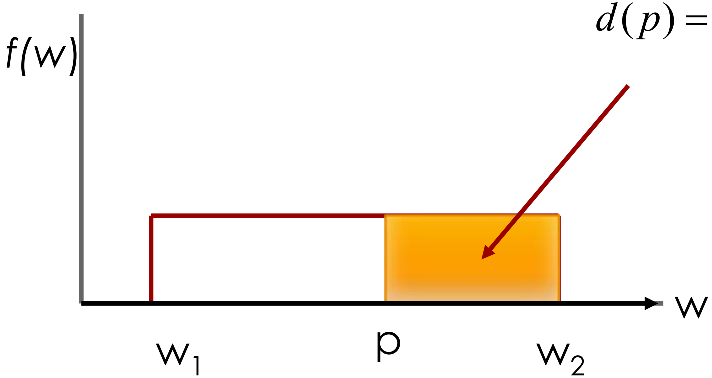


$$(w_2 - w_1) f(w) = \frac{w_2 - w_1}{w_2 - w_1} = 1$$

Demand Function

- The demand at a given price p is given by the proportion of customers who have a wtp > p
- It is given by the area of the region under the wtp density to the right of p, i.e., 1-F(p)

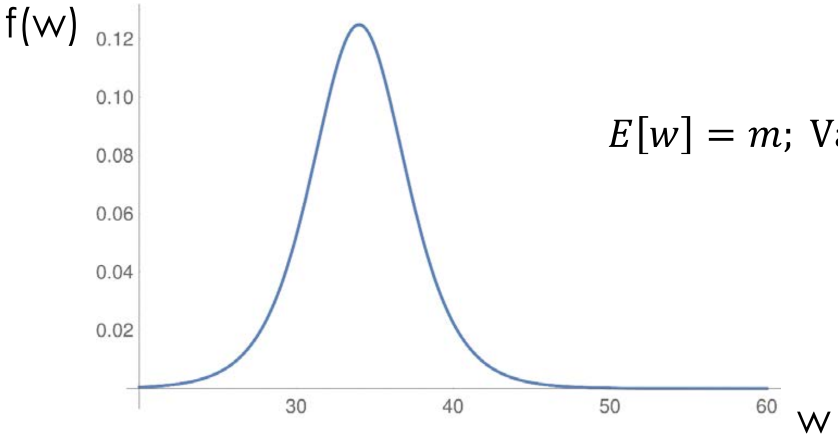
$$d(p) = \frac{(w_2 - p)}{(w_2 - w_1)}$$



Logistic WTP

- The p.d.f of a logistic is given by the formula

$$f(w) = \frac{\exp(-(w-m)/s)}{s(1+\exp(-(w-m)/s))^2}; \quad s > 0$$

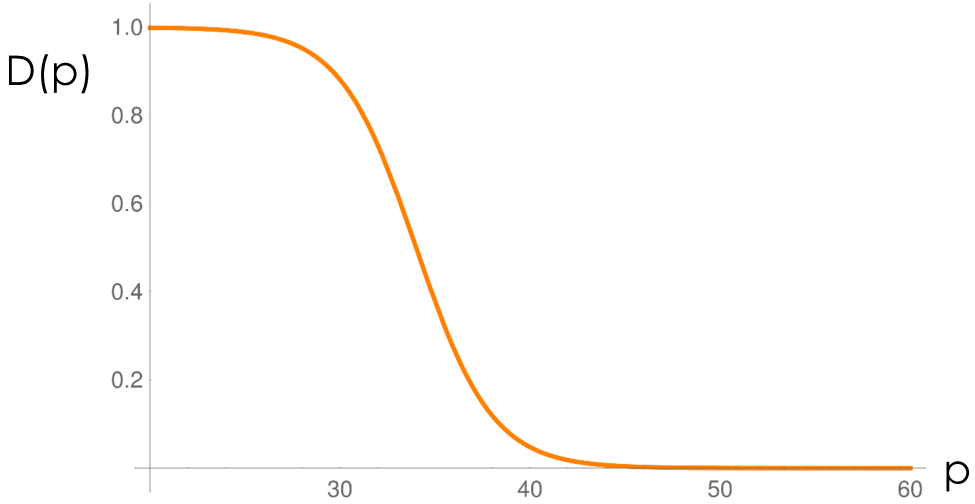


$$E[w] = m; \; \mathrm{Var}[w] = \frac{s^2\pi^2}{3}$$

Logistic Demand Curve

- The demand curve is given by

$$D(p) = \frac{1}{1 + \exp((p - m)/s)}$$



Logistic Distribution: Estimation

- We will fit a logistic distribution to the distribution of willingness to pay
- The p.d.f of the logistic distribution is

$$f(w) = \frac{\exp(-(w-m)/s)}{b(1+\exp(-(w-m)/s))^2}; \quad s > 0$$

- We can use either method of moments or maximum likelihood estimation to estimate the parameters *m* and s.

Logistic WTP: Estimation

- We can use the method of moments (MOM) to estimate the two parameters, *m* and *s*.
- Let sample mean be $\hat{m}$ and sample standard deviation be $\hat{\sigma}$.
- Equating sample moments to theoretical moments, we have

$$m = \hat{m}, \text{ and } s^2 = \frac{3\,\hat{\sigma}^2}{\pi^2}$$

$$s^2 = \frac{3\hat{\sigma}^2}{\pi^2}$$

$$Var(W) = \frac{s^2\pi^2}{3} = \hat{\sigma}^2$$

Likelihood and Log-Likelihood function

- For given values of the parameters, the likelihood for the i^th observation w_i, is the value of the pdf $f(w_i; m, s)$

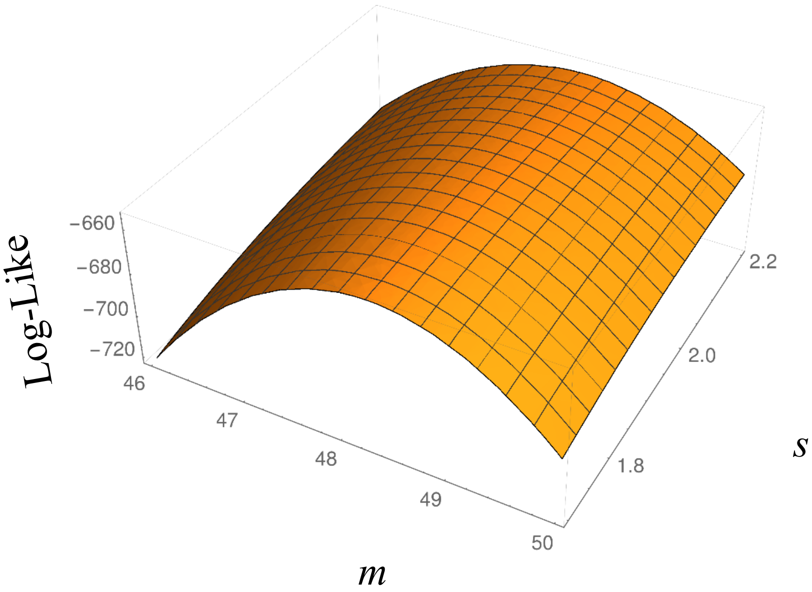
- The likelihood for the entire dataset is the product of the likelihoods for each observation

$$L(\vec{w}; m, s) = \prod_{i=1}^{n} f(w_i; m, s)$$

- We estimate the parameters by maximizing the log-likelihood

$$LL(\vec{w}; m, s) = \sum_{i=1}^{n} \log f(w_i; m, s)$$

Maximum Likelihood Estimates



Logistic Parameter Estimates

- Uncertainty in parameter estimates results in uncertainty about the true demand curves and the true optimal prices

Parameter	Mean	Std. Error
m	49.39	0.539
s	2.42	0.264

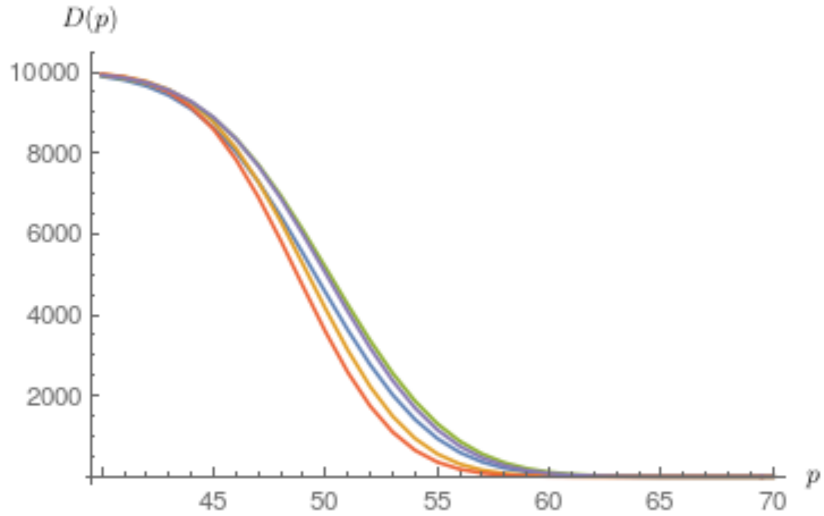
- Uncertainty intervals can be estimated for complicated quantities, via the bootstrap

Bootstrapping

- We can consider our sample of n=60 observations as the population
  - Randomly select n observations with replacement
  - Get MLE or MOM estimates for sample and compute demand curve
- Repeat the above two steps many times (1000) to get many bootstrap values and demand curves

Bootstrapped Demand Curves

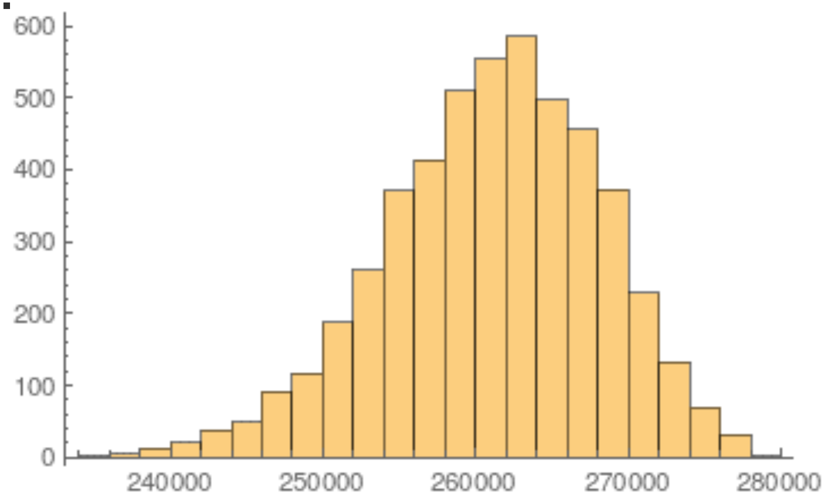
- The true demand curve is uncertain. Could be any of these



- The optimal price is also uncertain.

Range of Actual Profits

- We can compute the uncertainty about actual realized profits when we set the price to the "optimal value" that is computed from the sample.



Realized Profit

Contingent Valuation

- Direct estimates of WTP values may not be available
- Can estimate the WTP distribution using indirect means (via Yes, No questions for particular prices)
  - Double-bounded dichotomous choice

Double-Bounded Dichotomous Choice

- First Question: The gym payment plan will require $45 monthly. Would you be willing to join the gym?

    1. Agree
    2. Disagree

- If agree to first question-
    - Second Question: If the amount was $50, will you agree?
- If disagree to first question
    - Second Question: If the amount was $40, will you agree?

Double-Bounded Dichotomous Choice

- Imagine a wtp survey with three conditions (treatments)
  - **Survey 1**: Low =40, mid=45, high=50
  - **Survey 2**: Low =45, mid=50, high=55
  - **Survey 3**: Low =50, mid=55, high=60
- People are randomly assigned to one of these conditions
- People’s responses to the first and second questions are recorded

Willingness to Pay and Responses

Low=40 | **Mid=45** | **High=50**

N,N	N,Y	Y,N	Y,Y
$Pr(W < 40)$ $= F(40)$	$Pr(40 < W < 45)$ $= Pr(W < 45) - Pr(W < 40)$ $= F(45) - F(40)$	$Pr(45 < W < 50)$ $= Pr(W < 50) - Pr(W < 45)$ $= F(50) - F(45)$	$Pr(W > 50)$ $= 1 - F(50)$

Wtp (W)

Willingness to pay and Responses

- Let true wtp be w
  - If w < low, then response is No-No
  - If w > high, then response is Yes-Yes
  - If low < w < mid, then response is No-Yes
  - If mid < w < high, then response is Yes-No
- Hence, a No-No response is consistent with
  - Prob(w < low) = F(low)
- A No-Yes response is consistent with
  - Prob(low < w < mid) = Prob(w < mid)-Prob(w < low) = F(mid)-F(low)

Inferring WTP distribution

- We can use the responses across individuals to estimate the wtp distribution
- We assume that the wtp values follow a normal distribution with some unknown mean (μ) and standard deviation (σ)
- We write the probability of each response, and then maximize the product of the response probabilities across individuals.
- In reality, we look for the parameter estimates that maximize the log-probability of the sample responses
- These are called Maximum Likelihood Estimates (MLE)

Inferring WTP Distributions

- The DBDC method gives us a distribution of wtp values without ever asking people for wtp values directly
- Such an approach is preferred as it reduces hypothetical bias

Summary

- Wtp distributions can be used to estimate demand curves
- Demand curves can be derived based on wtp distributions
- Uncertainty about demand parameters translates into uncertainty about optimal prices and realized profits