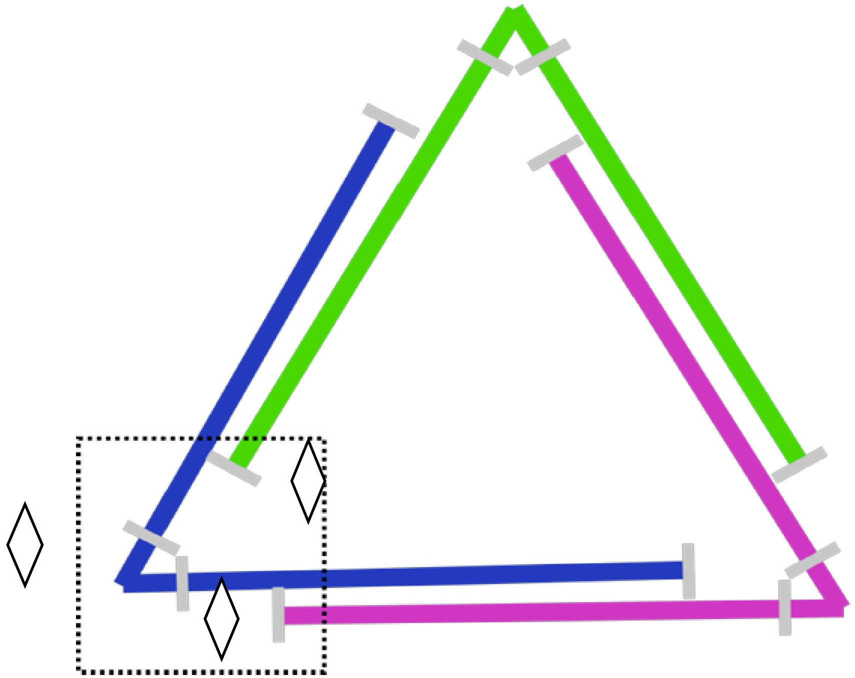
Seismometer Position Optimization for Newtonian Noise Mitigation in the Einstein Telescope

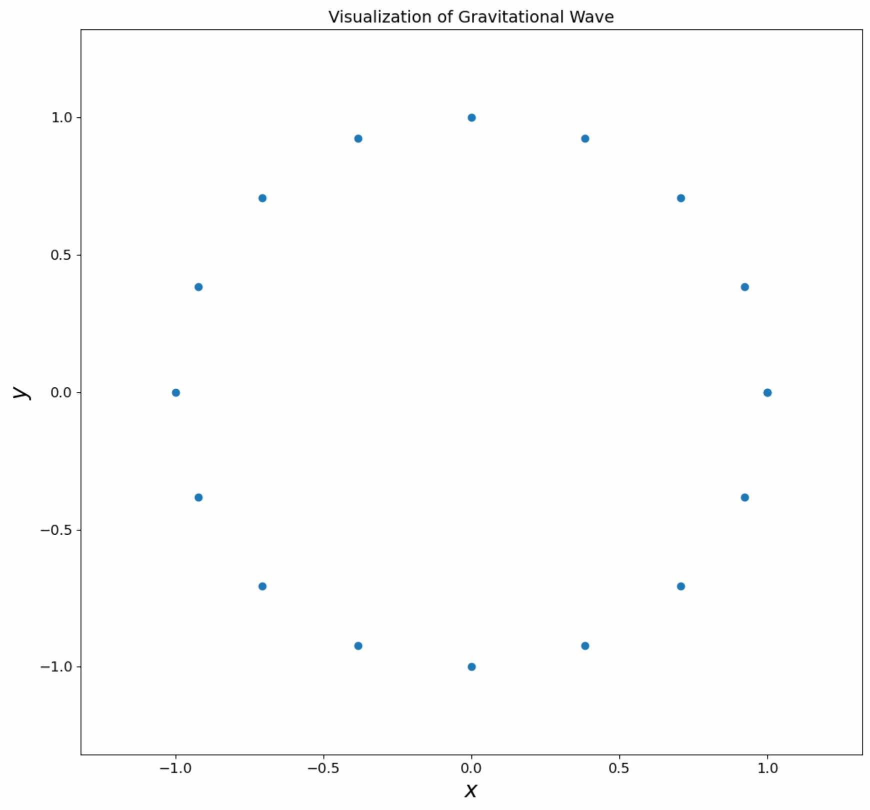


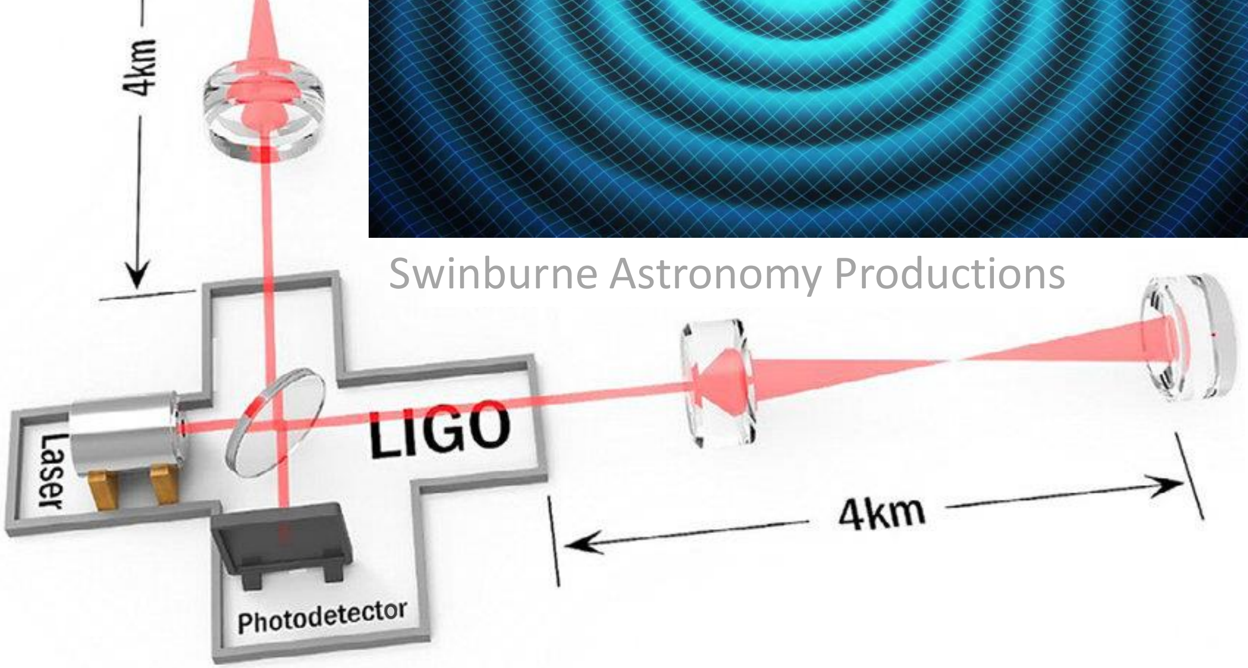
Patrick Schillings

Introduction

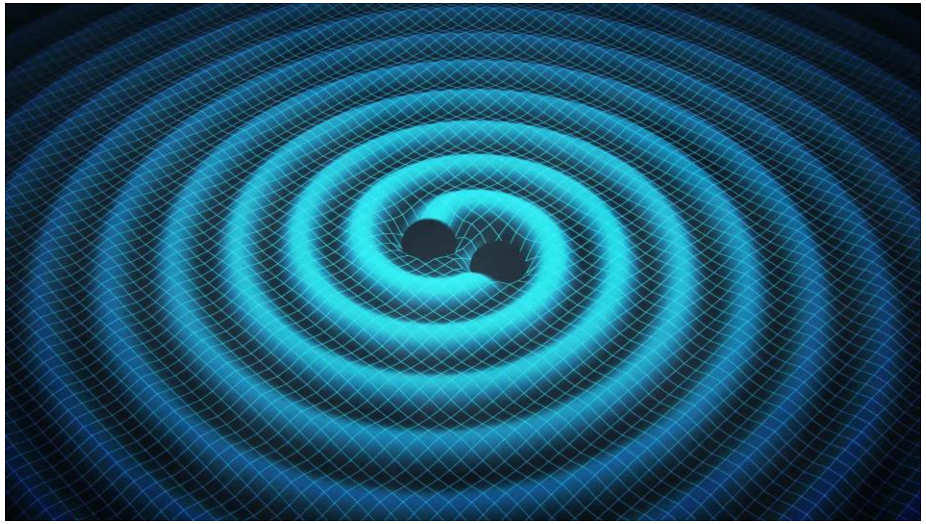
What is the Einstein Telescope and why do we want to place seismometers?

How to Measure Gravitational Waves





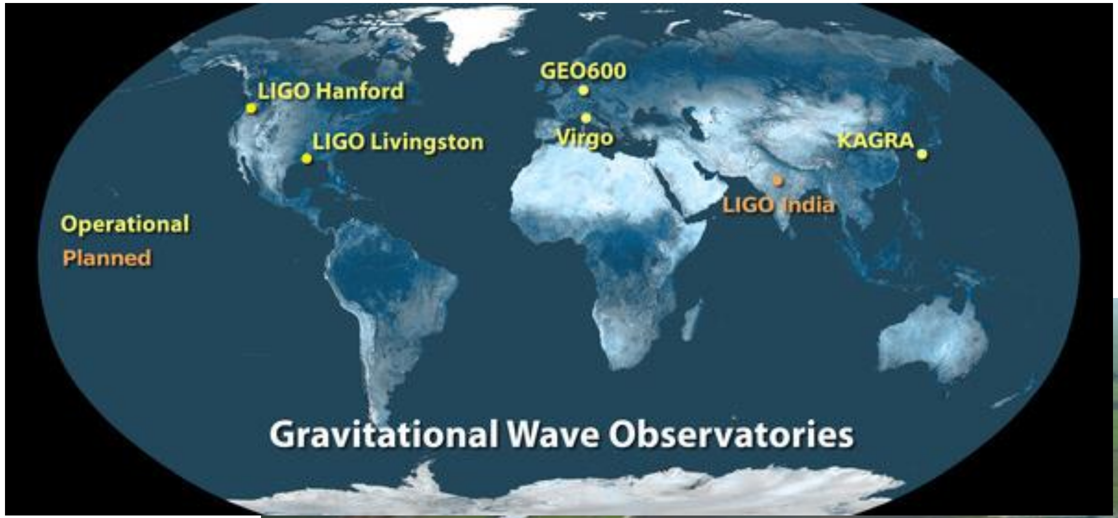
Chao Zuo et a. (2020)



Swinburne Astronomy Productions

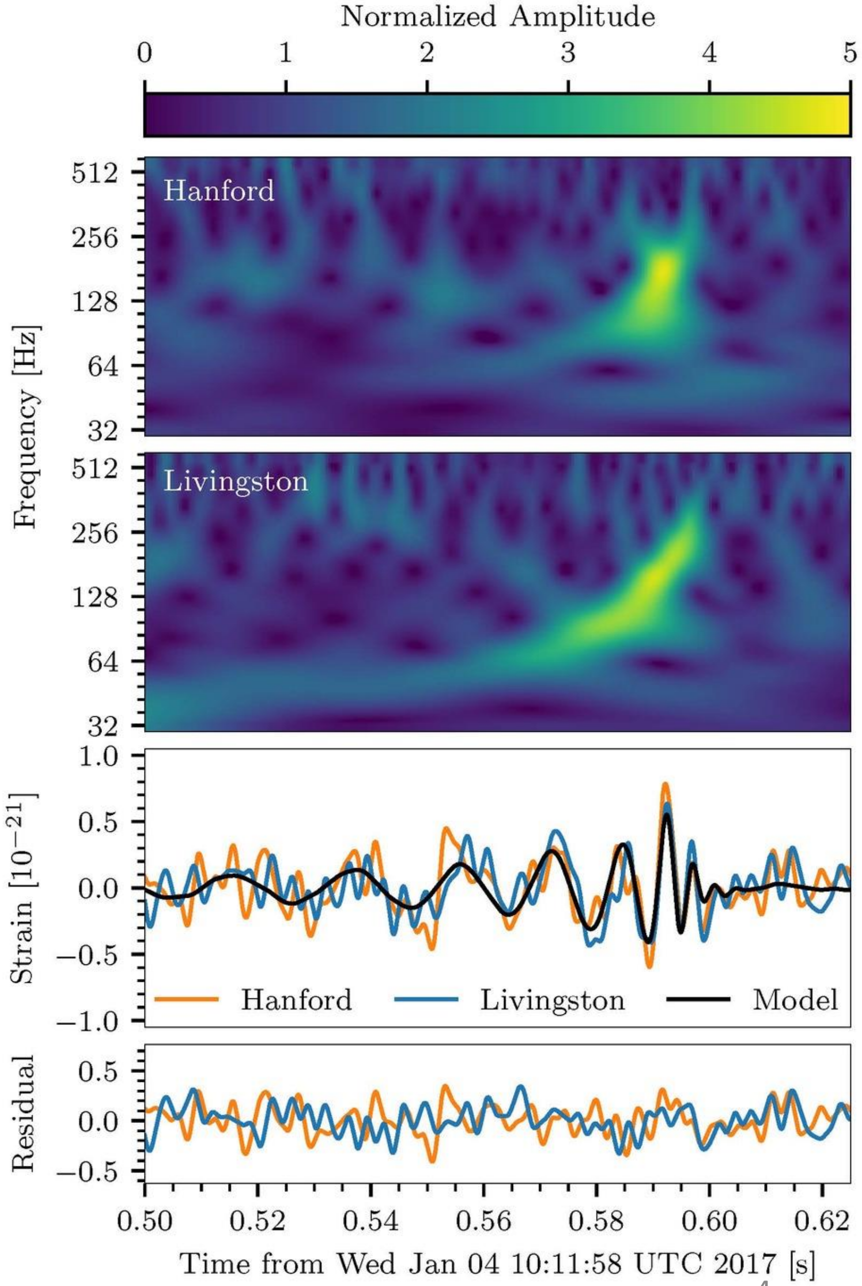
Strain $h = \frac{\Delta L}{L}$

LIGO, Virgo and Kagra



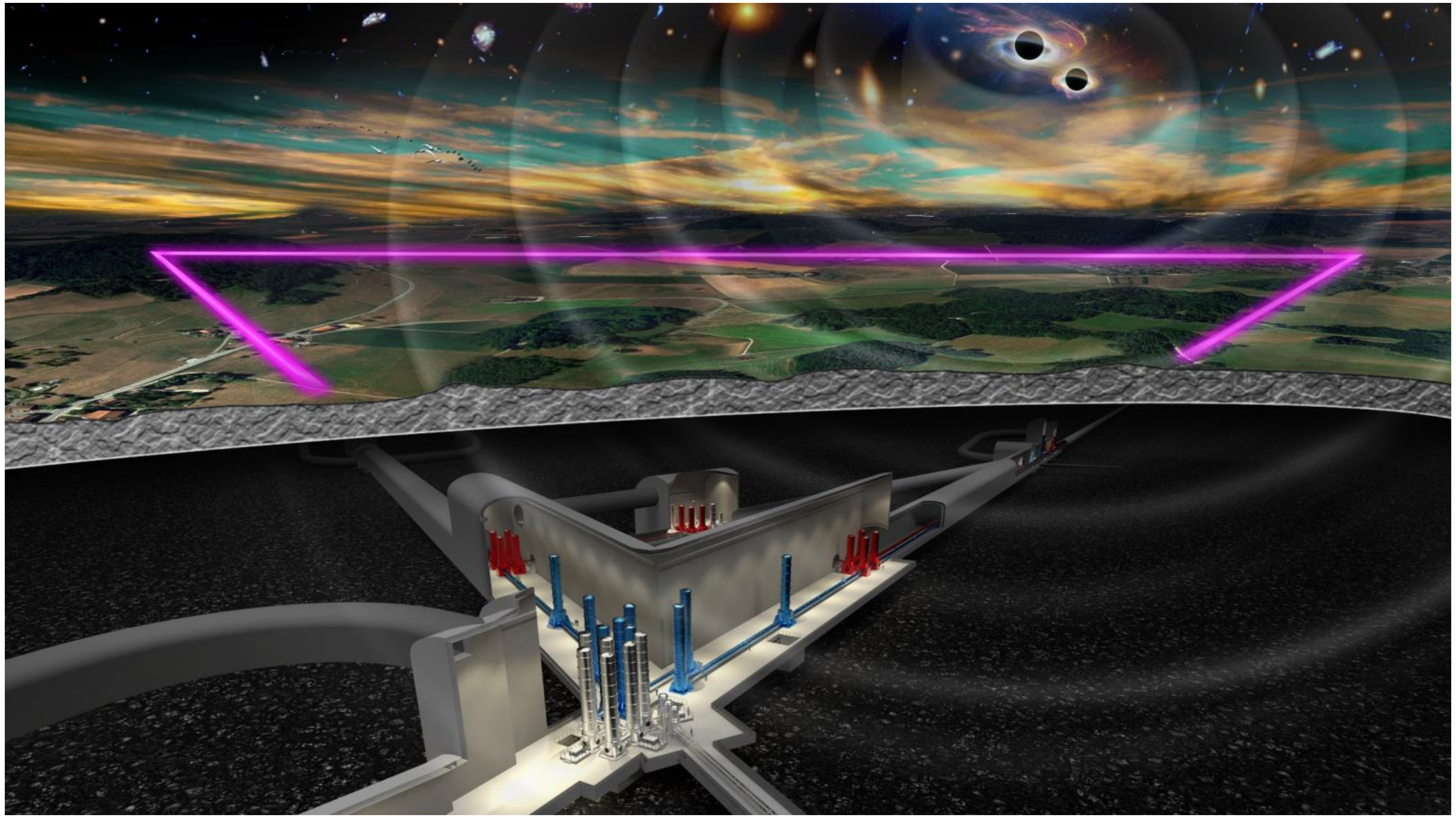
Caltech/MIT/LIGO Lab

Virgo collaboration/CCO 1.0

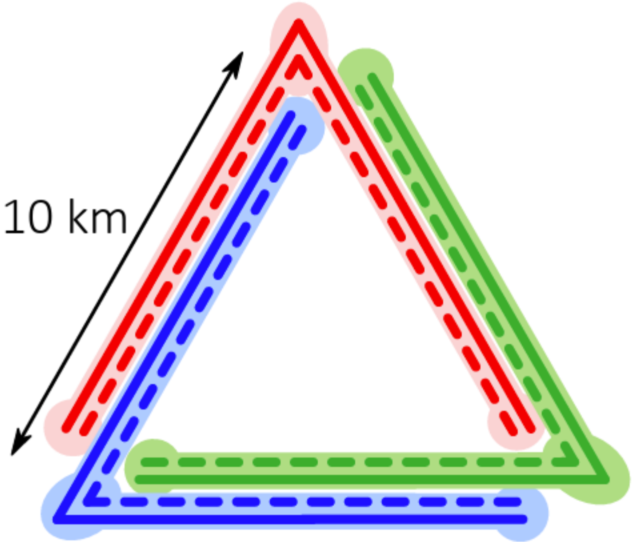


LIGO/Caltech/MIT

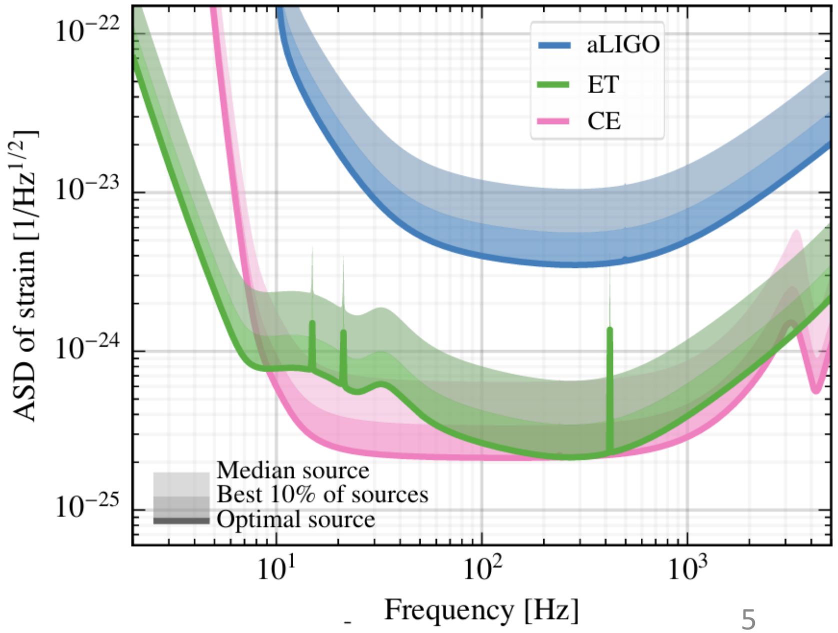
The Einstein-Telescope



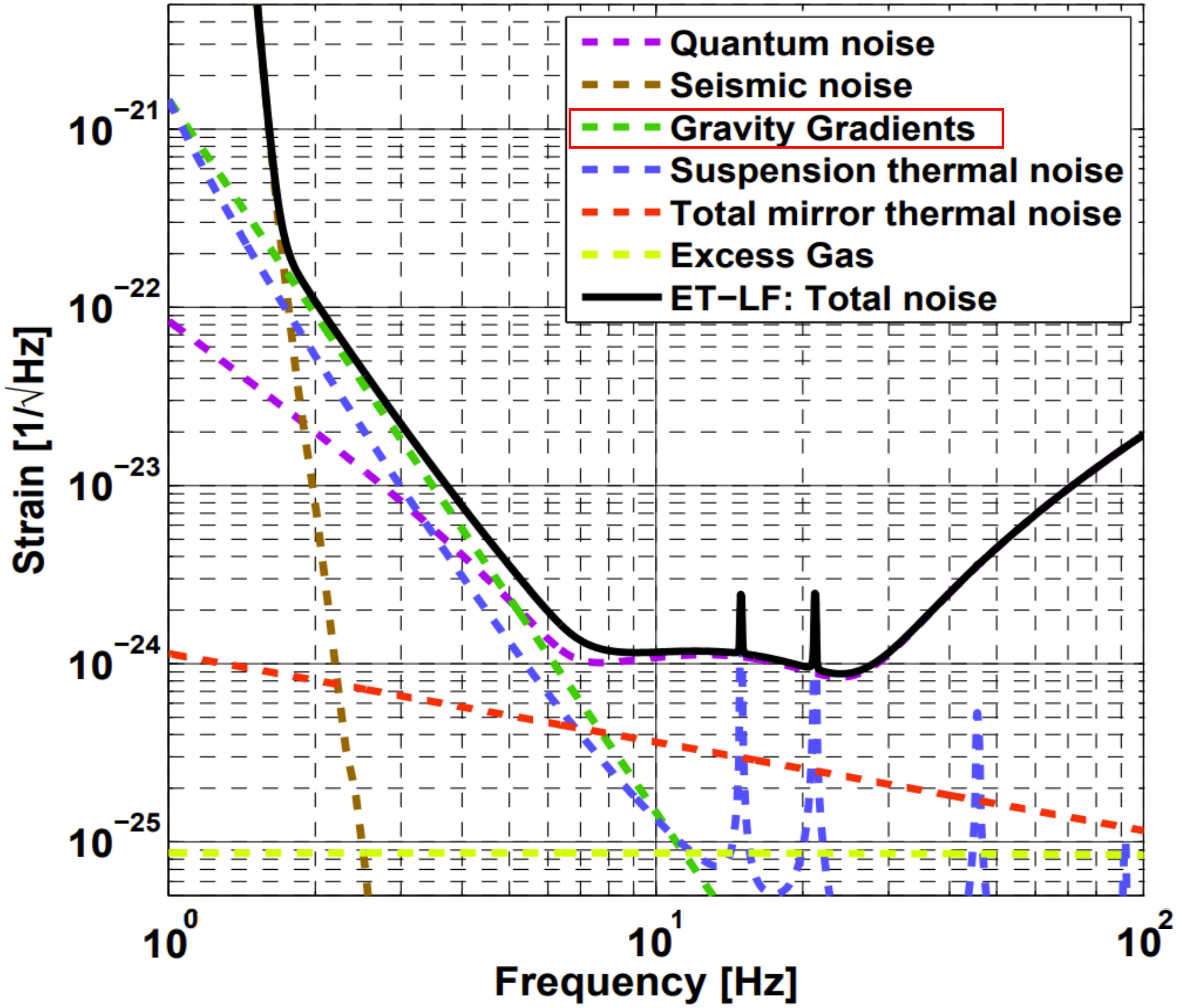
Marco Kraan, Nikhef



ET design report update (2020)

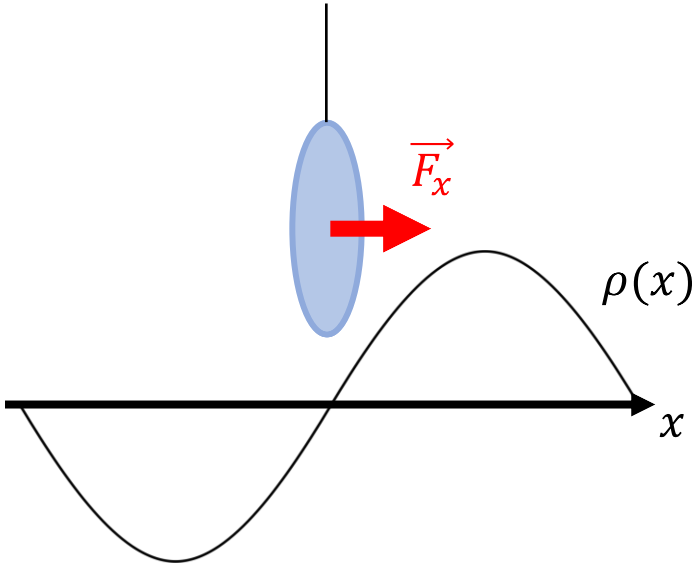


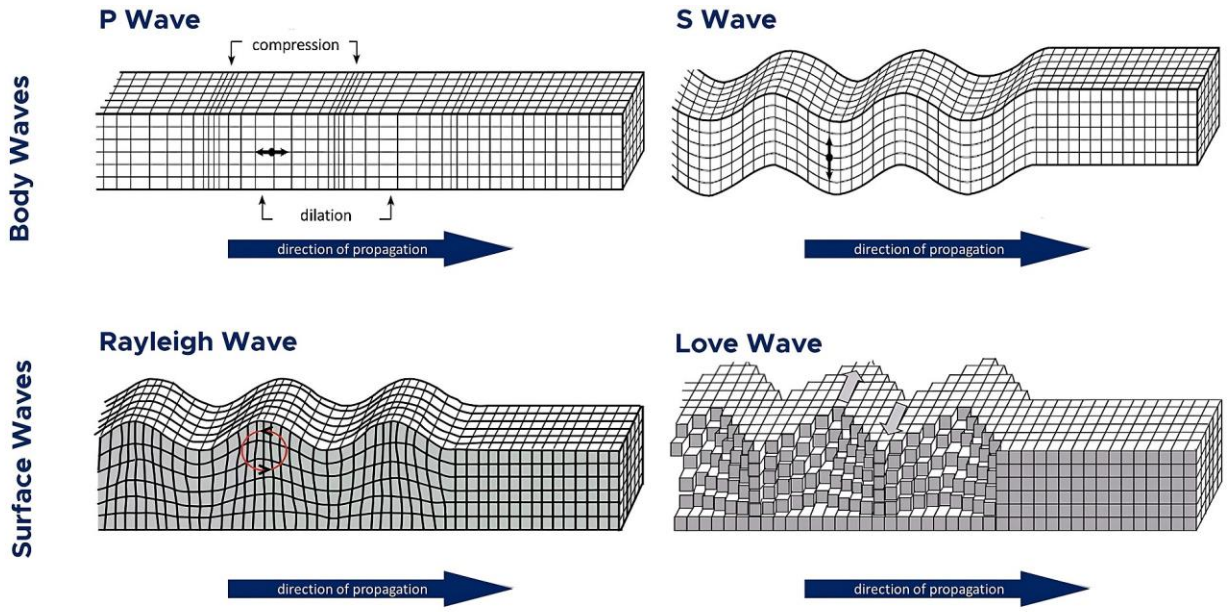
Einstein telescope‘s limiting noise curve



Hild et al., arXiv:1012.0908

Newtonian Noise





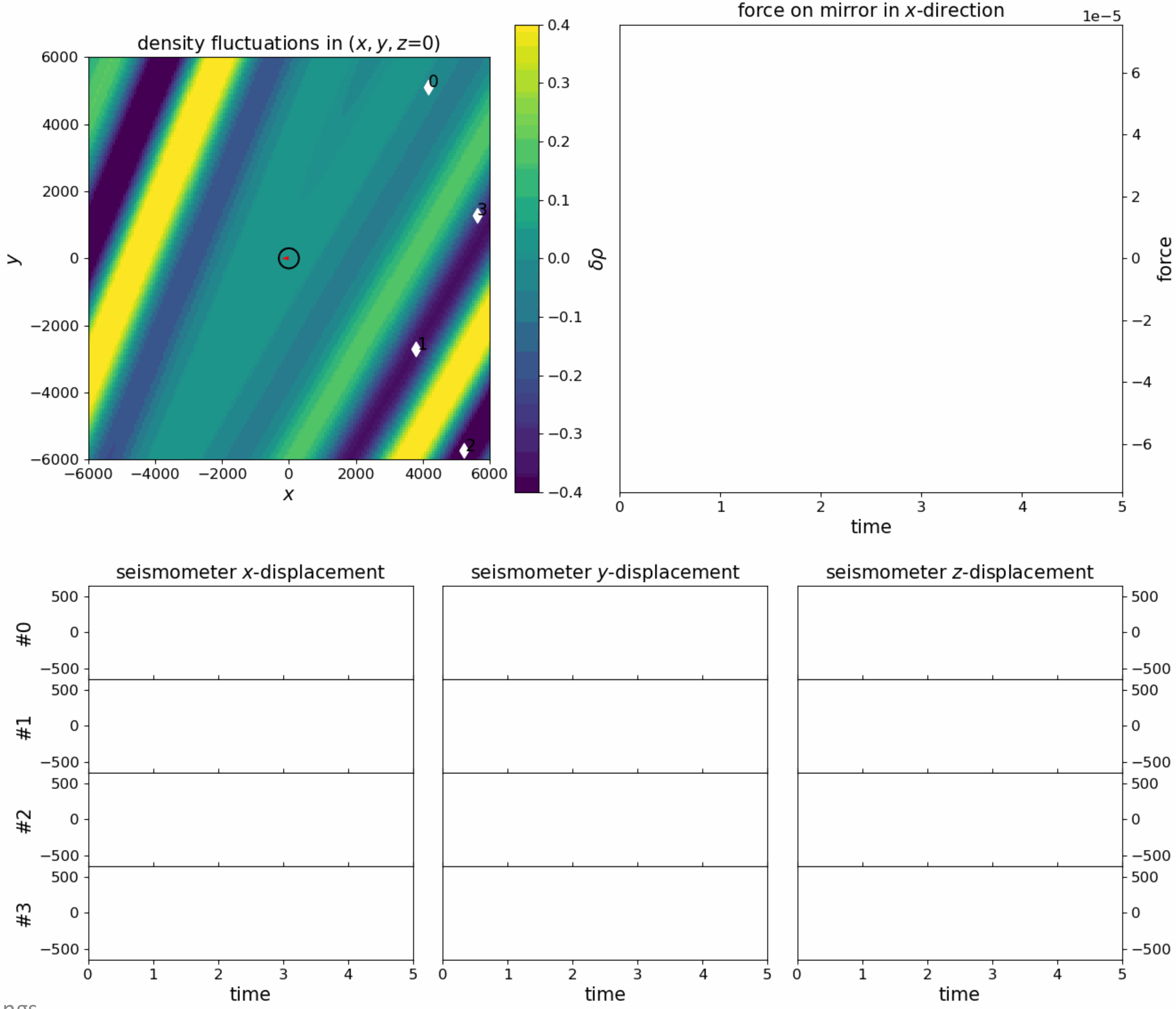
B.A. Bolt, Textbook (1982)

- Density fluctuations in the Earth couple gravitationally to the ET mirror

- Cannot be shielded: Must be predicted and subtracted
- Boreholes for seismometers are expensive

→Optimize seismometer positions

Density Fluctuations, Force and Seismometer Data at $t = 0.0$



One Mirror: F. Badaracco & J. Harms, arXiv:1903.07936

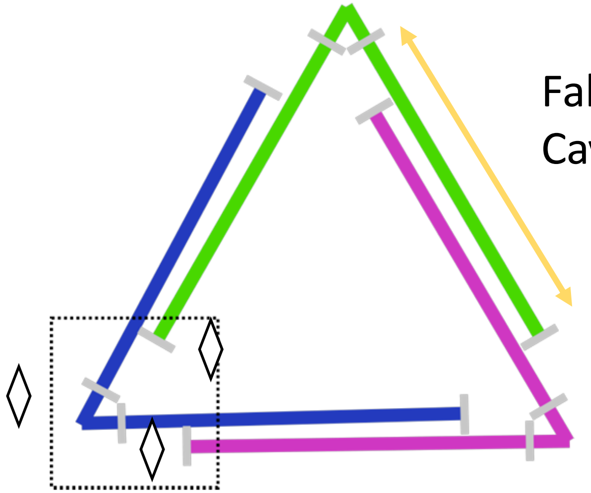
Joint Corner: F. Badaracco et al., arXiv:2310.05709

Wiener Filter

Signal estimate

$$\tilde{s} = W \cdot d \qquad \text{with} \quad W = \langle s d^\dagger \rangle \langle d d^\dagger \rangle^{-1}$$

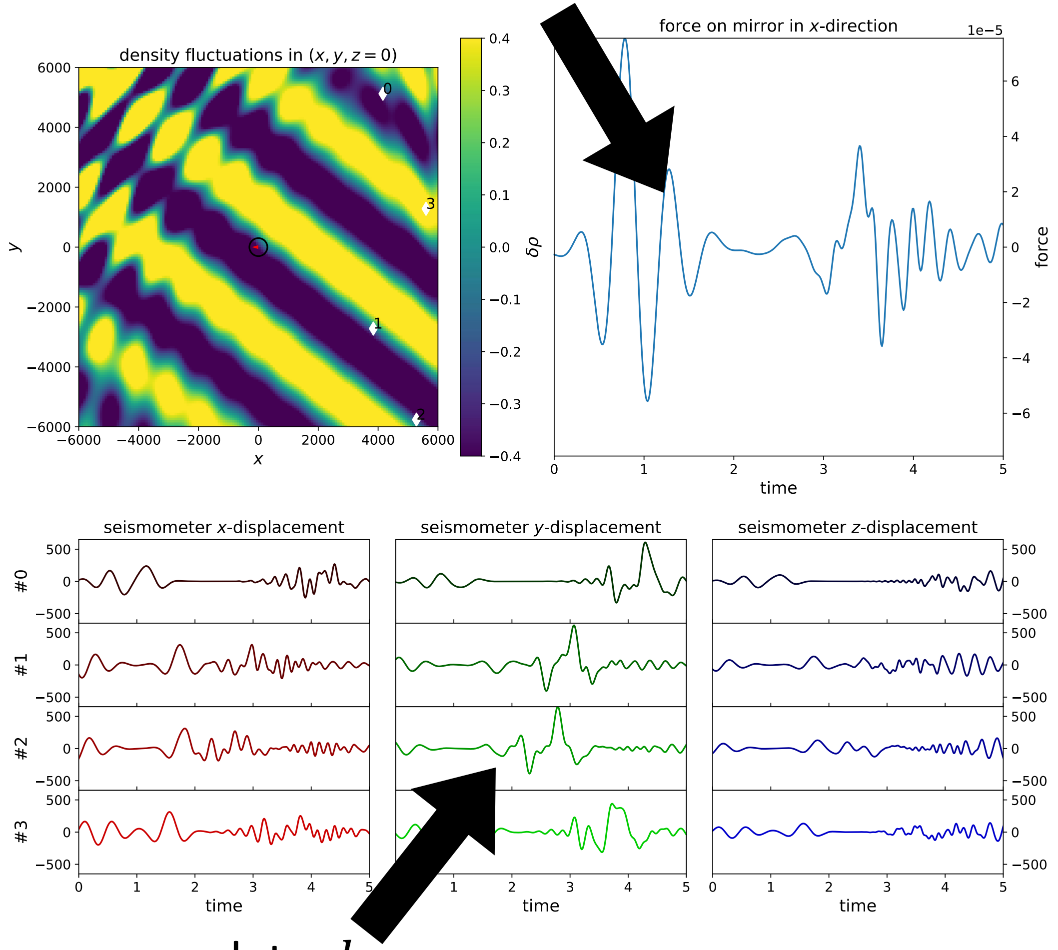
Residual $R(\omega) \equiv \frac{E[(s-\tilde{s})^2]}{E[s^2]} = 1 - \frac{\vec{c}_{ds}^{\,\dagger} \cdot C_{dd}^{-1} \cdot \vec{c}_{ds}}{c_{ss}}(\omega)$



Fabry-Perot-Cavity

Joint Corner

signal s



data d

Assumptions: homogeniety, isotropy, plain waves

→ analytical result

Minimize $\max(R(\omega))$ for the four mirrors

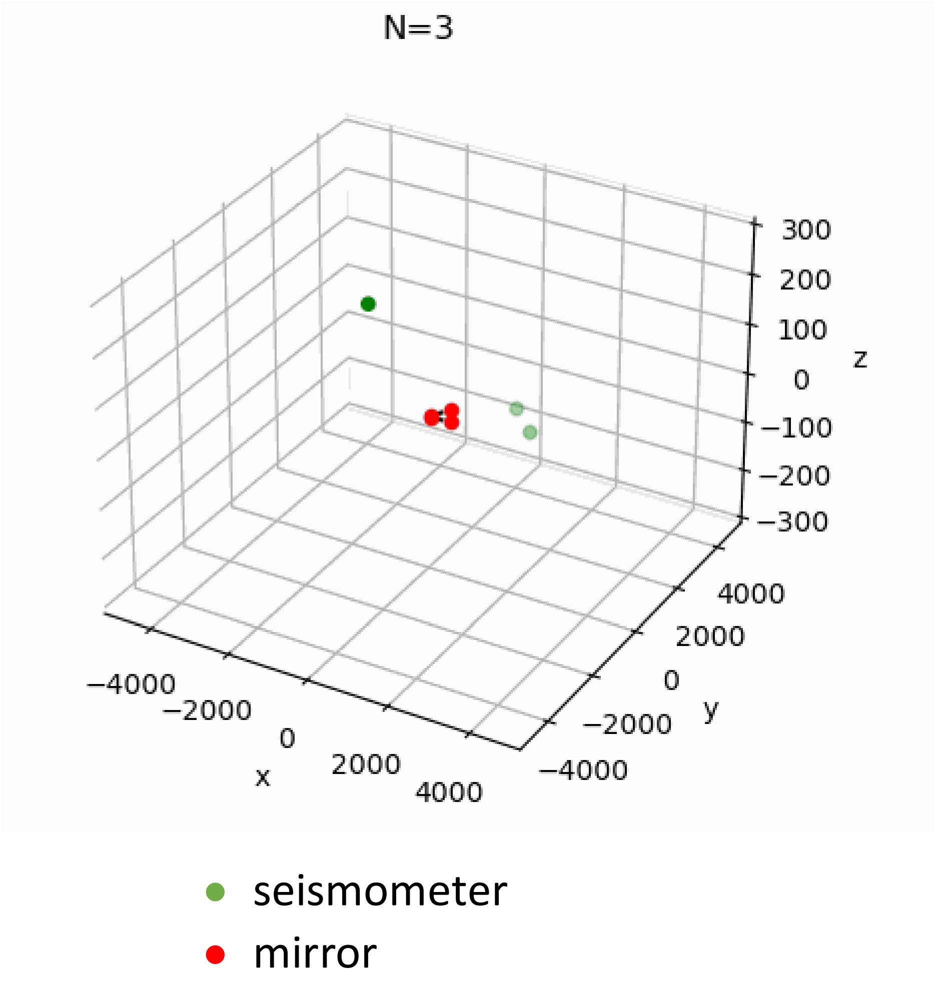
Optimization Algorithm

→Particle Swarm Optimization

(general, fast and reliable for small *N*)

Parameter (benchmark value/range):

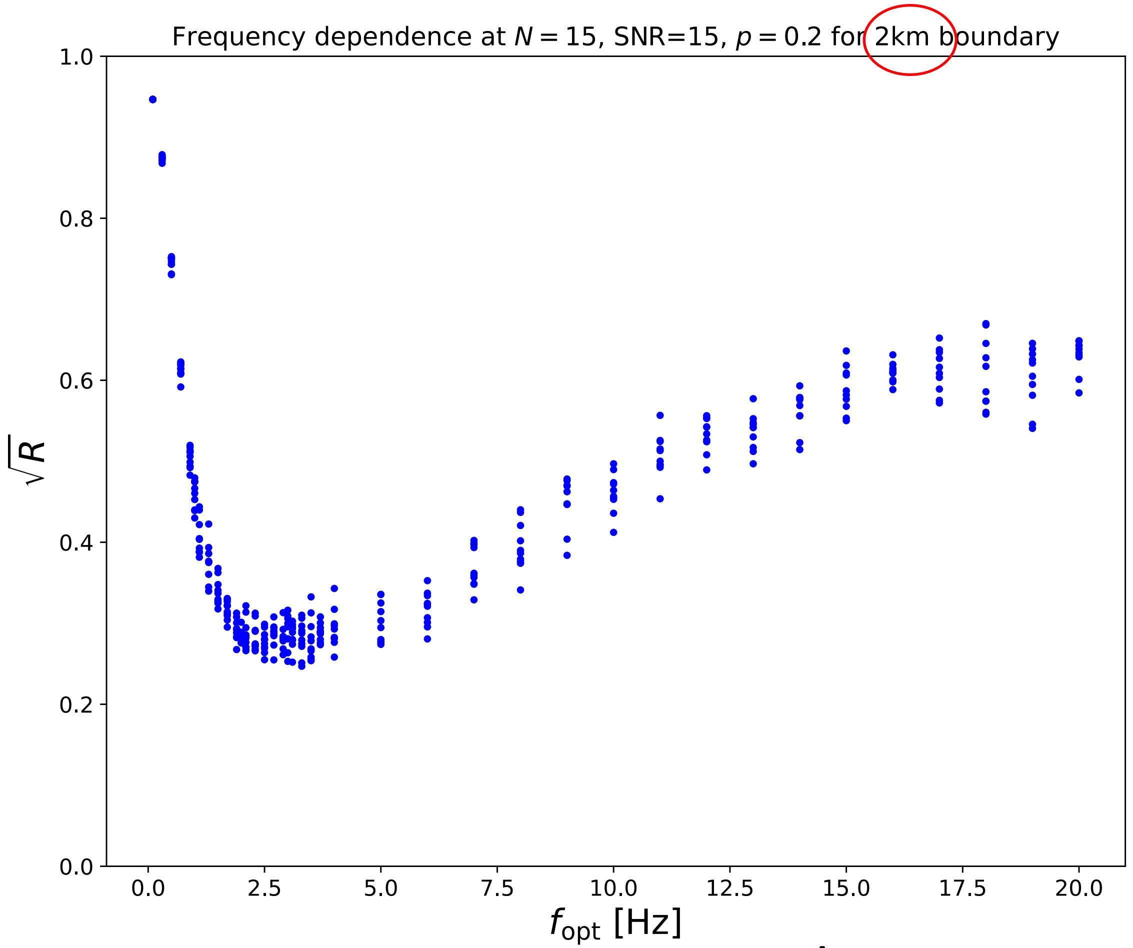
- Number of Seismometers *N* (15/1-100)
- Signal-to-Noise-Ratio of operating seismometers SNR (15/1-30)
- Optimization frequency *f* (1 Hz/1-20 Hz)
- Ratio of P- and S-waves *p* (0.2/0-1)

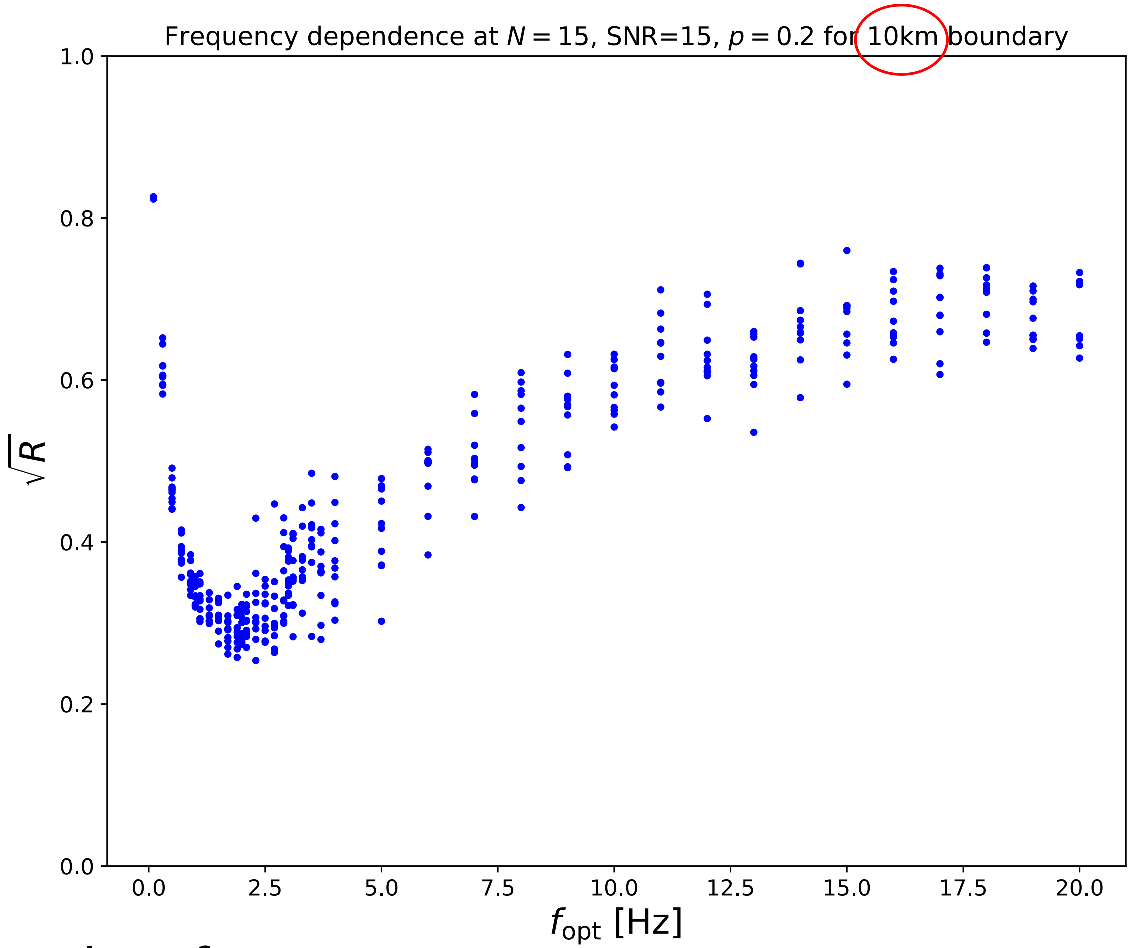


Parameter Studies

What influence do the different parameters have?

Boundary Dependency

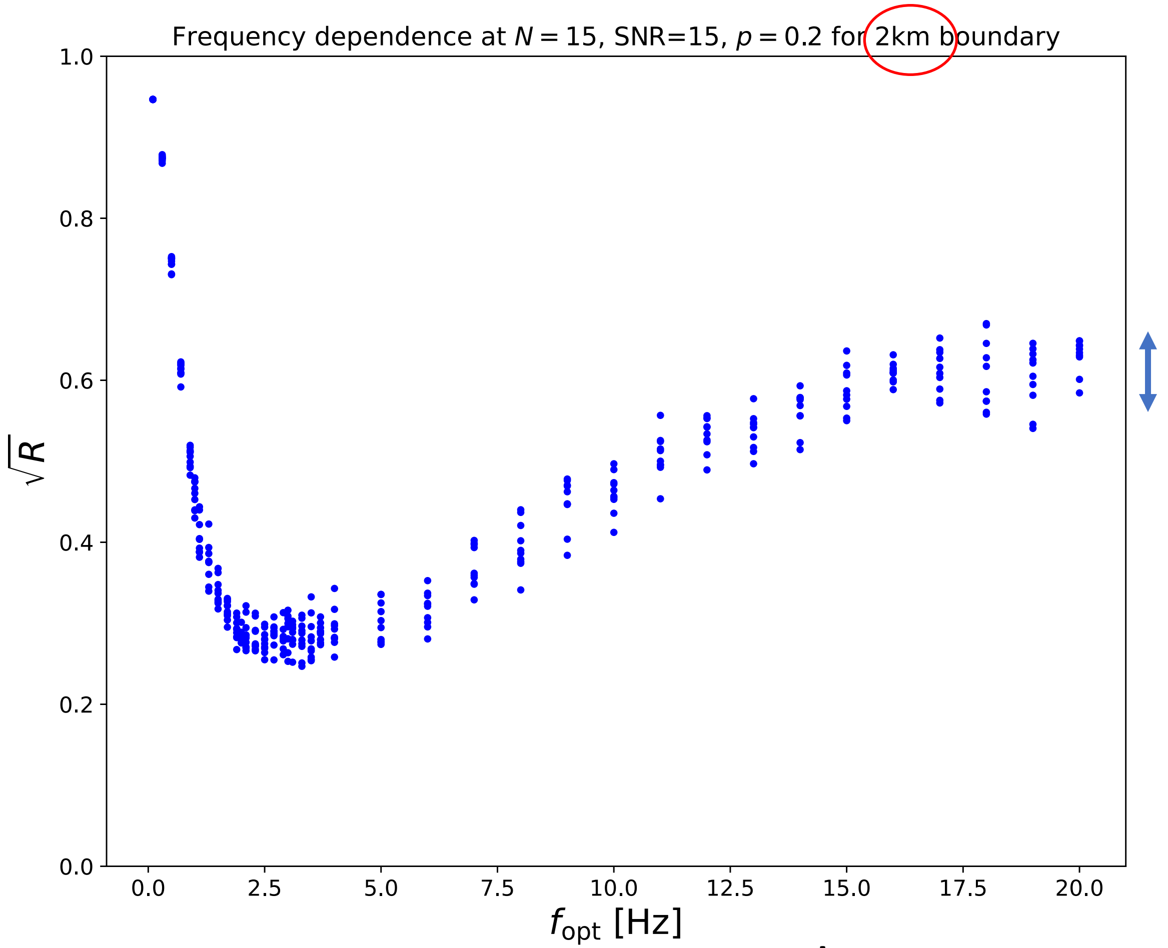


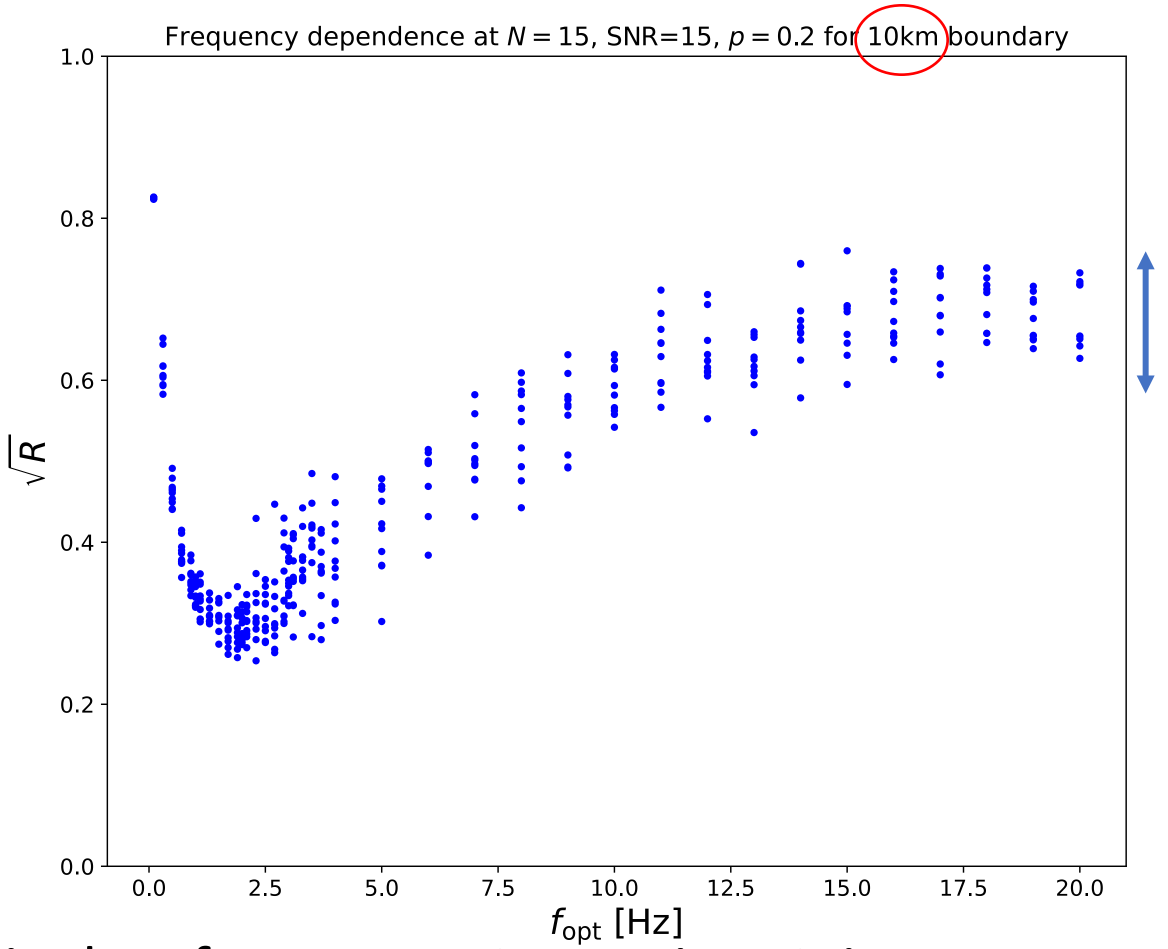


- Seismometers need space to predict low frequency Newtonian Noise

Larger spread and higher residuals due to worse convergence of PSO in larger volume…

Boundary Dependency





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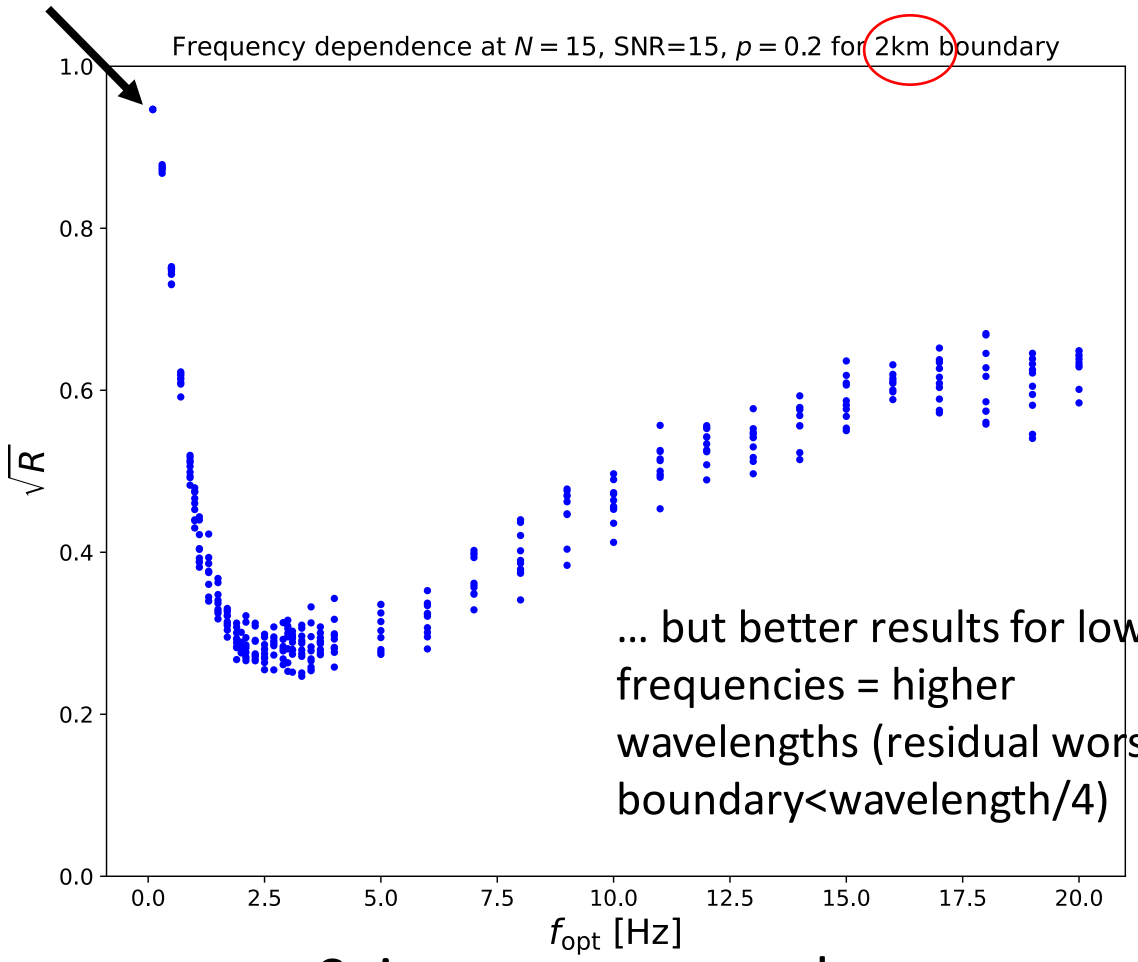
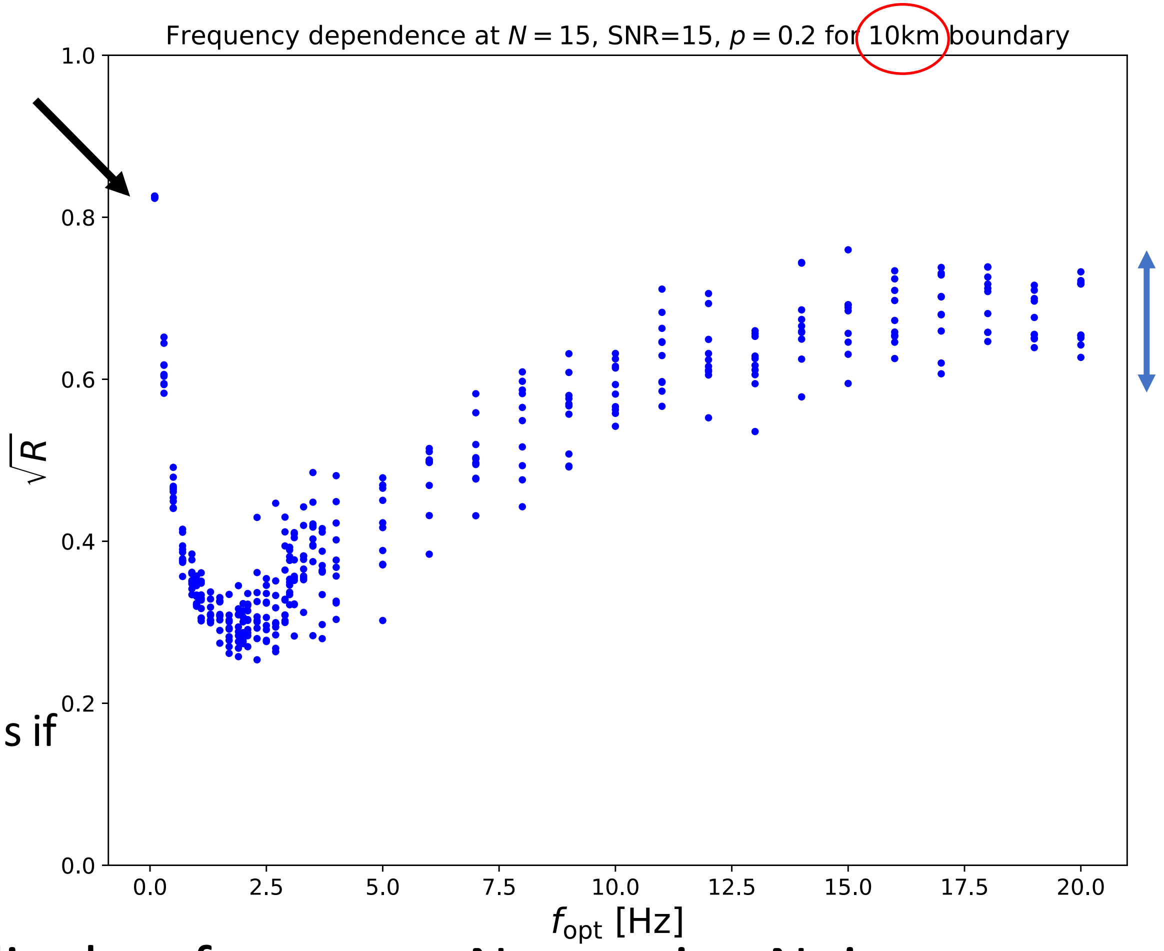
Boundary Dependency

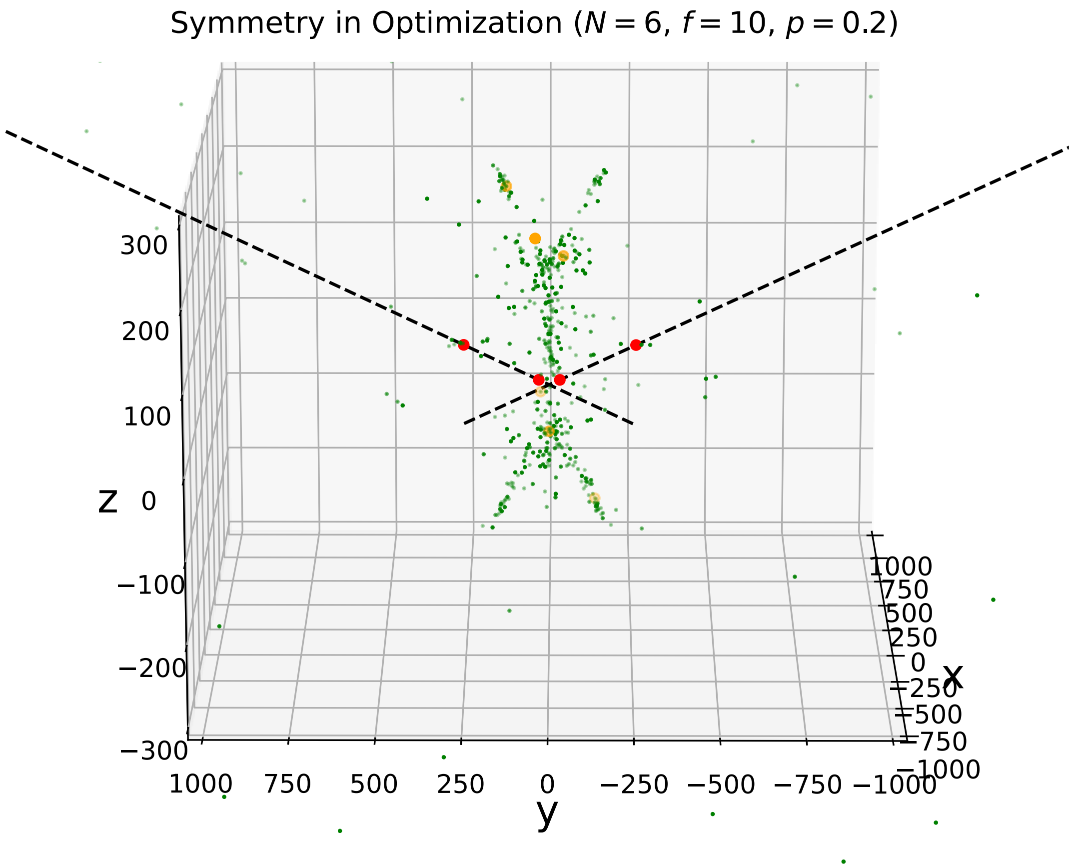


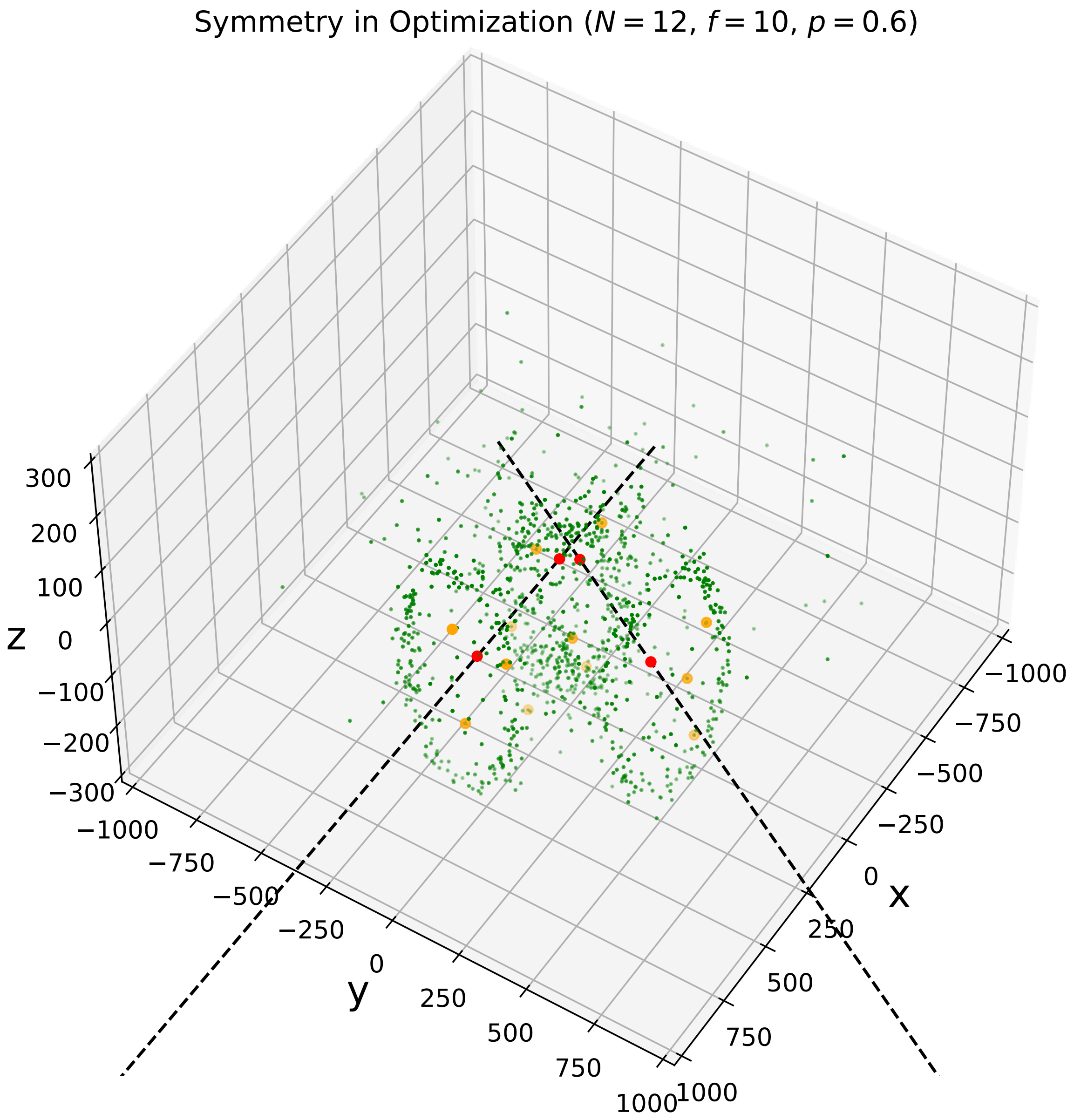
… but better results for low frequencies = higher wavelengths (residual worsens if boundary<wavelength/4)



- Seismometers need space to predict low frequency Newtonian Noise

Optimal Positions

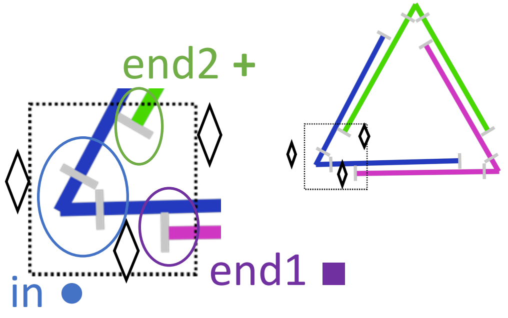


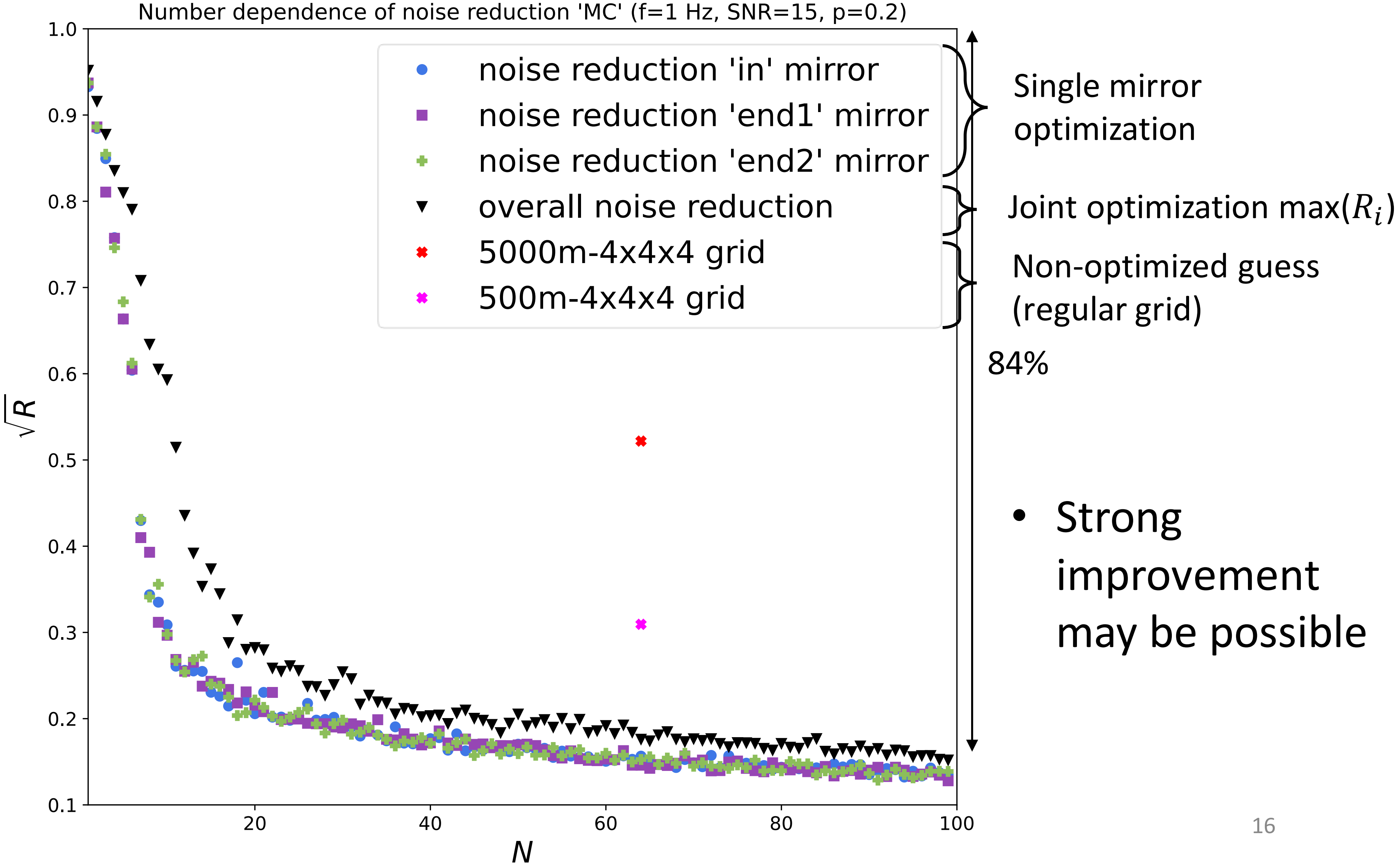


- There is a natural occuring symmetry in the optimized positions

(see also F. Badaracco et al., arXiv:2310.05709)

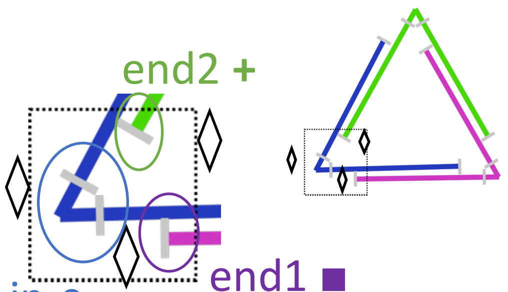
Dependence on Number of Seismometers



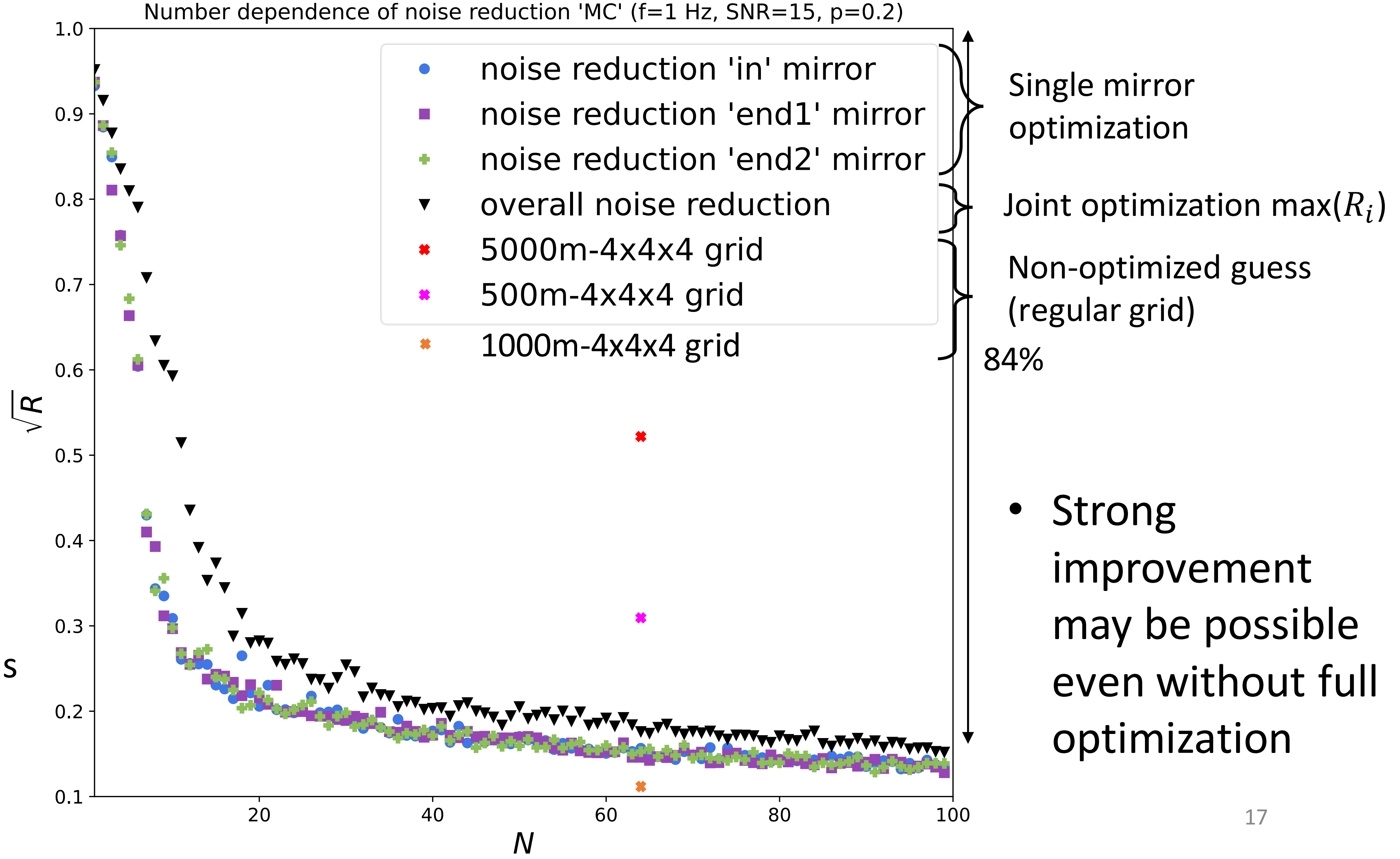


- Strong improvement may be possible

Dependence on Number of Seismometers

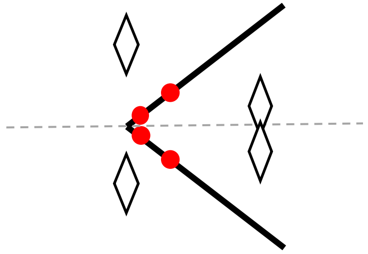


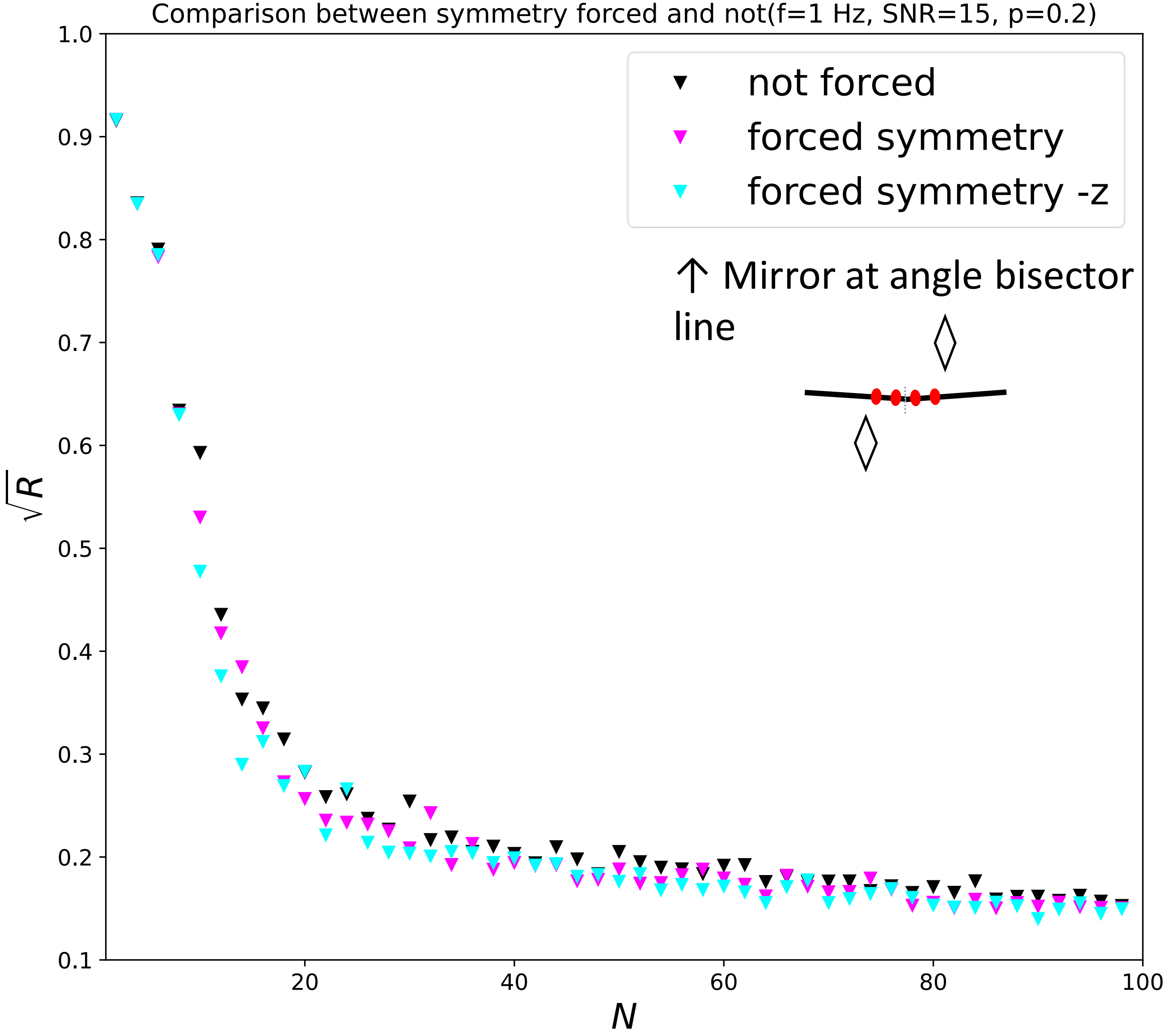
- Limited optimization with PSO for $N > 20$
- In the process of testing alternative optimization algorithms (e.g. LBFGS)



- Strong improvement may be possible even without full optimization

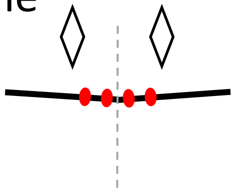
Dependence on Number of Seismometers





← As before

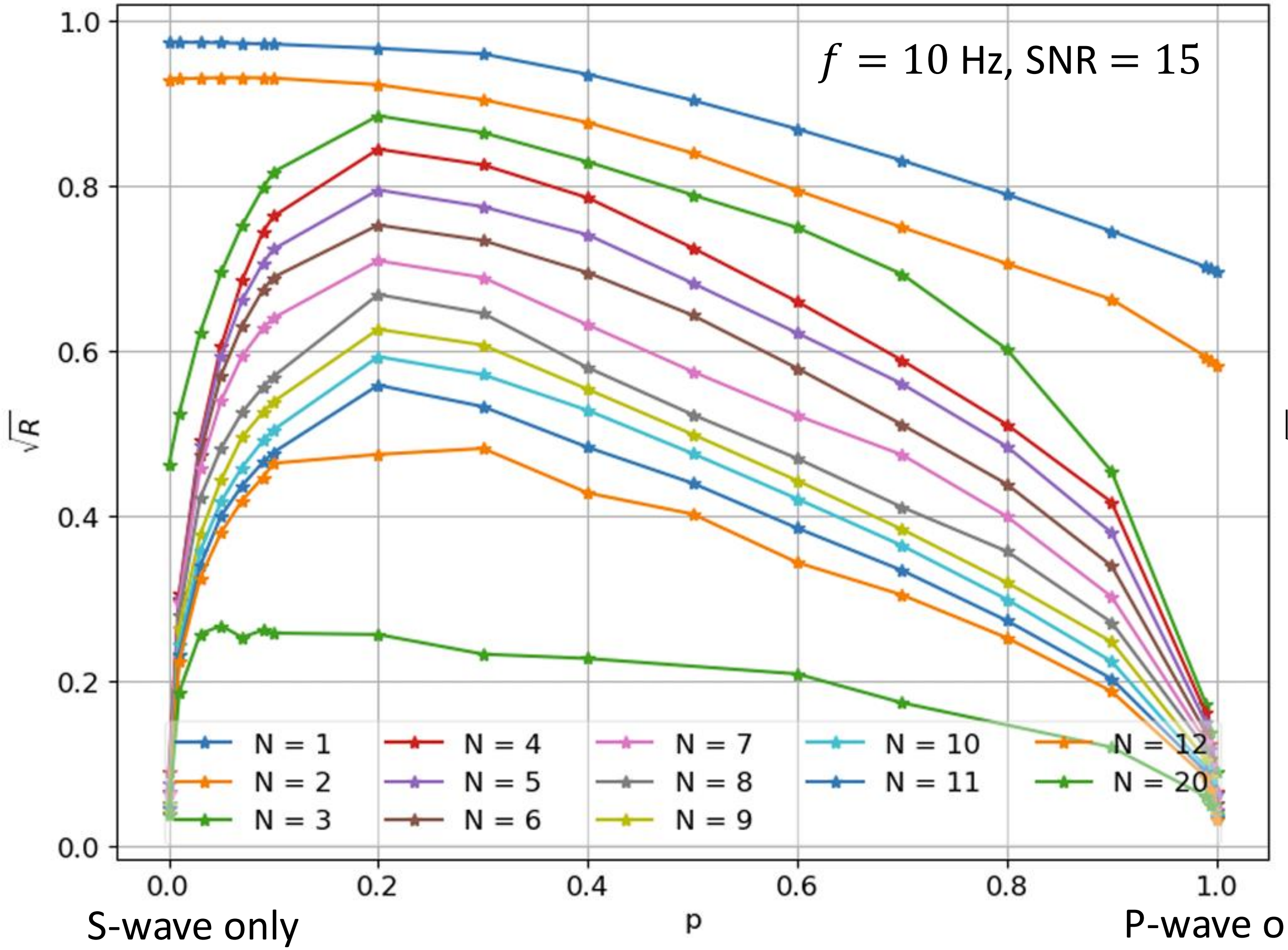
← Mirror at angle bisecting plane



- Exploiting the inherent symmetry can improve the optimization

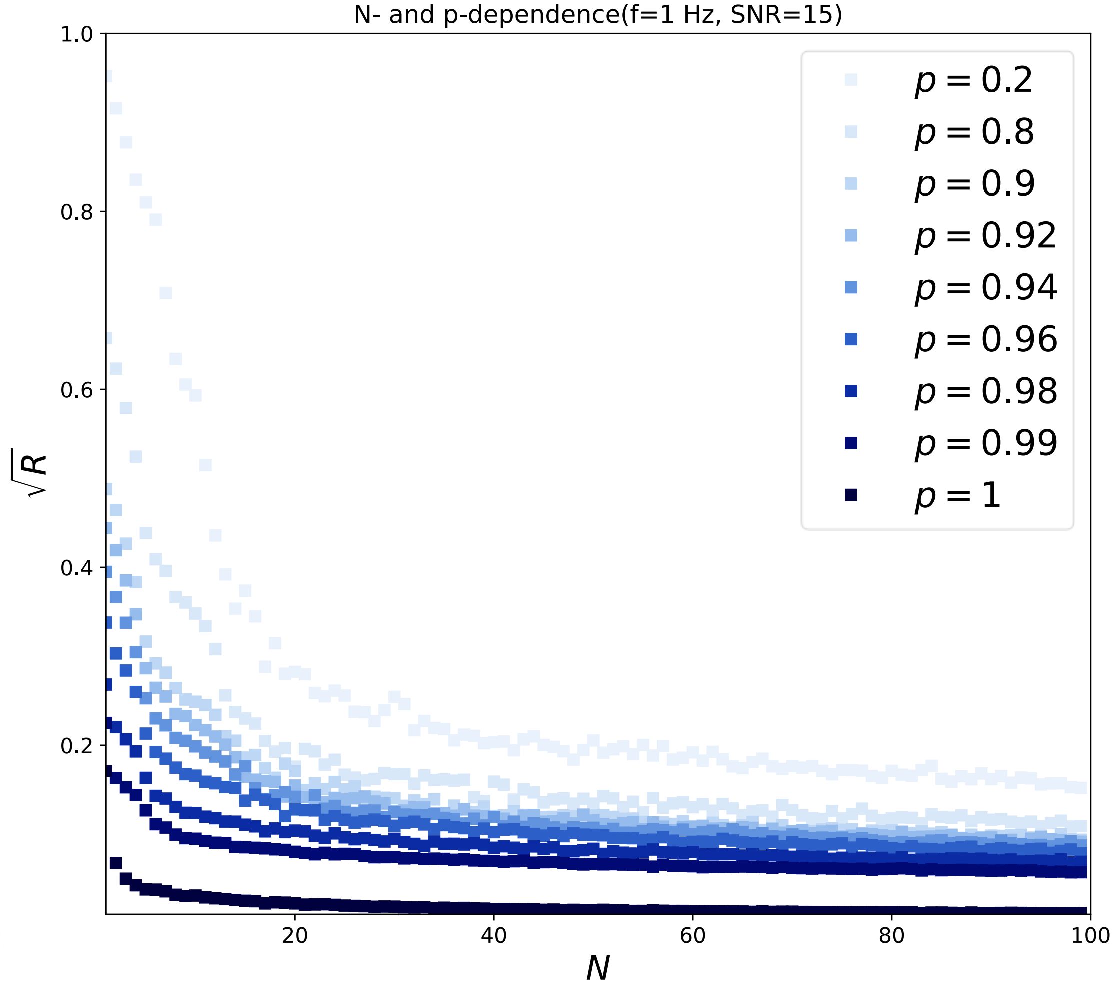
p Dependency

- The first x seismometers optimize the translation gain of certain features (1. Triangulation, 2. P- and S-differentiation, 3. Noise)



S-wave only

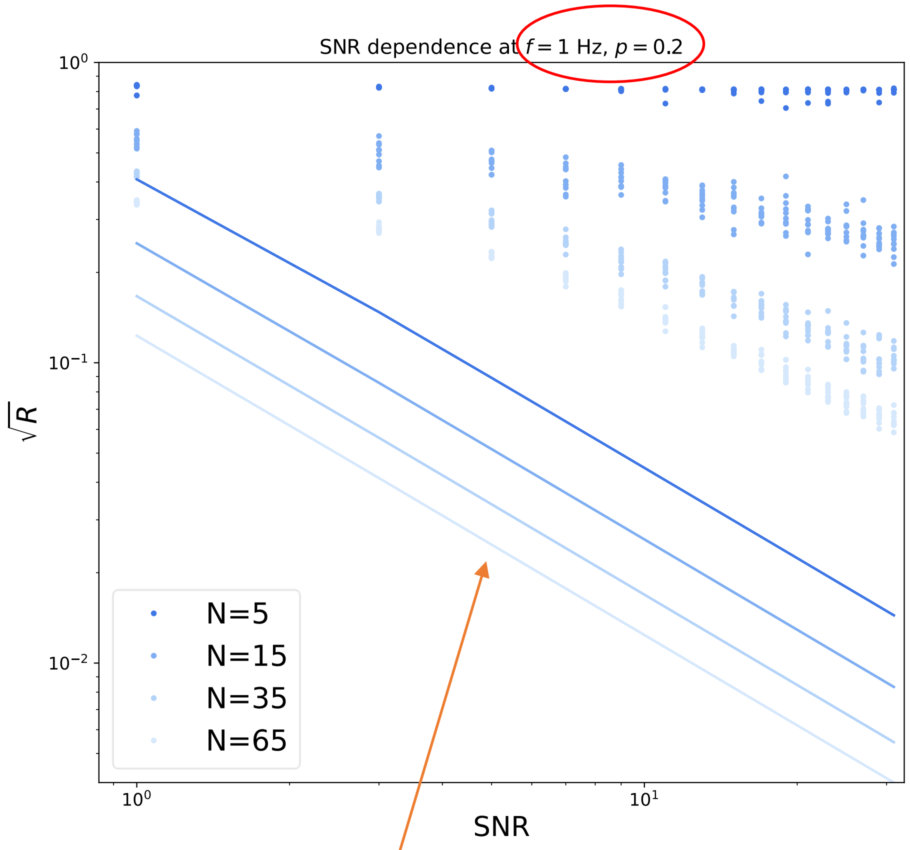
P-wave only

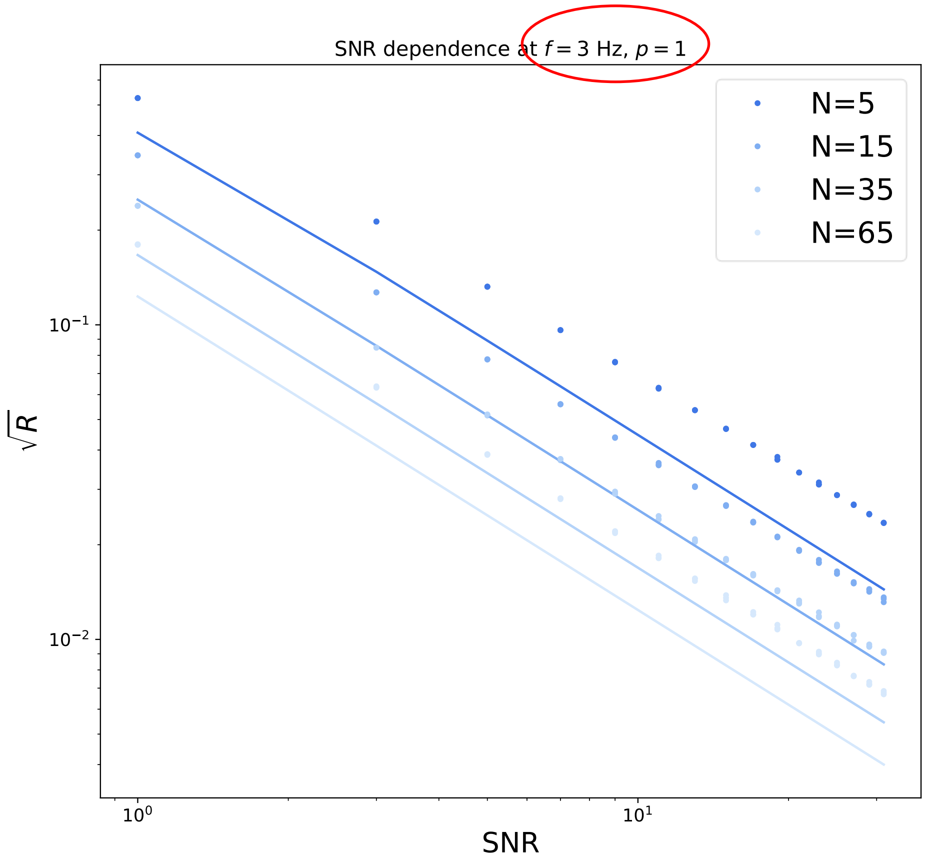


F. Badaracco et al., arXiv:2310.05709

SNR Dependency

- R decreases with N and SNR as expected

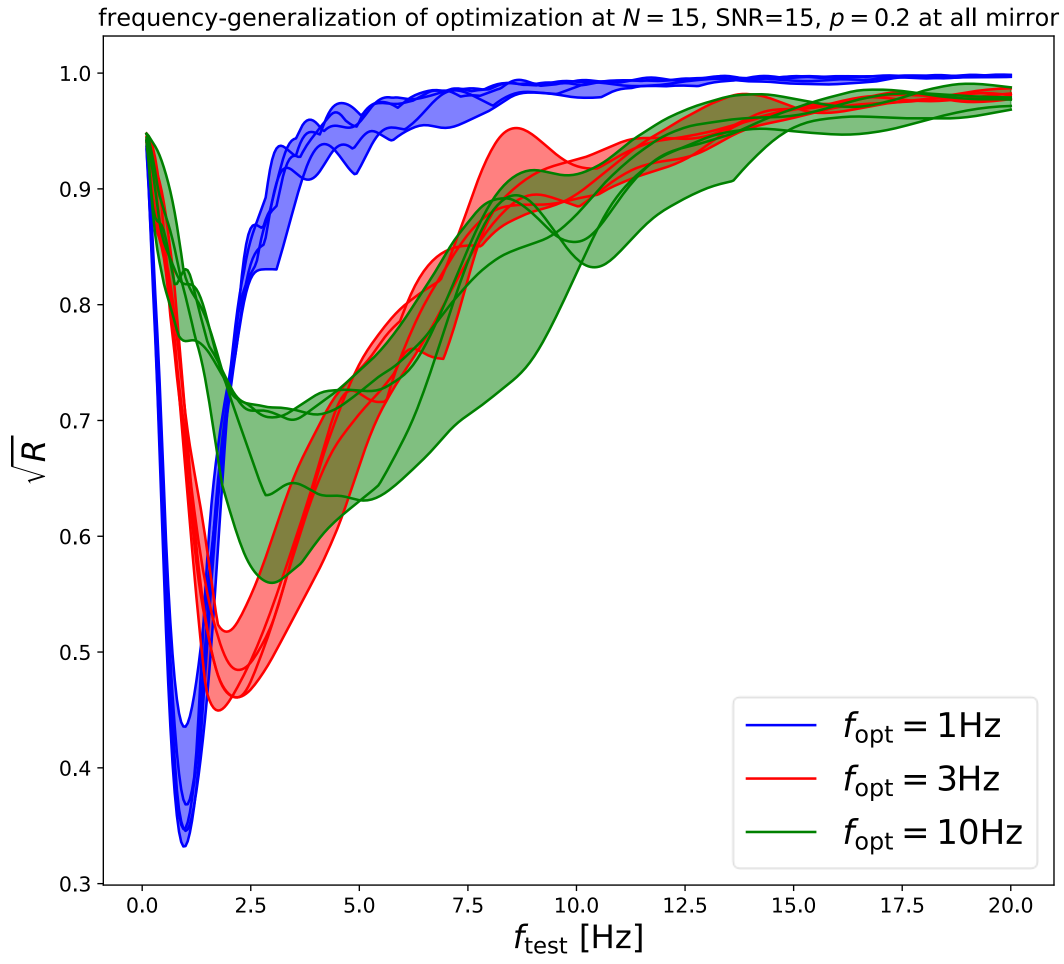


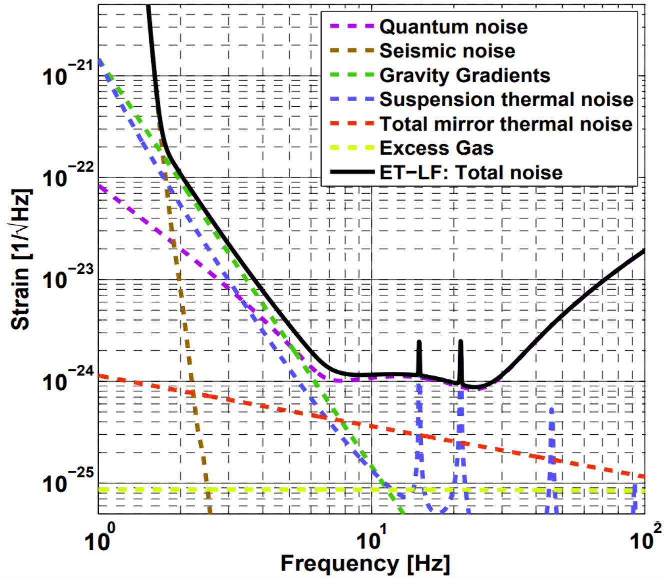


- Well above SNR-limit (mitigation only limited by noise, not translation) when p!=1 or 0

$$R = \frac{1}{1 + N \cdot \mathrm{SNR}^2}$$

Frequency Generalization

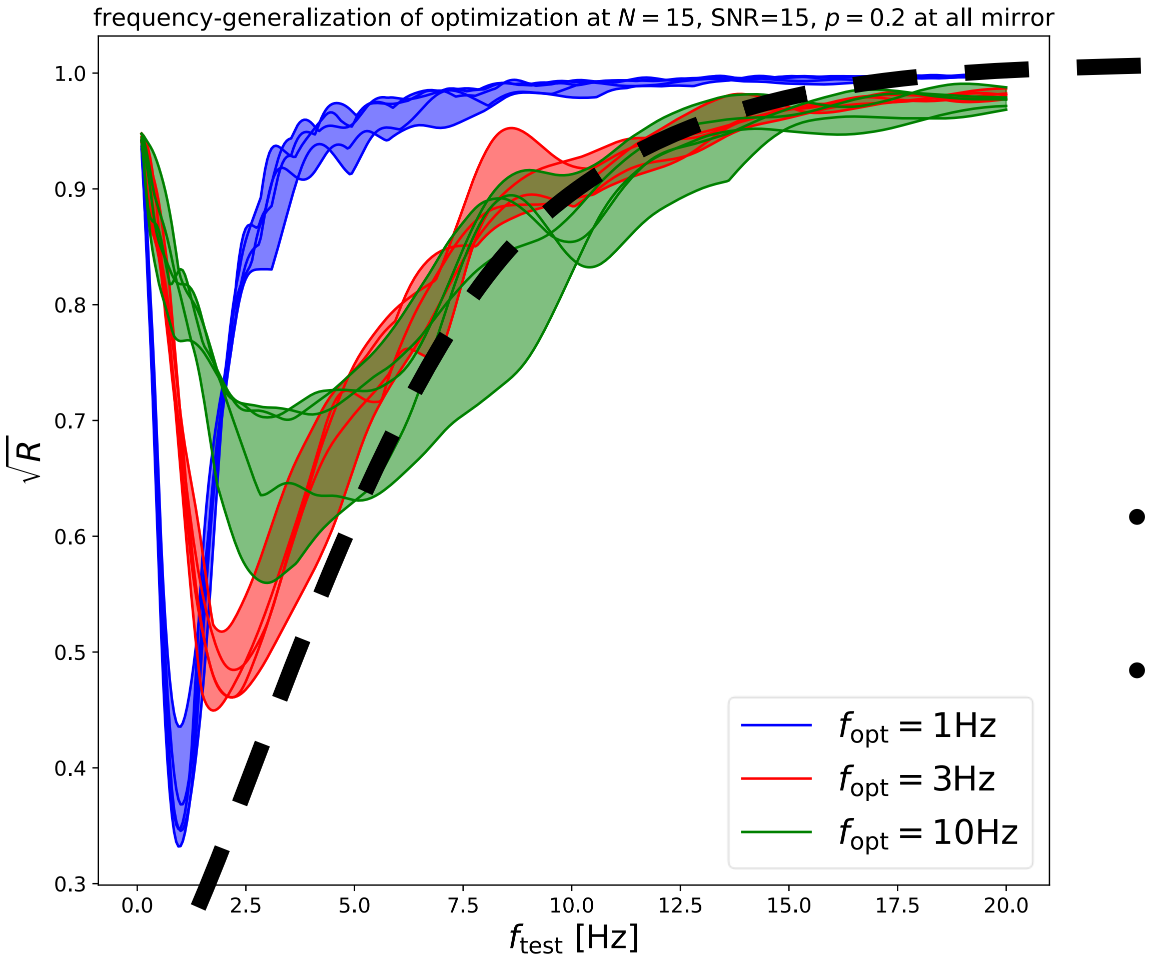


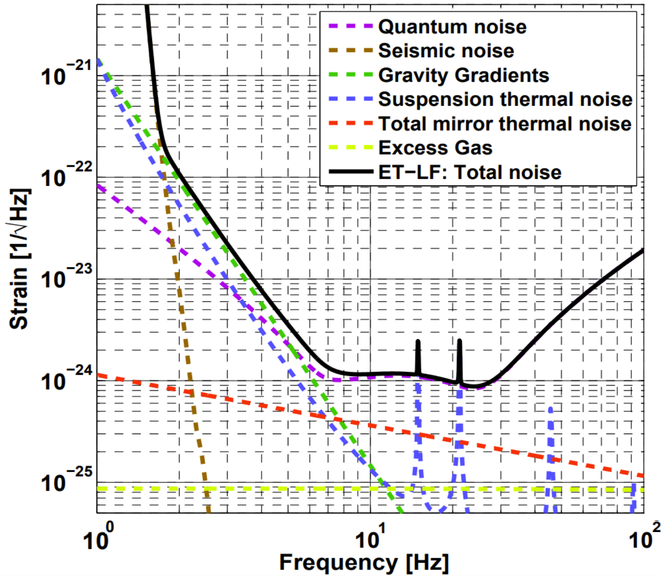


Hild et al., arXiv:1012.0908

- Depends strongly on actual seismometer positions

Frequency Generalization





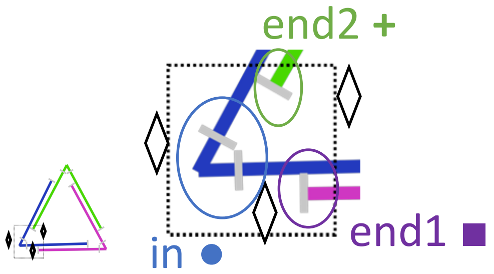
Hild et al., arXiv:1012.0908

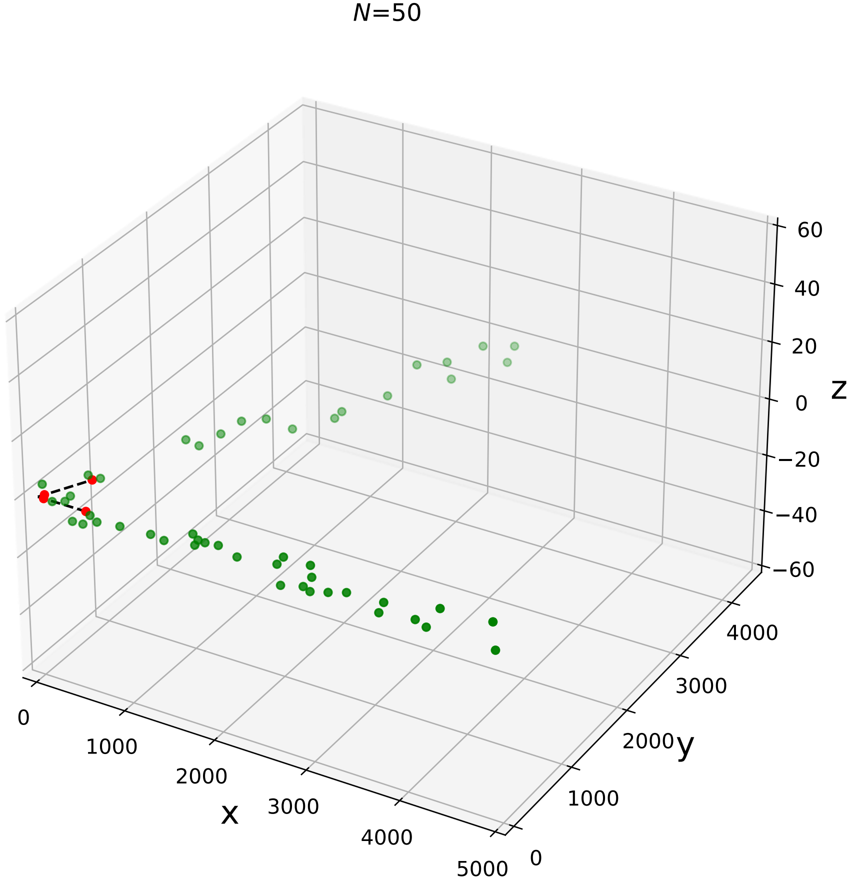
- Depends strongly on actual seismometer positions
- Works differently well for different optimization frequencies

In-Tunnel Optimization

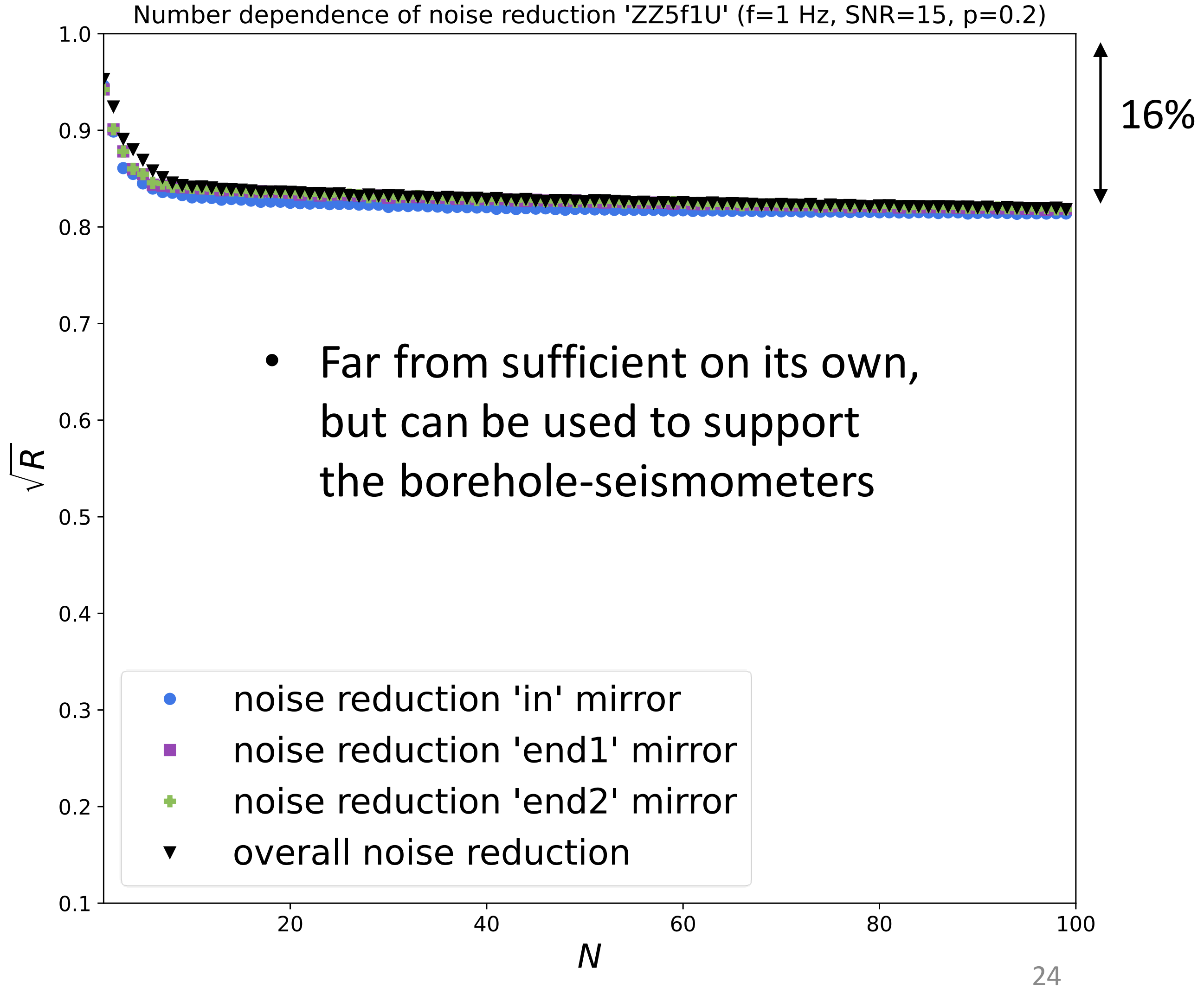
How much improvement can we still archieve if we limit the optimization procedure to the tunnel surface?

Two-Cylinder-Model



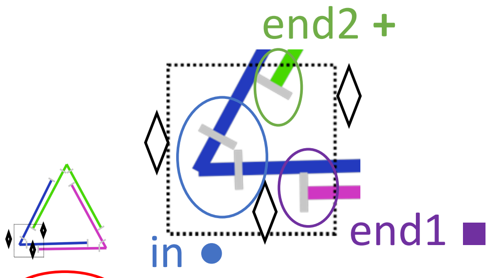


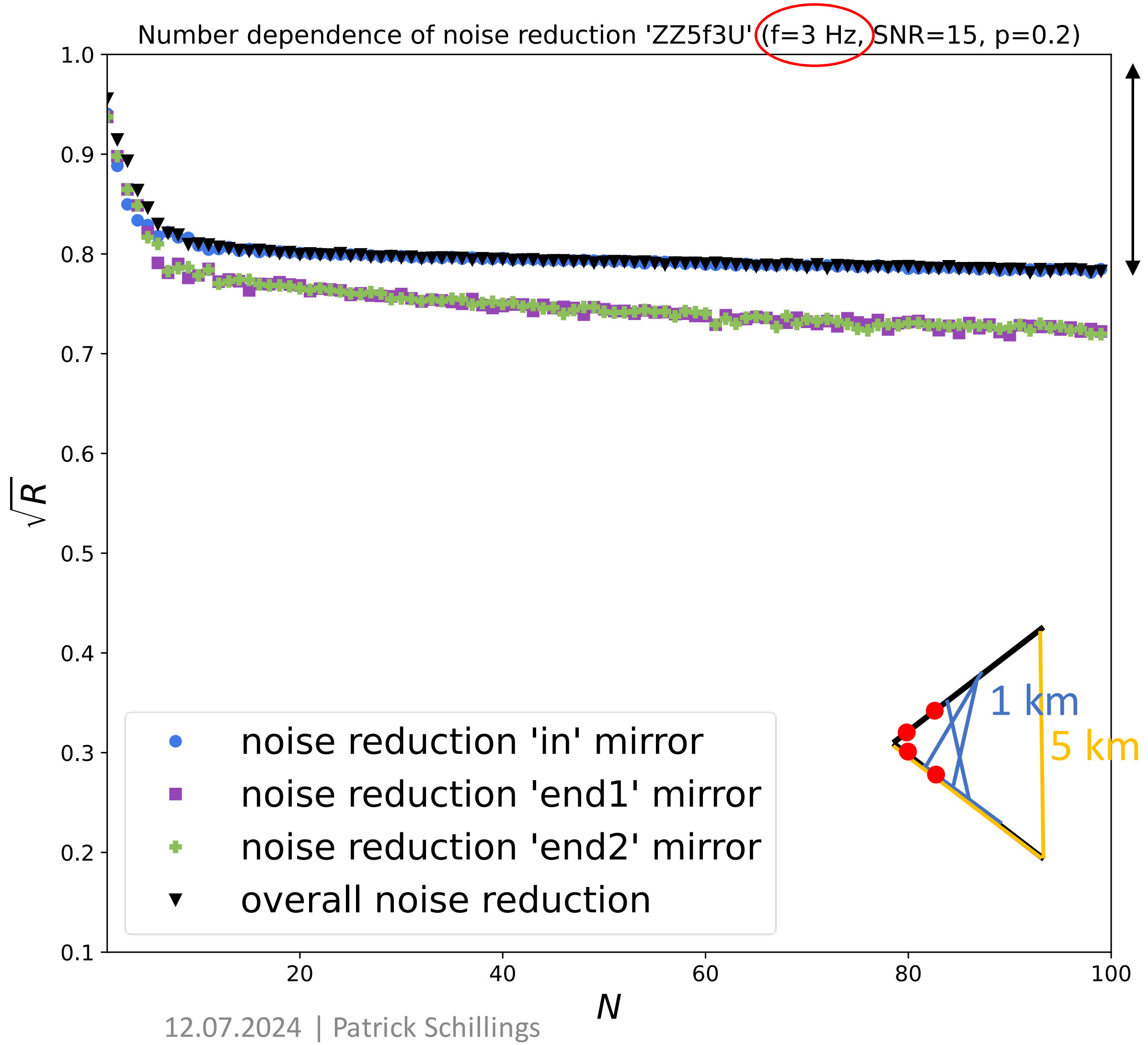
Tunnel radius = 5m

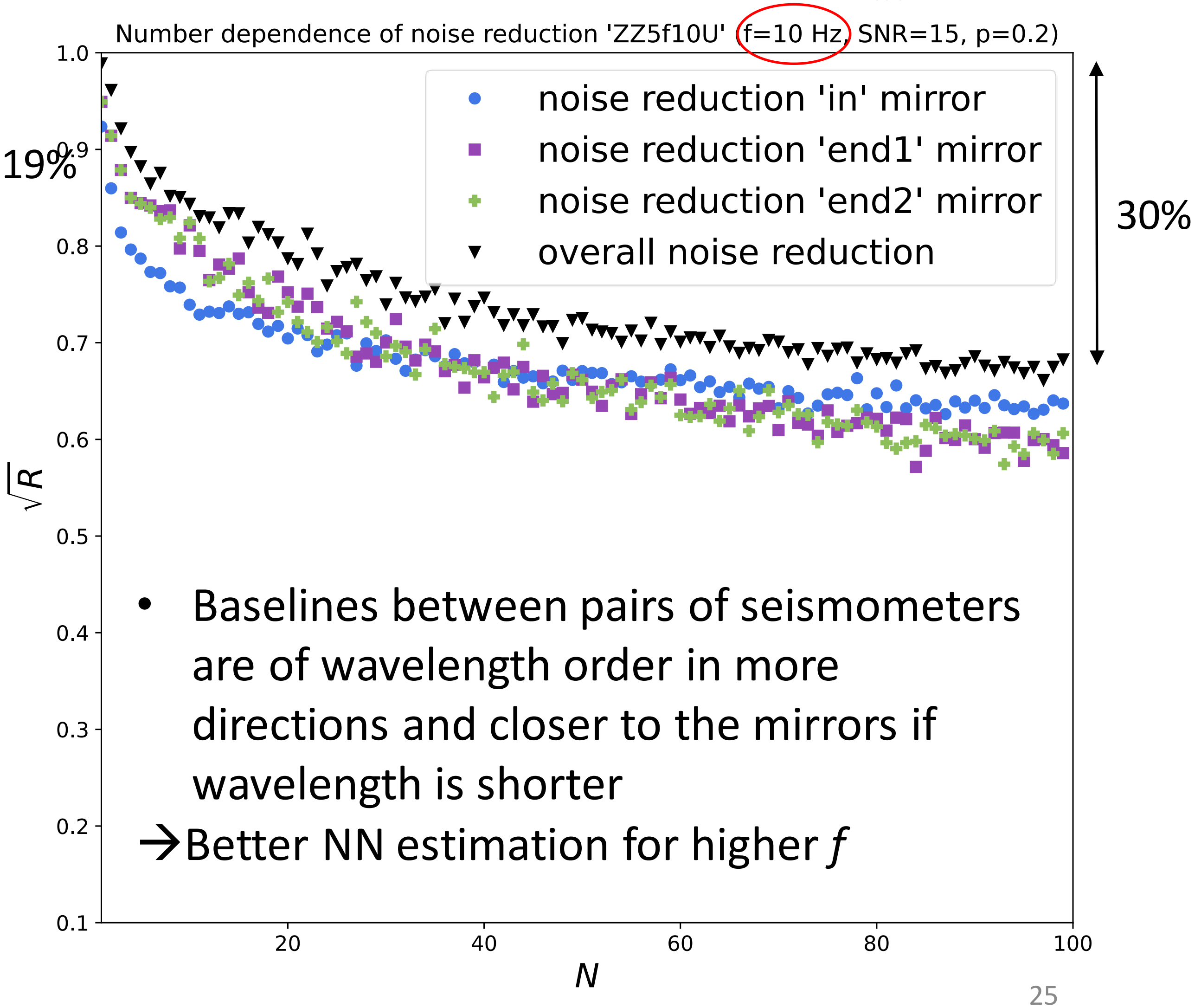


- Far from sufficient on its own, but can be used to support the borehole-seismometers

...for Different Frequencies



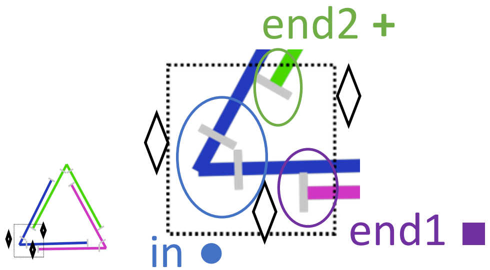


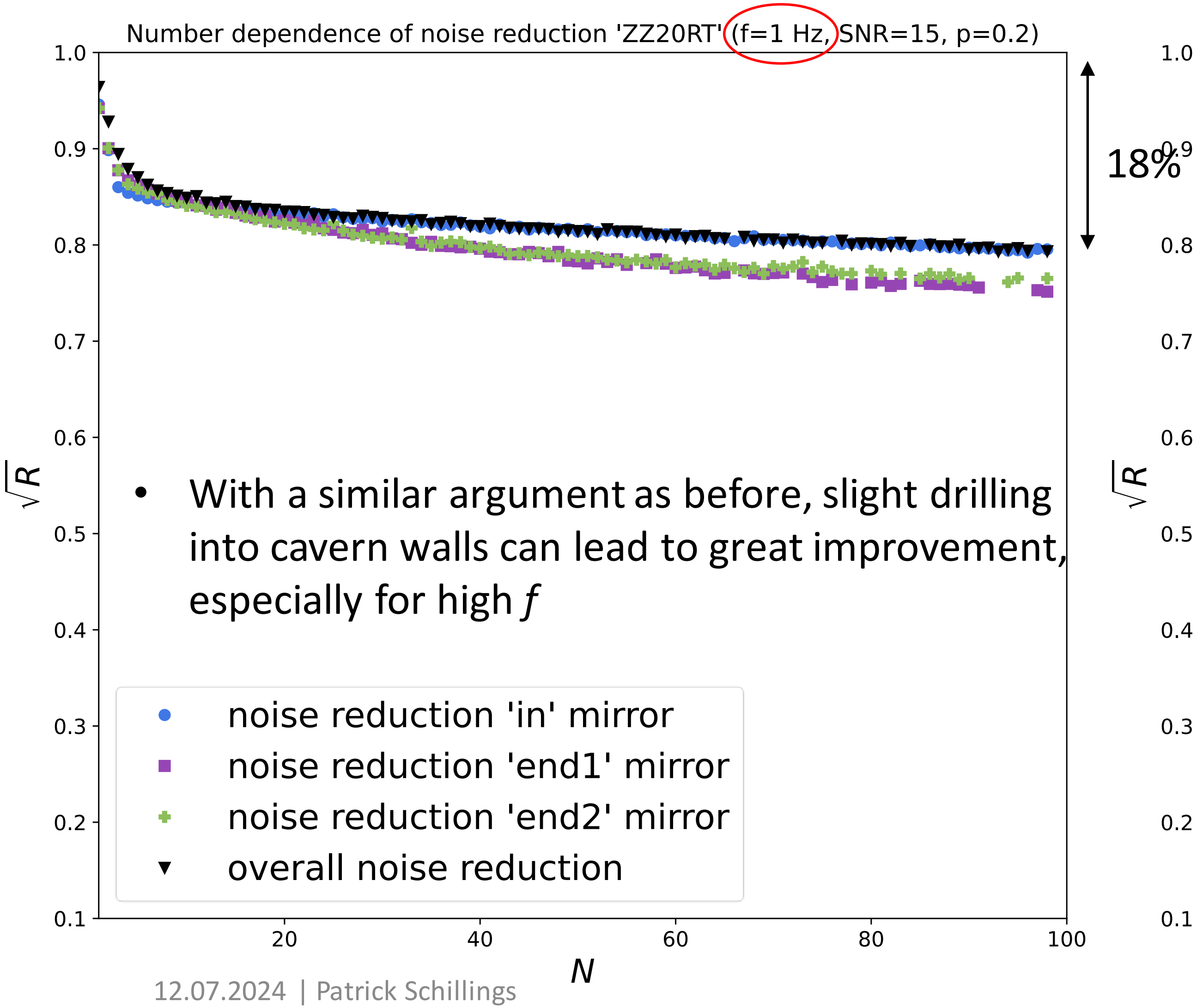


- Baselines between pairs of seismometers are of wavelength order in more directions and closer to the mirrors if wavelength is shorter

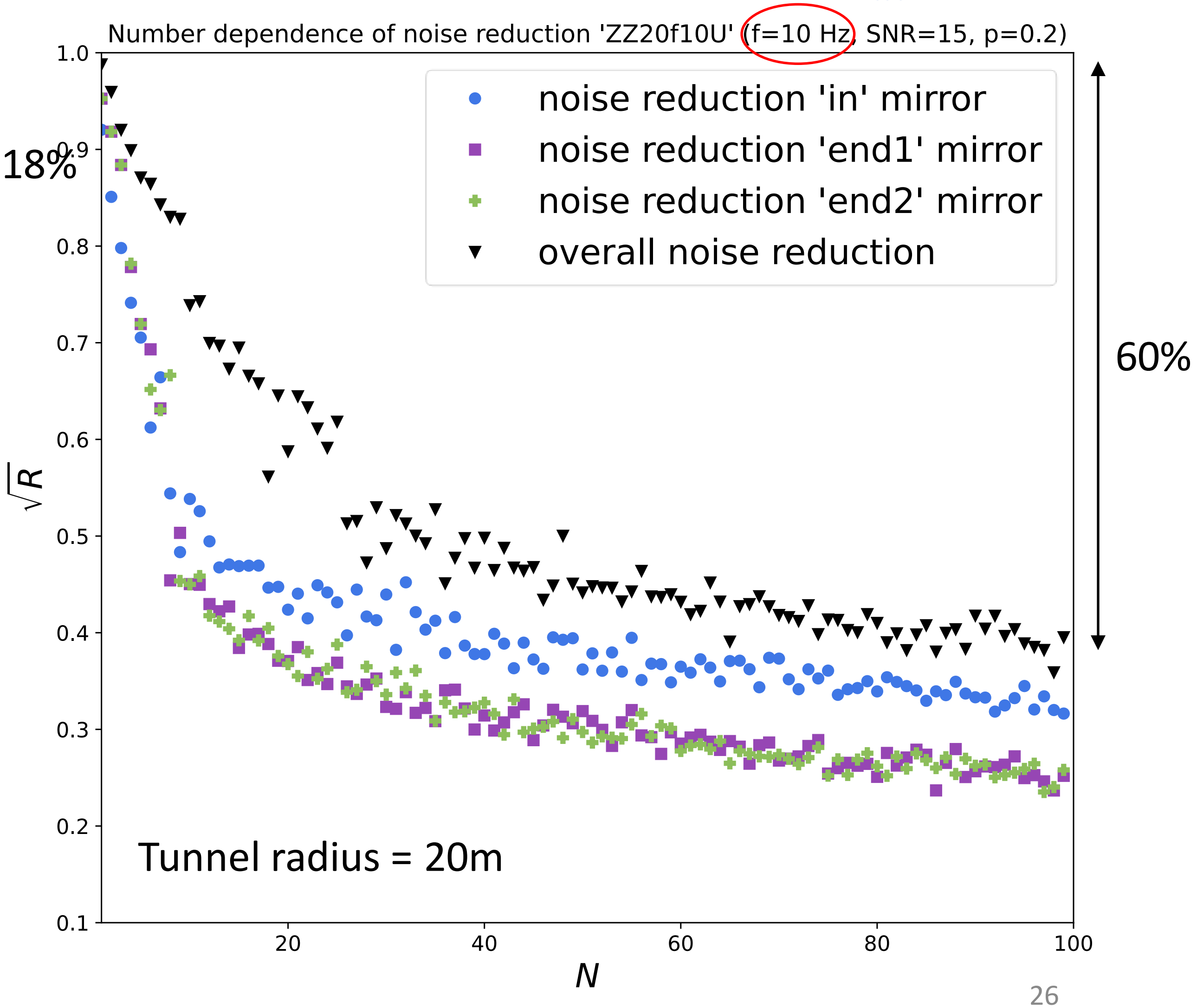
→Better NN estimation for higher *f*

…for Different Tunnel Radii

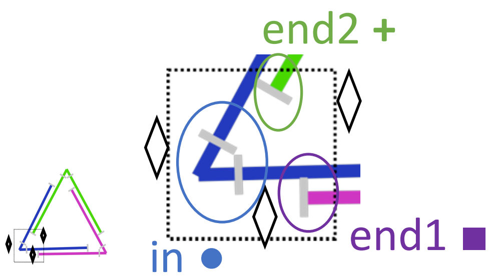




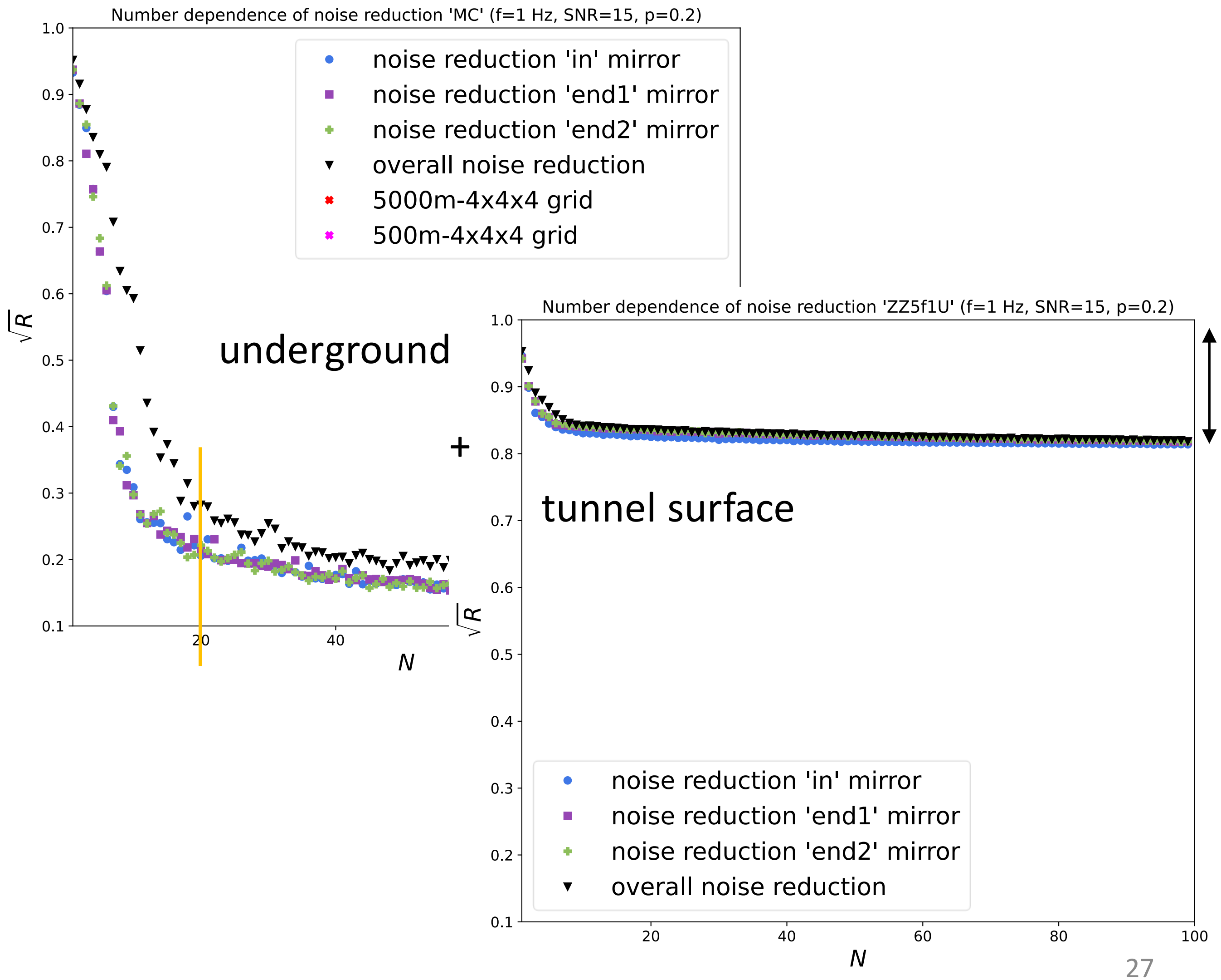
- With a similar argument as before, slight drilling into cavern walls can lead to great improvement, especially for high *f*



Gain from in-Tunnel Optimization



- 20 → 17 boreholes with additional in-tunnel seismometers
- Similar effects with already placed seismometers
- Combined in-tunnel and volume optimization with regard to actual cost (€) is in progress



Simulations

Towards a more realistic optimization

Seismic Simulation

- Real-time prediction of Newtonian Noise for wave packages, moving away from the analytic calculation

Simulation of density fluctuations generated by p waves from far away, time-limited sources

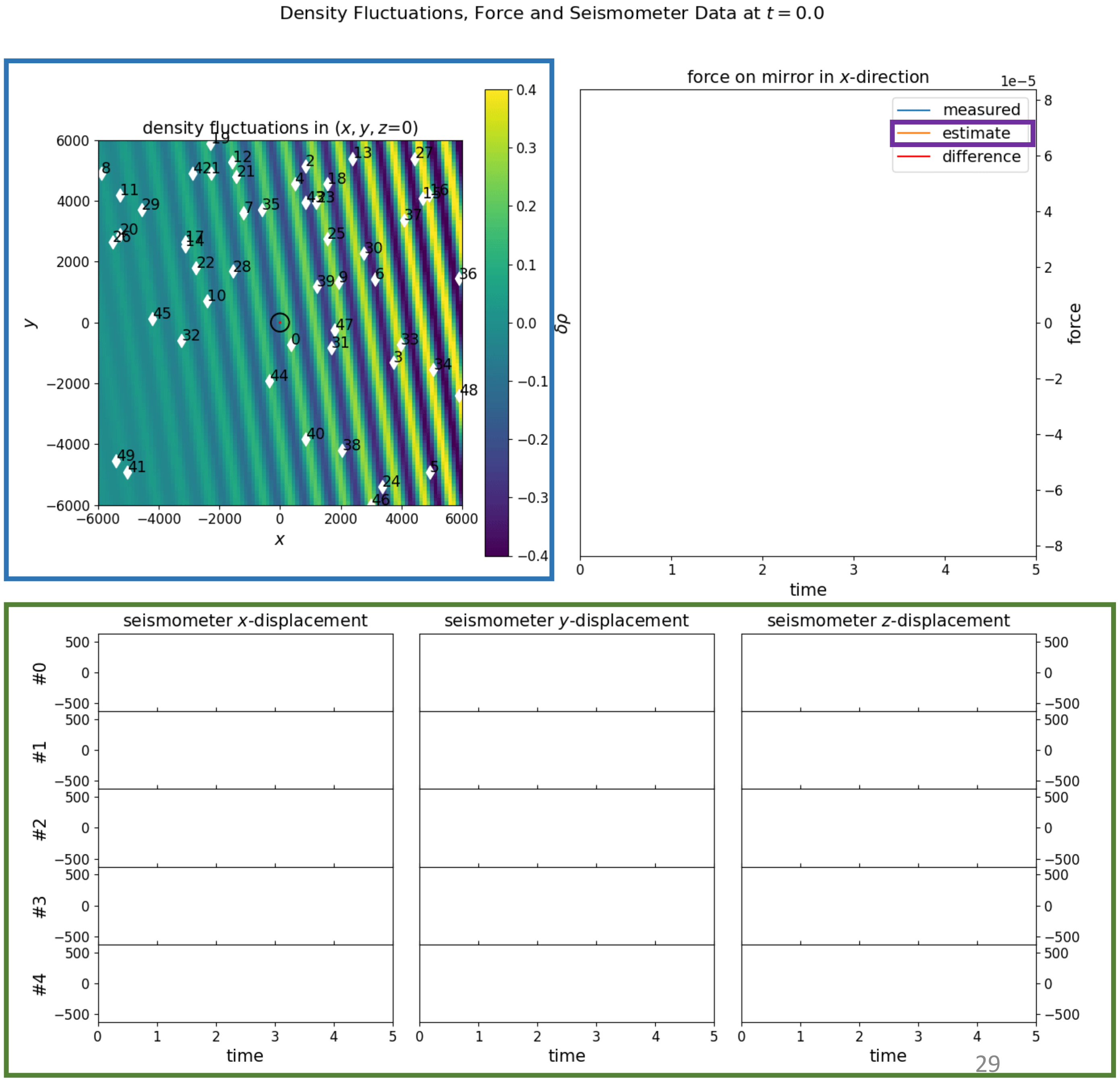
Seismometer projection in z=0-plane

Cavern

Force on mirror (x-direction)

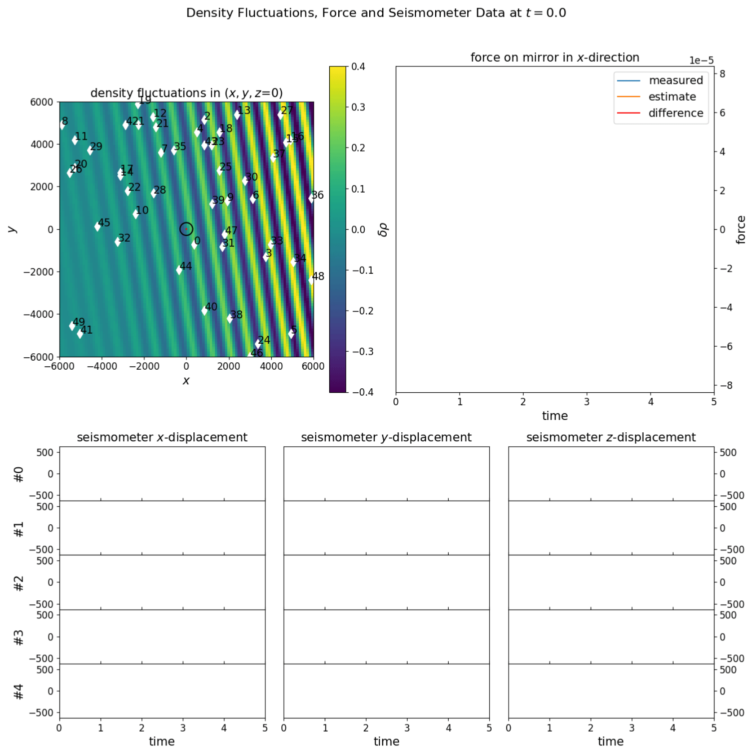
Example seismometer data

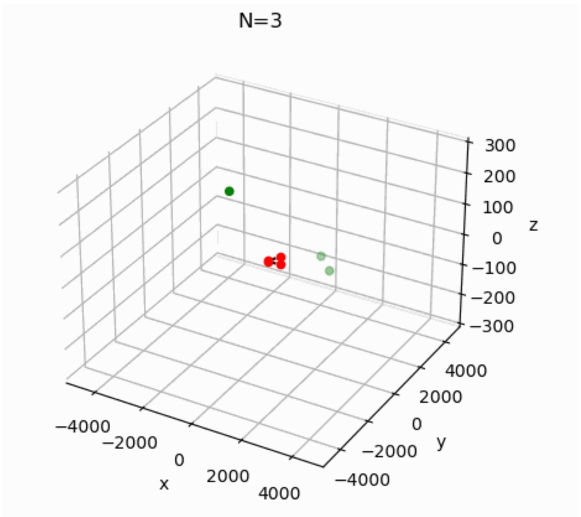
WF*seismometer data (WF from correlations of large number of previous simulations)



Summary

- Many results by Francesca, Jan et al. reproduced for < 20 seismometers
- Optimization for > 20 seismometers now possible but probably needs other optimization methods → ongoing work with gradient-based methods
- Performed studies of adding seismometers in tunnels → can reduce number of necessary boreholes
- Working towards a more realistic optimization based on simulations



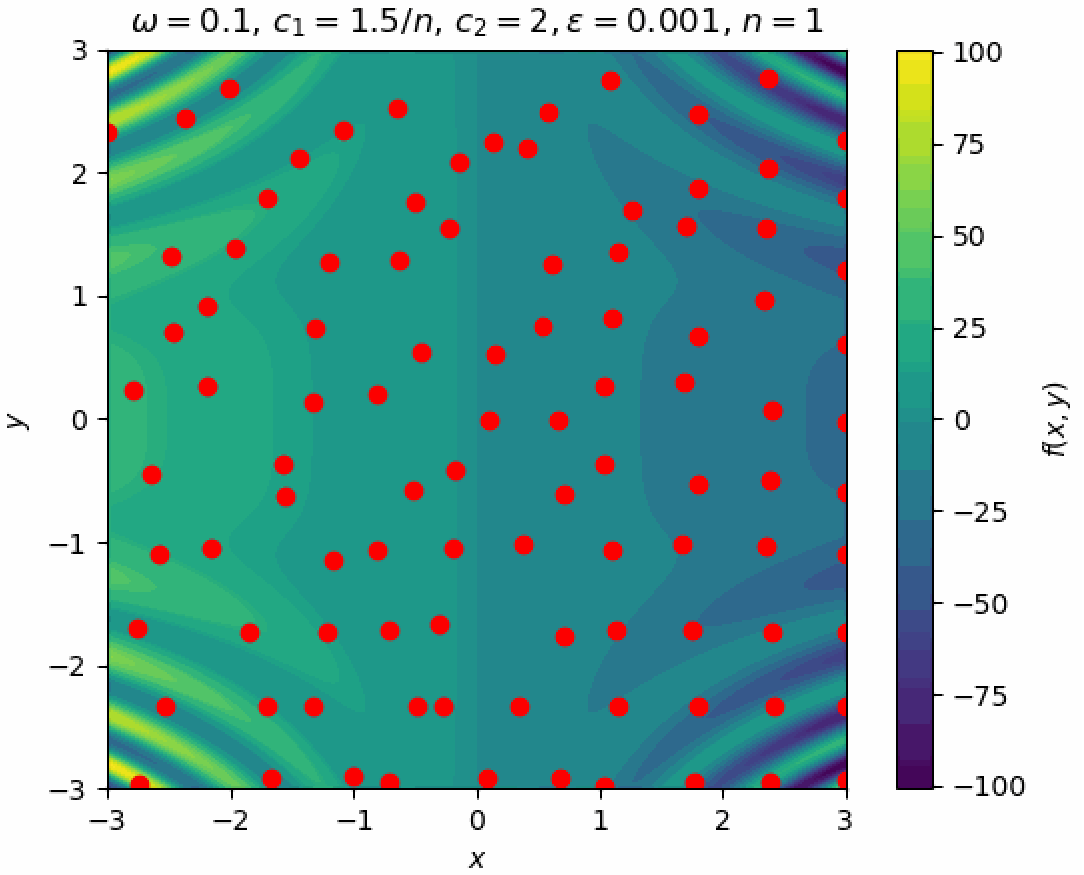


Backup

Particle Swarm Optimization

For each particle:

$$\vec{v}_{n+1} = \omega\vec{v}_n + c_1 r_1(\vec{r}_{pb} - \vec{r}_n) + c_2 r_2(\vec{r}_{gb} - \vec{r}_n)$$



$\vec{v}_n$ - change in particle position in step n

$\vec{r}_n$ - particle position in step n

$\vec{r}_{pb}$ - position of the personal best value, this particle has seen

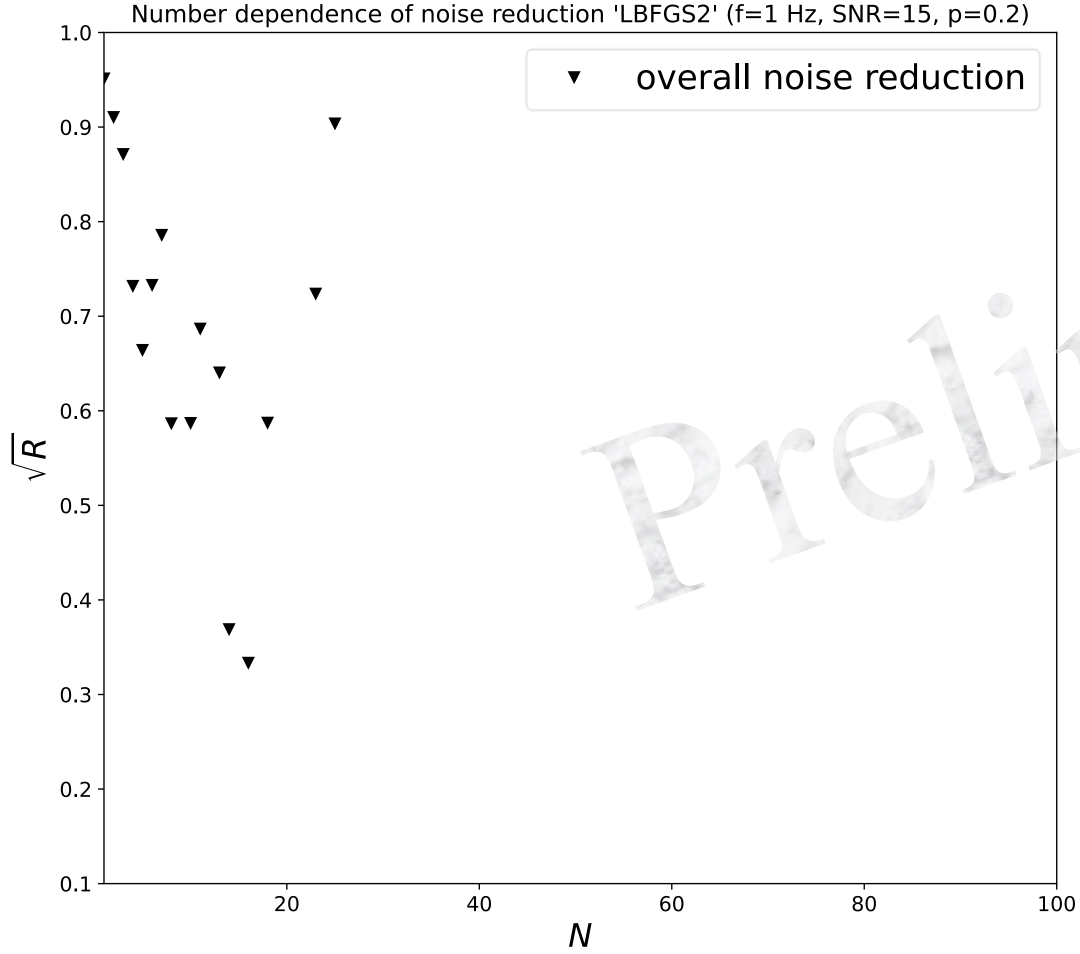
$\vec{r}_{gb}$ - position of the global best value, one of the particles has seen

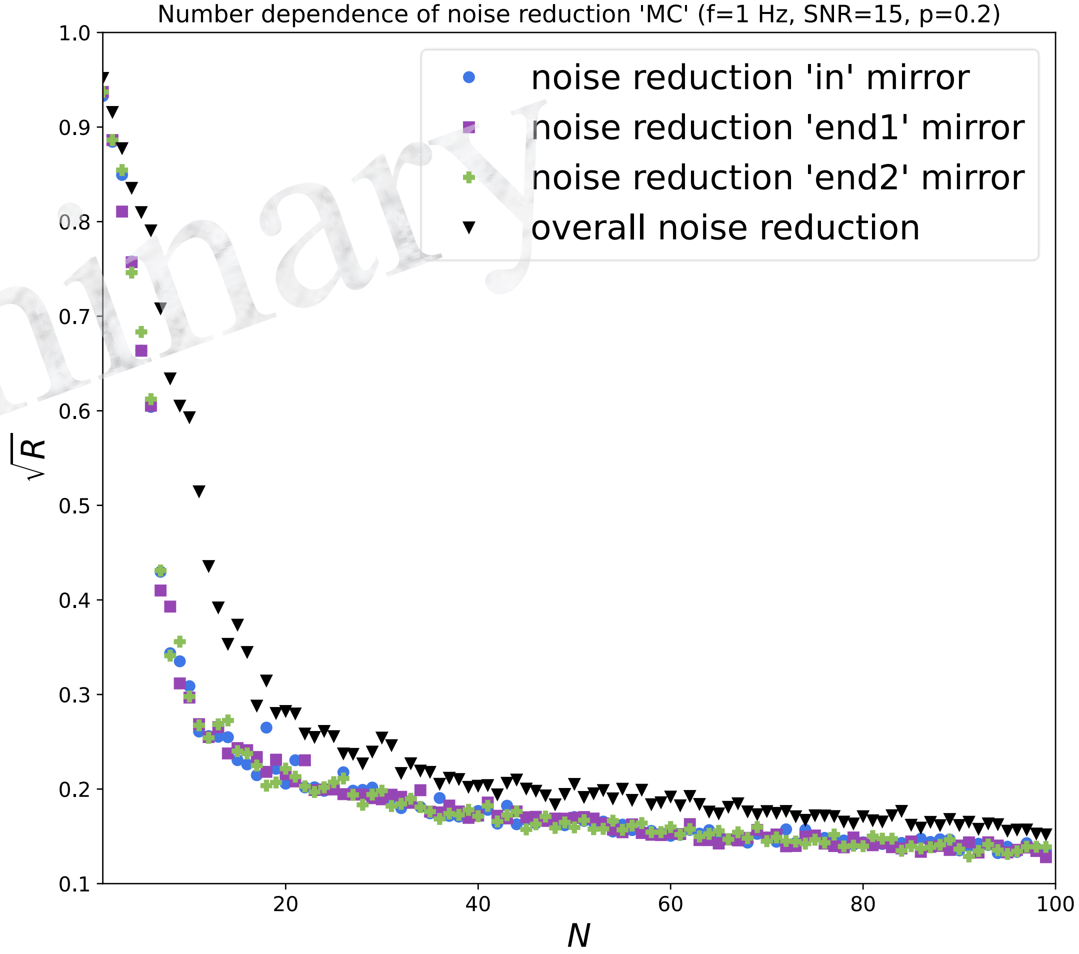
ω, c_1, c_2 - parameters „inertia weight“, „cognitive coefficient“ and „social coefficient“

r_1, r_2 - random numbers $\in [0,1]$

Idea: J. Kennedy & R. Eberhart, *Particle swarm optimization* (1995)

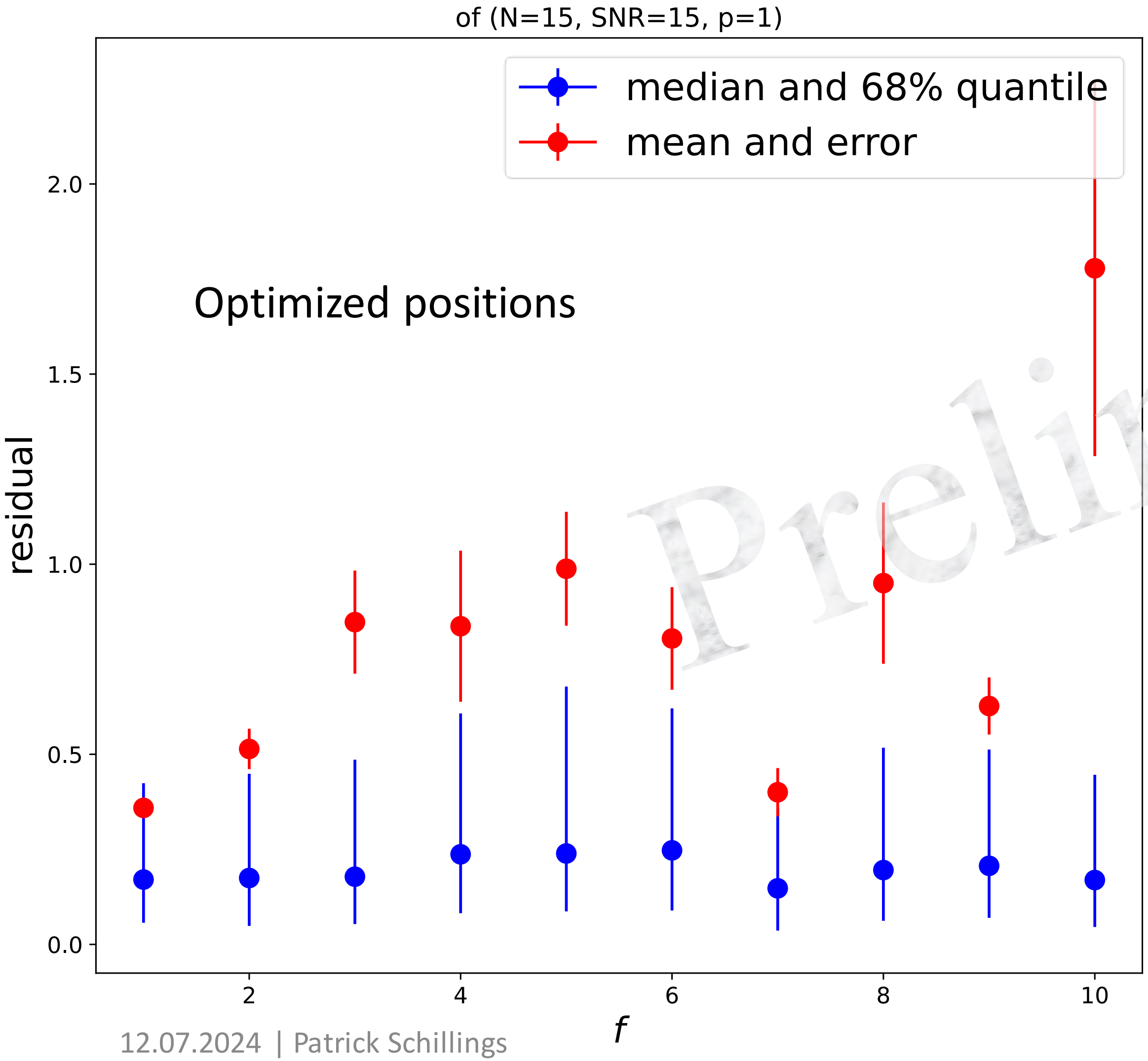
LBFGS-Results

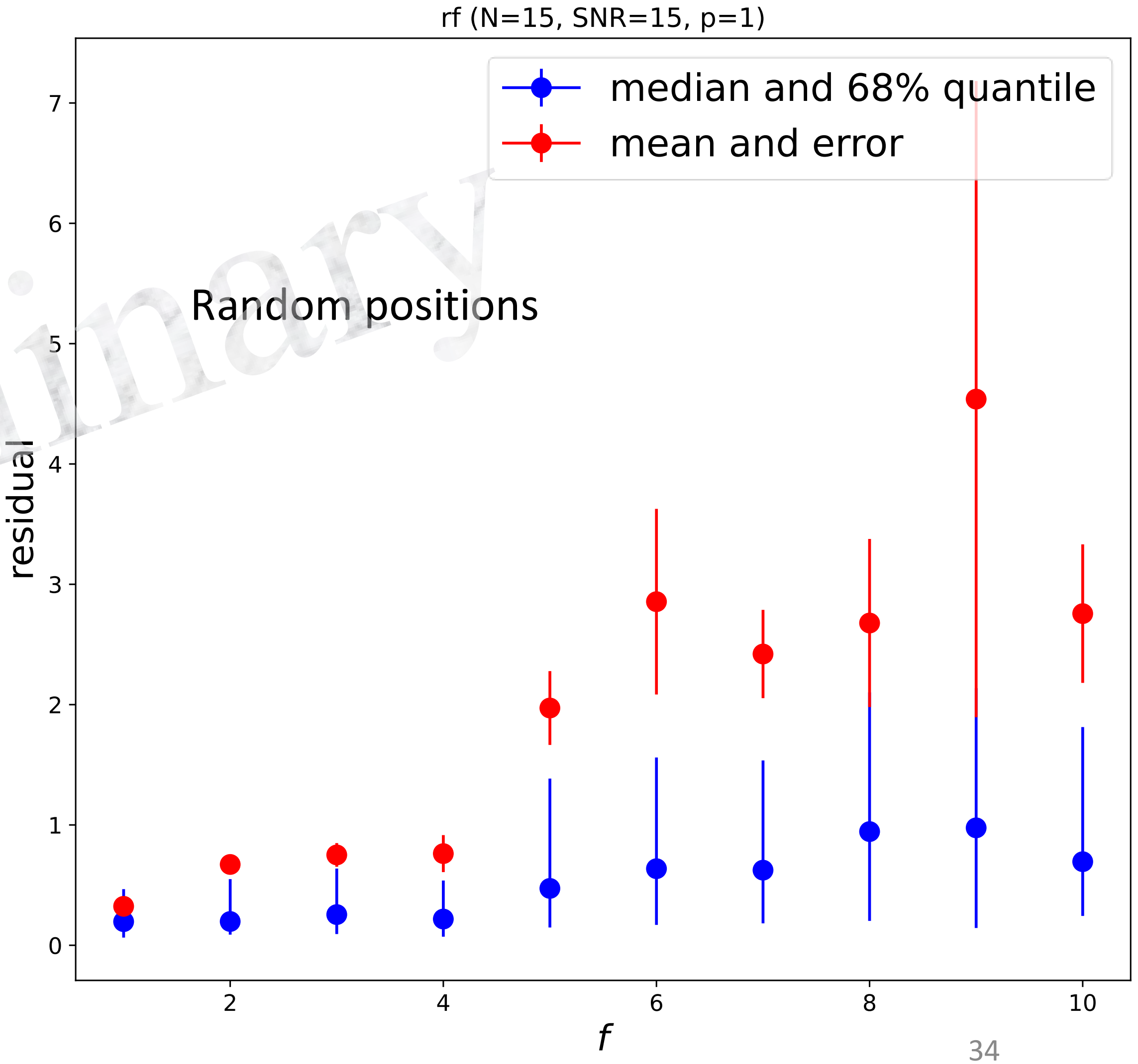




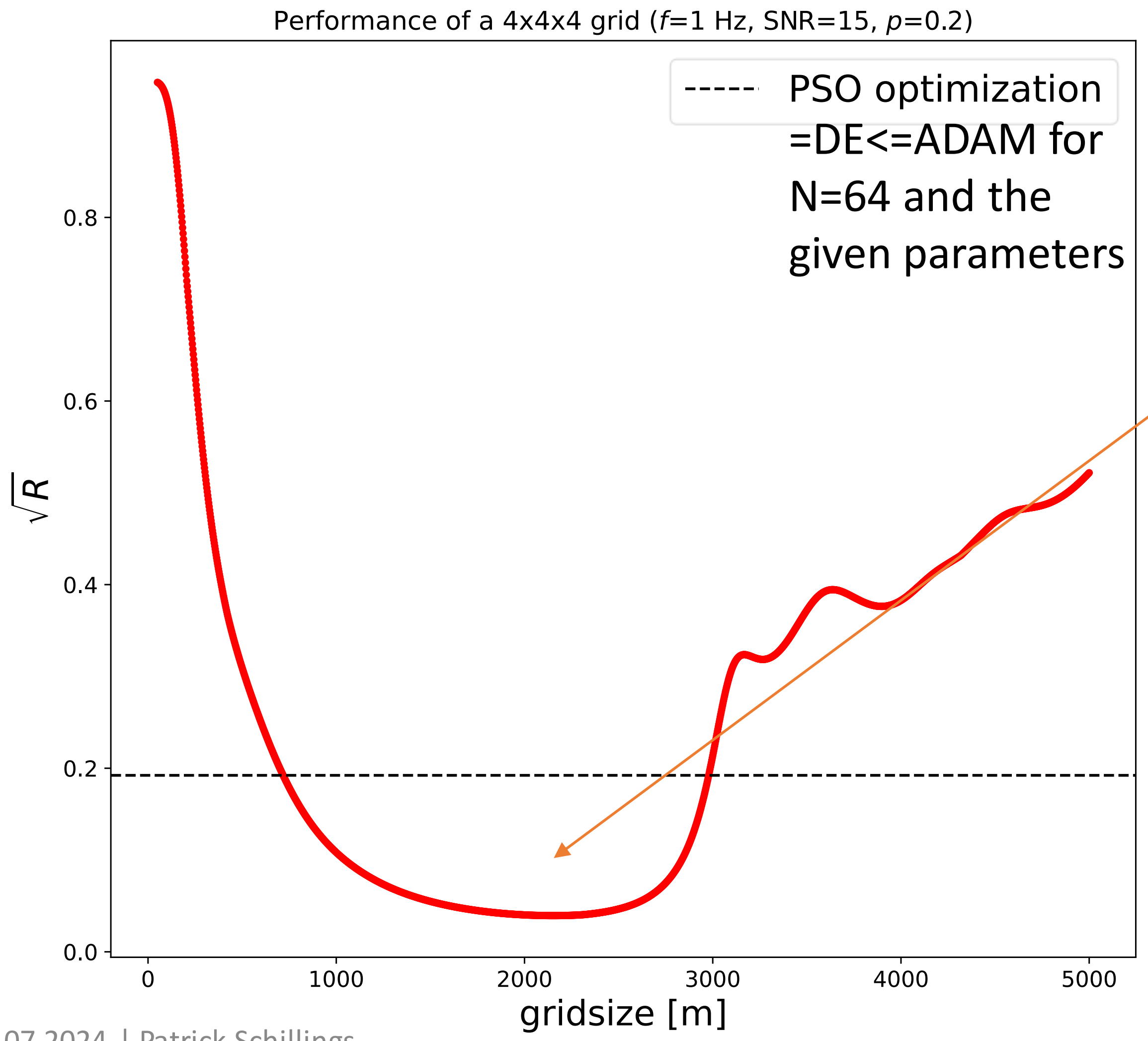
Confidence for Analytical Results

Plain waves, WF in frequency space
4500 training, 500 test





Performance of a simple grid



Residual from taking the positions as a 4x4x4 regular cubic grid with a total size of 2*gridsize centered at (0,0,0) – the corner of ET

For distances between seismometers close to a quarter of the wavelength, performance is best – even better than all optimization attempts. That‘s depressing. ☹

Complicated optimization algorithms may not really find the global minimum. A regular grid isn‘t even the best one can imagine.

→ Is this good news, because in a 1d-minimization it is much, much simpler?

→ Is this bad news, because if a grid is not a good choice, will the optimizers be? Can we find good positions in all geometries?

→ Is this no big deal, because we do not need the global minimum, but are happy with a local one?