BE4 Computational Neuroscience: Problem set 7

Synaptic Plasticity

Exercise 1: Bienenstock Cooper Munro (BCM) Rule

Implement a simulation of the competition between two input patterns $\mathbf{x}_1 = (20, 0)$ and $\mathbf{x}_2 = (0, 20)$. At each timestep, one of the two patterns is presented to the neuron, the pattern is chosen randomly with a probability 0.5 of being pattern $\mathbf{x}_1$ and 0.5 of being pattern $\mathbf{x}_2$. The weights follow the BCM rule. The output of the neuron at each timestep is given by

$$y(t) = \mathbf{w}^T \mathbf{x}(t), \tag{1}$$

where $\mathbf{x}(t)$ is the input presented at time t.

Recall from the lecture that the weight update for the BCM rule is:

$$\frac{d}{dt}\mathbf{w} = \eta \mathbf{x} y (y - \theta), \tag{2}$$

where θ is the sliding threshold: $\theta = \frac{\langle y^2 \rangle}{y_0}$. Note that the average $\langle y \rangle$ must be computed online. The threshold will therefore obey the rule:

$$\tau \frac{d}{dt}\theta = -\theta + \frac{y(t)^2}{y_0} \tag{3}$$

Implement this simulation using a total time $T = 10$ s, $\eta = 10^{-6}$ ms^{-1}, $y_0 = 10$, $\tau = 50$ ms. Use the Euler method with a timestep of 1ms. You should also put a hard bound for the weights at 0 (when $w_i < 0$, set it to 0).

Plot the weights $\mathbf{w}$, the sliding threshold θ and the output y. What do you expect to see? Run the code a few times. What do you see? Zoom on the first 500 time steps, what do you see?

Exercise 2: Spike-Timing Dependent Plasticity (STDP)

Implement a model of a synapse between two neurons with Spike Timing Dependent Plasticity.

STDP can be implemented online assuming that each presynaptic spike leaves a trace x and each postsynaptic spike leaves a trace y following the update rules:

$$\tau_+ \frac{d}{dt} x^{pre} = -x^{pre} + \delta(t - t^{pre}) \tag{4}$$

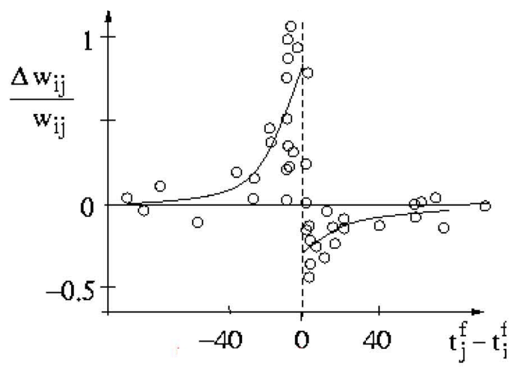
$$\tau_- \frac{d}{dt} y^{post} = -y^{post} + \delta(t - t^{post}). \tag{5}$$

The weight change of the synapse is:

$$\frac{d}{dt}w = A_+ x^{pre} \delta(t - t^{post}) - A_- y^{post} \delta(t - t^{pre}) \tag{6}$$

Implement STDP for a single synapse between two neurons with $A_+ = A_- = 1$, $\tau_+ = 10ms$ and $\tau_- = 20ms$. Use Euler method with a timestep of 1ms. Simulate 60 pre-post pairings repeated every second (i.e. 1Hz, time interval 1000ms), for different lags (i.e. time of the presynaptic spike t^{pre} - time of the postsynaptic spike t^{post}). Simulate for lags from -50ms to 50ms every 5ms.

Plot the weight change as a function of the time difference between presynaptic and postsynaptic spikes, you should get something similar to the following experimental results:



If you are late with the problem set, you can use the code structure stdp_code_structure.m, and only write the code for the STDP updates. But best is to write the code from scratch.