Computational Neuroscience: Problem set 1

Single Neuron Models

Exercise 1: Passive Membrane

The voltage accross a passive membrane can be described by the equation

$$\tau \frac{du}{dt} = -(u - u_{rest}) + R\, I(t)\,. \tag{1}$$

1.1 Step Current

Consider a current $I(t) = 0$ for $t < t_0$ and $I(t) = I_0$ for $t > t_0$. Calculate the voltage $u(t)$.

1.2 Pulse Current

Consider a current pulse

$$I(t) = \begin{cases} 0 & \text{for } t < t_0 \text{ and } t > t_0 + \Delta \\ q/\Delta & \text{for } t \geq t_0 \text{ and } t < t_0 + \Delta\,, \end{cases} \tag{2}$$

where Δ is a short time and q is the total electrical charge.
Consider first $\Delta = 0.1\tau$, and then $\Delta = 0.05\tau$, $\Delta = 0.025\tau$. Draw the input current pulse and the voltage response. What happens in the limit $\Delta \to 0$? (Hint: use $e^{-x} \approx 1 - x$ for small x)

1.3 Delta Function

The delta-function can be defined by the limit of a short pulse:

$$\delta(t - t_0) = \lim_{\Delta \to 0} f_\Delta(t) \quad \text{where} \quad f_\Delta(t) = \begin{cases} 1/\Delta & \text{for } t_0 \leq t < t_0 + \Delta \\ 0 & \text{otherwise}\,. \end{cases} \tag{3}$$

Convince yourself that the integral $\int_{t_1}^{t_2} \delta(t - t_0)\, dt$ is equal to one if $t_1 \leq t_0 < t_2$ and vanishes otherwise.
Write equation (1) using the δ-function for the case that an extremely short current pulse arrives at time t^f. Pay attention to the units!

Exercise 2: Integrate-And-Fire Model

Consider the model of Eq.(1) with a threshold at u = ϑ. If the membrane potential reaches the threshold, it is reset to the resting potential. Assume that you start with a neuron at rest.

2.1 Apply a constant current $I(t) = I_0$. What is the minimal current to reach the threshold?

2.2 What is the time it takes to reach the threshold, for any current I_0 larger than the minimal current of question 2.1?

2.3 Calculate the firing frequency f as a function of I_0.

The function $g(I_0)$ which gives the firing frequency as a function of the constant applied current is called gain function.