Computational Neuroscience: Problem set 2

Hodgkin-Huxley and Phase Plane Analysis

Exercise 1: Model of an Ion Channel

Consider the following model for an ion channel: the electrical current I_{ion} through the channel is given by

$$I_{ion} = g_{ion} r^{n_1} s^{n_2} (u - u_{ion})$$

where u is the membrane potential of the neuron, g_{ion} and u_{ion} are two constants, and $n_1 = 2$, $n_2 = 1$. The quantities r and s obey the equations

$$\begin{aligned} \frac{dr}{dt} &= -\frac{r - r_0(u)}{\tau_r(u)} \\ \frac{ds}{dt} &= -\frac{s - s_0(u)}{\tau_s(u)} \end{aligned}$$

with r_0, s_0, τ_r and τ_s as shown in Fig.1.

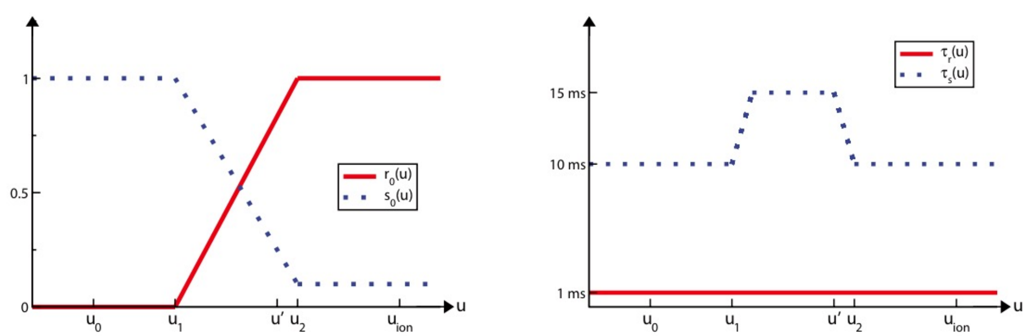

Figure 1 Graphical representation of the variables r_0, s_0, τ_r and τ_s.

1.1 What is the *biological* interpretation of the following parameters :

r : ....

s : ....

g_{ion} : ....

u_{ion} : ....

1.2 How does the channel react (in terms of partial or full opening/closing) to a step change in membrane potential? Suppose that for $t < 0$, the membrane potential is clamped at a value u_0, and that at $t = 0$ it instantaneously jumps to a value $u' = u_2(1 - \varepsilon)$ with $\varepsilon \ll 1$ where it is maintained for all $t \geq 0$ (see figure 1 for the values of u_0, u', u_2 and u_{ion}).

- For $t < 0$, the channel is .... because ....
- At $t = 1$ ms, the channel is .... because ....
- At $t = 3$ ms, the channel is .... because ....
- At $t = 20$ ms, the channel is .... because ....
- At $t = 100$ ms, the channel is .... because ....

Exercise 2: Phase Plane Stability Analysis

2.1 Linear System

Consider the following linear system:

$$\left[\begin{array}{ll} \frac{du}{dt} & = \alpha u - w \\ \\ \frac{dw}{dt} & = \beta u - w\,, \end{array}\right. \tag{1}$$

These equations can be written in matrix form as $dx/dt = Ax$ where $x = \begin{pmatrix} v \\ w \end{pmatrix}$ and $A = \begin{pmatrix} \alpha & -1 \\ \beta & -1 \end{pmatrix}$. Determine the conditions for stability of the point (u=0, w=0) in the case $\beta > \alpha$ by studying the eigenvalues of the above matrix. (Hint: distinguish the cases of real and complex eigenvalues.)