BE4 Computational Neuroscience: Problem set 8

Synaptic Plasticity

Exercise 1: The Perceptron

Implement the perceptron and check whether it is able to learn the input patterns.

Recall that the perceptron takes inputs from a set of p patterns encoded as vectors of length n. Let $\mu = 1, ..., p$ index the patterns, and let the associated input vectors be denoted by $\mathbf{x}^\mu$. The output for a given input pattern is then

$$y^\mu = \Theta(\mathbf{w} \cdot \mathbf{x}^\mu - b), \tag{1}$$

where Θ is the Heaviside step function – i.e., $\Theta(\xi) = 0$ if $\xi < 0$, $\Theta(\xi) = 1$ otherwise – and $\mathbf{w}$ is the weight vector (of length n). At each iteration, the weight change is calculated for each input pattern μ as

$$\Delta \mathbf{w} = \alpha \mathbf{x}^\mu (y_t^\mu - y^\mu), \tag{2}$$

where y_t^μ is target output for pattern μ and α is the learning rate of the algorithm.

You will need to iterate the weight change until either every input pattern is learned successfully or the maximum number of iterations is reached. In all of the following exercises, plot the error $E = \sum_\mu (y_t^\mu - y^\mu)^2$ as a function of the iteration number.

1.1

First, apply the perceptron to the following dataset with $p = 4$ and $n = 2$:

$$x = \{(0,0), (0,1), (1,0), (1,1)\}, \qquad y_t = \{0,1,1,1\} \qquad (OR).$$

Then, use the perceptron on the same dataset, but with different labels:

$$y_t = \{0,1,1,0\} \qquad (XOR).$$

As parameters, use $b = 1$, $\alpha = 1$, maximum number of iterations 10. Plot the total error as a function of the iterations. For which target set y_t does the perceptron converge? Why (or why not)?

1.2

Now generate a random dataset of $p = 50$ binary patterns of length $n = 100$. The respective labels should also be generated at random (*hint*: the elements of the patterns and the labels should be taken to be either 1 or 0, with equal probability). Use parameters $b = 1$, $\alpha = 1$ and maximum number of iterations = 50.

Use the perceptron on the dataset, plot the error and check that it converges.

Try to change $p = 100$ and $p = 200$. Does it still converge? Why?