Imperial College London

IMPERIAL COLLEGE LONDON

DEPARTMENT OF BIOENGINEERING

Biomedical Instrumentation Lab Report

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1 Lab 1 - Passive Filters

Question 1.1. *What function does this circuit perform? Can you identify any possible sub—circuits within it? Write and elaborate on the basic relations needed for deriving the transfer function of this block.*

This circuit performs a band-pass filter action. It is composed of four passive filters, two first order low-pass RC (Resistor and Capacitor) filters with a cut-off frequency of 4.08 Hz on the left-hand side of the circuit and two first-order high-pass RC filters with a cut-off frequency of 80 mHz. The cut-off frequency of such filters can be calculated from:

$$f_{c-L} = f_{c-H} = \frac{1}{2\pi RC}$$

To derive the transfer function of any of these filters we can first take the impedance of the components we are dealing with:

$$Z_{capacitor} = \frac{1}{Cs}$$

$$Z_{resistor} = R$$

We can easily see how the circuit for every filter [comprised by one capacitor and one resistor at a time] is a voltage divider. Where,

$$\frac{V_{out}}{V_{in}} = \frac{Z_{capacitor}}{Z_{capacitor} + Z_{resistor}} \qquad \text{for the low-pass filter.}$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_{resistor}}{Z_{capacitor} + Z_{resistor}} \qquad \text{for the high-pass filter.}$$

Through further expanding and simplifying these equations the transfer function of a low-pass filter of the kind we use in the circuit is found to be:

$$L(s) = \frac{1}{RCs + 1}$$

And the transfer function of a high-pass filter of the kind we use in the circuit is:

$$H(s) = \frac{RCs}{RCs + 1}$$

, where s is the Laplace variable (for the purpose of our studies s = jω [angular frequency]).

The transfer function of the whole circuit results in the multiplication of the transfer function of the four sub-circuits that compose it.

Measurements 1 and 2

Frequency [Hz]	Output Voltage (M1)	Output Voltage (M2)	dB drop(M1)	dB drop(M2)
0.5	0.936	0.896	-0.574	-0.954
1	0.84	0.792	-1.514	-2.025
2	0.672	0.588	-3.453	-4.612
3	0.568	0.432	-4.913	-7.290
4	0.488	0.332	-6.232	-9.577
5	0.44	0.276	-7.131	-11.18
6	0.408	0.224	-7.787	-12.99
7	0.368	0.192	-8.683	-14.33
8	0.348	0.138	-9.168	-15.70
9	0.332	0.138	-9.577	-17.20
10	0.31	0.122	-10.17	-18.27

Question 2-b *If the circuit was extended as shown in the diagram below how many-dBs/Oct attenuation would you expect between the frequencies of 12Hz and24Hz?*

In the scenario where we are considering frequencies of 12 Hz and 24 Hz, it is safe to assume that the dB drop of the high-pass filter will be 0 (i.e. there will be no signal attenuation by the high-pass filter). Hence our transfer function can simplify the product of the low-pass filter sub-circuits transfer functions.
By adding a third LP filter in our circuit we get:

$$TF = \frac{V_{out}}{V_{in}} = \left(\frac{1}{1+RCs}\right)^3$$

We could calculate the decibel drop in amplitude by first calculating the gain(magnitude) of this transfer function:

$$\left\|\frac{V_{out}}{V_{in}}\right\| = \left\|\left(\frac{1}{1+RCs}\right)^3\right\|$$

$$\Leftrightarrow \left\|\frac{V_{out}}{V_{in}}\right\| = \left\|\left(\frac{1}{1+RCs}\right)\right\|^3$$

Now, we substitute s for jω and take the magnitude at the numerator and denominator:

$$\Leftrightarrow \left\|\frac{V_{out}}{V_{in}}\right\| = \sqrt{\frac{1^2}{1^2+\left(\frac{\omega}{\omega_o}\right)^2}}^{\,3} \text{ ,by taking } \omega_o = \frac{1}{RC}$$

In the next step we convert the gain $\left\|\frac{V_{out}}{V_{in}}\right\|$ into decibels:

$$dB = 20\,log_{10}\left(\left\|\frac{V_{out}}{V_{in}}\right\|\right)$$

$$\Leftrightarrow dB = 20\,log_{10}\left(\sqrt{\frac{1}{1+\left(\frac{\omega}{\omega_o}\right)^2}}^{\,3}\right)$$

$$\Leftrightarrow dB = 60\, log_{10}(1) - 60 log_{10}\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}$$

$$\Leftrightarrow dB = -30\, log_{10}\left(1 + \left(\frac{\omega}{\omega_o}\right)^2\right)$$

By changing ω for 2πf, and finally the frequency f for 12 Hz and 24 Hz and R and C for their respective values we get:

$$\omega_o = \frac{1}{39 \cdot 10^{-}3}\, rad \cdot s^{-1} = 25.6\, rad \cdot s^{-1}$$

$$f = 12Hz \qquad dB_1 = -30\, log_{10}\left(1 + \left(\frac{2\pi \cdot 12}{25.6}\right)^2\right)$$

$$\Leftrightarrow dB_1 = -29.6\, dBs$$

$$f = 24Hz \qquad dB_2 = -30\, log_{10}\left(1 + \left(\frac{2\pi \cdot 24}{25.6}\right)^2\right)$$

$$\Leftrightarrow dB_2 = -46.6\, dBs$$

The increase in frequency from 12 Hz to 24 Hz is the equivalent to one octave hence we expect an attenuation between the frequencies of 12 Hz and 24 Hz of:

$$dB\, drop = \frac{dB_2 - dB_1}{1} = 17\, dBs/Oct$$

2 Basics

Some guidelines:

- Always use vector graphics (scale free)
- In graphs, label the axes
- Make sure the font size (labels, axes) is sufficiently large
- When using colors, avoid red and green together (color blindness)
- Use different line styles (solid, dashed, dotted etc.) and different markers to make it easier to distinguish between lines

2.0.1 Equations

Here are a few guidelines regarding equations

- Please use the `align` environment for equations (`eqnarray` is buggy)
- Please number all equations: It will make things easier when we need to refer to equation numbers. If you always use the `align` environment, equations are numbered by default.
- Vectors are by default column vectors, and we write

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \tag{1}$$

- Note that the same macro (`\colvec`) can produce vectors of variable lengths, as

$$\boldsymbol{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \tag{2}$$

- Matrices can be created with the same command. The & switches to the next column:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix} \tag{3}$$

- Determinants. We provide a simple macro (`\matdet`) whose argument is just a matrix array:

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 2 & 2 & 2 \end{vmatrix} \tag{4}$$

- If you do longer manipulations, please explain what you are doing: Try to avoid sequences of equations without text breaking up. Here is an example: We consider

$$U_1 = [\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}] \subset \mathbb{R}^4, \quad U_2 = [\begin{bmatrix}-1\\1\\2\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}] \subset \mathbb{R}^4. \tag{5}$$

To find a basis of $U_1 \cap U_2$, we need to find all $\boldsymbol{x} \in V$ that can be represented as linear combinations of the basis vectors of U_1 and U_2, i.e.,

$$\sum_{i=1}^{3} \lambda_i \boldsymbol{b}_i = \boldsymbol{x} = \sum_{j=1}^{2} \psi_j \boldsymbol{c}_j , \tag{6}$$

where $\boldsymbol{b}_i$ and $\boldsymbol{c}_j$ are the basis vectors of U_1 and U_2, respectively. The matrix $\boldsymbol{A} = [\boldsymbol{b}_1|\boldsymbol{b}_2|\boldsymbol{b}_3| - \boldsymbol{c}_1| - \boldsymbol{c}_2]$ is given as

$$\boldsymbol{A} = \begin{bmatrix}1 & 0 & 0 & 1 & 0\\1 & 1 & 0 & -1 & -1\\0 & 1 & 1 & -2 & 0\\0 & 0 & 1 & 0 & 0\end{bmatrix}. \tag{7}$$

By using Gaussian elimination, we determine the corresponding reduced row echelon form

$$\begin{bmatrix}1 & 0 & 0 & 1 & 0\\0 & 1 & 0 & -2 & 0\\0 & 0 & 1 & 0 & 0\\0 & 0 & 0 & 0 & 1\end{bmatrix}. \tag{8}$$

We keep in mind that we are interested in finding $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}$ and/or $\psi_1, \psi_2 \in \mathbb{R}$ with

$$\begin{bmatrix}1 & 0 & 0 & 1 & 0\\0 & 1 & 0 & -2 & 0\\0 & 0 & 1 & 0 & 0\\0 & 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\\lambda_2\\\lambda_3\\\psi_1\\\psi_2\end{bmatrix} = \boldsymbol{0}. \tag{9}$$

From here, we can immediately see that $\psi_2 = 0$ and $\psi_1 \in \mathbb{R}$ is a free variable since it corresponds to a non-pivot column, and our solution is

$$U_1 \cap U_2 = \psi_1 \boldsymbol{c}_1 = [\begin{bmatrix}-1\\1\\2\\0\end{bmatrix}], \quad \psi_1 \in \mathbb{R}. \tag{10}$$

2.1 Gaussian elimination

We provide a template for Gaussian elimination. It is not perfect, but it may be useful:

$$
\begin{aligned}
&\left[\begin{array}{rrrrr|r}
1 & -2 & 1 & -1 & 1 & 0 \\
0 & 0 & -1 & 1 & -3 & 2 \\
0 & 0 & 0 & -3 & 6 & -3 \\
0 & 0 & -1 & -2 & 3 & a
\end{array}\right]
\begin{array}{l} \\ \\ \\ -R_2 \end{array} \\
\rightsquigarrow &\left[\begin{array}{rrrrr|r}
1 & -2 & 1 & -1 & 1 & 0 \\
0 & 0 & -1 & 1 & -3 & 2 \\
0 & 0 & 0 & -3 & 6 & -3 \\
0 & 0 & 0 & -3 & 6 & a-2
\end{array}\right]
\begin{array}{l} \\ \\ \\ -R_3 \end{array} \\
\rightsquigarrow &\left[\begin{array}{rrrrr|r}
1 & -2 & 1 & -1 & 1 & 0 \\
0 & 0 & -1 & 1 & -3 & 2 \\
0 & 0 & 0 & -3 & 6 & -3 \\
0 & 0 & 0 & 0 & 0 & a+1
\end{array}\right]
\begin{array}{l} \\ \cdot(-1) \\ \cdot(-\frac{1}{3}) \\ \\ \end{array} \\
\rightsquigarrow &\left[\begin{array}{rrrrr|r}
1 & -2 & 1 & -1 & 1 & 0 \\
0 & 0 & 1 & -1 & 3 & -2 \\
0 & 0 & 0 & 1 & -2 & 1 \\
0 & 0 & 0 & 0 & 0 & a+1
\end{array}\right]
\end{aligned}
$$

The arguments of this environment are:

1. Number of columns (in the augmented matrix)
2. Number of free variables (equals the number of columns after which the vertical line is drawn)
3. Column width
4. Stretch factor, which can stretch the rows further apart.

3 Answer Template

1) Discrete models
   c)
   d)
   e)
2) Differentiation
   a)
   b)
   d)
   e)
3) Continuous Models
   a)
   b)
   c)
   d)
   e)
   f)
   g)
4) Linear Regression
   a)
   b)
   c)
   d)
5) Ridge Regression
   a)
   b)
   c) i)
      ii)
6) Bayesian Linear Regression
   b)
   c)
   d)
   e) (bonus)