Electrical Engineering II Coursework

Exercise 1:

a) The current flowing out the BJT device is going to be small, generally in the range of mA, and in addition to that we have a small resistance of 10Ω. Hence the voltage drop across the resistor is going to be small and V_{CE} can be assumed to be the same as V_{CC}. With the help of OrCAD we have been able to print the I_C-V_{CE} characteristics for DC analysis for a varying base current I_{IN} (usually called I_B) by steps of 10 µA, starting from 10 µA up to 100 µA. This can be seen in **figure 1**.

The circuit used to derive figure 1 is the one on **figure 2**.

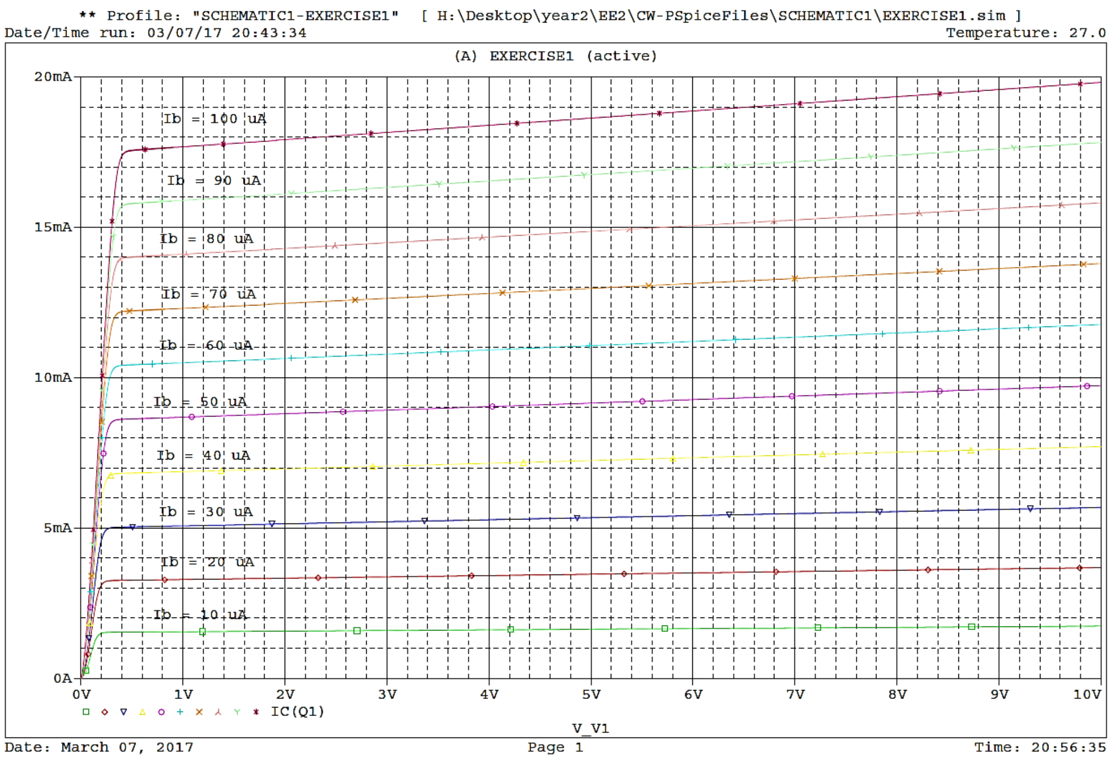
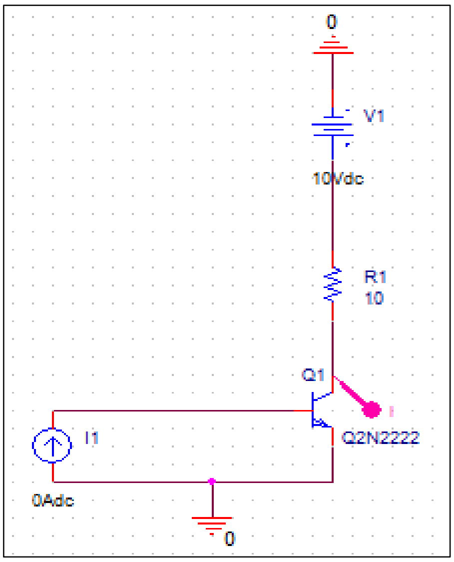


Figure 1: I_C-V_{CE} characteristics for a Q2N2222 BJT



← **Figure 2:** Circuit involving the Q2N2222 BJT

b) As the active region of the I_C-V_{CE} characteristics graph (see **figure 3** for reference) varies linearly, the value we get for β is going to be the proportional to where we take our I_C at (i.e. the V_{CE} at which we take the data), as I_B stays the same for every line of the characteristic graph. In addition I_C is also dependent on the Early voltage V_A that we are going to calculate later on. Where β can be obtained from:

$$\beta = \frac{I_C}{I_B}$$

where I_C stand for the current through the collector (going into the BJT) and I_B for the current to the base (as the Q2N2222 transistor is a NPN transistor).

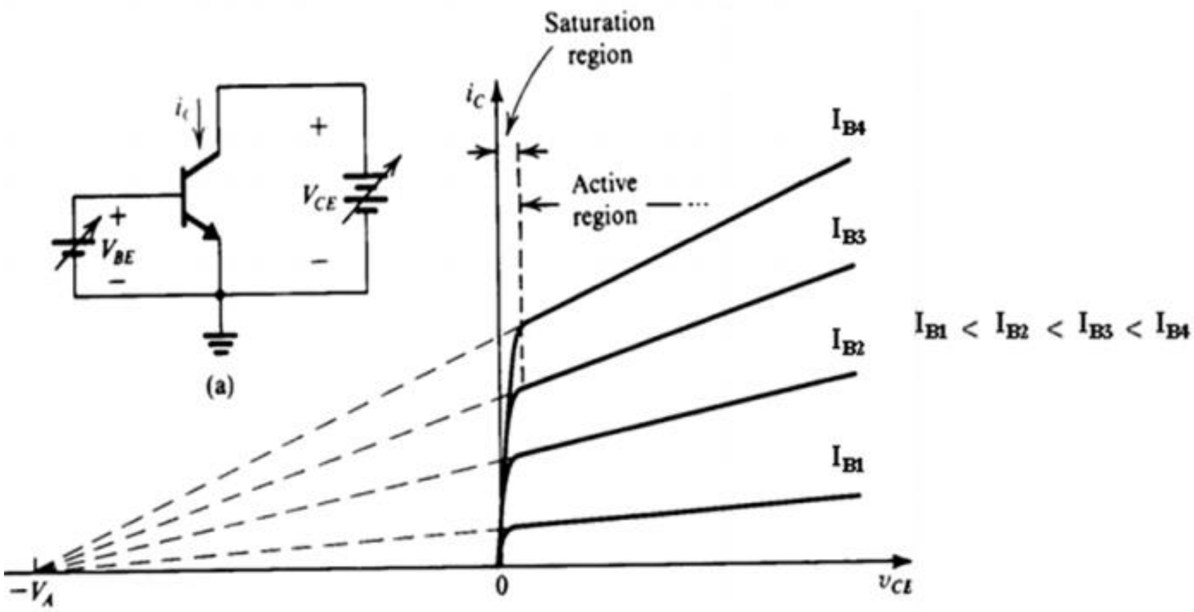


Figure 3: General BJT I_C-V_{CE} characteristics graph, showing linearity of the active region

This is the data we get from the OrCAD simulation, by taking the values at V_{CE} = 1V:

I_{IN} (or I_B) [μA]	I_C [mA]	β [unitless]
10	1.54	154
20	3.28	164
30	5.06	169
40	6.87	172
50	8.68	174
60	10.5	175
70	12.3	176
80	14.1	176
90	15.9	177
100	17.7	177

Figure 4: β values for different values of I_C [linearly changing]

It is good to mention that as I_{IN} gets bigger the value of β gets stable.

c) By increasing V_{CE}, which following Kirkoff Voltage Law can also be written as V_{CB}+V_{BE}, we directly increase V_{CB}, as V_{BE} is constant.
This causes an increase of the reverse biasing in the collector-base junction of the BJT, increasing the space-charge region width and decreasing the width of the part of the base which is active.
As we know the current is equal to the flow of positive charges (i.e. protons) through a region of

space. Mathematically this can be interpreted as: $\frac{dn_p}{dx}$. As we have decreased the active base width, we have decreased dx, which implies an increase of the collector current I_C. This generates the tilt of the characteristics of the BJT device.

d) The current flows from the collector to the emitter, the collector being the current output. The output resistance a BJT has can be found from:

$r_O = \frac{dV_{CE}}{di_C}$ or, $r_O = \frac{\Delta V_{CE}}{\Delta i_C}$ in non-continuous analysis as we have it.

For accuracy we are going to take much separated values of V_{CE}, 1 Volt and 9.5 Volts.
The following table (**figure 5**) displays different values of the output resistance r_O for different values of I_{IN} and for a constant ΔV_{CE} = 8.5V.

I_{IN} (or I_B) [μA]	Δi_C [mA]	r_O[kΩ]
10	0.18	48.1
20	0.37	22.7
30	0.58	14.7
40	0.79	10.8
50	0.99	8.56
60	1.20	7.08
70	1.41	6.03
80	1.61	5.27
90	1.82	4.68
100	2.02	4.21

Figure 5: Output resistances for different values of base current I_{IN}

e) The value of the Early voltage value can be found from:

$r_0 = \frac{V_A}{I_C}$ → $V_A = r_0 I_C$

By calculating every value of V_A for different r_0 values and their respective I_C values, and then taking the average of that we find that **V_A = 74.3V**.

f) As done in part **b)** we plot the I_C-V_{CE} characteristics graph for different values of I_{IN}, although now we take a broader range of values.

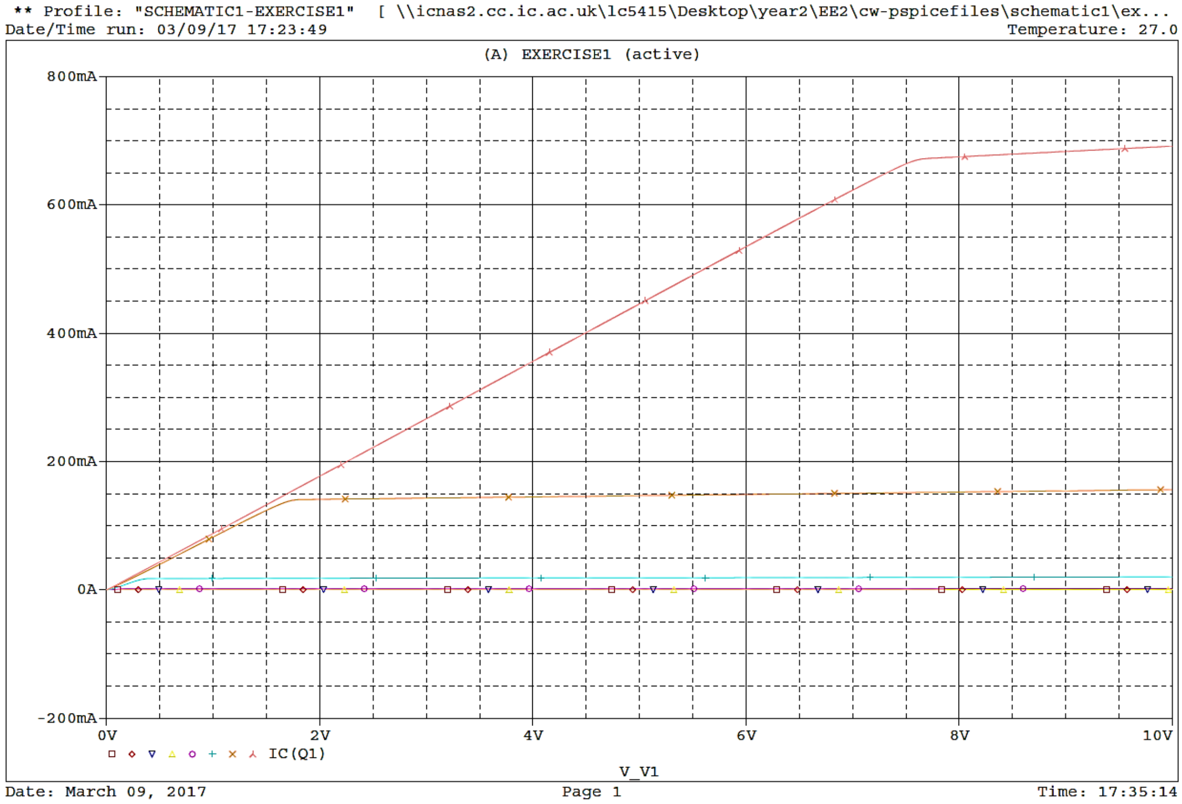


Figure 6: I_C-V_{CE} characteristics for a Q2N2222 BJT for V_{CE} ϵ [1nA;

This is what we get when we take values of I_C at V_{CE} = 8V

I_{IN} (or I_B) [μA]	I_C [mA]	β [unitless]
10^{-3}	$2.94 \cdot 10^{-5}$	29.4
10^{-2}	$5.24 \cdot 10^{-4}$	52.4
10^{-1}	$8.62 \cdot 10^{-3}$	86.2
1	$1.28 \cdot 10^{-1}$	128
10	1.69	169
10^{2}	$1.93 \cdot 10^{1}$	193
10^{3}	$1.52 \cdot 10^{2}$	152
10^{4}	$6.75 \cdot 10^{-5}$	67.5

Figure 7: β values for different values of I_C [changing by decades]

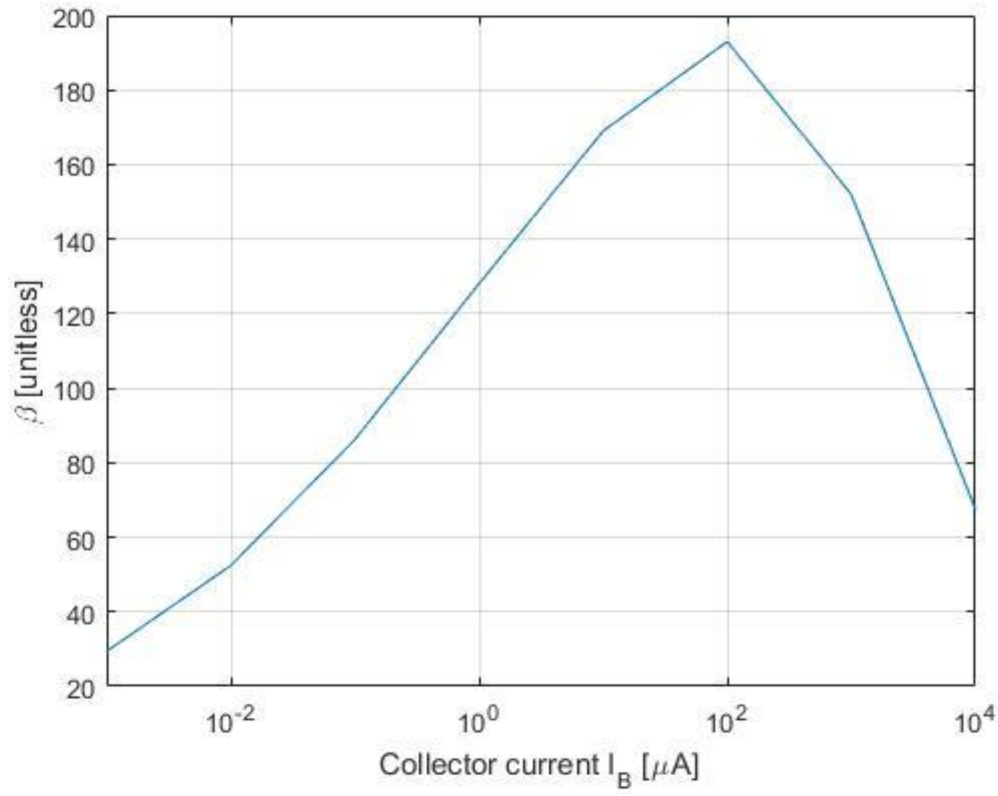


Figure 8: β dependence on I_B (semilog scale) for V_{CE}=8V

We observe that β increases with I_B until it reaches a top limit value of 193 at I_B equal to 19.3 Ma, for higher values of I_B β decreases.

The first part of the graph, where β increases, can be explained by the normal way of working of a BJT device. Specifically this BJT is of type NPN, by the injection of electrons through the emitter, most are transmitted to the collector, some others go to the base of the device and some recombined with holes in the p-regions of the BJT. The ratio of the current (flow of positive charges) that goes to the collector to the current that goes to the base is effectively β.

by increasing I_B we are increasing the amount of electrons we take away from the base, which we are originally taking from the emitter, hence also increasing I_C. As I_C increases faster than I_B, because more electrons flows into the collector, β increases with IB (or IC).

Once I_B reaches the value of 100 µA, the value of β drops, as there are no more free holes for entering electrons to recombine with, therefore creating an overload of electrons, quickly reducing the value of I_C and hence decreasing β.

Exercise 2:

a)

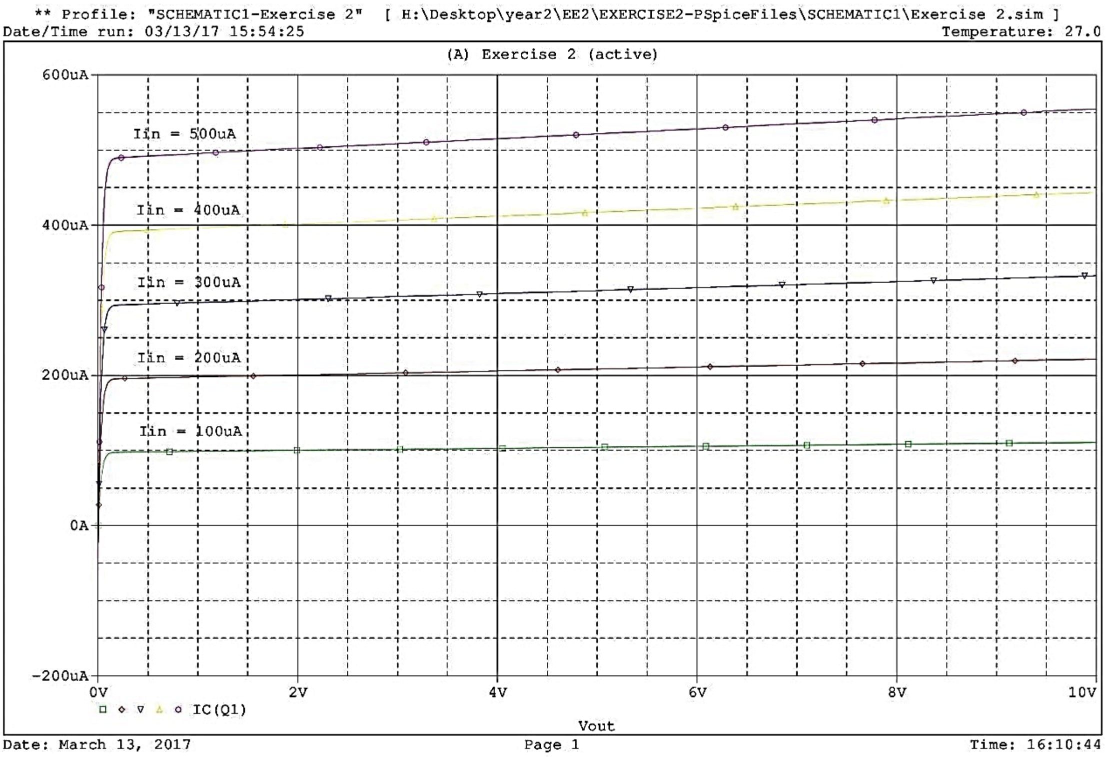


Figure 9: I_{out}-V_{out} plot of a simple current mirror for standard OrCAD β value (255.9)

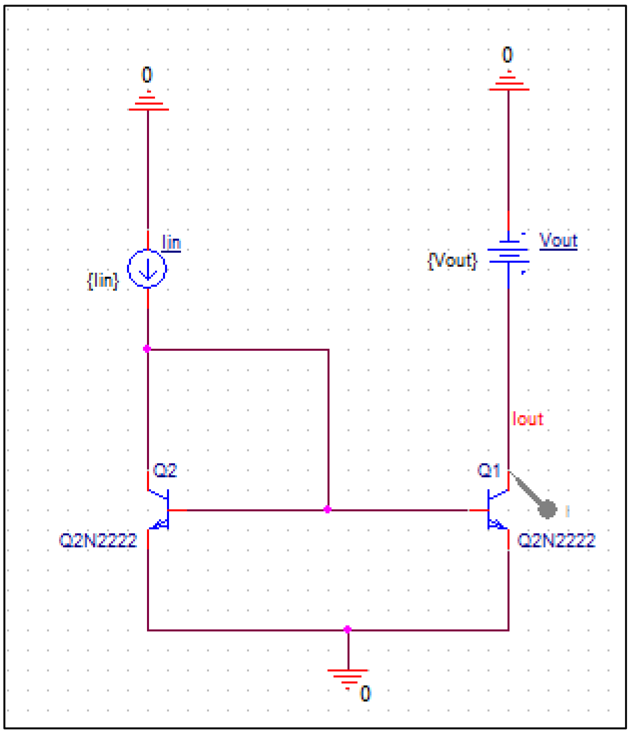


Figure 10: Simple Current Mirror used for this exercise

Above the parametric analysis of a simple current mirror can be seen.

b) In a similar to the operation we did in exercise 1 question d, we can now find the output resistance r_0 from the following:

$$r_O = \frac{\Delta V_{out}}{\Delta i_{out}}$$

We take values of I_{out} at V_1 = 1V and V_2 = 9.5V having ΔV_{out} = 8.5.

I_{IN} [μA]	Δi_{out} [μA]	r_O[kΩ]
100	10.6	805
200	22.6	375
300	33.8	252
400	45.0	189
500	56.4	151

Figure 11: Output resistances values for different values of I_{IN} for a simple CM

c)

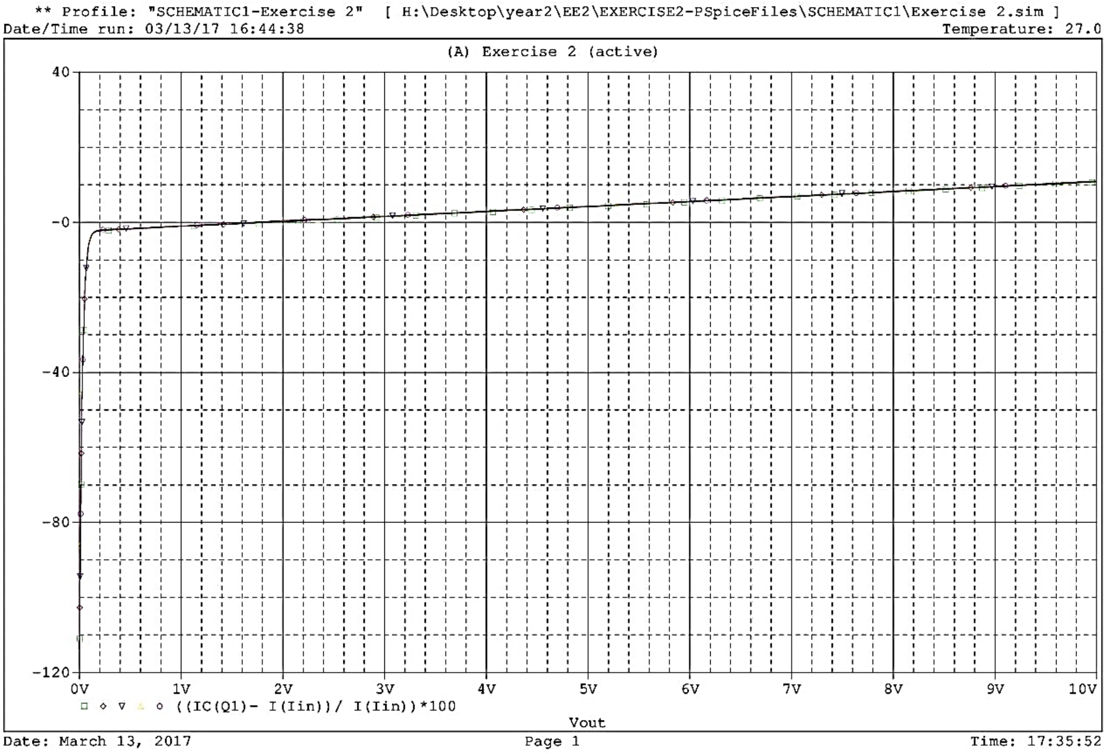


Figure 12: Relative error of the mirroring action against V_{OUT}

The relative error values for V_{OUT} = 1V and for different values of I_{IN} can be seen on **figure 13**.

I_{IN} [μA]	$Relative\ error$[%]
100	-1.13
200	-1.10
300	-1.04
400	-0.94
500	-0.91

Figure 13: Relative error of the mirroring action at V_{OUT} = 1V

d)

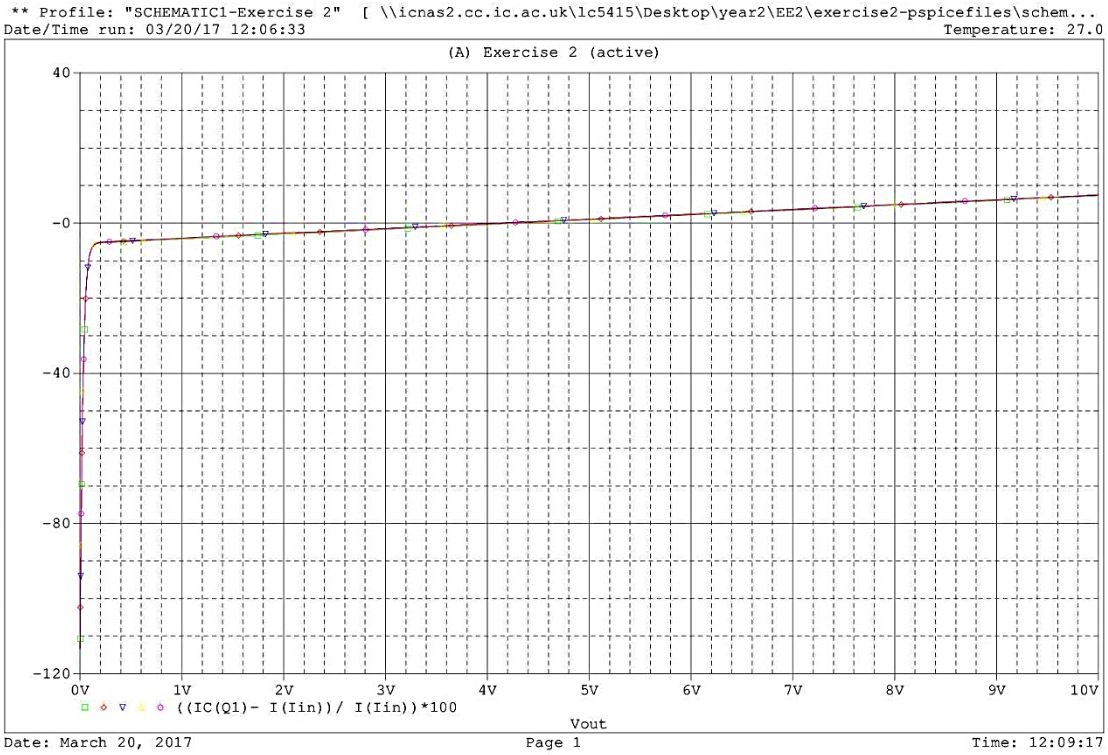


Figure 14: Relative error of the mirroring action against V_{OUT} for BJT's with β = 50

I_{IN} [μA]	$\boldsymbol{Relative\ error}[\%]$
100	-4.25
200	-4.08
300	-4.04
400	-4.04
500	-3.97

Figure 15: Relative error of the mirroring action at V_{OUT} = 1V for BJT's with β = 50

We find higher error values for a lower β value. We can say higher β values ensure more adequate behaviour of the current mirror at V_{out} equal 1V. The voltage at which the relative error is zero has shifted to 4V. If we were intended to implement a certain system with a certain range of voltages it would be interesting to select BJTs with a specific β value in dependence of those voltages so that our mirroring action has a minimum error.

e)

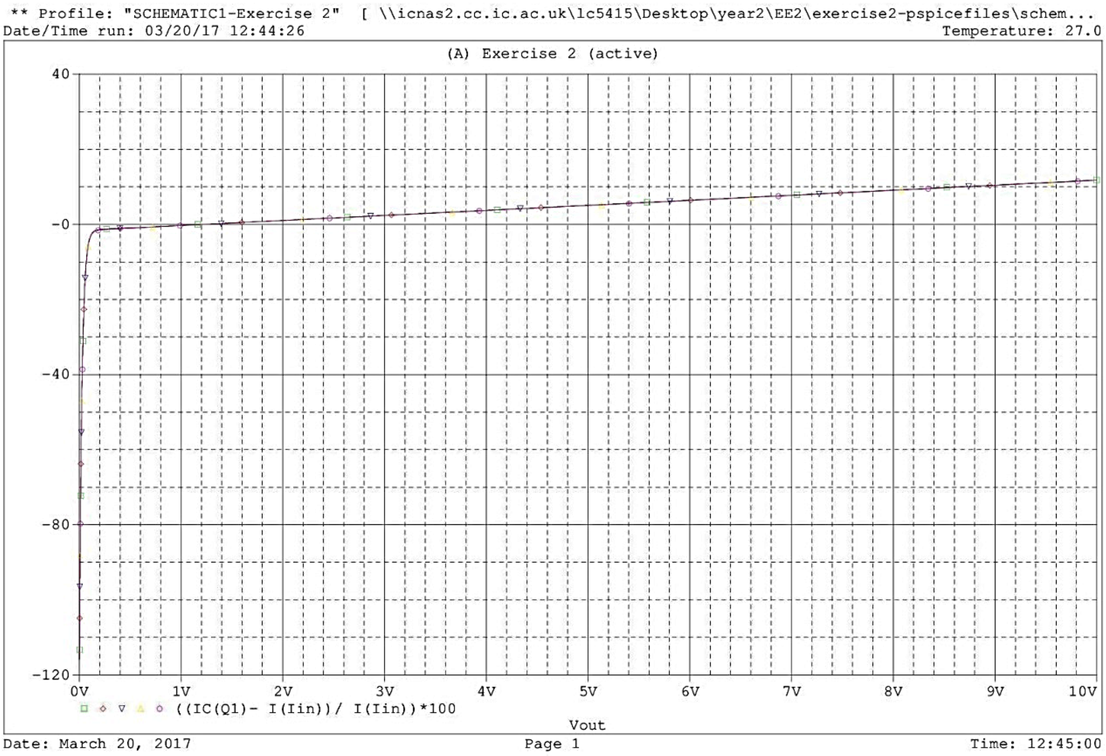


Figure 16: Relative error of the mirroring action against V_{OUT} for a β compensated current mirror

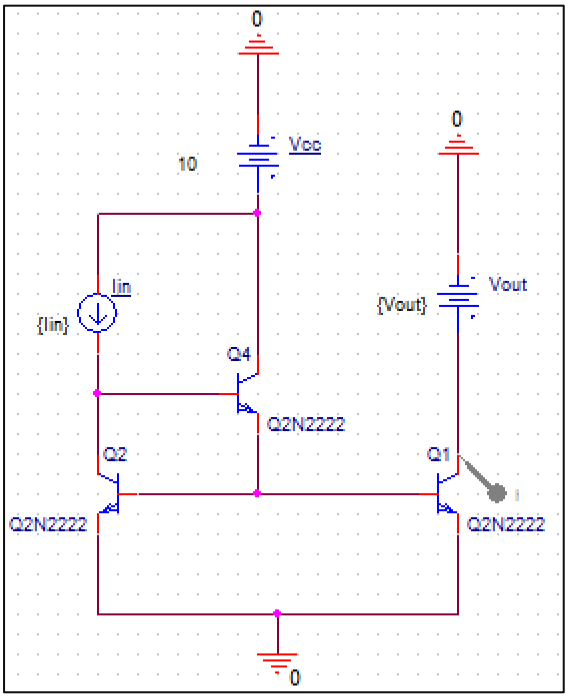


Figure 17: β compensated current mirror circuit

I_{IN} [μA]	$Relative\ error[\%]$
100	$-250 \cdot 10^{-3}$
200	$-289 \cdot 10^{-3}$
300	$-309 \cdot 10^{-3}$
400	$-328 \cdot 10^{-3}$
500	$-341 \cdot 10^{-3}$

Figure 18: Relative error of the mirroring action at V_{OUT} = 1V for a β compensated current mirror

We observe a much reduced error with the β compensated current mirror circuit at V_{out} equal 1V, even though we have used a β of 50.

f)

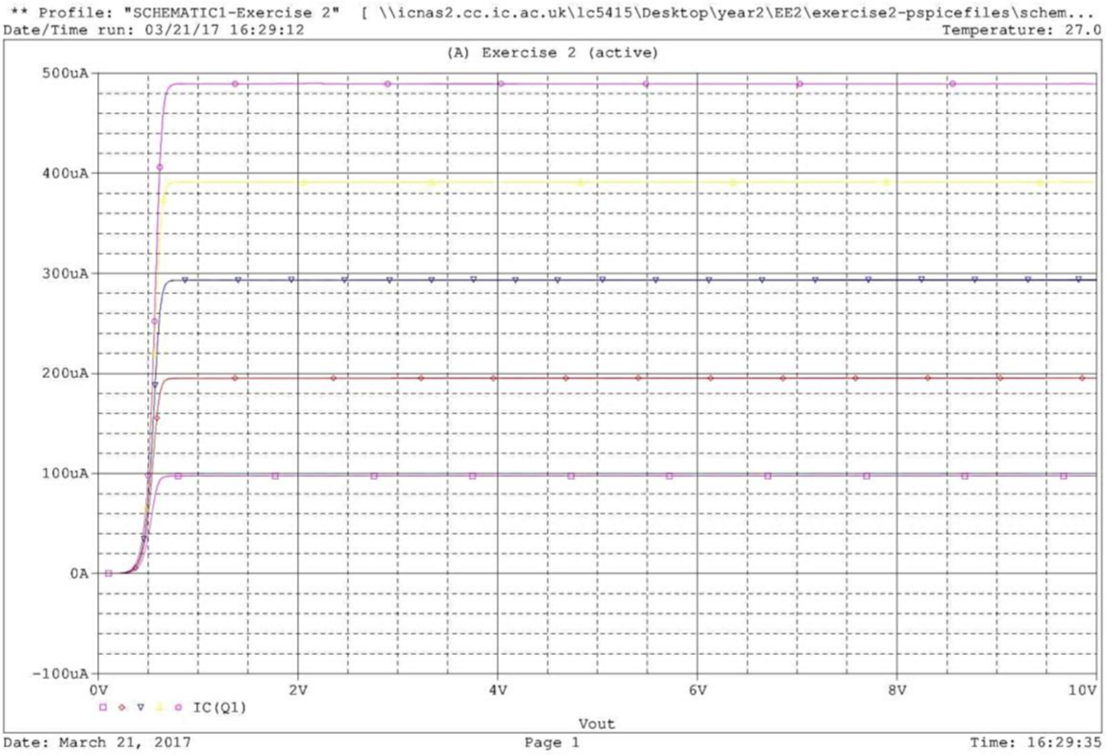


Figure 19: I_{out}-V_{out} plot of a cascode current mirror

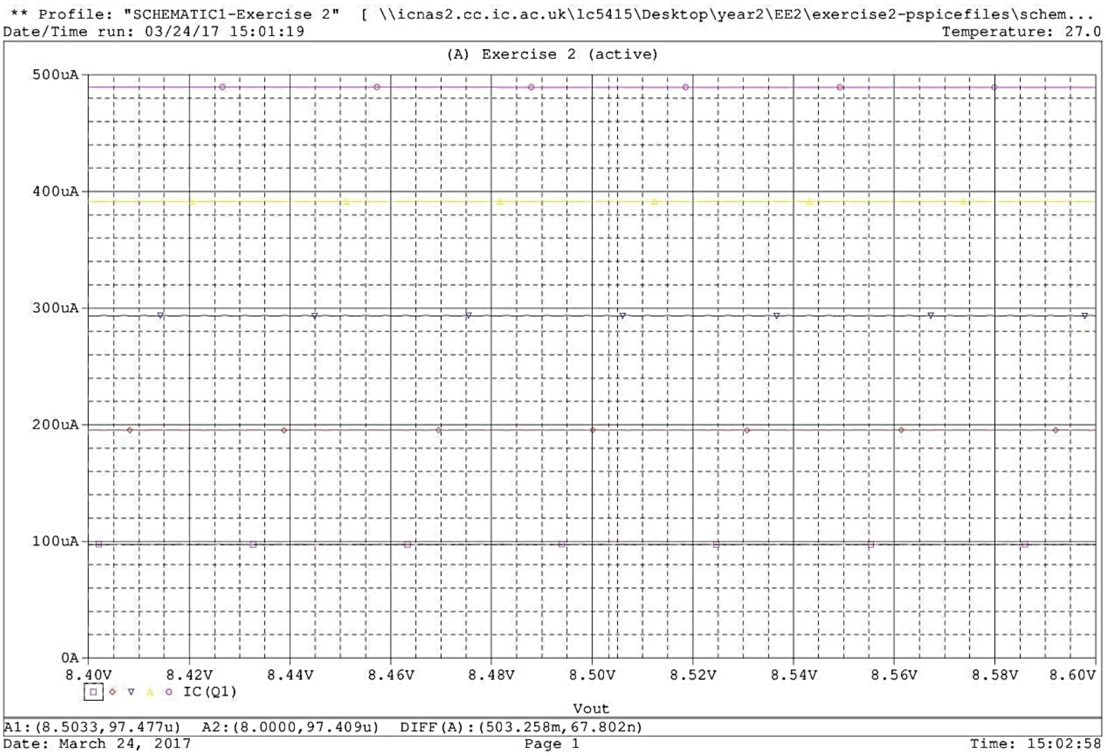


Figure 20: I_{out}-V_{out} plot of a cascode current mirror zoomed for $V_{out} \in [8.4V, 8.6V]$

Figure 20 show the inaccuracy and low reliability of OrCAD as the slope of all plots oscillates constantly whereas theoretically they should be perfectly straight or in any case have a sliglty increasing value of I_{out} for an increasing V_{out} .This is not accurate enough for a correct calculation of an exact value of r_o. But at least we can ensure a good enough range of values for r_0.

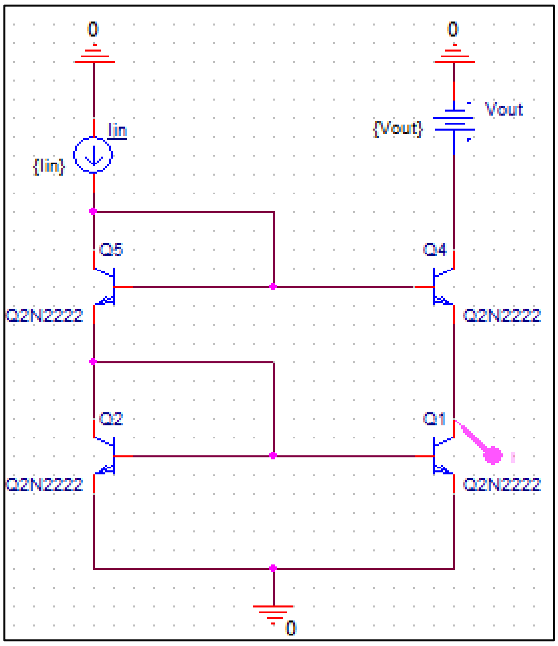


Figure 21: Cascode CM circuit

$$r_O = \frac{\Delta V_{out}}{\Delta i_{out}}$$

We take values of I_{out} at V_1 = 1V and V_2 = 9.999V having ΔV_{out} = 8.999.

I_{IN} [µA]	Δi_{out} [nA]	r_O[MΩ]
100	79	108
200	147	57.8
300	212	40.1
400	138	61.6
500	64	133

Figure 22: Output resistances values for different values of I_{IN} for a cascode CM

As mentioned previously the OrCAD plot is not very accurate as the slope of the curve is almost 0 on the active region of the BJTs so that I_{out} values are not reliable enough, hence r_o are not very reliable either.
Any way we know that the output resistance r_O is in the range of 40 to 133 mega-ohms, while for the simple current mirror case we had a range roughly between 100 and 800 kilo-ohms. The cascode current mirror output resistance is 3 orders of magnitude than the simple current mirror output resistance, which shows how powerful and reliable the cascode current mirror is.

g)

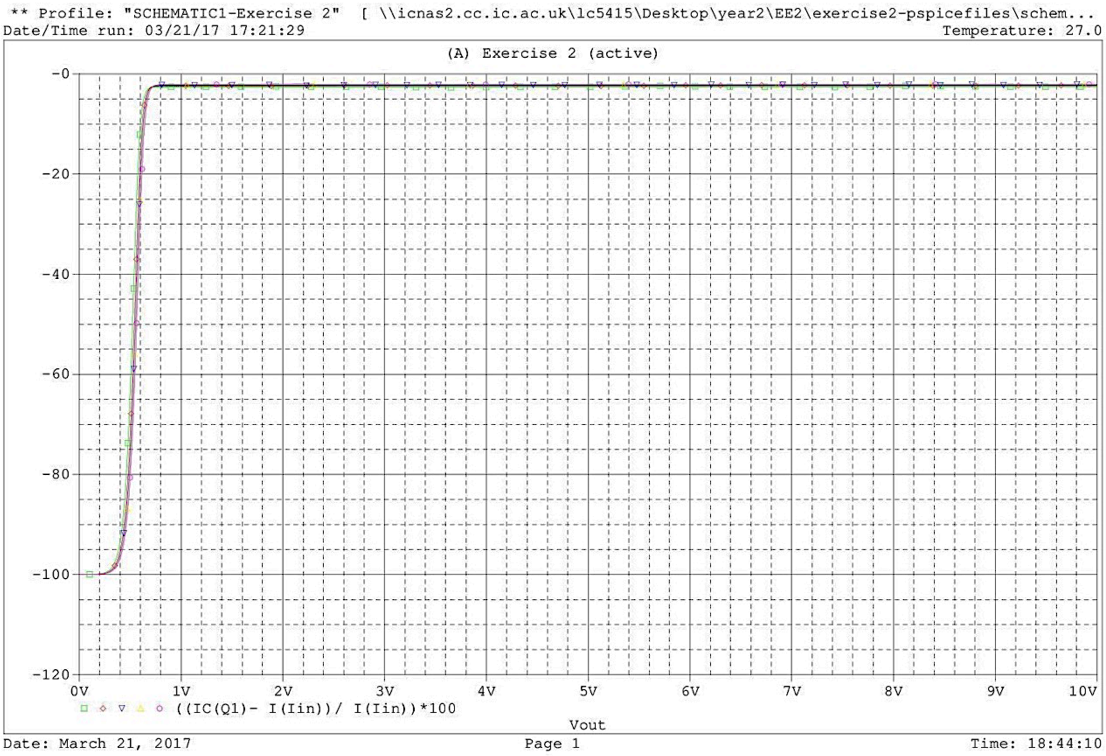


Figure 23: Relative error of the cascade current mirror for different values of I_{IN}

We find relative error values two times greater than the values for a simple current mirror and very high values of the error for V_{out} less than 0.8V. The error remains stable for increasing values of V_{out}.

I_{IN} [µA]	$Relative\ error$[%]
100	-2.66
200	-2.36
300	-2.25
400	-2.18
500	-2.13

Figure 24: Relative error of the mirroring action of cascade Cm at V_{out} = 1V

h) The simple CM has a much lower output impedance than the cascode CM, therefore the output current I_{out}, out of a simple current mirror would be less stable than the I_{out} from a cascode CM.

On the other hand the cascode CM comes with a greater relative error at low voltage values in comparison with the simple CM, which is not interesting if we want to precisely mirror a current.

In addition for a cascade CM the relative error is almost 100% for values of V_{out} between 0 and 400mV, then quickly rises to approximately a value of relative error of -2.5% for V_{out} equal to 800 mV and then remains stable at that value for higher values V_{out}.

For the simple CM, the value of the relative error quickly rises to negative units of error. For higher values of V_{out} instead of the relative error value remaining stable as it happens with the cascode CM, the simple CM relative error increases linearly as V_{out} increases.#

In conclusion, the simple CM has lower relative error if we work with small values of V_{out}, whereas the cascode CM is no reliable for small values of V_{out} as the relative error is too high but it is indeed more reliable than the simple CM for higher values of V_{out}. The cascode CM also comes with much higher (3 orders of magnitude greater) values of the output impedance than the simple CM, which is going to send out much more stable signals.

i)

Figure 25: Transfer characteristic of an emitter degenerated CM for degenerated resistance values of 1Ω and 400Ω

We find that for a high value of the degenerated resistance (400 Ω), the transfer characteristic of the current mirror is almost perfectly linear, giving values of I_{out} almost identical to the I_{IN} values.

The addition of resistor at the emitter of the BJTs is then beneficial to the mirroring action.

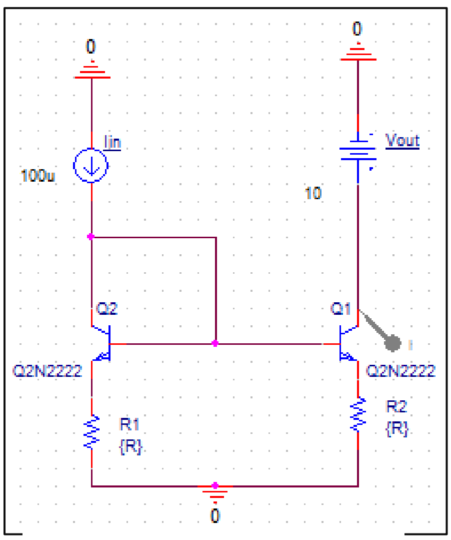


Figure 26: Emitter degenerated current CM with values of R of 1Ω and 400Ω