BE4 Computational Neuroscience: Problem set 3

Single Neuron Models

Piecewise linear Fitzhugh-Nagumo Model

The Fitzhugh-Nagumo model is defined by the equations

$$\left[\begin{array}{ll} \frac{du}{dt} & = F(u,w) = f(u) - w + I \\ \\ \frac{dw}{dt} & = G(u,w) = bu - w\,, \end{array}\right. \tag{1}$$

Here, u(t) is the membrane potential and w(t) is a second, time-dependent variable. I stands for the injected current. A simplified model is obtained by considering a piecewise linear f(u):

$$f(u) = \begin{cases} -u & \text{if } u < 1 \\ \frac{(u-1)}{a} - 1 & \text{if } 1 < u < 1 + 2a \\ 2(1+a) - u & \text{if } u > 1 + 2a \end{cases} \tag{2}$$

with 0<a<1, b>1/a.

(i) Sketch the nullclines in a (u,w)-plot. Consider the case I=0. How does the fixed point move as I is varied? Sketch the form of the flow (i.e., the vector (du/dt, dw/dt)) along the nullclines and deduce qualitatively the shape of the trajectories.

(ii) Calculate the Jacobian matrix evaluated at the fixed point,

$$J = \begin{pmatrix} \partial F/\partial u & \partial F/\partial w \\ \\ \partial G/\partial u & \partial G/\partial w \end{pmatrix} \tag{3}$$

• Determine, by studying the eigenvalues of J, the linear stability of the fixed point when varying the constant I.

• What happens if the fixed point destabilizes?

(We won't consider the situations where the fixed point coincides with one of the discontinuities at $u = 1$ and $u = 1 + 2a$, as this analysis is beyond the scope of the course).