Regularized Estimation of Structural Instability in Factor Models: The US Macroeconomy and the Great Moderation☆

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Abstract

This paper shows that the parsimoniously time-varying methodology of Callot and Kristensen (2015) can be applied to factor models. We apply this method to study macroeconomic instability in the US from 1959:1 to 2006:4 with a particular focus on the Great Moderation.

Models with parsimoniously time-varying parameters are models with an unknown number of break points at unknown locations. The parameters are assumed to follow a random walk with a positive probability that an increment is exactly equal to zero so that the parameters do not vary at every point in time. The vector of increments, which is high dimensional by construction and sparse by assumption, is estimated using the Lasso.

We apply this method to the estimation of static factor models and factor augmented autoregressions using a set of 190 quarterly observations of 144 US macroeconomic series from Stock and Watson (2009). We find that the parameters of both models exhibit a higher degree of instability in the period from 1970:1 to 1984:4 relative to the following 15 years. In our setting the Great Moderation appears as the gradual ending of a period of high structural instability that took place in the 1970s and early 1980s.

JEL codes: C01, C13, C32, C38, E32.

Keywords: Parsimoniously time-varying parameters, factor models, structural break, Lasso.

1. Introduction

The Great Moderation is a period of macroeconomic stability in the United States thought to have begun in the 1980s (Kim and Nelson, 1999; McConnell and Perez-Quiros, 2000; Stock and Watson, 2003), or even in the 1950s (Blanchard and Simon, 2001) with an interruption in the 1970s and early 1980s. This period is marked by a decline of inflation and by a relative stabilisation of the business cycle and of monetary policy which can be attributed either to a decline in output volatility or to changes in the dynamics of macroeconomic variables (Stock and Watson, 2009). The decline in output volatility is well documented by the authors cited previously, but this does not preclude the possibility of changes in the dynamics of macroeconomic variables. This paper proposes to quantify parameter instability in factor

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models before and during the Great Moderation using the parsimoniously time-varying (ptv) framework of Callot and Kristensen (2015) as it imposes minimal assumptions on the dynamics of the parameters.

The contributions of this paper take both a methodological and an empirical form. From a methodological point of view, we show that the ptv framework proposed by Callot and Kristensen (2015) can be readily used with factor models. In this framework, the parameters are assumed to follow a random walk with a positive probability that an increment is exactly equal to zero, and the resulting parameter paths are estimated using the Lasso. The empirical contribution of this paper consists in estimating a large number of factor models using US macroeconomic data to get a picture of the parameter instability in the last 50 years. We document that the instability is widespread, but that the majority of the breaks occur before the Great Moderation. We also document that allowing for moderate time variation in the parameters can substantially improve the fit of the model in a forecasting context suggesting that improvements in forecasting performance could be possible by taking parameter instability into account.

Factor models have been investigated and applied for more than a decade (for a general review, see Stock and Watson (2011); Bai and Ng (2008)). The problem of breaks and structural instability in the parameters of factor models remains a field open for exploration though. The seminal work of Stock and Watson (2002) provides insights into the problem, they show that the principal components (PC) factor estimator remains consistent if faced with moderate structural instability in the factor loadings. More recently the literature has seen a number of contributions related to testing for structural breaks in the loadings. The first formal test was proposed by Breitung and Eickmeier (2011) who considered the problem of testing the loadings associated with the individual variables. This has since been followed up by Chen, Dolado, and Gonzalo (2014) and Han and Inoue (2014) who propose tests for breaks in all loadings jointly, and Yamamoto and Tanaka (2013) who proposed a modified version of the Breitung and Eickmeier (2011) test.

More closely related to the present paper, Cheng, Liao, and Schorfheide (2014) use shrinkage methods to determine both the break points and the number of factors, and Corradi and Swanson (2014) propose a test for the joint hypothesis of breaks in the factor loadings and in the parameters of a factor augmented forecasting model. Although these contributions take different approaches to the issue of structural instability, they do have characteristics in common: They only consider a fixed number of breaks (often a single one) which are typically assumed to be common to all factors and large in magnitude. Breitung and Eickmeier (2011) show how such a setup will lead to overestimation of the number of factors. This is, in various ways, exploited in the mentioned papers to detect breaks. For example, Cheng et al. (2014) determine the break by minimizing the sum of the numbers of pre- and post-break factors instead of using a traditional sum-of-squared residuals criterion; and Corradi and Swanson (2014) utilize that the information criterion of Bai and Ng (2002) will overestimate the number of factors when the loadings are subject to breaks in order to construct a test statistic.

Our approach to the problem is different from existing papers. Instead of considering only a single or fixed number of breaks we allow for more general forms of instability in the parameters of the model. The parameters associated with the factors, and those associated with potential autoregressive terms, are allowed to change independently across variables and time. Bates, Plagborg-Møller, Stock, and Watson (2013) shows that the PC factor estimator is consistent under general forms of structural instability, including the forms of instability assumed in the ptv framework. The benefit of this is that we can rely on standard results regarding estimation of the factors and that e.g. the information criterion of Bai and Ng (2002) for selecting the number of factors is still valid.

Callot and Kristensen (2015) introduced the idea of ptv parameters for VAR models. This framework allows for an unknown number of break points at unknown locations by estimating the vector of changes in the parameters, which is high dimensional by construction and sparse by assumption, using the Lasso. In this paper we show that static factor models as well as factor augmented autoregressive (FAAR) models can be estimated in the ptv framework. The ptv models are well suited for our purpose of investigating the Great Moderation in terms of parameter stability in factor models. The parameters are modelled non parametrically, the estimation of the number of breaks and of their locations is data driven, and the parameters can remain stable for any duration or even the whole sample in which case our estimator is equivalent to OLS. This allows for the parameters to remain constant, experience a few changes (as in the structural breaks literature), or exhibit much more unstable paths, independently across variables.

We conduct an empirical study of the US macroeconomy using the ptv methodology with both static factor models and FAAR forecasting models. In particular we follow up on, and expand upon, the empirical investigation of the Great Moderation by Stock and Watson (2009). Stock and Watson (2009) finds that the Great Moderation was indeed associated with breaks in both factor loadings and the parameters of FAAR forecasting models. Using the ptv methodology of Callot and Kristensen (2015) we investigate structural instability throughout the sample with a focus on the period of the Great Moderation, and investigate whether accounting for structural instability in this fashion is helpful for forecasting.

The remainder of the paper is organized as follows, in the next section we present our analytical framework and discuss the estimation of the time-varying parameters and of the factors. Section 3 is dedicated to our empirical investigation of the Great Moderation and is followed by a conclusion summarizing our findings.

2. Analytical framework

This section presents the theoretical framework for our investigation of the Great Moderation. We begin by discussing the estimation of the static factor model, and FAAR model, with time-varying parameters used in the empirical section. We introduce the process assumed to drive the time-varying parameters, the parsimonious random walk, and present the results established in Callot and Kristensen (2015) for the estimation of models with parsimoniously time-varying parameters. This is followed by a discussion of the estimation of common factors when part of their loadings on the observed variables is assumed to follow a parsimonious random walk, and on the implications of using estimated factors instead of the unobserved factors.

2.1. Models

We make use of two models in this paper. A static factor model is used to estimate the factors and perform structural analysis. A FAAR model is also considered, with the primary purpose of forecasting. In the static factor model we assume that the variables of interest, X_{it}, $t = 1,\dots,T$, $i = 1,\dots,n$, are generated by a factor model with r_F unknown factors, F_t:

$$X_{it} = \lambda'_{it} F_t + e_{it}, \tag{1}$$

where $\lambda_{it} \in \mathbb{R}^{r_F}$ is the vector of factor loadings at time t for variable i. We also use this model with a large number of variables n to estimate the factors using PC. We provide consistency results for the estimated factors below, and discuss the data used to estimate the factors in the data section.

Factor models are frequently used for macroeconomic forecasting, in which context the estimated factors are used as predictors in the forecasting models. In the case of linear regression models, this is referred to as FAAR model. Such a model for a h-step ahead direct forecast can be written as

$$X_{it+h}^{(h)} = \mu_i + \beta'_{it} F_t + \sum_{j=0}^{p-1} \gamma_{ijt} X_{it-j} + \varepsilon_{it}. \tag{2}$$

To simplify the discussion on the estimation of these models, we write both of them in the same compact form and drop the variable subscript i:

$$y_t = \xi'_t Z_t + \epsilon_t. \tag{3}$$

Z_t is a vector of dimension $r \times 1$ containing the factors and, in the case of the FAAR model, the lags of the dependent variable. The $1 \times r$ vector of parameters ξ_t is assumed to be time varying, and y_t is either X_{it} or $X_{it+h}^{(h)}$.

If the parameters of both these models were assumed to be constant over time, and the factors known or estimated, we could consistently estimate the models by OLS. Instead we let the parameters of the models vary over time and, to be specific, we assume that the parameters follow parsimonious random walks. This process is formalized in the following assumption.

Assumption 1 (Parsimonious random walk)**.** *Assume that the parameters follow a parsimonious random walk with ξ_0 given.*

$$\xi_t = \xi_{t-1} + \zeta_t \odot \eta_t.$$

η_t and ζ_t are vectors of length r with the following properties:

$$\begin{aligned}
\alpha_T &= k_\alpha T^{-a}, \;\; 0 \le a \le 1, \;\; k_\alpha > 0 \\
\zeta_{jt} &= \begin{cases} 1, & w.p.\;\; \alpha_T \\ 0, & w.p.\;\; 1-\alpha_T \end{cases} \qquad j \in 1, \dots, r \\
\eta_t &= \mathcal{N}(0, \Omega_\eta) \\
E(\eta'_t \eta_u) &= 0 \, if \, t \neq u \\
E(\eta'_t \zeta_u) &= 0 \;\forall\, t, u \in 1, \dots, T
\end{aligned}$$

The vector of increment to the parameters is given by the element-by-element product of the two vectors of mutually independent random variables ζ_t and η_t. η_t is a set of $i.i.d.$ random variables with mean zero and bounded variance. ζ_t is a vector of binary variables in which each element takes value 1 with probability α_T and zero otherwise.

The parsimonious random walk assumption implies that many of the increments to the parameter vectors are equal to zero, the probability of a non zero increment is controlled by $\alpha_T = k_\alpha T^{-a}$. The constant k_α scales the probability α_T and must be such that $0 \le \alpha_T \le 1$. If k_α satisfies this restriction for some T_0, it will satisfy it for any $T \ge T_0$ since $a \ge 0$. Consistency requirements for the Lasso estimator will impose a tighter lower bound on a. Note that we do not set or estimate a (or α_T) but simply assume that a is larger than some quantity which we make explicit later.

2.2. Estimation of the parsimoniously time-varying parameter models

Assumption 1 implies that the vector of increments to the parameter vector is sparse, it contains many zero, but is also high dimensional since it is at least as large as the sample size, the number of parameters to estimate is of the order of rT.

Recall the compact model,

$$y_t = \xi_t' Z_t + \epsilon_t.$$

Define the following matrices:

$$Z^D = \begin{bmatrix} Z_1 & 0 & \cdots & 0 \\ 0 & Z_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Z_T \end{bmatrix}, W = \begin{bmatrix} I_r & 0 & \cdots & 0 \\ I_r & I_r & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I_r & I_r & \cdots & I_r \end{bmatrix}, Z^D W = \begin{bmatrix} Z_1 & 0 & \cdot & 0 \\ Z_2 & Z_2 & \cdot & 0 \\ \vdots & \vdots & \ddots & \vdots \\ Z_T & Z_T & \cdots & Z_T \end{bmatrix}.$$

We can write the parsimoniously time-varying model (3) as a simple regression model

$$y = Z^D W \theta + \epsilon$$

where the parameter vector $\theta' = [\xi_0' + \zeta_1' \odot \eta_1', \zeta_2' \odot \eta_2', ..., \zeta_T' \odot \eta_T']$ has length rT, and $y = (y_1, ..., y_T)'$, $\epsilon = (\epsilon_1, ..., \epsilon_T)$'. The matrix $Z^D W$ contains T observations for rT covariates constructed from the original r variables. The first r elements of θ are the sum of the initial value of the parsimonious random walk ξ_0 and the first increment $\zeta_1 \odot \eta_1$. The subsequent elements of θ are the increments of the parsimonious random walk $\zeta_t \odot \eta_t,\ t > 1$ so that by cumulating the entries of θ we can recover the full path of the parameters.

The parameter vector we seek to estimate, θ, is sparse and high dimensional, we estimate θ using the Lasso as in Callot and Kristensen (2015). We make extra assumptions below regarding the distribution of the innovations and of the factors, in particular to ensure that all the variances involved are finite.

Assumption 2 (Covariates and innovations)**.** *Assume that:*

i) $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ *is a sequence of* $i.i.d$ *innovation terms,* $\sigma_\epsilon^2 < \infty$.

ii) $F_t \sim \mathcal{N}(0, \Omega_F^2)$. *For all* $k = 1, ..., r_F$, $\mathrm{Var}(F_{kt}) = \sigma_{Fk}^2 < \infty$.

iii) $E(\epsilon' F) = 0$.

iv) $\mathrm{Var}(y_t) \leq M < \infty$ *for all* t *and some positive constant* M.

Assumption 2 *(iv)* ensures that the variance of y_t is bounded from above at all points in time. It is a high level assumption that we give here, a lower level assumption on the dynamics of the model is used instead in Callot and Kristensen (2015) to which the interested reader is referred.

Similarly we state the following assumption, the restricted eigenvalue condition, which is a standard assumption in the Lasso literature, introduced by Bickel, Ritov, and Tsybakov (2009). Define κ_T^2 as the restricted eigenvalue, we then assume the following restricted eigenvalue condition to hold:

Assumption 3 (Restricted eigenvalue condition)**.** *Assume that:*

i) $\kappa_T^2 > 0$.

ii) $\kappa_T^2 \in \Omega_p(T^{d-1})$ *for some* $d \in (0,1]$.[1]

The rate of decrease of κ_T^2 stems from technical assumptions on the rate at which the distance between breaks increases asymptotically; the detail of the assumptions and proof of the result can be found in Callot and Kristensen (2015).

We can now state the first theorem from Callot and Kristensen (2015) which provides upper bounds on the prediction and parameter estimation errors. Let $\left[\sigma_1^2, ..., \sigma_{rT}^2\right] = \text{diag}(\text{Var}(Z^D W))$ and $\sigma_T^2 = \max\left(\sigma_\epsilon^2, \max_{1\le k\le rT} \sigma_k^2\right)$ where σ_ϵ^2 is the variance of ϵ and σ_k^2 is the variance of the k^{th} column in $Z^D W$. Define the active set $\mathscr{S}_T$ as the set of indices corresponding to non-zero parameters in θ, $\mathscr{S}_T = \left\{j \in (1,...,rT) | \theta_j \neq 0\right\}$, and its cardinality $|\mathscr{S}_T| = s$. Finally, let $\tilde{\lambda}_T$ be the Lasso penalty parameter. We then have the following result:

Theorem 1. *For* $\tilde{\lambda}_T = \sqrt{\frac{8\ln(1+T)^5 \ln(1+r)^2 \ln(r(T-r+1))\sigma_T^4}{T}}$ *and some constant* $A > 0$, *under assumptions 1, 2, and 3, and on the set* $\mathscr{B}_T$ *with probability at least equal to* $1-\pi_T^{\mathscr{B}}$ *we have the following inequalities:*

$$\frac{1}{T}\left\| Z^D W(\hat{\theta}-\theta)\right\|^2 \le \frac{16 s \tilde{\lambda}_T^2}{\kappa_T^2}, \tag{4}$$

$$\left\|\hat{\theta}-\theta\right\|_{\ell_1} \le \frac{16 s \tilde{\lambda}_T}{\kappa_T^2}, \tag{5}$$

with $\pi_T^{\mathscr{B}} = 2(1+T)^{-1/A} + 2(r(T-r+1))^{1-\ln(1+T)}$.

The bounds given in theorem 1 are upper bounds on the ℓ_1 norm of the parameter estimation error and on the mean squared estimation error of the Lasso. These bounds hold on a set that has probability at least $1-\pi_T^{\mathscr{B}}$ for a given value of $\tilde{\lambda}_T$. Nonetheless these bounds are valid for any value of the penalty parameter as long as $\left\| T^{-1}\epsilon' Z^D W\right\|_\infty \le \tilde{\lambda}_T/2$ is satisfied. Holding everything else constant the probability of this inequality being satisfied decreases with $\tilde{\lambda}_T$ as do the upper bounds in theorem 1; there is a trade-off between the tightness of the bounds and the probability with which they hold.

Theorem 2 below provides an asymptotic counterpart to theorem 1.

Theorem 2. *Let a and d be scalars with* $a, d \le 1$, $1-a+d \le 1$, *and* $\frac{3}{2}-a-d < 0$.*Then under assumptions 1, 2, and 3, and as* $T \to \infty$ *we have:*

$$\frac{1}{T}\left\| Z^D W(\hat{\theta}-\theta)\right\|^2 \to^p 0 \tag{6}$$

$$\left\|\hat{\theta}-\theta\right\|_{\ell_1} \to^p 0 \tag{7}$$

This theorem establishes the consistency of the Lasso for our models. The parameter estimation error, and the prediction error, both tend to zero with probability tending to one.

[1] $f(T) \in \Omega_p(g(T))$ means that there exists a constant $c > 0$ such that $f(T) \ge c g(T)$ for $T \ge T_0$ for a certain T_0 onwards with probability approaching one.

Callot and Kristensen (2015) shows that, as is usual with the Lasso, the estimator correctly sets many parameters to zero without setting non zero parameters to zero under the condition that the smallest non zero parameter is not too small. They also show that the adaptive Lasso, a second stage estimator using an adaptive penalty constructed using the Lasso estimates, is sign consistent. This means that the adaptive Lasso set all zero parameters to zero while retaining the non zero parameters with probability tending to 1. We refer to Callot and Kristensen (2015) for details.

2.3. Factor estimation with parsimoniously time-varying loadings.

Until this point we have assumed the factors to be known, in this section we discuss the issue of estimating the factors under the assumption that their loadings can be parsimoniously time varying. We then discuss the effect of using estimated factors instead of the true factors in the model.

If we assume that the factor loadings do not vary over time, then it is well known that the factors can be consistently estimated by means of PC, see e.g. Bai and Ng (2002). Bates et al. (2013) gives conditions under which the same holds in the case of time varying loadings. We use the results of Bates et al. (2013) to show that factors can be consistently estimated when the loadings a follow parsimonious random walk as in assumption 1. We follow the notation of Bates et al. (2013) closely (which also corresponds to the notation of Bai and Ng (2002)).

Whereas Bates et al. (2013) give general results in the case of time-varying loadings we will specifically make the following assumption:

Assumption 4 (Time-varying Factor Loadings)**.** *The loadings in* (1) *must satisfy the following:*

i) With probability $\pi_{n,T}$ the loadings for variable i, λ_{it}, follow a parsimonious random walk as defined in Assumption 1 with fixed initial value λ_{i0}. Alternatively, with probability $1-\pi_{n,T}$ the loadings are constant.

ii) The probability $\pi_{n,T}$ must satisfy: $\pi_{n,T} = \mathcal{O}(1/\min(n^{1/2}T^{1-a}, T^{3/2-a}))$ where a is the parameter controlling α_T as defined in Assumption 1 with $a \geq 1/2$.

iii) For all (i, j, s, t), e_{it}, the factor model innovations, are independent of $(\zeta_{js} \odot \eta_{js}, F_s)$, the factor loading innovations and the factors. Furthermore, the factor loading innovations, $\zeta_{ip,t}\eta_{ip,t}$, are independent across i, t, and p.

We should note that assumption 4*(iii)* implies that breaks occur independently across variables. Although this does not necessarily correspond well with empirical observations, where series tend to co-break, this assumption is needed to ensure that the factors can be consistently estimated. Note, however, that independence is only needed for the factor estimation, the results of theorem 1 hold under the more general requirements of assumption 1.

In addition to this we make the following standard assumptions (corresponding the assumptions 1–3 in Bates et al. (2013) or assumptions A–C in Bai and Ng (2002)):

Assumption 5 (Factors)**.** $\mathbb{E}\|F_t\|^4 \leq M$ *and* $T^{-1}\sum_{t=1}^{T} F_t F_t' \to^p \Sigma_F$ *as* $T \to \infty$ *for some positive definite matrix* Σ_F.

Assumption 6 (Initial Factor Loadings)**.** $\|\lambda_{i0}\| \leq \bar{\lambda} < \infty$, *and* $\|\Lambda_0'\Lambda_0/n - D\| \to 0$ *as* $n \to \infty$ *for some positive definite matrix* $D \in \mathbb{R}^{r_F \times r_F}$.

Assumption 7 (Idiosyncratic Errors)**.** *The following conditions hold for all n and T.*

1. $\mathbb{E}(e_{it}) = 0$, $\mathbb{E}|e_{it}|^8 \leq M$.

2. $\gamma_n(s,t) = \mathbb{E}(e_s' e_t / n)$ *exists for all* (s,t). $|\gamma_n(s,s)| \leq M$ *for all* s, *and* $T^{-1}\sum_{s,t=1}^{T}|\gamma_n(s,t)| \leq M$.

3. $\tau_{ij,ts} = \mathbb{E}(e_{it}e_{js})$ *exists for all* (i,j,s,t). $|\tau_{ij,tt}| \leq |\tau_{ij}|$ *for some* τ_{ij} *and for all* t, *while* $n^{-1}\sum_{i,j=1}^{n}|\tau_{ij}| \leq M$. *In addition,* $(nT)^{-1}\sum_{i,j=1}^{n}\sum_{s,t=1}^{T}|\tau_{ij,ts}| \leq M$.

4. *For every* (s,t), $\mathbb{E}\left|n^{-1/2}\sum_{i=1}^{n}[e_{is}e_{it} - \mathbb{E}(e_{is}e_{it})]\right|^4 \leq M$.

Under these assumptions we have the usual consistency result, the proof of which can be found in the appendix.

Theorem 3. *Let assumptions 4–7 hold, and* $n, T \to \infty$ *with* $T^{1-a}/n^{1/2} \to k$ *for some constant* $k \geq 0$, *then*

$$C_{nT}^2 \left(T^{-1} \sum_{t=1}^{T} \left\| \widehat{F}_t - H' F_t \right\|^2 \right) = \mathcal{O}_p(1) \tag{8}$$

where $C_{nT}^2 = \min(n,T)$ *and* H *is the usual rotation matrix as defined in e.g. Bates et al. (2013).*

Note that Assumption 4 puts restrictions on the extent to which the variables are allowed to have time-varying loadings through the probability $\pi_{n,T}$. This is in contrast to the literature on large breaks where it is typically assumed that the break affects all variables. However, the results are comparable to Example 3 in Bates et al. (2013) which treats the case of a single large break. The comparable case in terms of our results would be that of a fixed number of breaks which occurs if we have $a = 1$ which implies that $\pi_{n,T} = \mathcal{O}(1/\min(n^{1/2}, T^{1/2}))$. Likewise, Example 3 in Bates et al. (2013) requires that at most $\mathcal{O}(n^{1/2})$ variables undergo a break. In cases where $a < 1$ we have a trade-off between the (expected) number of breaks in the loadings (which is now increasing in T) and the (expected) number of series which may be associated with breaks as controlled by $\pi_{n,T}$. It is important to note that in order to obtain the result we must restrict the relative growth of n and T. However, by doing so we are able to recover the usual rate of convergence. This is important because it ensures that we can apply the IC_p criterion of Bai and Ng (2002) to determine the number of factors (Bai and Ng, 2002, Corollary 2).

2.4. Estimation of the ptv model with estimated factors

From assumption 2 one can see that the covariates are required to be normally distributed with finite variance. Since the estimated factors $\widehat{F}$ are a linear combination of the data X, it suffice to ensure that X_t is Gaussian to ensure that the estimated factors satisfy assumption 2. Bai and Ng (2006) show that using estimated factors does not invalidate consistency and asymptotic normality of the OLS estimator provided $\sqrt{T}/n \to 0$. Given that the objective function used to estimate the ptv models is a penalized least squares criterion, and that theorem 3 shows that the factor are estimated with the usual efficiency, we conjecture that the same holds true for the ptv model.

3. Empirical Results

Stock and Watson (2009) set out to investigate the effect of a structural break on factor models and in particular their ability to forecast. They consider a very specific case, namely the Great Moderation, which they argue could have caused a break in the mid 1980s. Specifically, they test for a structural break in 1984:1 in both the loadings of the factor model and the parameters of the FAAR forecasting model. We take the analysis, and the data, of Stock and Watson (2009) as our starting point and investigate parameter instability over the entire sample period.

This section contains a description of the data we use and practical details regarding the implementation of the ptv method described above. We then present our empirical results; first focusing on aggregate parameter instability, second investigating more closely some particular variables of interest, and finally looking at the improvements in fit of the ptv model relative to OLS.

3.1. Data

We use the same dataset as Stock and Watson (2009). It consists of 144 quarterly time series for the United States spanning the period 1959:1–2006:4. As is customary in the literature the series are transformed to be stationary and standardized prior to estimation. Appendix B provides details on the transformations and a complete variable list. Due to the transformations we lose the first two observations, hence the effective sample period is 1959:3–2006:4, i.e. a total of $T = 190$ quarterly observations.

A unique feature of this dataset, as discussed in Stock and Watson (2009), is the treatment of disaggregation. The full dataset contains both aggregate and sub-aggregate series, and as argued in their paper the inclusion of series related by identities (being the sum of sub-aggregates) does not add useful information to the estimation problem. For this reason when estimating the factors we only use a subset of the data consisting of 109 series that excludes higher level aggregates related by identities to the lower level sub-aggregates. However, for the analysis of the structural stability of the loadings below, we use all 144 series as they are all related to the factors. Hence, in the first step we estimate the factors using the 109 series, and in the second step we use the methodology described above to estimate the time-varying loadings for all 144 series.

3.2. Estimation

Although we have described the estimation procedure in general there are a number of details we must address before the estimation can be carried out in practice. As noted, we estimate the factors by PC, however to do this we must decide on how many factors to include in the model. A number of methods have been proposed for this, and one of the most commonly used is undoubtedly the IC_p information criterion of Bai and Ng (2002). The authors provide three variants of their criterion, and the results for our dataset differ substantially depending on which is used. Specifically, the criteria choose either 2, 4, or 10 factors. One could argue that the safe choice would then be to use 10 factors, but this could lead to undesirable over-fitting, and hence we follow Stock and Watson (2009) and use 4 factors throughout the paper.

The 4 estimated factors we will use throughout this application are plotted in figure 1. The first factor appears to decrease before, and reach a minimum during, each recession while the second factor has an almost symmetrical pattern with droughts during recessions and rapid increases at the end of each recession and during the recovery period. These factors could be interpreted as capturing the business cycle. The third and fourth factors are much more volatile than the first two, factor 3 exhibits a trend while factor 4 seems to have a higher variance in the middle of the sample than on either end. These last two factors do not lend themselves to an unambiguous economic interpretation. For a possible explanation of this recall that one of the information criteria of Bai and Ng (2002) suggested that there may be only two factors implying that factors 3 and 4 contain little if any relevant information.

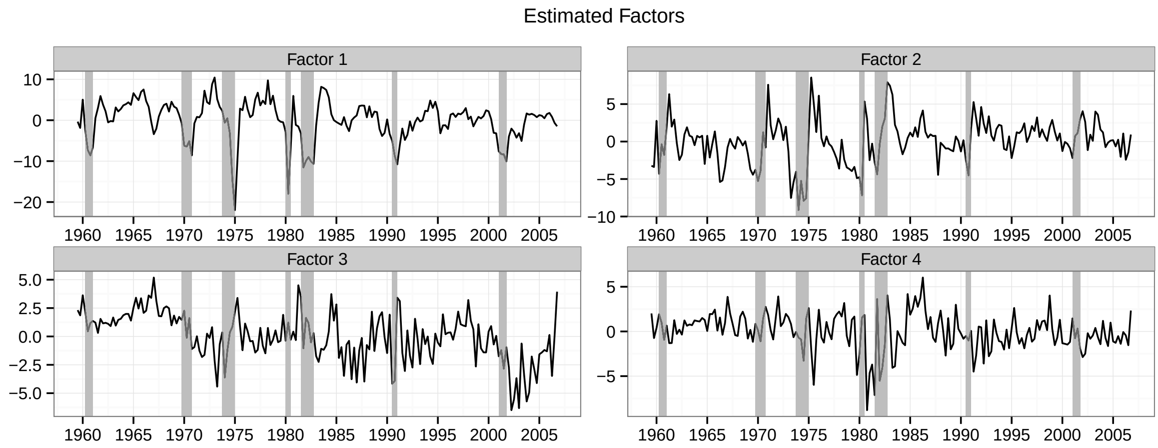

Figure 1: Estimated factors, NBER recessions in grey.

For the FAAR forecasting model we must be specific about the forecast target. We focus on the stability of a four-quarter ahead relationship. For real activity variables the target, $X^{(4)}_{it+4}$, is growth over the next four quarters. For inflation it is average quarterly inflation over the next four quarters minus last quarter's inflation. For variables in levels it is simply the value of the variable four quarters ahead. For details, see Appendix B. Further, in order to fully specify (2) we must choose the lag length p. As we are not concerned with computing actual forecasts, this choice is of less importance, and we simply fix it at $p = 4$.

The actual estimation using the procedure of Callot and Kristensen (2015) is easily implemented using the `parsimonious` package in `R`.[2] As is generally the case the Lasso requires selection of the penalty parameter. This can be done in a number of ways, however, we will follow Callot and Kristensen (2015) and select it using the Bayesian Information Criterion (BIC). As already noted all variables have been standardized, however, the estimated factors will likely have a much larger variance making it difficult for the Lasso to simultaneous detect breaks in the parameters associated with the factors and the parameters associated with the autoregressive lags in (2) if they are all penalized equally. To avoid having to introduce different penalty parameters for the factors and the autoregressive lags, we instead also standardize the estimated factors before the Lasso estimation. We will do this in both the case of the factor model (1) and the FAAR forecasting model (2). The estimation procedure also easily allows for the possibility of having a parsimoniously time-varying intercept by simply including dummies. The need for taking into account instability in the mean of the variables was documented by Stock and Watson (2012). They subtracted a local mean from the variables prior to estimation and found that this mean changed substantially over the sample period. All the presented results are based on models with parsimoniously time-varying intercepts. Our results are, however, robust to this choice and qualitatively similar results are obtained if the intercept is assumed constant. Finally, in the interest of space we only report results for the Lasso and do not consider the adaptive Lasso.

3.3. Results

We begin by examining the stability of the loadings in the factor model (1). Figure 2 plots the number of breaks in the loadings at every point in time for each factor. Recall that at each

[2]Replication files can be found at `https://github.com/lcallot/ptv-fac`.

point in time for a given factor the vector of loadings is of length $n = 144$, hence if we observe, say, 10 breaks, then that means that 10 out of the 144 variables experience a break.

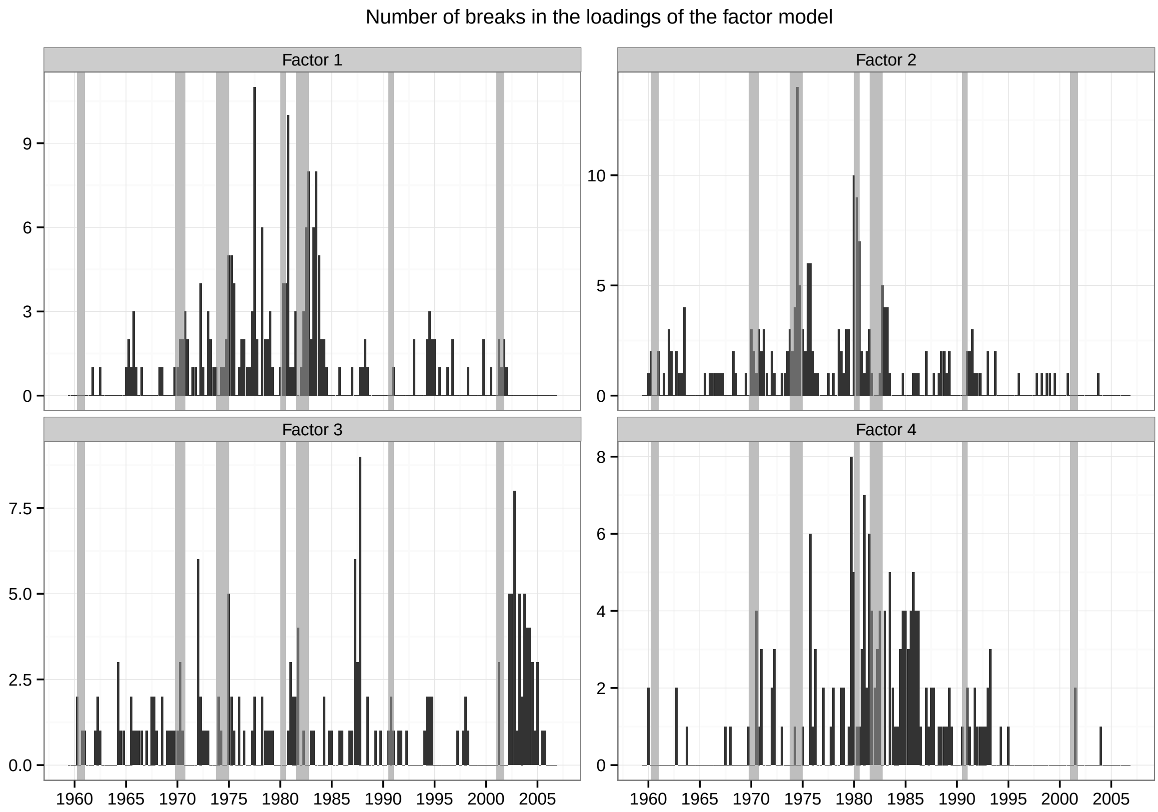

Figure 2: Breaks in the factor model, $n = 144$. The grey areas represent the NBER recessions.

The first impression given by figure 2 is that there is some degree of structural instability over the entire sample period. Closer inspection reveals that in every panel there exists clusters with a large number of breaks, and that outside of these clusters the loadings appear to be relatively stable. This is in particular the case for the parameters associated with the first and second factors where many breaks are detected from the early 1970s to the early 1980s, and much fewer breaks outside this period. From this plot it is not clear whether the instability is greater during recessions.

The parameters associated with the third factor experience breaks relatively uniformly throughout the sample, with a surge in 1987 and after 2000. The breaks in the loadings associated with the fourth factor are concentrated in the 1975 to early 1990s period. Figure C.9 (appendix) displays the number of breaks in the intercept at every point in time for the factor and forecasting models. For both models the breaks in the intercept appear to be uniformly spread across the sample in contrast to the clusters observed in the factor loadings.

In figure 3 we consider mean absolute change in the loadings defined as $n^{-1}\sum_{i=1}^{n}\left|\lambda_{it}-\lambda_{it-1}\right|$ for a given point in time t where the mean is taken across all $n = 144$ variables. This illustrates the large differences in the sizes of the breaks. It appears that most of the breaks are (relatively) small, but few large changes occur from the mid 1970s to the mid 1980s in the loadings of factors 1, 2, and 4. This plot provides further evidence in favour of the Great Moderation as a period of stability following a period of structural instability in the 1970s and early 1980s.

In order to illustrate how allowing for breaks affects the fit of the model we turn to the FAAR forecasting model. Figure 4 shows the cross-sectional root mean squared error (RMSE),

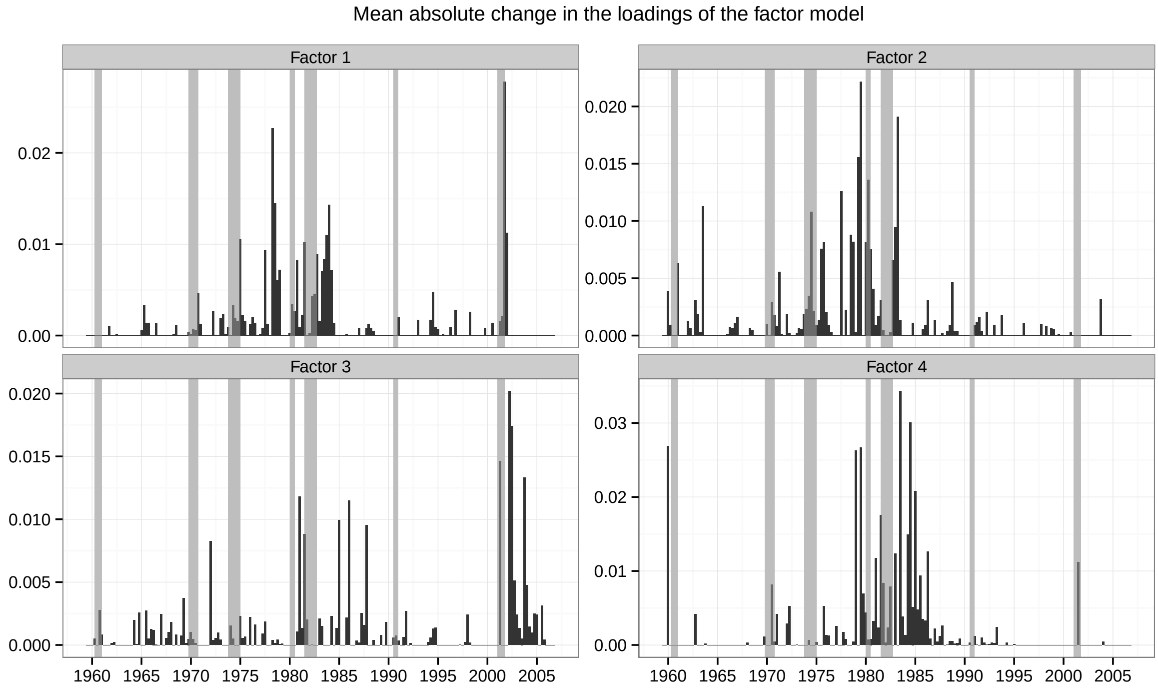

Figure 3: Size of breaks in the factor model, $n = 144$. The grey areas represent the NBER recessions.

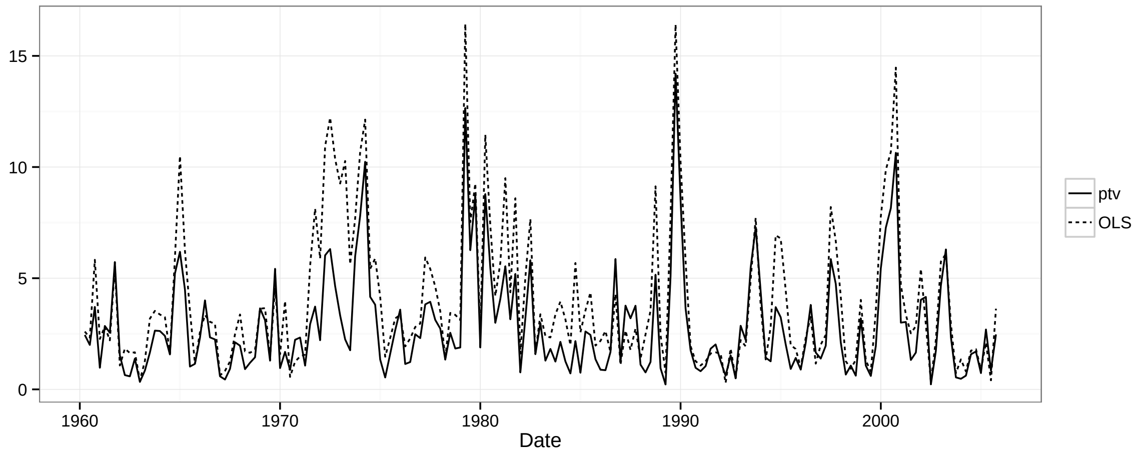

Figure 4: Cross-sectional RMSE in the FAAR model.

that is, $\sqrt{\frac{1}{144}\sum_{i=1}^{144}\hat{\epsilon}_{it}^2}$ for all $t = 1, \ldots, 190$. The gain in RMSE from allowing parsimoniously time-varying parameters (relative to OLS) is particularly large in the early 1970s, corresponding to the region with more and larger breaks in Figures 2 and 3 for the factor model. We will return to results on the fit of the FAAR model in more detail later.

First, we now look into the detail of which variables are associated with the breaks in table 1. In the table we consider both the factor model (1) and the FAAR forecasting model (2). We report the number of breaks over the entire sample (denoted All), the period preceding the Great Moderation from 1970:1 to 1984:4, and the period of the Great Moderation from 1985:1 to 1999:4. For the forecasting model we separately consider the number of breaks in the parameters associated with the factors, β, and the parameters associated with the autoregressive lags, γ. In the interest of space the table only includes a subset of the variables,

Series	Factor model			Forecasting model					
				Coef. on factors			Coef. on AR lags		
	All	1970:1 1984:4	1985:1 1999:4	All	1970:1 1984:4	1985:1 1999:4	All	1970:1 1984:4	1985:1 1999:4
RGDP				1	1				
Cons				4	3		5	4	
GPDInv	1	1		1	1				
Exports									
Imports									
Gov									
IP: total	5	3		1	1				
NAPM prodn	4	3	1						
Capacity Util	18	9	2	11	10	1	8	2	
Emp: total				2	2				
Help wanted indx	1	1							
Emp CPS total				2	2				
U: all				2	2				
HStarts: Total	28	15	4	17	11		11	11	
BuildPermits	45	17	11						
PMI									
NAPM new ordrs									
NAPM vendor del	6	6		9	6		7	4	
NAPM Invent	1	1		2	2		6	6	
PGDP				17	13	3	8	8	
PCED				8	7	1	1	1	
CPI-All				3	3				
PCED-Core				4	4				
CPI-Core				41	29	1	15	15	
PGPDI				24	18	6	5	5	
PFI				23	18	5	4	4	
PEXP				14	12	1	4	4	
PIMP				17	16	1	5	5	
PGOV				3	3				
Com: spot price (real)									
OilPrice (Real)	6	2	4	43	28	13	38		29
NAPM com price	10	5	3	2	2		3	3	
Real AHE: goods	12	7	3	2	2		6	6	
Labor Prod				1		1			
Real Comp/Hour				1	1				
Unit Labor Cost	2	2		7	7		3	3	
FedFunds	4	4		7	5	1	8	8	
6 mo T-bill	3	3		4	4		6	6	
5 yr T-bond	2	2		4	3	1	2	2	
Aaabond	1	1		1	1		3	3	
Baa bond				2	2		6	5	1
M1				2	2				
MZM	30	20	8	12	7	2	4	4	
M2	1	1		9	5	3	4	3	1
MB				2	1	1	2		1
Reserves tot	29	11	7	5	3	1	4		
Bus loans									
Cons credit							1	1	
Ex rate: avg	3	2	1						
S&P 500	9	5	2	12	6	4	4	1	3
DJIA	1		1	2		2			
Consumer expect	2	2		1	1				

Table 1: Number of breaks in the loadings of the factor model and the parameters of the forecasting model detailed by variable.

most sub-aggregates have been removed, results for the remaining variables can be found in Appendix C.

Regarding the timing of the breaks, table 1 clearly shows that the majority of the breaks, all of them in some cases, occur in the period preceding the Great Moderation with both models. There is a striking difference in the number of breaks between the two models though; many more breaks are selected in the forecasting model than in the factor model, and the variables with a large number of breaks are not systematically the same across models. Three possible explanations for the difference between models are, first, the different target variables as explained in section *3.2*, second, the doubling of the number of variables, and third, the potentially high degree of persistence of the extra variables.

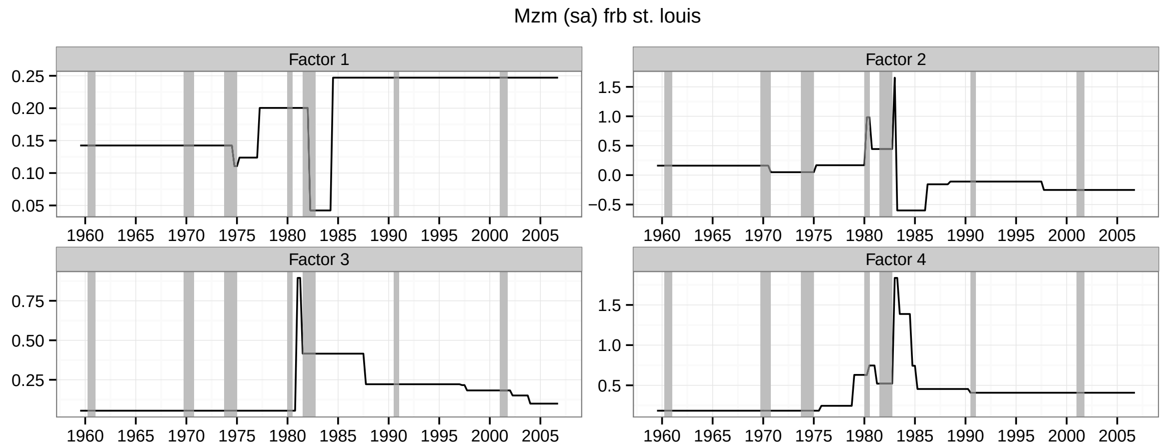

Figure 5: Time-varying loadings of the factor model, Money Zero Maturity.

Table 1 gives some interesting information as to which variables are subject to instability, despite the differences between the two models. In the static factor model the breaks are concentrated among monetary and financial variables: MZM (Money Zero Maturity) has 30 breaks and the total amount of reserve 29, interest rates (4, 3, and 2 breaks for the Fed fund rate, the 6 month T-bill, and the 5 year T-bond respectively), exchange rates (3), and the Dow Jones Industrial Average (1). Many breaks are also found among variables from the real sector of the economy, housing start (28 breaks) and build permits (45), industrial production (5), and the manufacturing indices (NAPM) with 6 and 1 breaks in two out of four variables.

The pattern is different for the FAAR models that have more breaks overall, mostly in the factor loadings, and occurring within different groups of variables. Price variables experience many breaks, up to 56 for CPI Core and 81 for Oil Price. Breaks are also frequently detected in monetary and financial variables, up to 15 for the 5 year T-bond and around 10 for the other variables of this type.

To illustrate the results discussed above we now take a close look at the estimated loadings of four variables. While we will focus on models in which the estimated parameters vary over time, it should be kept in mind that in a large fraction of loadings no break is found. Furthermore, and as the plots below will make apparent, within models exhibiting breaks the parameters associated with certain variables are still found to be constant.

Figure 5 shows the estimated loadings of the static factor model for Money Zero Maturity, which is one of the variables with which the largest number of breaks is associated. Most of these breaks, in particular the large ones, occur between the late 1970s and the early 1980s which is consistent with important changes in monetary policy during that period. The few

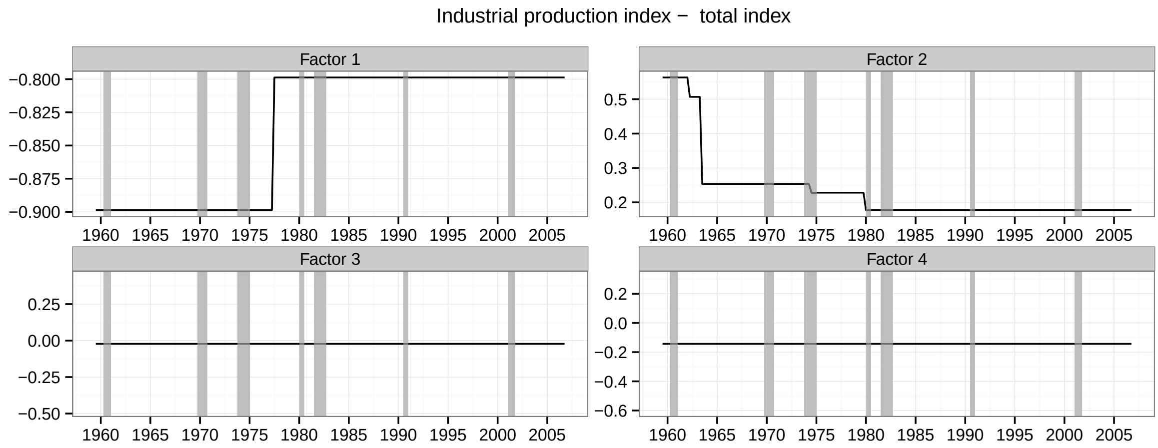

Figure 6: Time-varying loadings of the factor model, Industrial Production.

breaks occurring outside this period are for the most part very small changes to the estimated loading.

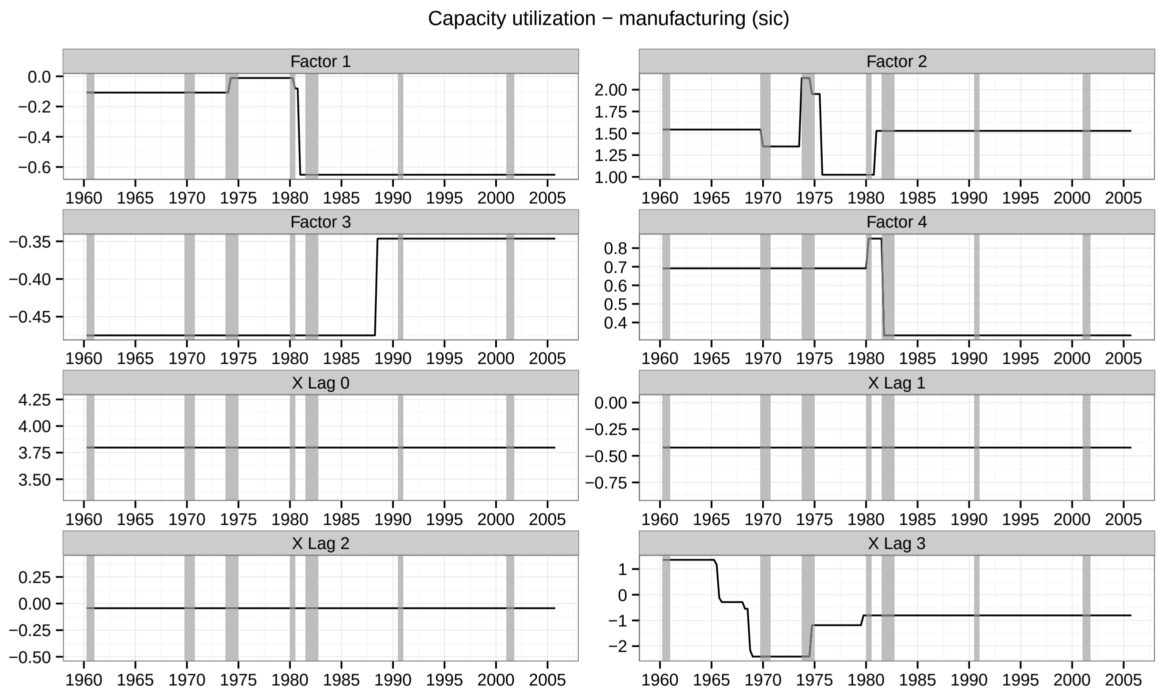

Figure 7: Time-varying loadings of the forecasting model, Capacity Utilization.

Figure 6 shows the estimated loadings for industrial production, for which 5 breaks are found. The loading on the first factor has a single large upward break in 1977 which could be interpreted as indicating that industrial production reacts less to downturns from that point onwards. The second factor has 2 large downward breaks in the early 1960s and two smaller breaks in the second half of the 1970s, which could be interpreted as indicating that industrial production recovers more slowly. The loadings for the last two factors are found to be constant.

Figure 7 shows the estimated parameters of a forecasting model for the Capacity Utilization variable, where 11 breaks are found in the factor loadings and 8 in the AR lags. Figure 7 shows

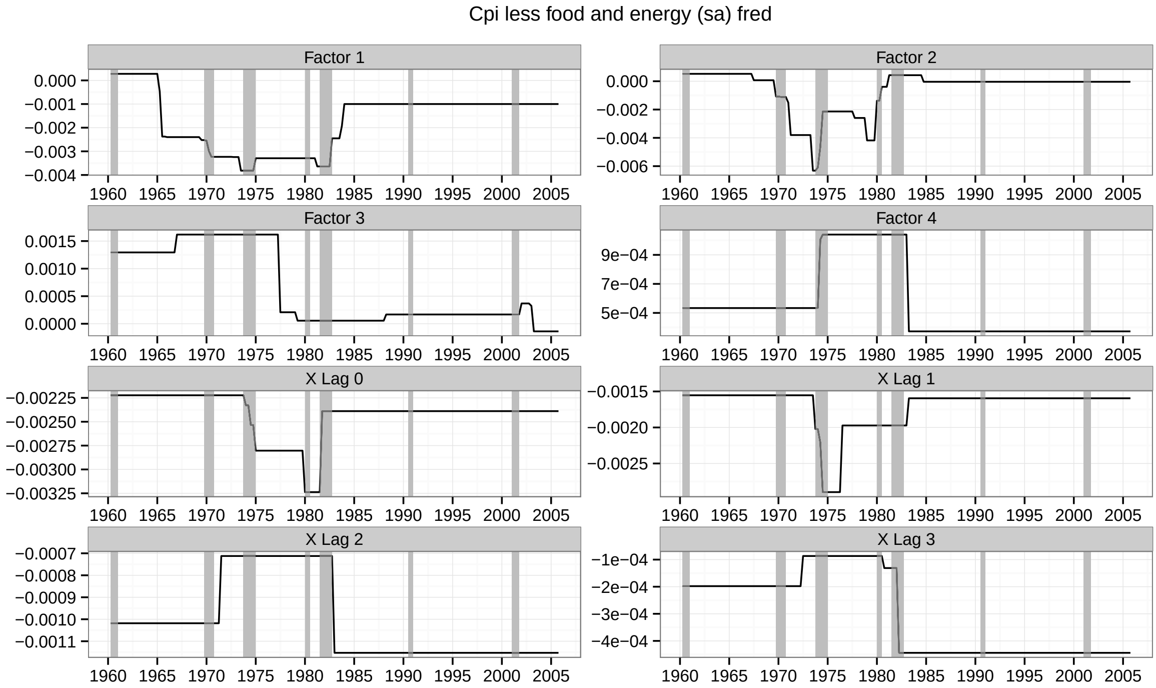

Figure 8: Time-varying loadings of the forecasting model, CPI Core.

that 5 out of the 8 parameters experience a break and that the instability typically takes the form of few large breaks with very little gradual adjustment.

Figure 8 is an example of a forecasting model with a large number of breaks with a total of 56, 41 in the factor loadings and 15 in the AR parameters, 44 of these breaks are located in the pre Great Moderation period. In this model, the breaks lead to many substantial and persistent changes in the parameter value. The factor loadings have more breaks than the AR parameters, this results in more jagged paths with several small adjustments in the parameter path. The loadings on the factors could be interpreted as implying that the business cycle has had little effect on CPI Core except for the middle of the 1970s, a period of low economic growth and high inflation in the US.

In a forecasting context model instability is a concern, and as table 1 illustrates many forecasting relationships do not appear to be stable over time. We do not view the applied methodology as a means to obtain better forecasts, but as a tool to illustrate that there might be issues when neglecting instabilities or breaks. Nonetheless it is still of interest to see how much the fit of the forecasting model is improved when we allow for a moderate amount of time variation in the parameters. For this purpose table 2 reports a number of useful statistics for a subset of variables, the remaining results can be found in Appendix C.

The first column of the table gives the standard deviation of the series being forecast. The second column gives the RMSE of the residuals of the forecasting model (2) when estimated by the ptv methodology, i.e. allowing for breaks. The third column gives the RMSE of the residuals of the forecasting model (2) when simply estimated by OLS, i.e. not allowing for breaks. The fourth column gives the relative RMSE of these two approaches and the last column the number of breaks in the corresponding model. The upper bound for the relative RMSE statistic is 1 and corresponds to the case where no breaks were selected, in which case our estimator is equal to OLS. Allowing for breaks must improve the fit to compensate for the penalty in the Lasso estimation, therefore when breaks are estimated the relative RMSE is

Series	Std. dev. of $X_{it}^{(4)}$	RMSE, ptv	RMSE, OLS	Relative RMSE	# Breaks
RGDP	0.0217	0.0164	0.0174	0.9421	1
Cons	0.0175	0.0118	0.0144	0.8160	9
GPDInv	0.0979	0.0713	0.0736	0.9689	1
Exports	0.0608	0.0517	0.0517	1.0000	0
Imports	0.0694	0.0509	0.0509	1.0000	0
Gov	0.0245	0.0211	0.0211	1.0000	0
IP: total	0.0433	0.0293	0.0317	0.9237	1
NAPM prodn	6.9244	5.7070	6.1090	0.9342	0
Capacity Util	4.5537	1.6163	2.5634	0.6305	19
Emp: total	0.0202	0.0126	0.0134	0.9415	2
Help wanted indx	12.2287	8.2756	8.9425	0.9254	0
Emp CPS total	0.0133	0.0090	0.0098	0.9258	2
U: all	0.9700	0.6328	0.6604	0.9581	2
HStarts: Total	0.2094	0.0862	0.1579	0.5456	28
BuildPermits	0.2467	0.1747	0.1747	1.0000	0
PMI	6.5564	5.1061	5.7118	0.8940	0
NAPM new ordrs	7.3697	5.8167	6.4712	0.8989	0
NAPM vendor del	10.3401	5.2072	8.7598	0.5944	16
NAPM Invent	6.2786	3.6914	5.1907	0.7112	8
PGDP	0.0029	0.0012	0.0022	0.5716	25
PCED	0.0032	0.0022	0.0026	0.8514	9
CPI-All	0.0043	0.0029	0.0031	0.9434	3
PCED-Core	0.0025	0.0019	0.0021	0.9278	4
CPI-Core	0.0038	0.0009	0.0028	0.3157	56
PGPDI	0.0052	0.0021	0.0038	0.5614	29
PFI	0.0053	0.0023	0.0039	0.5984	27
PEXP	0.0102	0.0051	0.0080	0.6420	18
PIMP	0.0190	0.0097	0.0158	0.6167	22
PGOV	0.0051	0.0031	0.0032	0.9479	3
Com: spot price (real)	0.1136	0.0932	0.0932	1.0000	0
OilPrice (Real)	0.2485	0.0464	0.2396	0.1935	81
NAPM com price	14.2807	10.3370	12.1543	0.8505	5
Real AHE: goods	0.0144	0.0082	0.0113	0.7300	8
Labor Prod	0.0165	0.0123	0.0149	0.8216	1
Real Comp/Hour	0.0140	0.0115	0.0130	0.8819	1
Unit Labor Cost	0.0311	0.0152	0.0194	0.7814	10
FedFunds	2.2061	1.3908	1.8203	0.7640	15
6 mo T-bill	1.6417	1.1463	1.4092	0.8135	10
5 yr T-bond	1.3442	1.0934	1.2524	0.8730	6
Aaabond	1.0090	0.7926	0.9385	0.8445	4
Baa bond	1.1242	0.8188	1.0039	0.8156	8
M1	0.0095	0.0069	0.0072	0.9700	2
MZM	0.0183	0.0076	0.0117	0.6434	16
M2	0.0070	0.0035	0.0050	0.7044	13
MB	0.0066	0.0044	0.0050	0.8809	4
Reserves tot	0.0238	0.0125	0.0150	0.8345	9
Bus loans	0.0148	0.0118	0.0118	1.0000	0
Cons credit	0.0114	0.0091	0.0093	0.9770	1
Ex rate: avg	0.0660	0.0595	0.0595	1.0000	0
S&P 500	0.1435	0.1015	0.1357	0.7481	16
DJIA	0.1400	0.1163	0.1282	0.9067	2
Consumer expect	10.7688	8.5925	9.2908	0.9248	1

Table 2: Root mean squared errors (RMSE) of the residuals of the forecasting model when allowing for breaks of not, and the relative RMSE of these two approaches.

always below 1. We should stress that this is purely an in-sample comparison of the forecasting models and not a pseudo out-of-sample forecasting experiment.

For certain variables the reduction in RMSE by allowing for breaks is quite substantial, for example a reduction of roughly 70% for CPI-Core (the parameters of this model are plotted in figure 8), and 80% for Oil Price. The presence of numerous relatively large breaks (despite the small magnitude of the loadings) explains this reduction in the RMSE. The results in table 2 shows that reduction in RMSE relative to OLS is closely related, but not directly proportional, to the number of breaks. Again this is to be expected as the penalty associated with each extra parameter must be compensated with improvement in fit.

4. Conclusion

We have applied the parsimoniously time varying parameter framework of Callot and Kristensen (2015) to factor models estimated using a set of data from Stock and Watson (2009) containing 144 US macroeconomic variables observed from 1959:1 to 2006:4. The ptv framework allows for an unknown number of breaks at unknown locations to be consistently estimated using the Lasso. We take advantage of this flexibility to study the stability of the parameters of macroeconomic models, and in particular we focus our investigation on the Great Moderation.

We find that for a large share of variables the parameters of either the static factor model or the dynamic forecasting FAAR model are unstable. The number of breaks, their locations, and the resulting parameter paths are very diverse. Nonetheless common patterns emerge, in particular a concentration of parametric instability in the period between 1970 and the middle of the 1980s, and a relative stability of those parameters in the 15 years following. Within our ptv framework, the Great Moderation appears to be a period of stability following a period of instability of the process driving macroeconomic variables. Adequately modelling this instability in time-varying parameter models appears to require more than a single break towards the beginning of the Great Moderation.

Further research could usefully complement our results. On the methodological side, obtaining confidence bands for the estimated parameters is a priority and refining break detection by the introduction of thresholding a possibility. On the empirical side, it would be useful to systematically assess the effect of allowing general forms of parametric instability on the choice of the number of factors and lags, both in terms of model specification and of forecasting performance.

Appendix A. Proofs.

Proof of Theorem 3. The result follows by checking the conditions of Corollary 1 in Bates et al. (2013). Define a random variable v_{ip} that takes on the value 1 with probability $\pi_{n,T}$ and the value 0 otherwise, and is independent from η and ζ and across i, p. Then the loadings for variable i and factor p follows the process:

$$\lambda_{ip,t} = \lambda_{ip,t-1} + v_{ip}\zeta_{ip,t}\eta_{ip,t} \tag{A.1}$$

or rewritten:

$$\lambda_{ip,t} - \lambda_{ip,0} = \xi_{ip,t} \tag{A.2}$$

where $\xi_{ip,t} = v_{ip}\sum_{s=1}^{t}\zeta_{ip,s}\eta_{ip,s}$ and ζ, η satisfy the conditions in Assumption 1. Note that compared to the expressions in Bates et al. (2013) we have explicitly set $h_{nT} = 1$.

We need to bound two expressions, the first:

$$\sup_{s,t\leq T}\sum_{i,j=1}^{n}\left|\mathbb{E}(\xi_{ip,s}\xi_{jq,t})\right| = \sup_{s,t\leq T}\sum_{i,j=1}^{n}\left|\mathbb{E}(v_{ip}\sum_{u=1}^{s}\zeta_{ip,u}\eta_{ip,u}v_{jq}\sum_{v=1}^{t}\zeta_{jq,v}\eta_{jq,v})\right| \tag{A.3}$$

Due to independence across factors the expression is trivially bounded for $p \neq q$, hence we only need consider the case of $p = q$ (and drop the factor index to ease notation). Furthermore, due to independence across variables we get:

$$\begin{aligned}
&= \pi_{n,T} n \sup_{s,t\leq T}\left|\mathbb{E}(\sum_{u=1}^{\min(s,t)}\zeta_{i,u}\eta_{i,u}^{2})\right| \\
&= \pi_{n,T} n\alpha_T \sup_{s,t\leq T}\min(s,t)\,\mathbb{E}(\eta_{i,1}^{2}) \\
&= \pi_{n,T} n\alpha_T T\,\mathbb{E}(\eta_{i,1}^{2}) \\
&= \pi_{n,T}\alpha_T \mathcal{O}(nT) = Q_1(n,T)
\end{aligned} \tag{A.4}$$

The second expression:

$$\sum_{t,s=1}^{T}\sum_{i,j=1}^{n}\left|\mathbb{E}(\xi_{ip_1,s}\xi_{jq_1,s}\xi_{ip_2,t}\xi_{jq_2,t}\right| \tag{A.5}$$

$$= \sum_{t,s=1}^{T}\sum_{i,j=1}^{n}\left|\mathbb{E}\left(v_{ip_1}\sum_{u=1}^{s}\zeta_{ip_1,u}\eta_{ip_1,u}v_{jq_1}\sum_{v=1}^{s}\zeta_{jq_1,v}\eta_{jq_1,v}v_{ip_2}\sum_{k=1}^{t}\zeta_{ip_2,k}\eta_{ip_2,k}v_{jq_2}\sum_{l=1}^{t}\zeta_{jq_2,l}\eta_{jq_2,l}\right)\right|$$

Again, due to independence across factors the expression is trivially bounded if all factor indices differ or one index differs from the rest. The non-trivial cases are thus when all indices are equal, i.e. $p_1 = p_2 = q_1 = q_2$, and when they are equal in pairs, say, $p_1 = p_2 \neq q_1 = q_2$. We start with the latter case:

$$
\begin{aligned}
&= \sum_{t,s=1}^{T} \sum_{i,j=1}^{n} \left| \mathbb{E}\left(\nu_{ip_1} \sum_{u=1}^{s} \zeta_{ip_1,u}\eta_{ip_1,u} \sum_{k=1}^{t} \zeta_{ip_1,k}\eta_{ip_1,k} \right) \mathbb{E}\left(\nu_{jq_1} \sum_{v=1}^{s} \zeta_{jq_1,v}\eta_{jq_1,v} \sum_{l=1}^{t} \zeta_{jq_1,l}\eta_{jq_1,l} \right) \right| \\
&= \sum_{t,s=1}^{T} \sum_{i,j=1}^{n} \left| \pi_{n,T} \min(s,t) \alpha_T \mathbb{E}(\eta^2_{ip_1,1}) \times \pi_{n,T} \min(s,t) \alpha_T \mathbb{E}(\eta^2_{jq_1,1}) \right| \\
&= \sum_{t,s=1}^{T} \sum_{i,j=1}^{n} \pi^2_{n,T} \alpha^2_T \mathcal{O}(T^2) = \pi^2_{n,T} \alpha^2_T \mathcal{O}(n^2 T^4) \qquad \text{(A.6)}
\end{aligned}
$$

Now consider the case where all factor indices are equal (again omitting the index to ease notation):

$$
\sum_{t,s=1}^{T} \sum_{i,j=1}^{n} \left| \mathbb{E}\left(\nu_i \sum_{u=1}^{s} \zeta_{i,u}\eta_{i,u} \nu_j \sum_{v=1}^{s} \zeta_{j,v}\eta_{j,v} \nu_i \sum_{k=1}^{t} \zeta_{i,k}\eta_{i,k} \nu_j \sum_{l=1}^{t} \zeta_{j,l}\eta_{j,l} \right) \right| \tag{A.7}
$$

Now, due to independence across variables, whenever $i \neq j$ we can use the same argument as above to show that the summand is $\pi^2_{n,T}\alpha^2_T\mathcal{O}(T^2)$. However, when $i = j$ the summand becomes

$$
\left| \mathbb{E}\left(\nu_i \sum_{u=1}^{s} \zeta_{i,u}\eta_{i,u} \sum_{v=1}^{s} \zeta_{i,v}\eta_{i,v} \sum_{k=1}^{t} \zeta_{i,k}\eta_{i,k} \sum_{l=1}^{t} \zeta_{i,l}\eta_{i,l} \right) \right| \tag{A.8}
$$

which is non-zero whenever all four time indices are equal, or they are equal in pairs. There are $\min(s,t)$ cases where they are all equal and the expression becomes $\mathbb{E}(\nu_i\zeta_{i,1}\eta^4_{i,1}) = \pi_{n,T}\alpha_T\mathcal{O}(1)$. When they are equal in pairs we get: $\mathbb{E}(\nu_i)\mathbb{E}(\zeta_{i,1}\eta^2_{i,1})\mathbb{E}(\zeta_{i,1}\eta^2_{i,1}) = \pi_{n,T}\alpha^2_T\mathcal{O}(1)$. In total there are $st + 2\min(s,t)^2$ non-zero summands and we get that (A.8) is $\pi_{n,T}\alpha_T\mathcal{O}(T) + \pi_{n,T}\alpha^2_T\mathcal{O}(T^2)$ implying that (A.7) is $\pi_{n,T}\alpha_T\mathcal{O}(nT^3) + \pi_{n,T}\alpha^2_T\mathcal{O}(nT^4) + \pi^2_{n,T}\alpha^2_T\mathcal{O}(n^2T^4) = Q_3(n,T)$.

According to Bates et al. (2013, Corollary 1) we need $Q_1(n,T) = \mathcal{O}(n)$ and $C^2_{nT}Q_3(n,T) = \mathcal{O}(n^2T^2)$. From (A.4) we have

$$
\begin{aligned}
Q_1(n,T) &= \pi_{n,T}\alpha_T\mathcal{O}(nT) \qquad \text{(A.9)} \\
&= \mathcal{O}(1/\min(n^{1/2}T^{1-a}, T^{3/2-a}))\mathcal{O}(T^{-a})\mathcal{O}(nT) \\
&= \mathcal{O}(n)
\end{aligned}
$$

We further have

$$
\begin{aligned}
C^2_{nT}Q_3(n,T) &= C^2_{nT}[\pi_{n,T}\alpha_T\mathcal{O}(nT^3) + \pi_{n,T}\alpha^2_T\mathcal{O}(nT^4) + \pi^2_{n,T}\alpha^2_T\mathcal{O}(n^2T^4)] \qquad \text{(A.10)} \\
&= \min(n,T)\mathcal{O}(1/\min(n^{1/2}T^{1-a}, T^{3/2-a}))\mathcal{O}(T^{-a})\mathcal{O}(nT^3) \qquad \text{(A.11)} \\
&+ \min(n,T)\mathcal{O}(1/\min(n^{1/2}T^{1-a}, T^{3/2-a}))\mathcal{O}(T^{-2a})\mathcal{O}(nT^4) \qquad \text{(A.12)} \\
&+ \min(n,T)\mathcal{O}(1/\min(nT^{2-2a}, T^{3-2a}))\mathcal{O}(T^{-2a})\mathcal{O}(n^2T^4) \qquad \text{(A.13)}
\end{aligned}
$$

Under the assumption that $a \geq 1/2$ and $T^{1-a}/n^{1/2} \to k \geq 0$ all three terms are $\mathcal{O}(n^2T^2)$ and the results follows. $\square$

Appendix B. Data Description

The dataset used is from Stock and Watson (2009) and can be downloaded from Mark Watson's homepage. The full list of variables along with descriptions from Stock and Watson (2009) has been reproduced below in table B.1. The majority of the variables are from the Global Insights Basic Economics Database. The remaining variables are either from The Conference Boards Indicators Database (TCB) or calculated by the authors using Global Insights or TCB data (AC). Transforming the variables to be stationary is done according to the transformation codes (TC), see table B.2 for details as well as details on how the h-quarter ahead version of the variable used in the factor-augmented forecasting regressions is constructed. In addition to this the following abbreviations are used: sa, seasonally adjusted; nsa, not seasonally adjusted; saar, seasonally adjusted at an annual rate. The E.F. column whether the variable was used to estimate the factors (= 1).

Short name	Mnemonic	TC	E.F.	Description
RGDP	GDP251	5	0	Real gross domestic product, quantity index (2000=100) , saar
Cons	GDP252	5	0	Real personal consumption expenditures, quantity index (2000=100) , saar
Cons-Dur	GDP253	5	1	Real personal consumption expenditures - durable goods , quantity index (2000=
Cons-NonDur	GDP254	5	1	Real personal consumption expenditures - non-durable goods, quantity index (200
Cons-Serv	GDP255	5	1	Real personal consumption expenditures - services, quantity index (2000=100) ,
GPDInv	GDP256	5	0	Real gross private domestic investment, quantity index (2000=100) , saar
FixedInv	GDP257	5	0	Real gross private domestic investment - fixed investment, quantity index (200
NonResInv	GDP258	5	0	Real gross private domestic investment - nonresidential , quantity index (2000
NonResInv-Struct	GDP259	5	1	Real gross private domestic investment - nonresidential - structures, quantity
NonResInv-Bequip	GDP260	5	1	Real gross private domestic investment - nonresidential - equipment & software
Res.Inv	GDP261	5	1	Real gross private domestic investment - residential, quantity index (2000=100
Exports	GDP263	5	1	Real exports, quantity index (2000=100) , saar
Imports	GDP264	5	1	Real imports, quantity index (2000=100) , saar
Gov	GDP265	5	0	Real government consumption expenditures & gross investment, quantity index (2
Gov Fed	GDP266	5	1	Real government consumption expenditures & gross investment - federal, quantit
Gov State/Loc	GDP267	5	1	Real government consumption expenditures & gross investment - state & local, q
IP: total	IPS10	5	0	Industrial production index - total index
IP: products	IPS11	5	0	Industrial production index - products, total
IP: final prod	IPS299	5	0	Industrial production index - final products
IP: cons gds	IPS12	5	0	Industrial production index - consumer goods
IP: cons dble	IPS13	5	1	Industrial production index - durable consumer goods
IP: cons nondble	IPS18	5	1	Industrial production index - nondurable consumer goods
IP: bus eqpt	IPS25	5	1	Industrial production index - business equipment

Table B.1: Data description.

Short name	Mnemonic	TC	E.F.	Description
IP: matls	IPS32	5	0	Industrial production index - materials
IP: dble mats	IPS34	5	1	Industrial production index - durable goods materials
IP: nondble mats	IPS38	5	1	Industrial production index - nondurable goods materials
IP: mfg	IPS43	5	1	Industrial production index - manufacturing (sic)
IP: fuels	IPS306	5	1	Industrial production index - fuels
NAPM prodn	PMP	1	1	Napm production index (percent)
Capacity Util	UTL11	1	1	Capacity utilization - manufacturing (sic)
Emp: total	CES002	5	0	Employees, nonfarm - total private
Emp: gds prod	CES003	5	0	Employees, nonfarm - goods-producing
Emp: mining	CES006	5	1	Employees, nonfarm - mining
Emp: const	CES011	5	1	Employees, nonfarm - construction
Emp: mfg	CES015	5	0	Employees, nonfarm - mfg
Emp: dble gds	CES017	5	1	Employees, nonfarm - durable goods
Emp: nondbles	CES033	5	1	Employees, nonfarm - nondurable goods
Emp: services	CES046	5	1	Employees, nonfarm - service-providing
Emp: TTU	CES048	5	1	Employees, nonfarm - trade, transport, utilities
Emp: wholesale	CES049	5	1	Employees, nonfarm - wholesale trade
Emp: retail	CES053	5	1	Employees, nonfarm - retail trade
Emp: FIRE	CES088	5	1	Employees, nonfarm - financial activities
Emp: Govt	CES140	5	1	Employees, nonfarm - government
Help wanted indx	LHEL	2	1	Index of help-wanted advertising in newspapers (1967=100;sa)
Help wanted/emp	LHELX	2	1	Employment: ratio; help-wanted ads:no. unemployed clf
Emp CPS total	LHEM	5	0	Civilian labor force: employed, total (thous.,sa)
Emp CPS nonag	LHNAG	5	1	Civilian labor force: employed, nonagric.industries (thous.,sa)
Emp. Hours	LBMNU	5	1	Hours of all persons: nonfarm business sec (1982=100,sa)
Avg hrs	CES151	1	1	Avg wkly hours, prod wrkrs, nonfarm - goods-producing
Overtime: mfg	CES155	2	1	Avg wkly overtime hours, prod wrkrs, nonfarm - mfg
U: all	LHUR	2	1	Unemployment rate: all workers, 16 years & over (%,sa)
U: mean duration	LHU680	2	1	Unemploy.by duration: average(mean)duration in weeks (sa)
U < 5 wks	LHU5	5	1	Unemploy.by duration: persons unempl.less than 5 wks (thous.,sa)
U 5–14 wks	LHU14	5	1	Unemploy.by duration: persons unempl.5 to 14 wks (thous.,sa)
U 15+ wks	LHU15	5	1	Unemploy.by duration: persons unempl.15 wks + (thous.,sa)
U 15–26 wks	LHU26	5	1	Unemploy.by duration: persons unempl.15 to 26 wks (thous.,sa)
U 27+ wks	LHU27	5	1	Unemploy.by duration: persons unempl.27 wks + (thous,sa)
BuildPermits	HSBR	4	0	Housing authorized: total new priv housing units (thous.,saar)
HStarts: Total	HSFR	4	0	Housing starts:nonfarm(1947-58);total farm&nonfarm(1959-)(thous.,sa
HStarts: NE	HSNE	4	1	Housing starts:northeast (thous.u.)s.a.
HStarts: MW	HSMW	4	1	Housing starts:midwest(thous.u.)s.a.
HStarts: South	HSSOU	4	1	Housing starts:south (thous.u.)s.a.
HStarts: West	HSWST	4	1	Housing starts:west (thous.u.)s.a.
PMI	PMI	1	1	Purchasing managers' index (sa)

Table B.1: Data description.

Short name	Mnemonic	TC	E.F.	Description
NAPM new ordrs	PMNO	1	1	Napm new orders index (percent)
NAPM vendor del	PMDEL	1	1	Napm vendor deliveries index (percent)
NAPM Invent	PMNV	1	1	Napm inventories index (percent)
Orders (ConsGoods)	MOCMQ	5	1	New orders (net) - consumer goods & materials, 1996 dollars (bci)
Orders (NDCapGoods)	MSONDQ	5	1	New orders, nondefense capital goods, in 1996 dollars (bci)
PGDP	GDP272A	6	0	Gross domestic product price index
PCED	GDP273A	6	0	Personal consumption expenditures price index
CPI-All	CPIAUCSL	6	0	Cpi all items (sa) fred
PCED-Core	PCEPILFE	6	0	Pce price index less food and energy (sa) fred
CPI-Core	CPILFESL	6	0	Cpi less food and energy (sa) fred
PCED-Dur	GDP274A	6	0	Durable goods price index
PCED-motorveh	GDP274_1	6	1	Motor vehicles and parts price index
PCED-hhequip	GDP274_2	6	1	Furniture and household equipment price index
PCED-oth dur	GDP274_3	6	1	Other price index
PCED-nondur	GDP275A	6	0	Nondurable goods price index
PCED-food	GDP275_1	6	1	Food price index
PCED-clothing	GDP275_2	6	1	Clothing and shoes price index
PCED-energy	GDP275_3	6	1	Gasoline, fuel oil, and other energy goods price index
PCED-oth nondur	GDP275_4	6	1	Other price index
PCED-services	GDP276A	6	0	Services price index
PCED-housing	GDP276_1	6	1	Housing price index
PCED-hhops	GDP276_2	6	0	Household operation price index
PCED-elect & gas	GDP276_3	6	1	Electricity and gas price index
PCED-oth hhops	GDP276_4	6	1	Other household operation price index
PCED-transport	GDP276_5	6	1	Transportation price index
PCED-medical	GDP276_6	6	1	Medical care price index
PCED-recreation	GDP276_7	6	1	Recreation price index
PCED-oth serv	GDP276_8	6	1	Other price index
PGPDI	GDP277A	6	0	Gross private domestic investment price index
PFI	GDP278A	6	0	Fixed investment price index
PFI-nonres	GDP279A	6	0	Nonresidential price index
PFI-nonres struc Price Index	GDP280A	6	1	Structures
PFI-nonres equip	GDP281A	6	1	Equipment and software price index
PFI-residential	GDP282A	6	1	Residential price index
PEXP	GDP284A	6	1	Exports price index
PIMP	GDP285A	6	1	Imports price index
PGOV	GDP286A	6	0	Government consumption expenditures and gross investment price index
PGOV-Federal	GDP287A	6	1	Federal price index
PGOV-St & loc	GDP288A	6	1	State and local price index
Com: spot price (real)	PSCCOMR	5	1	Real spot market price index:bls & crb: all commodities(1967=100) (psccom/pcepilfe)
OilPrice (Real)	PW561R	5	1	Ppi crude (relative to core pce) (pw561/pcepilfe)
NAPM com price	PMCP	1	1	Napm commodity prices index (percent)
Real AHE: goods	CES275R	5	0	Real avg hrly earnings, prod wrkrs, nonfarm - goods-producing (ces275/pi071)
Real AHE: const	CES277R	5	1	Real avg hrly earnings, prod wrkrs, nonfarm - construction (ces277/pi071)
Real AHE: mfg	CES278 R	5	1	Real avg hrly earnings, prod wrkrs, nonfarm - mfg (ces278/pi071)
Labor Prod	LBOUT	5	1	Output per hour all persons: business sec(1982=100,sa)

Table B.1: Data description.

Short name	Mnemonic	TC	E.F.	Description
Real Comp/Hour	LBPUR7	5	1	Real compensation per hour,employees:nonfarm business(82=100,sa)
Unit Labor Cost	LBLCPU	5	1	Unit labor cost: nonfarm business sec (1982=100,sa)
FedFunds	FYFF	2	1	Interest rate: federal funds (effective) (% per annum,nsa)
3 mo T-bill	FYGM3	2	1	Interest rate: u.s.treasury bills,sec mkt,3-mo.(% per ann,nsa)
6 mo T-bill	FYGM6	2	0	Interest rate: u.s.treasury bills,sec mkt,6-mo.(% per ann,nsa)
1 yr T-bond	FYGT1	2	1	Interest rate: u.s.treasury const maturities,1-yr.(% per ann,nsa)
5 yr T-bond	FYGT5	2	0	Interest rate: u.s.treasury const maturities,5-yr.(% per ann,nsa)
10 yr T-bond	FYGT10	2	1	Interest rate: u.s.treasury const maturities,10-yr.(% per ann,nsa)
Aaabond	FYAAAC	2	0	Bond yield: moody's aaa corporate (% per annum)
Baa bond	FYBAAC	2	0	Bond yield: moody's baa corporate (% per annum)
fygm6-fygm3	SFYGM6	1	1	fygm6-fygm3
fygt1-fygm3	SFYGT1	1	1	fygt1-fygm3
fygt10-fygm3	SFYGT10	1	1	fygt10-fygm3
fyaaac-fygt10	SFYAAAC	1	1	fyaaac-fygt10
fybaac-fygt10	SFYBAAC	1	1	fybaac-fygt10
M1	FM1	6	1	Money stock: m1(curr,trav.cks,dem dep,other ck'able dep)(bil$,sa)
MZM	MZMSL	6	1	Mzm (sa) frb st. louis
M2	FM2	6	1	Money stock:m2(m1+o'nite rps,euro$,g/p&b/d mmmfs&sav&sm time dep(bil$,sa)
MB	FMFBA	6	1	Monetary base, adj for reserve requirement changes(mil$,sa)
Reserves tot	FMRRA	6	1	Depository inst reserves:total,adj for reserve req chgs(mil$,sa)
Reserves nonbor	FMRNBA	6	1	Depository inst reserves:nonborrowed,adj res req chgs(mil$,sa)
Bus loans	BUSLOANS	6	1	Commercial and industrial loans at all commercial banks (fred) billions $ (sa)
Cons credit	CCINRV	6	1	Consumer credit outstanding - nonrevolving(g19)
Ex rate: avg	EXRUS	5	1	United states;effective exchange rate(merm)(index no.)
Ex rate: Switz	EXRSW	5	1	Foreign exchange rate: switzerland (swiss franc per u.s.$)
Ex rate: Japan	EXRJAN	5	1	Foreign exchange rate: japan (yen per u.s.$)
Ex rate: UK	EXRUK	5	1	Foreign exchange rate: united kingdom (cents per pound)
EX rate: Canada	EXRCAN	5	1	Foreign exchange rate: canada (canadian $ per u.s.$)
S&P 500	FSPCOM	5	1	S&P's common stock price index: composite (1941-43=10)
S&P: indust	FSPIN	5	1	S&P's common stock price index: industrials (1941-43=10)
S&P div yield	FSDXP	2	1	S&P's composite common stock: dividend yield (% per annum)
S&P PE ratio	FSPXE	2	1	S&P's composite common stock: price-earnings ratio (%,nsa)
DJIA	FSDJ	5	1	Common stock prices: dow jones industrial average
Consumer expect	HHSNTN	2	1	U. of Mich. index of consumer expectations(bcd-83)

Table B.1: Data description.

TC	Transformation (X_{it})	h-quarter ahead variable $X^{(h)}_{it+h}$
1	$X_{it} = Y_{it}$	$X^{(h)}_{it+h} = Y_{it+h}$
2	$X_{it} = \Delta Y_{it}$	$X^{(h)}_{it+h} = Y_{it+h} - Y_{it}$
3	$X_{it} = \Delta^2 Y_{it}$	$X^{(h)}_{it+h} = h^{-1}\sum_{j=1}^{h} \Delta Y_{it+h-j} - \Delta Y_{it}$
4	$X_{it} = \log Y_{it}$	$X^{(h)}_{it+h} = \log Y_{it+h}$
5	$X_{it} = \Delta \log Y_{it}$	$X^{(h)}_{it+h} = \log Y_{it+h} - \log Y_{it}$
6	$X_{it} = \Delta^2 \log Y_{it}$	$X^{(h)}_{it+h} = h^{-1}\sum_{j=1}^{h} \Delta \log Y_{it+h-j} - \Delta \log Y_{it}$

Table B.2: Variable transformations.

Appendix C. Additional Results

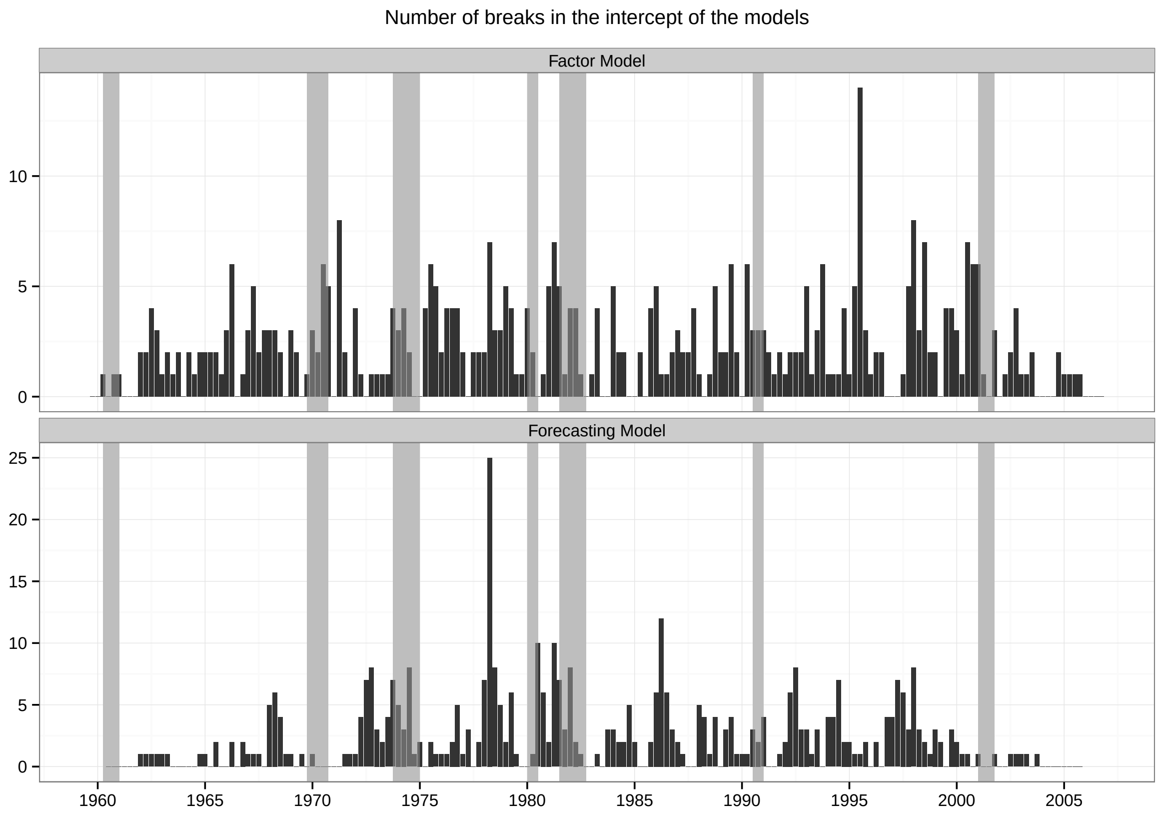

Figure C.9: Breaks in the intercepts of the models. The grey areas represent the NBER recessions.

Series	Factor model			Forecasting model					
				Coef. on factors			Coef. on AR lags		
	All All	1970:1 1984:4	1985:1 1999:4	All All	1970:1 1984:4	1985:1 1999:4	All All	1970:1 1984:4	1985:1 1999:4
Cons-Dur				6	3				
Cons-NonDur									
Cons-Serv				1	1		1	1	
FixedInv				1	1				
NonResInv				4	4		1		1
NonResInv-Struct	2	2		2	2				
NonResInv-Bequip									
Res.Inv	1	1							
Gov Fed				53	17	12	48	15	14
Gov State/Loc				12	5	5	2	1	1
IP: products	1	1		2	2				
IP: final prod	2	2		4	3				
IP: cons gds				4	4				
IP: cons dble				1	1				
IP:cons nondble									
IP:bus eqpt									
IP: matls	42	20	8						
IP: dble mats	11	6							
IP:nondble mats	4	4							
IP: mfg	6	3							
IP: fuels				2	2				
Emp: gds prod	4	3	1	3	3		2	1	1
Emp: mining	51	32	12	48	26	14	15	12	2
Emp: const	4	3							
Emp: mfg	1	1		4	4		1	1	
Emp: dble gds				2	2		1	1	
Emp: nondbles	2	2		15	9	3	5	5	
Emp: services	11	7	1	1	1				
Emp: TTU	2	2		1	1				
Emp: wholesale	2	2							
Emp: retail				1	1				
Emp: FIRE	27	12	11				1		1
Emp: Govt	1	1					2		
Help wanted/emp	18	12							
Emp CPS nonag									
Emp. Hours	1			1	1		1	1	
Avg hrs	9	7							
Overtime: mfg				3	2		2	1	1
U: mean duration				2	2				
U < 5 wks									
U 5-14 wks				6	5		2	2	
U 15+ wks	1	1		3	2				
U 15-26 wks	1	1		1	1				
U 27+ wks									
HStarts: NE	4	1	3	4	4				
HStarts: MW	12	8	1	14	6	2	24	24	
HStarts: South	11	10	1						
HStarts: West	38	22	6	31	17	2	22	12	3
Orders (ConsGoods)									
Orders (NDCapGoods)				2	2				
PCED-Dur				20	13	2	3	3	
PCED-motorveh				12	8		2	2	

Table C.3: Number of breaks in the loadings of the factor model and the parameters of the forecasting model detailed by variable.

Series	Factor model			Forecasting model					
				Coef. on factors			Coef. on AR lags		
	All All	1970:1 1984:4	1985:1 1999:4	All All	1970:1 1984:4	1985:1 1999:4	All All	1970:1 1984:4	1985:1 1999:4
PCED-hhequip				4	2	2	1	1	
PCED-oth dur				1	1				
PCED-nondur									
PCED-food				5	4		5	5	
PCED-clothing				3	3				
PCED-energy				2	1	1			
PCED-oth nondur				17	12	2	5	5	
PCED-services									
PCED-housing				1	1				
PCED-hhops									
PCED-elect & gas				7	1	4	5		
PCED-oth hhops				1	1				
PCED-transport	47	27	18	20	17	3	7	7	
PCED-medical				10	6	1	7	7	
PCED-recreation									
PCED-oth serv				3	2	1			
PFI-nonres				25	16	6	14	10	3
PFI-nonres struc				21	16	4	5	2	2
PFI-nonres equip	3	2	1	32	17	10	17	12	5
PFI-residential							1	1	
PGOV-FED									
PGOV-SL				3	1	2			
Real AHE: const	3	2	1	6	5		5	5	
Real AHE: mfg	9	5	3	2	2		3	2	1
3 mo T-bill	8	8		6	5	1	6	6	
1 yr T-bond	3	2	1	3	3				
10 yr T-bond	3	3		32	17	9	30	13	16
fygm6-fygm3	24	18	1	14	9	3	7	4	3
fygt1-fygm3	27	19	6	8	7	1	2	1	1
fygt10-fygm3	27	17	8	3	2	1			
fyaaac-fygt10	10	5	3						
fybaac-fygt10	15	7	5				1		
Reserves nonbor	30	19	6	4	3		3	1	2
Ex rate: Switz									
Ex rate: Japan									
Ex rate: UK	4	2	2						
EX rate: Canada	2			4	2	1			
S&P: indust	11	6	3	1		1			
S&P div yield	18	9	7	4	2	2	1	1	
S&P PE ratio	13	2	6	2		2			

Table C.3: Number of breaks in the loadings of the factor model and the parameters of the forecasting model detailed by variable.

Series	Std. dev. of $X_{it}^{(4)}$	RMSE, ptv	RMSE, OLS	Relative RMSE	# Breaks
Cons-Dur	0.0627	0.0440	0.0497	0.8856	6
Cons-NonDur	0.0166	0.0148	0.0148	1.0000	0
Cons-Serv	0.0122	0.0085	0.0102	0.8356	2

Table C.4: Root mean squared errors (RMSE) of the residuals of the forecasting model when allowing for breaks of not, and the relative RMSE of these two approaches.

Series	Std. dev. of $X_{it}^{(4)}$	RMSE, ptv	RMSE, OLS	Relative RMSE	# Breaks
FixedInv	0.0664	0.0484	0.0506	0.9579	1
NonResInv	0.0686	0.0445	0.0505	0.8821	5
NonResInv-Struct	0.0790	0.0583	0.0634	0.9193	2
NonResInv-Bequip	0.0729	0.0531	0.0531	1.0000	0
Res.Inv	0.1310	0.0978	0.0978	1.0000	0
Gov Fed	0.0445	0.0047	0.0378	0.1250	101
Gov State/Loc	0.0216	0.0124	0.0173	0.7184	14
IP: products	0.0374	0.0242	0.0274	0.8851	2
IP: final prod	0.0368	0.0231	0.0277	0.8325	4
IP: cons gds	0.0318	0.0177	0.0231	0.7666	4
IP: cons dble	0.0756	0.0504	0.0553	0.9120	1
IP:cons nondble	0.0206	0.0162	0.0181	0.8951	0
IP:bus eqpt	0.0720	0.0504	0.0504	1.0000	0
IP: matls	0.0523	0.0379	0.0390	0.9701	0
IP: dble mats	0.0799	0.0573	0.0573	1.0000	0
IP:nondble mats	0.0494	0.0326	0.0387	0.8423	0
IP: mfg	0.0483	0.0342	0.0349	0.9781	0
IP: fuels	0.0449	0.0384	0.0432	0.8899	2
Emp: gds prod	0.0349	0.0210	0.0238	0.8839	5
Emp: mining	0.0706	0.0142	0.0636	0.2227	63
Emp: const	0.0477	0.0338	0.0338	1.0000	0
Emp: mfg	0.0361	0.0224	0.0254	0.8830	5
Emp: dble gds	0.0457	0.0292	0.0314	0.9290	3
Emp: nondbles	0.0239	0.0108	0.0179	0.6060	20
Emp: services	0.0132	0.0070	0.0077	0.9039	1
Emp: TTU	0.0180	0.0103	0.0116	0.8883	1
Emp: wholesale	0.0201	0.0129	0.0136	0.9484	0
Emp: retail	0.0189	0.0102	0.0125	0.8200	1
Emp: FIRE	0.0165	0.0094	0.0102	0.9171	1
Emp: Govt	0.0163	0.0094	0.0104	0.9060	2
Help wanted/emp	0.2971	0.2281	0.2281	1.0000	0
Emp CPS nonag	0.0138	0.0094	0.0100	0.9439	0
Emp. Hours	0.0238	0.0164	0.0174	0.9458	2
Avg hrs	0.5211	0.3246	0.3246	1.0000	0
Overtime: mfg	0.3996	0.2414	0.2873	0.8401	5
U: mean duration	2.0219	0.9627	1.0998	0.8753	2
U < 5 wks	0.0860	0.0713	0.0713	1.0000	0
U 5-14 wks	0.1710	0.1125	0.1279	0.8795	8
U 15+ wks	0.3188	0.1859	0.1968	0.9446	3
U 15-26 wks	0.2806	0.1892	0.1920	0.9856	1
U 27+ wks	0.3888	0.2247	0.2247	1.0000	0
HStarts: NE	0.3044	0.1628	0.1843	0.8833	4
HStarts: MW	0.2541	0.0909	0.1915	0.4743	38
HStarts: South	0.2433	0.1628	0.1628	1.0000	0
HStarts: West	0.2800	0.0626	0.1959	0.3198	53
Orders (ConsGoods)	0.0662	0.0467	0.0501	0.9310	0
Orders (NDCapGoods)	0.1274	0.0962	0.0992	0.9699	2
PCED-Dur	0.0050	0.0025	0.0041	0.6201	23
PCED-motorveh	0.0085	0.0041	0.0061	0.6662	14
PCED-hhequip	0.0046	0.0034	0.0038	0.9021	5
PCED-oth dur	0.0062	0.0049	0.0051	0.9779	1
PCED-nondur	0.0069	0.0049	0.0049	1.0000	0
PCED-food	0.0063	0.0038	0.0045	0.8401	10
PCED-clothing	0.0065	0.0038	0.0041	0.9425	3

Table C.4: Root mean squared errors (RMSE) of the residuals of the forecasting model when allowing for breaks of not, and the relative RMSE of these two approaches.

Series	Std. dev. of $X_{it}^{(4)}$	RMSE, ptv	RMSE, OLS	Relative RMSE	# Breaks
PCED-energy	0.0540	0.0347	0.0363	0.9564	2
PCED-oth nondur	0.0052	0.0025	0.0042	0.6086	22
PCED-services	0.0026	0.0021	0.0021	1.0000	0
PCED-housing	0.0026	0.0020	0.0021	0.9512	1
PCED-hhops	0.0074	0.0054	0.0054	1.0000	0
PCED-elect & gas	0.0158	0.0083	0.0114	0.7343	12
PCED-oth hhops	0.0061	0.0045	0.0046	0.9836	1
PCED-transport	0.0202	0.0050	0.0084	0.5906	27
PCED-medical	0.0037	0.0023	0.0032	0.7357	17
PCED-recreation	0.0036	0.0025	0.0025	1.0000	0
PCED-oth serv	0.0066	0.0047	0.0051	0.9135	3
PFI-nonres	0.0052	0.0019	0.0042	0.4432	39
PFI-nonres struc	0.0076	0.0035	0.0062	0.5756	26
PFI-nonres equip	0.0056	0.0019	0.0045	0.4241	49
PFI-residential	0.0087	0.0045	0.0046	0.9858	1
PGOV-FED	0.0087	0.0042	0.0042	1.0000	0
PGOV-SL	0.0044	0.0032	0.0035	0.9251	3
Real AHE: const	0.0224	0.0115	0.0155	0.7441	11
Real AHE: mfg	0.0135	0.0092	0.0115	0.7970	5
3 mo T-bill	1.7145	1.1599	1.4675	0.7904	12
1 yr T-bond	1.7022	1.3617	1.4975	0.9093	3
10 yr T-bond	1.1865	0.3193	1.1098	0.2878	62
fygm6-fygm3	0.1847	0.1177	0.1770	0.6650	21
fygt1-fygm3	0.4063	0.2564	0.3702	0.6925	10
fygt10-fygm3	1.2283	0.8486	0.9174	0.9250	3
fyaaac-fygt10	0.4817	0.3101	0.3255	0.9527	0
fybaac-fygt10	0.6662	0.4236	0.4471	0.9476	1
Reserves nonbor	0.0397	0.0178	0.0220	0.8075	7
Ex rate: Switz	0.1122	0.1078	0.1078	1.0000	0
Ex rate: Japan	0.1071	0.0951	0.0992	0.9588	0
Ex rate: UK	0.0954	0.0905	0.0905	1.0000	0
EX rate: Canada	0.0466	0.0367	0.0407	0.9014	4
S&P: indust	0.1491	0.1374	0.1405	0.9779	1
S&P div yield	0.5481	0.4234	0.4616	0.9173	5
S&P PE ratio	3.9634	3.1941	3.2898	0.9709	2

Table C.4: Root mean squared errors (RMSE) of the residuals of the forecasting model when allowing for breaks of not, and the relative RMSE of these two approaches.

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