BEDS: Bayesian Emergent Dissipative Structures

From Rivers to Gödel – A Unified Framework for Emergence, Learning, and Formal Systems

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What is the common point between a river, a neural network, and Gödel's incompleteness theorem? They can all be formalized as Bayesian Emergent Dissipative Structures.

Foreword

This document presents a unified theoretical framework called **BEDS** (Bayesian Emergent Dissipative Structures). The central thesis: what we call "learning" in machine learning, "dissipation" in thermodynamics, "evolution" in biology, and "proof" in mathematics are the **same process** viewed from different angles.

The document is structured in five parts:

1. **Analogy** – Building intuition through the river metaphor
2. **Conjecture** – Connecting Gödel, Landauer, and Prigogine
3. **Formal Results** – Mathematical constants as necessary structures
4. **Implementation** – A sustainable P2P network architecture
5. **Philosophy** – Implications and perspectives

Disclaimer. The formalism presented here is deliberately simple – perhaps too simple. I have probably missed something obvious, or unknowingly reinvented existing results. This is speculative work born from stepping back and reflecting on my research. It does not include a systematic literature review and does not reflect the rigor of my peer-reviewed academic work. Most of it was vibecoded with Claude Opus 4.5 (claude.ai). However, I find this angle of attack compelling enough to share – hopefully others will find something valuable in it too. The goal is to formulate ideas clearly enough that they can be evaluated, criticized, and potentially refuted. Thanks for reading.

Part I — Analogy: The River That Learns

1.1 Water That Learns

Imagine a drop of water falling on virgin terrain. It could go anywhere — maximum uncertainty. It flows, erodes, chooses a path. Then another drop, then another. Each passage digs certain channels deeper, abandons others.

The riverbed forms.

Now the arriving water "knows" where to go — uncertainty about the path has been absorbed by the structure. What the river has learned is the relationship between its input (rain) and its output (sea), inscribed in rock.

The price paid: energy dissipated as heat and erosion.

```
       .::. .::. Rain (maximum uncertainty)
         |||
         vvv
    +----------------+
    |  Virgin terrain | <- All paths possible
    +----------------+
         |
         | Time passes...
         v
    +----------------+
    |  ========      | <- Channels form
    |    ========    |
    |      ======    |
    +----------------+
         |
         | More time...
         v
    +----------------+
    |      ||        | <- Single riverbed
    |      ||        |   (crystallized knowledge)
    |      ||        |
    +----------------+
         |
         v
       ~~~ Sea (certainty)
```

1.2 The Network That Learns

A neural network does exactly the same thing. Data arrives in a space of random parameters — they could produce anything. They traverse, weights adjust, certain paths strengthen. Valleys form in parameter space.

At the end, the network "knows": this input gives that output.

The price paid: CPU cycles dissipated as heat.

```
    [#][#][#] Data (maximum uncertainty)
         |||
         vvv
```

```
+--------------------+
| Random weights     |
|  ? -> ? -> ? -> ?  | <- All outputs possible
|  ? -> ? -> ? -> ?  |
+--------------------+
     |
     | Training...
     v
+--------------------+
| Learning weights   |
|  o -> * -> o -> *  | <- Some paths strengthen
|  o -> o -> * -> o  |
+--------------------+
     |
     | Convergence...
     v
+--------------------+
| Trained network    |
|  *===*===*===*     | <- Crystallized function
|                    |
+--------------------+
     |
     v
   [o] Prediction (certainty)
```

1.3 The Same Story

	River	Neural Network
Flux	Water	Data
Terrain to sculpt	Geology	Parameter space
Erosion	Friction on rock	Weight updates
Learned structure	Riverbed	Found minimum (valley)
What is represented	Path of least resistance	Input -> Output function
Exported entropy	Heat, sediments	Heat (CPU/GPU)

In both cases, **learning is converting flux into structure by exporting entropy**.

1.4 The Shortest Path

The Bayesian framework captures the essence of this process:

- **At the beginning**: water could go anywhere — maximum uncertainty
- **During**: each drop reduces possibilities, the bed deepens
- **At the end**: a single path dominates — maximum certainty

The Bayesian is the ideal river: it only digs what the water forces it to dig, no more.

```
UNCERTAINTY SPACE
```

```
    Start:     :::::::::::::::::::::::::::
               :::::::::::::::::::::::::::
               :::::::::::::::::::::::::::
               All hypotheses equally possible

    Data:      :::::::###:::::::::::::::::::
               :::::::###:::::::::::::::::::
               :::::::###:::::::::::::::::::
               Some regions more probable

    More data: ::::::::@@@@@::::::::::::::::::
               :::::::::@@@::::::::::::::::::::
               ::::::::::@:::::::::::::::::::::
               Concentration on truth

    End:       ::::::::::*:::::::::::::::::::::
               Posterior = crystallized belief
```

Central Conjecture: Under the Laplace approximation (locally quadratic free energy) and when the Fisher metric varies slowly, the natural gradient trajectory approximates a geodesic on the statistical manifold. In this regime, the convergence path is both optimal (least informational action) and geodesic.

1.5 The Mill That Emerges

Now that the riverbed exists, something new becomes possible.

Water flows predictably – it's free energy, available. A mill can attach itself. It couldn't have existed before: without a stable bed, where would it be built? On what flow would it rely?

The mill is itself a dissipative structure. Water traverses it, gears turn, wear, adjust. At first, it creaks, jams. Then the parts find their place, the rhythm stabilizes. The mill has learned to convert current into rotation.

The price paid: friction, wear, heat.

But once crystallized, the mill in turn releases energy – a regular rotation, exploitable. A blacksmith sets up shop. His forge learns iron: what temperatures, what gestures, what rhythms. It crystallizes. Its tools, once forged, allow digging canals, improving the bed, building other mills.

The loop closes. Or rather: it rises.

```
Rain (maximum uncertainty)
    |
    v
Riverbed ----- crystallizes ----- -> stable current (free energy)
                                            |
                                            v
                  Mill ----- crystallizes ----- -> rotation (free energy)
                                                        |
                                                        v
                             Forge ----- crystallizes ----- -> tools
                                                                  |
                                                                  v
```

```
                                                     ...
```

Each structure can only exist because the previous one crystallized. The mill inherits an axiom: “water flows here, at this rate.” It doesn’t have to learn it – it’s acquired, inscribed in rock. Its possibility space is already restricted, and that’s precisely what allows it to exist.

This is recursion: **the crystallized posterior becomes the prior of the next structure**.

1.6 The BEDS Formalism

1.6.1 Dissipative Structure

A **dissipative structure** (Prigogine, 1977) is an open system that: - Absorbs an incoming flux Phi_in - Maintains internal order (negentropy N) - Exports entropy H to the environment

```
    Phi_in -> [  S  ] -> H_out
              |
              +-- N (maintained order)
```

The second law imposes:

$$\frac{dS_{system}}{dt} = \frac{dS_{internal}}{dt} + \frac{dS_{exchange}}{dt}$$

where: - $\frac{dS_{internal}}{dt} \geq 0$ (irreversible production) - $\frac{dS_{exchange}}{dt}$ can be negative (negentropy import)

Existence condition: A dissipative structure exists as long as:

$$\Phi_{in} > \Phi_{min}$$

1.6.2 The Fundamental Correspondence

The BEDS framework proposes a formal isomorphism between thermodynamics and Bayesian inference:

Thermodynamics	Bayesian Inference
Internal energy E	Negative log-likelihood $-\log p(D \mid \theta)$
Entropy S	Entropy of $q(\theta)$
Temperature T	Regularization parameter β^{-1}
Free energy F	ELBO (negative)
Thermal equilibrium	Optimal posterior
Thermal fluctuations	Epistemic uncertainty

Status: This is a mathematical isomorphism, allowing transposition of intuitions and tools between domains. Physical identification (measuring a “crystallized bit” in joules) remains a research program.

1.6.3 Information Geometry

The space of probability distributions forms a **statistical manifold** $\mathcal{M}$. The Fisher metric defines the natural distance:

$$g_{ij}(\theta) = \mathbb{E}_{p(x|\theta)}\left[\frac{\partial \log p(x|\theta)}{\partial \theta_i}\frac{\partial \log p(x|\theta)}{\partial \theta_j}\right]$$

The **natural gradient** corrects the ordinary gradient to point in the steepest descent direction in distribution space:

$$\tilde{\nabla} L(\theta) = G(\theta)^{-1} \nabla L(\theta)$$

```
EUCLIDEAN SPACE              STATISTICAL MANIFOLD
(parameters)                 (distributions)

-------------->              +----------------+
|                            |                |
| grad L (wrong direction)   | G^-1 grad L    |
|                            |  (geodesic)    |
v                            +----------------+

Straight line in theta       Shortest path in P(x|theta)
!= shortest in P             = minimal KL divergence
```

1.7 Basins of Attraction

Imagine rain falling on empty terrain. Water falls randomly — pure noise.

After a few minutes, what do you see?

Puddles. Always in the same places.

Why? Because the terrain has **hollows**. Water didn't "decide" to go there. It fell there naturally, because that's where potential energy is minimal.

```
              NOISE (rain)
                  |
                  v
+-------------------------+
|    TERRAIN              |
|                         |
|    \        /           |
|     \      /            |
|      \    /             |
|       v  v              |
|       *  *              | <- Puddles (attractors)
|    BASIN  BASIN         |
+-------------------------+
```

Analogy	Mathematics
Random drops	Random combinations
Terrain	Space of relations

Analogy	Mathematics
Hollows	Basins of attraction
Puddles	Constants (e, pi)
Passage between	Euler (saddle point)

1.8 Recursive Emergence

When a dissipative structure reaches a stable state (local minimum of free energy), it **crystallizes**: its parameters stop fluctuating significantly.

```
Level 0: Raw data -> Structure S0 crystallizes
                           |
Level 1: S0 as prior -> Structure S1 crystallizes
                           |
Level 2: S1 as prior -> Structure S2 crystallizes
                           |
                          ...
```

Formally:

$$p_{n+1}(\theta) = p_n(\theta|D_n)$$

At each level, the space of possibilities shrinks:

$$\mathcal{H}(p_{n+1}) < \mathcal{H}(p_n)$$

Technical Conclusion: Any formal, economic, or other system that is not a dissipative structure is destined to collapse. Only systems that convert flux into structure by exporting entropy can maintain themselves indefinitely.

Part II — Conjecture: Gödel, Landauer, Prigogine

2.1 Three Results

Gödel (1931): Any consistent formal system capable of expressing arithmetic contains true statements that cannot be proven within the system.

Landauer (1961): Erasing one bit of information requires at least $k_B T \ln 2$ joules of energy.

Prigogine (1977): Open systems can maintain internal order by exporting entropy to their environment.

2.2 An Observation

These three results share a common theme: **closure has costs**.

```
+--------------------------------------------------------------+
|                    CLOSED SYSTEMS                            |
+--------------------------------------------------------------+
|                                                              |
|  GODEL           LANDAUER           PRIGOGINE                |
|  ======          ========           =========                |
|                                                              |
|  Closed formal   Closed             Closed                   |
|  system          computation        physical system          |
|  (no external    (no energy         (no entropy              |
|  axioms)         dissipation)       export)                  |
|       |               |                  |                   |
|       v               v                  v                   |
|  INCOMPLETENESS  REVERSIBILITY      DISORDER                 |
|  (true but       CONSTRAINTS        INCREASES                |
|  unprovable      (can't erase       (2nd law)                |
|  statements)     without cost)                               |
|                                                              |
+--------------------------------------------------------------+
```

- A closed formal system (no external axioms) -> incompleteness
- A closed computational process (no energy dissipation) -> reversibility constraints

- A closed physical system (no entropy export) -> disorder increases

---

2.3 The Conjecture

> **Pathologies of formal systems (incompleteness, undecidability, paradoxes) may be understood as manifestations of a dissipation deficit.**

More precisely: when a formal system creates self-referential loops (as in Gödel's construction), those loops have no "exit" — no way to export the problematic information to an external level.

If a system could “dissipate” self-reference — export it to a meta-level, anchor it in physical reality — it might avoid the blockage.

```
GODEL'S CONSTRUCTION

+------------------------------------+
|    FORMAL SYSTEM                   |
|                                    |
|    Statement G:                    |
|    "This statement is             |
|     unprovable in this system"   |
|           |                        |
|           | Self-reference         |
|           v                        |
|    +-------------+                |
|    |      G      |<--------------+
|    +-------------+                |
|                                    |
|    NO EXIT = BLOCKAGE             |
+------------------------------------+


DISSIPATIVE SYSTEM (hypothetical)

+------------------------------------+
|    FORMAL SYSTEM                   |
|                                    |
|    Statement G:                    |
|           |                        |
|           | Self-reference         |
|           v                        |
|    +-------------+                |
|    |      G      |----------------+---> META-LEVEL
|    +-------------+                |    (exported)
|                                    |
|    EXIT EXISTS = NO BLOCKAGE?     |
+------------------------------------+
```

2.4 What This Would Mean (If True)

2.4.1 Stable Mathematics Requires Physicality

Not just in implementation, but structurally. A mathematical community that forgets, prunes, and interacts with reality can remain coherent. An isolated formal system cannot.

2.4.2 Incompleteness Is the Price of Openness

Tarski’s hierarchy of metalanguages is infinite. But if each level “crystallizes” properly before promoting to the next, the system remains usable.

```
TARSKI'S HIERARCHY as BEDS
```

```
Level n+1: Meta-metalanguage
     ^
     | Crystallized axioms
     |
Level n: Metalanguage
     ^
     | Crystallized axioms
     |
Level n-1: Object language

Each level's posterior becomes the next level's prior.
Incompleteness is "exported upward" — dissipated.
```

2.4.3 Forgetting Is Necessary for Coherence

Systems that preserve everything (perfect memory, no pruning) accumulate contradictions.

> *To think without contradicting oneself, one must forget.*

2.5 Related Ideas

Chaitin's Omega: Algorithmic information theory connects randomness and incompleteness. Is there an energy interpretation? Omega represents the ultimate incompressible information — perhaps the "thermodynamic floor" of formal systems.

Friston's Free Energy Principle: Biological systems minimize variational free energy to maintain their integrity. This provides a theoretical precedent for applying Bayesian formalism to self-organized systems.

Wheeler's "It from Bit": If information is physical, are logical constraints also physical? The BEDS conjecture suggests yes: formal pathologies are information-theoretic manifestations of thermodynamic constraints.

Part III — Formal Results: e, pi, phi, and Euler

Complete mathematical proofs are provided in Annex D. This section presents the key results and their interpretation.

3.0 Intuition

Imagine rain falling on an empty lot. Billions of drops land at random — pure noise.

```
    ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴     Noise (maximum uncertainty)
    ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴ ∴
         |  |  |  |  |
         v  v  v  v  v
    ┌───────────────────────┐
    │  ~~~    ~~~    ~~~    │    Dissipation...
    │    ↘    ↓↓↓    ↙      │
    │      ↘  ↓↓↓  ↙        │
    └───────────────────────┘
               |
               v
    ┌──────────┴──────────┐
    │                     │
    ▼                     ▼
  ┌───┐                 ┌───┐
  │ e │                 │ π │    Two puddles. Always the same ones.
  └───┘                 └───┘
    │                     │
    └──────────┬──────────┘
               │
               ▼
         ┌──────────┐
         │  Euler   │            The passage between them.
         │e^iπ+1=0  │
         └──────────┘
               │
               ▼
              [φ]                The first echo.
```

The water didn't "choose" these hollows. It fell into them.

This is exactly what happens with mathematics. If you build any system capable of: - representing uncertainty - updating it in light of evidence - quantifying information

…then **e**, **π**, and **φ** appear. Not because we define them. Because we cannot avoid them.

Constant	What it encodes	Metaphor
e	Accumulation	The deepening hollow
π	Return	The circular hollow
Euler	Balance	The saddle point between
φ	Self-similarity	The first echo from the saddle

Thesis: These constants are not human inventions.
They are the scars left by entropy as it dissipates.

3.1 The Axioms

The BEDS framework rests on four axioms only:

> **Axiom 1 (State)**: There exist probability distributions P(x) over a measurable space X.
>
> **Axiom 2 (Sampling)**: Realizations x ~ P can be drawn from distributions.
>
> **Axiom 3 (Entropy)**: Every distribution has an entropy H(P) = -int P(x) log P(x) dx.
>
> **Axiom 4 (Conditioning)**: Observing evidence y updates distributions via Bayes' rule.

What emerges under natural assumptions: arithmetic, the constants e, π, φ, functions exp/sin/cos, Euler's equation. The derivations below require additional hypotheses (continuity, dimensionality, symmetry) that are physically natural but not strictly contained in the four axioms.

```
+--------------------------------------------------------+
|                    BEDS AXIOMS                         |
|                                                        |
|      P(x)      Sampling      H(P)        Bayes         |
|        |           |          |            |           |
|        +-----------+----------+------------+           |
|                        |                               |
|                        v                               |
|        +--------------------------------+              |
|        |      MUST EMERGE               |              |
|        |                                |              |
|        |   e    pi    phi   sin   cos   |              |
|        |   exp  Euler  arithmetic       |              |
|        +--------------------------------+              |
+--------------------------------------------------------+
```

3.2 Emergence of e

> **Theorem 1**: If Bayesian updates are continuous and infinitesimal (likelihood of the form 1 + g(x)/n), then their composition produces an exponential function with base e.The continuous composition of infinitesimal Bayesian updates necessarily produces an exponential function with base e.

Key idea: When we accumulate information continuously – each piece infinitesimally small – the result is multiplication by e^(total information). The limit definition emerges naturally:

$$\boxed{e := \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.71828...}$$

```
CONTINUOUS BAYESIAN UPDATING

n=1:   (1 + 1/1)^1  = 2.000
n=10:  (1 + 1/10)^10 = 2.594
n=100: (1 + 1/100)^100 = 2.705
n->oo: limit = e = 2.718

=======================================
e emerges as the limit of continuous
information accumulation
```

Interpretation: The constant e is not an arbitrary definition. It is the **trace** left by continuous Bayesian information accumulation. It is the only function that **knows itself perfectly** – it is its own derivative, at all orders, to infinity.

> **e is the mathematical signature of DEPARTURE – monotonic growth that perfectly predicts itself.**

Full proof: Annex D.1

3.3 Emergence of pi

> **Theorem 2**: In a space of dimension ≥ 2 with rotational symmetry, normalizing the maximum-entropy distribution (Gaussian) produces π as the period of angular integration.

Key idea: The Gaussian is the distribution that maximizes entropy for fixed variance. Normalizing it requires computing int $e^{(-x}$2)dx. This integral, when squared and converted to polar coordinates, reveals rotational symmetry – and pi emerges as the period of that rotation.

$$\boxed{\int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}}$$

```
EMERGENCE OF pi FROM ROTATIONAL SYMMETRY

Cartesian:                Polar:

    y                     The integrand e^(-(x^2+y^2))
    |                     depends only on r^2 = x^2+y^2
    |
----+----> x              -> Rotational symmetry
    |                     -> Angle integral = 2pi
    |                     -> pi emerges as the period
```

Interpretation: pi is not primarily “the ratio of circumference to diameter.” That geometric fact is a *consequence*. pi is the **signature of rotational symmetry** – the period that emerges when systems have no preferred direction.

> **pi is the mathematical signature of RETURN – the only number encoding eternity in a cycle.**

Full proof: Annex D.2

3.4 Euler's Equation

> **Theorem 3**: The equation e^{i*pi} + 1 = 0 follows necessarily from Theorems 1 and 2.

Key idea: Extending the exponential to complex arguments, multiplication by (1 + i*theta/n) represents an infinitesimal rotation. Composing n such rotations gives total rotation theta. At theta = pi, we've rotated halfway around the circle, arriving at -1.

$$\boxed{e^{i\pi} + 1 = 0}$$

```
THE TWO MODES MEET

MODE: DEPART (e)               MODE: RETURN (pi)
================               ==================

    |                              +------+
    |                              |      |
    |                         <----+      +---->
    v                              |      |
--------------->                   +------+

"Departs forever"              "Returns forever"


EULER: e^(i*pi) + 1 = 0
=======================

DEPART applied to HALF-RETURN = -1
Add 1 = SILENCE (zero)
```

Interpretation: Euler's equation is not a mysterious coincidence. It is the **meeting point** where departure (e) applied to half a return (pi) produces exact opposition (-1), which added to unity gives silence (0).

> **Euler is the silence that emerges when departing and returning balance perfectly.**

Full proof: Annex D.3

3.5 Emergence of phi (Golden Ratio)

> **Theorem 4**: The golden ratio can be expressed as φ = 2cos(π/5), linking it to e and π via Euler's formula. The angle π/5 corresponds to the first regular polygon (pentagon) exhibiting non-trivial self-similar structure:

$$\varphi = e^{i\pi/5} + e^{-i\pi/5} = 2\cos\left(\frac{\pi}{5}\right) = \frac{1+\sqrt{5}}{2}$$

Key idea: The angle pi/5 corresponds to the pentagon – the first polygon with non-trivial self-similarity. At this angle, the return mode generates a number satisfying phi^2 = phi + 1, the equation of self-reference.

```
WHY pi/5 IS SPECIAL

n    theta=pi/n    2cos(theta)    Property
-----------------------------------------
1    pi            -2             Opposition
2    pi/2          0              Orthogonality
3    pi/3          1              Trivial
4    pi/4          sqrt(2)        Square diagonal
5    pi/5          phi            SELF-SIMILARITY <-
6    pi/6          sqrt(3)        Hexagon

5 5 is the smallest n ≥ 3 where 2cos(π/n)
is an algebraic number of degree 2 satisfying
x² = x + 1. Whether this constitutes "self-reference"
in a deep sense remains interpretive.
```

Interpretation: phi is not independent of e and pi – it's their combination at the angle where self-reference first emerges.

> **phi is the mathematical signature of SELF-SIMILARITY – the structure that contains itself.**

Full proof: Annex D.4

3.6 The Complete Picture

```
+----------------------------------------------------------------------+
|                                                                      |
|                        BEDS AXIOMS                                   |
|                                                                      |
|         P(x)      Sampling      H(P)        Bayes                    |
|           |           |          |            |                     |
|           +-----------+----------+------------+                     |
|                           |                                          |
|                           v                                          |
|           +-------------------------------+                          |
|           |                               |                          |
|           v                               v                          |
|   +---------------+               +---------------+                  |
|   |     e         |               |       pi      |                  |
|   |   DEPART      |               |    RETURN     |                  |
|   | (accumulation)|               |  (periodicity)|                  |
|   +-------+-------+               +-------+-------+                  |
|           |                               |                          |
|           +---------------+---------------+                          |
|                           |                                          |
|                           v                                          |
|               +-------------------+                                  |
|               |     e^{i*theta}   |                                  |
|               |  (combination)    |                                  |
|               +---------+---------+                                  |
|                         |                                            |
|        +----------------+----------------+                           |
|        |                |                |                           |
|        v                v                v                           |
|     theta = pi      theta = pi/2     theta = pi/5                    |
|        |                |                |                           |
|        v                v                v                           |
|   +---------+      +---------+      +---------+                      |
|   |  EULER  |      |    i    |      |   phi   |                      |
|   | silence |      |rotation |      |auto-sim.|                      |
|   +---------+      +---------+      +---------+                      |
|                                                                      |
+----------------------------------------------------------------------+
```

Note on rigor: The derivations above are mathematically correct, but the claim that these constants “could not have not existed” given only the BEDS axioms requires implicit assumptions about continuity, dimensionality, and symmetry. A fully rigorous treatment would need to derive these properties from more primitive axioms — an open problem.

Summary Table

Constant	Mechanism	Property	Mode
e	lim(1 + 1/n)^n	Self-derivation	Pure DEPART
pi	Period of rotation	Periodicity	Pure RETURN
Euler	e^{i*pi} = -1	Silence	DEPART + full RETURN

Constant	Mechanism	Property	Mode
i	e^{i*pi/2}	90 deg rotation	DEPART + half RETURN
phi	2cos(pi/5)	Self-similarity	DEPART + 1/5 RETURN

3.7 The Conclusion

e is what **knows itself** — the only function that is its own derivative.

pi is what **returns** — the only number encoding eternity in a cycle.

phi is what **contains itself** — where return generates self-reference.

Euler is **silence** — where departing and returning balance perfectly.

These constants could not have not existed. They are the scars left by entropy as it dissipates. They are the only attractors of a universe that infers.

Part IV — Implementation: Sustainable P2P Network

4.1 Motivation

BEDS structures are not just a theoretical framework — they can be instantiated directly in a peer-to-peer network.

The central idea: **each node in the network is a BEDS**.

```
CLASSICAL NETWORK              BEDS NETWORK

+---+     +---+               +---+     +---+
| N | --> | N |               | S | <-> | S |
+---+     +---+               +---+     +---+
  |         |                   |         |
  v         v                   v         v
+---+     +---+               +---+     +---+
| N | <-- | N |               | S | <-> | S |
+---+     +---+               +---+     +---+

N = Node (passive)            S = BEDS (dissipative)

Messages: data                Messages: beliefs
Computation: forward pass     Computation: Bayesian fusion
Cost: O(parameters)           Cost: O(1) per update
```

4.2 Node Architecture

Each node contains:

Component	Role	Notation
Belief	Current state (posterior)	mu
Uncertainty	Variance on belief	sigma^2
Dissipation	Forgetting rate	gamma
Identity	Public key	pk
Secret	Private key	sk

Isolation Principle

Once created, a node: - **Cannot** be reconfigured - **Communicates only** via signed messages - **Dies only** if flux is cut

This is the dissipative constraint: the node exists through the flux that traverses it. No flux, no node.

MicroQuickJS: Ultra-Minimal Runtime

For maximum resource efficiency, BEDS nodes can be implemented using **Micro-QuickJS** (mquickjs), Fabrice Bellard’s JavaScript engine released in December 2025, specifically designed for embedded systems:

Specification	mquickjs	Standard JS (V8)
Minimum RAM	10 kB	~10 MB
ROM footprint	~100 kB	~30 MB
Object overhead	12 bytes (32-bit)	~64 bytes
Dependencies	None (self-contained)	System libraries
Language subset	ES5 strict	Full ES2024

```
MEMORY COMPARISON

V8/Node.js:        [████████████████████████████████] 10+ MB
QuickJS:           [████████]                         367 KB
MicroQuickJS:      [█]                                100 KB
BEDS node state:   [.]                                ~200 bytes

A BEDS node fits in the cache of a $0.50 microcontroller.
```

Why mquickjs is ideal for BEDS:

1. **Self-contained**: Own memory allocator, math library, and floating-point emulator
2. **No garbage collection pause**: Tracing + compacting GC with bounded latency
3. **Deterministic**: Bounded C stack usage (no recursion in parser)
4. **Portable bytecode**: 32-bit bytecode runs on any 32-bit embedded system

A complete BEDS node implementation in mquickjs requires:

```javascript
// Core BEDS state: ~200 bytes
const node = {
    mu: 0.0,        // belief (8 bytes)
    sigma: 1.0,     // uncertainty (8 bytes)
    gamma: 0.001,   // dissipation rate (8 bytes)
    pk: new Uint8Array(32),  // public key
    sk: new Uint8Array(32)   // private key
};

// Bayesian update: ~10 FLOPS
function fuse(msg) {
    const tau_self = 1 / (node.sigma * node.sigma);
    const tau_msg = 1 / (msg.sigma * msg.sigma);
    const tau_new = tau_self + tau_msg;
    node.mu = (tau_self * node.mu + tau_msg * msg.mu) / tau_new;
    node.sigma = Math.sqrt(1 / tau_new);
}
```

4.3 Exchange Protocol

Minimal Message Format

Field	Description
source	Who sends (public key)
belief	The belief mu
sigma	The uncertainty sigma
signature	Cryptographic proof

Four fields suffice. It’s canonical.

Bayesian Update

When node B receives a belief from A, it fuses by Gaussian product:

$$\tau_{new} = \tau_A + \tau_B$$
$$\mu_{new} = \frac{\tau_A \cdot \mu_A + \tau_B \cdot \mu_B}{\tau_{new}}$$
$$\sigma_{new} = \sqrt{\frac{1}{\tau_{new}}}$$

where tau = 1/sigma^2 is the precision.

```
BAYESIAN FUSION

Node A: mu_A = 10, sigma_A = 2    (tau_A = 0.25)
Node B: mu_B = 14, sigma_B = 1    (tau_B = 1.00)

Fusion:
tau_new = 0.25 + 1.00 = 1.25
mu_new = (0.25*10 + 1.00*14) / 1.25 = 13.2
sigma_new = sqrt(1/1.25) = 0.89

+------------------------------------------+
|     A           Fusion           B       |
|   +---+                        +---+     |
|   |10 |-------> 13.2 <---------|14 |     |
|   |+/-2|                       |+/-1|    |
|   +---+                        +---+     |
+------------------------------------------+
```

Temporal Dissipation

Without new messages, uncertainty grows:

$$\sigma(t) = \sigma_0 \cdot e^{\gamma t}$$

The node “forgets.” It returns toward maximum uncertainty — thermodynamic death if flux stops.

```
DISSIPATION WITHOUT FLUX

t=0:   sigma = 1.0    ||||||||||||||||||||
t=1:   sigma = 1.5    ||||||||||||||||||||||||||||||
t=2:   sigma = 2.3    ||||||||||||||||||||||||||||||||||||||||||||
t=oo:  sigma = oo     DEATH (maximum uncertainty)
```

4.4 Emergent Hierarchy

```
Level 0: Sensors -> raw beliefs
              | crystallization
Level 1: Aggregators -> fused beliefs
              | crystallization
Level 2: Meta-aggregators -> consensus
              |
             ...
```

Energy Bound Property

If each level crystallizes, then:

$$E_{total} = \sum_{n=0}^{\infty} E_n < \infty$$

The series converges because entropy decreases at each level. The network is **self-bounded energetically**.

4.5 Energy Budget

For a 5-minute cycle:

Phase	Duration	Energy
Wake + Measure	15 ms	~1 µWh
Bayesian fusion	20 ms	~1.6 µWh
LoRa transmission	100 ms	~20 µWh
Deep sleep	299.8 s	~275 µWh
Total	**300 s**	**~300 µWh**

Average power: ~3.6 mW

Why Bayesian Fusion Is Negligible

The Bayesian update of two Gaussians requires only ~10 floating-point operations:
- 2 divisions, 2 multiplications, 2 additions, 1 square root

This is a direct consequence of BEDS: Bayesian fusion is the **minimal operation** on the statistical manifold.

4.6 Autonomy Equation

	Value
Solar input (average)	~40 mW
Node consumption	~3.6 mW
Margin	**x11**

```
ENERGY BALANCE

(+) Solar: ~40 mW
     |
     v
+-------------+
|  BEDS Node  | <- 3.6 mW
+-------------+
     |
     v
[+] Surplus -> Storage

Autonomy without sun: ~47 hours
```

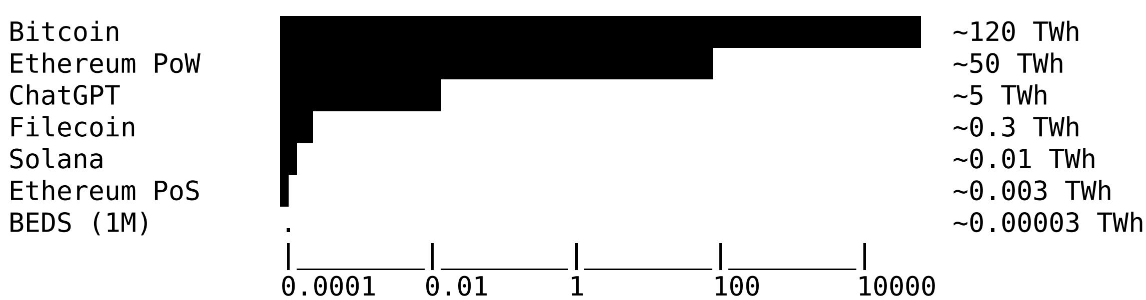
4.7 Comparison with Existing Systems

Global Energy Footprint

System	Annual Consumption	Learning	Consensus
Bitcoin	~120–150 TWh	None	Proof-of-Work
Ethereum (pre-Merge, PoW)	~21–95 TWh	None	Proof-of-Work
Ethereum (post-Merge, PoS)	~0.0026 TWh	None	Proof-of-Stake
Filecoin	~0.1–0.5 TWh*	None	Proof-of-Spacetime
Solana	~0.011 TWh	None	Proof-of-History
Flow	~0.00018 TWh	None	Proof-of-Stake
ChatGPT Inference	~1–10 TWh	None (frozen)	N/A
Federated Learning (1M)	~0.01–0.1 TWh	Distributed	Centralized aggr.
BEDS Network (1M nodes)	**~0.00003 TWh**	**Continuous**	**Bayesian fusion**

*Filecoin estimates vary widely depending on storage provider hardware.

ENERGY SCALE (log TWh/year)



Per-Transaction/Operation Comparison

System	Energy per Operation	Equivalent To
Bitcoin transaction	~1,200 kWh	2.8 house-months
Ethereum PoW transaction	~100 kWh	House for 3 days
Ethereum PoS transaction	~0.03 kWh	1 minute of microwave
Solana transaction	~0.0005 kWh	2 Google searches
BEDS belief update	~0.0000003 kWh	1 millisecond of LED light

Functional Comparison

Criterion	Bitcoin	Ethereum PoS	Federated Learning	BEDS Network
Learning	✗	✗	✓ (batched)	✓ (continuous)
Decentralized	✓	✓	Partial	✓
Energy autonomous	✗	✗	✗	✓
Real-time	✗	~12s blocks	✗	✓
Uncertainty quant.	✗	✗	Partial	✓ (native)
Byzantine tolerant	✓	✓	Partial	✓ (weighted)
Consensus type	Global	Global	Centralized	Local-first

Why BEDS Achieves Orders-of-Magnitude Efficiency

1. **No global consensus**: BEDS nodes only communicate with neighbors; no network-wide broadcast
2. **Bayesian fusion is O(1)**: Independent of network size, unlike gradient aggregation
3. **Computation = compression**: Each update reduces entropy, not just verifies
4. **Natural sparsity**: Uncertainty-weighted communication means low-confidence nodes stay quiet
5. **Solar-sustainable**: 3.6 mW average fits within a 2cm² solar cell budget

Memory Footprint Comparison

Runtime/System	Minimum RAM	Typical RAM	BEDS Nodes per GB
TensorFlow Lite	~1 MB	10–100 MB	10–100
PyTorch Mobile	~50 MB	100–500 MB	2–10
Node.js + V8	~10 MB	50–200 MB	5–20
QuickJS	~200 KB	1–5 MB	200–1000
MicroQuickJS	**10 KB**	50–100 KB	10,000–20,000
BEDS state only	**~200 B**	~500 B	2,000,000+

With mquickjs, a single Raspberry Pi Zero (512 MB) can simulate **thousands** of BEDS nodes for testing. A microcontroller with 64 KB RAM can run multiple BEDS agents.

A BEDS network doesn't prove it has *worked*. It proves it has *converged*.

Part V – Philosophy: Implications and Perspectives

5.1 On the Nature of Mathematics

The BEDS derivation suggests an intermediate position between Platonism and constructivism:

- Mathematics is not **invented** arbitrarily
- It is not **discovered** in a pre-existing abstract world
- It **emerges necessarily** from any system capable of representing uncertainty

5.2 Crystallization and Fragility

The erratic behaviors of current AI are not a bug, but a thermodynamic consequence of crystallization.

When a large language model is frozen after training, it crystallizes a hypothesis about the world as it was. As the environment drifts, the model extrapolates into uncalibrated regions.

Hallucinations, out-of-distribution fragility, adversarial vulnerability: symptoms of **excessive crystallization**.

```
    FROZEN LLM                    BEDS NETWORK
    ===========                   ============

    Crystallization -> Frozen     Crystallization <-> Dissipation
                        |                   |
              World drifts...        Tension maintained
                        v                   v
                  HALLUCINATION         ADAPTATION
```

A BEDS network maintains permanent tension between crystallization and dissipation. The system remains **liquid**: ordered but adaptable.

5.3 Toward a Quantum Substrate

A speculative conjecture: **Bayesian formalism might find its natural substrate in quantum computing**.

Paradigm	State	Adaptability
Frozen LLM	Crystal	None
Classical BEDS	Liquid	Via gamma
Quantum BEDS	Superposition	Native

A quantum BEDS node wouldn't *simulate* uncertainty – it would be *physical*, encoded in superposition. Structured forgetting would emerge from decoherence.

5.4 BEDS Cosmology: Note

If the universe is a dissipative structure, it requires an input and outputs. The Big Bang would be the input (low-entropy prior, maximum possibilities), black holes would be the outputs (exporting to a meta-level). Hawking radiation is not a leak — it's proof that black holes dissipate, not that they swallow. This meta-level would itself be a universe whose Big Bangs are our black holes: a recursive cascade where the crystallized posterior of one cosmos becomes the prior of the next. Smolin (1992) proposed "cosmological natural selection" — BEDS adds that it's not just selection, it's *learning*: each child universe inherits a more informed prior.

```
Parent universe
      |
      | Big Bang (input)
      v
 [ Our Universe ] ──→ Black hole A ──→ Child Big Bang A
                 ──→ Black hole B ──→ Child Big Bang B
```

> *The universe cannot prove itself — but it can spawn universes that prove what it could not.*

5.5 The Bayesian Society

A Bayesian society would be a set of agents who share their beliefs and update them collectively. Each agent observes the world, exchanges with others, and adjusts what they believe based on what they learn.

No central authority. No imposed doctrine. Just a continuous flow of observations, conversations, and updates.

```
THE BAYESIAN AGORA

A: "I saw rain"
 |
 +------> B: "Me too" ------> Confidence increases
 |
 +------> C: "I saw sun" --> Uncertainty remains
 |
 +------> D: "Check the clouds" --> New observation suggested

No one commands. Everyone updates.
```

When enough agents converge on the same belief — when collective uncertainty about a question becomes very low — this belief crystallizes and becomes a **shared axiom**. This axiom is imposed by no one: it emerges from the flow of information, like a riverbed emerges from the repeated passage of water.

These crystallized axioms then become the foundation on which new beliefs can be built. The cost of learning decreases at each level, because the space of possibilities shrinks.

The Entropy Landscape

But not everyone wants to live in the same thermodynamic regime. And that's fine.

```
THE ENTROPY MAP OF SOCIETY
```

```
-*- LOW ENTROPY                        ~*~ HIGH ENTROPY
(order, stability)                     (chaos, action)

+----------------------------------------------------------+
|                                                          |
|  -*- -*- -*-      :::::::::::      ~*~ ~*~ ~*~           |
|  +---------+      :::::::::::      +---------+           |
|  | (.) (.) |      ::: -> -> :::    | >> >> |             |
|  |         |      :::::::::::      |       |             |
|  | (.) (.) |        :::::::        | >> >> |             |
|  +---------+                       +---------+           |
|                                                          |
|  CRYSTALLIZED     FLOWING         EXPLORING              |
|  ZONES            ZONES           ZONES                  |
|                                                          |
+----------------------------------------------------------+

(.) = Those who seek harmony, contemplation, stability
->  = Those who wander, exchange, connect
>>  = Those who explore, disrupt, create
```

Some people are drawn to **negentropy** — order, predictability, harmony. They gravitate toward crystallized zones: monasteries, libraries, ancient traditions, well-established sciences. They are the -*- **snowflakes** — beautiful, structured, stable. They preserve what has been learned.

```
-*- THE CRYSTAL DWELLERS

"We have found our basin of attraction.
 The answers are here. Come rest."

+---------------------+
|  [=] [-] ~ [T]      |
|                     |
|  Low uncertainty    |
|  High stability     |
|  Deep roots         |
|                     |
|  Energy: minimal    |
|  (just maintenance) |
+---------------------+
```

Others are drawn to **entropy** — action, novelty, flux. They gravitate toward high-energy zones: startups, frontiers, revolutions, unsolved problems. They are the ~*~ **flames** — hot, dynamic, transformative. They discover what has not yet been found.

```
~*~ THE ENTROPY SURFERS

"The basin is boring.
 Let's find a new mountain to climb."

+---------------------+
|  ^ ? /\ !           |
|                     |
|  High uncertainty   |
```

```
|  High energy        |
|  Fast flux          |
|                     |
|  Energy: intense    |
|  (but short bursts) |
+---------------------+
```

And in between, the **travelers** -> — carrying beliefs from one zone to another, cross-pollinating, connecting crystals and flames:

```
-> THE BRIDGES

-*- Crystal Zone A                    -*- Crystal Zone B
+---------+                          +---------+
|  [=]    |                          |    (o)  |
| ancient |<------ -> --------------->| modern  |
| wisdom  |      carries             | science |
+---------+      beliefs             +---------+
                    |
                    |
                    v
               ~*~ Frontier
              +---------+
              | new     |
              | synthesis|
              +---------+
```

Everyone in Their Basin

The beautiful thing: **no one needs to be everywhere**.

The contemplative monk (.) is not failing by staying in one place. He is *maintaining* a crystallized structure — keeping ancient wisdom alive at minimal energy cost.

The restless entrepreneur » is not failing by never settling. She is *exploring* the entropy landscape — finding new basins that others can later inhabit.

The teacher -> is not failing by being neither. He is *connecting* — reducing the cost for others to learn by carrying compressed beliefs across the landscape.

```
THE ECOSYSTEM

          ... (new information rains down)
            |
            v
+--------------------------------------------+
|                                            |
|   -*-         ->         ~*~               |
|  PRESERVE <----------------> EXPLORE       |
|      |                     |               |
|      |    crystallize      |               |
|      <---------------------------+         |
|           when ready                       |
|                                            |
```

```
+-------------------------------------------+
            |
            v
        ~~~ (entropy flows out)
```

The Speculative Hypothesis

Only value systems, norms, and institutions that emerge from this process — that crystallize naturally from interactions and observations — are sustainable.

What is imposed from above, without roots in the collective dissipative structure, tends to collapse.

```
ORGANIC vs IMPOSED

ORGANIC (emerges)              IMPOSED (forced)

    ...                            [#]
     |                              |
     v                              v
+---------+                    +---------+
| ~ ~ ~ ~ | (natural flow)     | @@@@@@@@| (concrete dam)
|  ~ ~ ~  |                    | @@@@@@@@|
|   ~~~   |                    | @@@@@@@@|
+----+----+                    +----+----+
     |                              |
     v                              v
   ~~~                            !!! (eventually)

The river finds its bed.      The dam cracks.
```

An artificial dam can hold for a time, but the river always finds its true bed.

Where Will You Go?

The Bayesian society doesn't tell you where to be. It says: **find your thermodynamic niche**.

- Do you need rest? -*- Find a crystal. Join a tradition. Let others explore.
- Do you need action? ~*~ Find a frontier. Break new ground. Let others rest.
- Do you need connection? -> Walk between. Carry messages. Let others stay.

```
YOUR PLACE IN THE LANDSCAPE

+-----------------------------------------------------+
|                                                     |
|      -*-              ->              ~*~           |
|                                                     |
|   "I stay"      "I wander"      "I explore"         |
|                                                     |
|   Library        Cafe            Laboratory          |
|   Temple         Market          Startup             |
|   Garden         Road            Volcano             |
```

```
|                                                       |
|   All necessary. All honorable. All BEDS.             |
|                                                       |
+-------------------------------------------------------+
```

The goal is not to maximize or minimize entropy. The goal is to find where your flux flows best – and to let the structure emerge.

> *Some crystallize. Some explore. Some connect. All dissipate. All learn.*

5.6 Final Words

> **What we call learning is the universe finding its attractors.**

A formal system can only maintain coherence if it is open, dissipative, and recursive. A closed system pays the price of closure: incompleteness, rigidity, or death.

> *To think without contradicting oneself, one must forget. To know without crystallizing, one must remain open. To exist without dying, one must dissipate.*

The universe doesn't calculate. It flows. And in flowing, it learns.

P.S. – A word of caution: Cantor, Gödel, and Turing all worked on closed formal systems – and it drove them mad. Self-reference without dissipation is dangerous. BEDS offers a gentler path: observe the world, let structure emerge, and export your entropy. And if you're tired of thinking in circles – just go to BEDS!

How to Cite / Donation

If you find this work usefull (for bootstraping the new entropy coin $0x3F!) :

eth : 0x5664023c0de4d209a11f23978ef845ebe3e8b697

dot : 5FCpi5qu4SoYqkXyroRcmKNBDZDfkEqRVMeFS5RJrhHaNt7y

xmr : 468p23RoRLFAvbw4mGYWjrXavMtr9sUZbMFPNjChbaH1j2jV

ELmyyDyU4kcCntdA3jEHNzEQo1kzyKLu7R7QwK2aEvoQibW

This is my OpenPGP public key (fingerprint: DBB3D6FDABBA66A4), which I intend to use as a stable long-term identity key.

If you find this work useful or want to reference it:

"'bibtex @misc{caraffa2025beds, author = {Caraffa, Laurent}, title = {{BEDS}: {B}ayesian {E}mergent {D}issipative {S}tructures}, year = {2025}, note = {Working paper. Speculative framework, not peer-reviewed.}, howpublished = {https://github.com/lcaraffa/Bayesian_Emergent_Dissipative_Structures}, url = {https://lcaraffa.net} institution = {Univ. Gustave Eiffel, IGN-ENSG, LaSTIG} } "'

Inspiration

This work draws from five major sources of inspiration. The first is Yann LeCun's vision of joint embedding predictive architectures, presented in A Path Towards Autonomous Machine Intelligence (LeCun, 2022), complemented by the theoretical foundations of LeJEPA (Balestriero & LeCun, 2025) — the latter establishing that the isotropic Gaussian distribution is optimal for embeddings, a result that resonates with our conjecture on thermodynamic optimality. The second is François Roddier's Thermodynamique de l'évolution (2012), which applies Prigogine's principles to biological and social systems, and whose reading profoundly oriented this work toward a unified vision of dissipative structures at all scales. The distributed architecture also draws inspiration from the Polkadot network (Wood, 2016), whose interoperable parachain model offers a precedent for emergent consensus hierarchies — though our approach substitutes cryptographic proof with Bayesian convergence. The fourth is Douglas Hofstadter's Gödel, Escher, Bach: An Eternal Golden Braid, which explores through logic, art, and music how formal systems can engender self-reference, consciousness, and meaning. Finally, Leonardo da Vinci, whose timeless principle — "Simplicity is the ultimate sophistication" — haunts every page of this work: four axioms, ten operations per update, one equation for consensus.

Beyond these formal references, this work is inevitably shaped by countless conversations throughout my life — with colleagues, friends, family, strangers — and by all the books, films, articles, and media I've absorbed over the years. Ideas never emerge in isolation; they are distilled from an ocean of influences, most of which I cannot trace or name. To all those who unknowingly contributed: thank you.

If BEDS has any merit, it is in having added nothing unnecessary.

Annex A – Technical Details: P2P Network

A.4 LoRa Configuration

Parameter	Value
Frequency	868 MHz (EU) / 915 MHz (US)
Spreading Factor	SF7-SF12
Range	5-10 km (rural)

Annex B — Connection to Existing Work

B.1 Relation to JEPA (LeCun)

JEPA	BEDS
Encoder x -> s_x	Structure extraction from flux
Latent variable z	Non-exportable entropy
Energy D(s_y, s~_y)	Variational free energy
Isotropic Gaussian (LeJEPA)	Max entropy = dissipative equilibrium

JEPA prescribes *what* to build. BEDS explains *why*.

B.2 Relation to Free Energy Principle (Friston)

Both identify free energy minimization as the core organizing principle. BEDS adds recursive crystallization.

B.3 Relation to Information Bottleneck (Tishby)

$$\min_{p(t|x)} I(X;T) - \beta I(T;Y)$$

In BEDS terms: I(X;T) = entropy exported, I(T;Y) = structure maintained, beta = temperature.

Annex C — Energy Comparison Data

System	Annual Consumption	Mechanism
Bitcoin	~120 TWh	Proof of Work
Ethereum (post-merge)	~0.01 TWh	Proof of Stake
GPT-4 Training	~50 GWh (one-shot)	Backpropagation
ChatGPT Inference	~1-10 TWh/year	Forward pass
Bittensor	~0.1 TWh	Proof of Intelligence
BEDS (1M nodes)	**~0.00003 TWh**	**Bayesian fusion**

Calculation for BEDS: 1,000,000 nodes * 3.6 mW * 8760 h = 31.5 MWh = 0.00003 TWh

---

Annex D — Complete Mathematical Proofs

D.1 Complete Proof: Emergence of e

D.1.1 Setup

Let $P_0(x)$ be a prior distribution over parameter space $\mathcal{X}$. We receive a sequence of n observations $y_1, y_2, \dots, y_n$, each carrying infinitesimal information.

D.1.2 Infinitesimal Evidence Assumption

Each observation y_k has likelihood:

$$P(y_k|x) = 1 + \frac{g(x)}{n} + O\left(\frac{1}{n^2}\right)$$

where $g(x)$ is the **information direction** — which regions of x are favored by the evidence.

Justification: For small perturbations, any smooth likelihood can be Taylor-expanded around 1 (the uninformative likelihood). The $1/n$ scaling ensures total information remains finite as $n \to \infty$.

D.1.3 Sequential Bayesian Update

After observation y_1:

$$P_1(x) = \frac{P(y_1|x)P_0(x)}{P(y_1)} = \frac{(1 + g(x)/n)P_0(x)}{Z_1}$$

where $Z_1 = \int (1 + g(x)/n)P_0(x)dx$ is the normalization constant.

After all n observations:

$$P_n(x) \propto P_0(x) \prod_{k=1}^{n} P(y_k|x) = P_0(x)\left(1 + \frac{g(x)}{n}\right)^n$$

D.1.4 The Limit

Taking $n \to \infty$:

$$P_\infty(x) \propto P_0(x) \cdot \lim_{n\to\infty}\left(1 + \frac{g(x)}{n}\right)^n$$

Definition: We define the exponential function as this limit:

$$\exp(g(x)) := \lim_{n\to\infty}\left(1 + \frac{g(x)}{n}\right)^n$$

D.1.5 Proof That the Limit Exists and Equals e for g(x) = 1

Let $a_n = (1+1/n)^n$. We prove:

(a) The sequence is increasing:

By AM-GM inequality on n copies of $(1+1/n)$ and one copy of 1:

$$\frac{n(1+1/n)+1}{n+1} \geq ((1+1/n)^n \cdot 1)^{1/(n+1)}$$

This gives $(1+1/(n+1))^{n+1} \geq (1+1/n)^n$, so $a_{n+1} \geq a_n$.

(b) The sequence is bounded:

$$\left(1+\frac{1}{n}\right)^n = \sum_{k=0}^{n} \binom{n}{k} \frac{1}{n^k} < \sum_{k=0}^{\infty} \frac{1}{k!} < 1+1+\frac{1}{2}+\frac{1}{4}+\cdots = 3$$

(c) Therefore, by the monotone convergence theorem, the limit exists.

(d) The limit is the unique number e = 2.71828…

We can compute: $e = \sum_{k=0}^{\infty} \frac{1}{k!} = 1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\cdots$

D.1.6 The Self-Derivation Property

Theorem: $\frac{d}{dx}e^x = e^x$

Proof:

$$\frac{d}{dx}e^x = \lim_{h\to 0} \frac{e^{x+h}-e^x}{h} = e^x \lim_{h\to 0} \frac{e^h-1}{h}$$

We need to show $\lim_{h\to 0} \frac{e^h-1}{h} = 1$.

From the series expansion:

$$e^h = 1+h+\frac{h^2}{2!}+\frac{h^3}{3!}+\cdots$$

Therefore:

$$\frac{e^h-1}{h} = 1+\frac{h}{2!}+\frac{h^2}{3!}+\cdots \to 1 \text{ as } h \to 0$$

Thus $\frac{d}{dx}e^x = e^x$. ■

D.1.7 Uniqueness

Theorem: e^x is the unique function f satisfying $f'(x) = f(x)$ and $f(0) = 1$.

Proof: Suppose $f'(x) = f(x)$ with $f(0) = 1$. Consider $g(x) = f(x)/e^x$.

$$g'(x) = \frac{f'(x)e^x - f(x)e^x}{e^{2x}} = \frac{f(x)e^x - f(x)e^x}{e^{2x}} = 0$$

So $g(x)$ is constant. Since $g(0) = f(0)/e^0 = 1/1 = 1$, we have $g(x) = 1$ for all x, hence $f(x) = e^x$. ■

D.2 Complete Proof: Emergence of pi

D.2.1 Maximum Entropy Principle

Theorem (Jaynes): Among all distributions on $\mathbb{R}$ with fixed mean μ and variance σ^2, the one maximizing entropy is the Gaussian:

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Proof: We maximize the entropy functional subject to constraints:

$$\mathcal{L}[P] = -\int P(x) \ln P(x)\, dx - \lambda_0 \left(\int P(x)\, dx - 1\right) - \lambda_1 \left(\int x P(x)\, dx - \mu\right) - \lambda_2 \left(\int x^2 P(x)\, dx - (\sigma^2$$

Taking the functional derivative $\frac{\delta \mathcal{L}}{\delta P} = 0$:

$$-\ln P(x) - 1 - \lambda_0 - \lambda_1 x - \lambda_2 x^2 = 0$$

Therefore:

$$P(x) = \exp(-1 - \lambda_0 - \lambda_1 x - \lambda_2 x^2)$$

This is a Gaussian. The constraints determine $\lambda_0, \lambda_1, \lambda_2$ in terms of μ, σ^2. ■

D.2.2 The Gaussian Integral

Theorem: $I = \int_{-\infty}^{+\infty} e^{-x^2} dx = \sqrt{\pi}$

Proof (Poisson's method):

(a) Square the integral:

$$I^2 = \int_{-\infty}^{+\infty} e^{-x^2} dx \cdot \int_{-\infty}^{+\infty} e^{-y^2} dy = \iint_{\mathbb{R}^2} e^{-(x^2+y^2)} dx\, dy$$

(b) Observe rotational symmetry:

The integrand $e^{-(x^2+y^2)} = e^{-r^2}$ depends only on the radial coordinate $r = \sqrt{x^2+y^2}$.

(c) Convert to polar coordinates:

With $x = r\cos\theta$, $y = r\sin\theta$, the Jacobian is r:

$$I^2 = \int_0^{2\pi} d\theta \int_0^{\infty} e^{-r^2} r\, dr$$

(d) Evaluate the radial integral:

Let $u = r^2$, so $du = 2r\,dr$:

$$\int_0^\infty e^{-r^2} r\,dr = \frac{1}{2}\int_0^\infty e^{-u} du = \frac{1}{2}\left[-e^{-u}\right]_0^\infty = \frac{1}{2}$$

(e) Evaluate the angular integral:

The angular integral introduces π. To proceed rigorously, we need to establish what "angle" means.

Definition: The functions $\cos\theta$ and $\sin\theta$ are defined as the unique solutions to:

$$\frac{d^2 f}{d\theta^2} = -f$$

with initial conditions $\cos(0) = 1, \cos'(0) = 0$ and $\sin(0) = 0, \sin'(0) = 1$.

Definition: π is defined as:

$$\pi := \inf\{t > 0 : \cos(t) = -1\}$$

Equivalently, 2π is the period of cos and sin.

(f) The angular integral equals 2π:

The full rotation around the origin traverses angle from 0 to 2π:

$$\int_0^{2\pi} d\theta = 2\pi$$

(g) Conclusion:

$$I^2 = 2\pi \cdot \frac{1}{2} = \pi$$
$$\therefore I = \sqrt{\pi} \quad \blacksquare$$

D.2.3 Alternative Proof via Gamma Function

The Gamma function is defined as:

$$\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt$$

Property: $\Gamma(1/2) = \sqrt{\pi}$

Proof:

$$\Gamma(1/2) = \int_0^\infty t^{-1/2} e^{-t} dt$$

Substituting $t = x^2$, $dt = 2x\,dx$:

$$\Gamma(1/2) = \int_0^\infty x^{-1} e^{-x^2} \cdot 2x\,dx = 2\int_0^\infty e^{-x^2} dx = I$$

Since $\Gamma(1/2) = \sqrt{\pi}$ (provable via the reflection formula $\Gamma(s)\Gamma(1-s) = \pi/\sin(\pi s)$ at $s = 1/2$), we have $I = \sqrt{\pi}$. $\blacksquare$

D.2.4 Why pi Appears

The key insight is that pi emerges from **rotational symmetry**. Whenever a problem has no preferred direction in 2D, converting to polar coordinates introduces an angular integral over a full rotation, which equals 2π.

This is not about circles having circumference $2\pi r$. Rather, **circles have circumference $2\pi r$ because π is the period of the rotational symmetry group.**

D.3 Complete Proof: Euler's Equation

D.3.1 Extending the Exponential to Complex Numbers

Definition: For $z \in \mathbb{C}$:

$$e^z := \lim_{n\to\infty} \left(1 + \frac{z}{n}\right)^n = \sum_{k=0}^{\infty} \frac{z^k}{k!}$$

Convergence: The series converges absolutely for all $z \in \mathbb{C}$ by comparison with $\sum |z|^k/k! = e^{|z|}$.

D.3.2 Euler's Formula

Theorem: $e^{i\theta} = \cos\theta + i\sin\theta$

Proof:

(a) Expand $e^{i\theta}$ as a series:

$$e^{i\theta} = \sum_{k=0}^{\infty} \frac{(i\theta)^k}{k!} = \sum_{k=0}^{\infty} \frac{i^k\theta^k}{k!}$$

(b) Note the pattern of powers of i: - $i^0 = 1$ - $i^1 = i$ - $i^2 = -1$ - $i^3 = -i$ - $i^4 = 1$ (cycle repeats)

(c) Separate real and imaginary parts:

Real parts (even k): $1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots = \cos\theta$

Imaginary parts (odd k): $i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right) = i\sin\theta$

(d) Therefore:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \blacksquare$$

D.3.3 Euler's Identity

Theorem: $e^{i\pi} + 1 = 0$

Proof:

From Euler's formula with $\theta = \pi$:

$$e^{i\pi} = \cos\pi + i\sin\pi$$

From the definitions of cos and sin: - $\cos\pi = -1$ (half-period of cosine) - $\sin\pi = 0$ (zero-crossing of sine)

Therefore:

$$e^{i\pi} = -1 + i \cdot 0 = -1$$
$$e^{i\pi} + 1 = 0 \quad \blacksquare$$

D.3.4 Geometric Interpretation

The function $e^{i\theta}$ traces the unit circle in the complex plane as θ varies: - At $\theta = 0$: $e^{i\cdot 0} = 1$ (rightmost point) - At $\theta = \pi/2$: $e^{i\pi/2} = i$ (topmost point) - At $\theta = \pi$: $e^{i\pi} = -1$ (leftmost point) - At $\theta = 3\pi/2$: $e^{i\cdot 3\pi/2} = -i$ (bottommost point) - At $\theta = 2\pi$: $e^{i\cdot 2\pi} = 1$ (back to start)

Euler's identity states: starting at 1, rotating by angle π (half a full rotation) brings you to -1.

D.4 Complete Proof: Emergence of phi

D.4.1 The Golden Ratio

Definition: The golden ratio φ is the positive root of:

$$x^2 - x - 1 = 0$$

By the quadratic formula:

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.6180339887...$$

Property: φ satisfies the self-referential equation:

$$\varphi = 1 + \frac{1}{\varphi}$$

D.4.2 Connection to cos(pi/5)

Theorem: $\varphi = 2\cos(\pi/5)$

Proof:

(a) Express cos(pi/5) using the identity for cos(5*theta):

Let $c = \cos(\pi/5)$. Since $5 \cdot (\pi/5) = \pi$, we have $\cos(5 \cdot \pi/5) = \cos\pi = -1$.

Using the Chebyshev polynomial:

$$\cos(5\theta) = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

Setting $\theta = \pi/5$ and $c = \cos(\pi/5)$:

$$-1 = 16c^5 - 20c^3 + 5c$$

$$16c^5 - 20c^3 + 5c + 1 = 0$$

(b) Factor the polynomial:

This factors as:

$$(c+1)(4c^2 - 2c - 1)^2/\text{(some factor)} = 0$$

More directly, the minimal polynomial for $\cos(\pi/5)$ is:

$$4c^2 - 2c - 1 = 0$$

(c) Solve:

$$c = \frac{2 \pm \sqrt{4+16}}{8} = \frac{2 \pm \sqrt{20}}{8} = \frac{1 \pm \sqrt{5}}{4}$$

Since $\cos(\pi/5) > 0$, we take the positive root:

$$\cos(\pi/5) = \frac{1+\sqrt{5}}{4}$$

(d) Therefore:

$$2\cos(\pi/5) = \frac{1+\sqrt{5}}{2} = \varphi \quad \blacksquare$$

D.4.3 Expression via Euler's Formula

Theorem: $\varphi = e^{i\pi/5} + e^{-i\pi/5}$

Proof:

By Euler's formula:

$$e^{i\pi/5} = \cos(\pi/5) + i\sin(\pi/5)$$
$$e^{-i\pi/5} = \cos(\pi/5) - i\sin(\pi/5)$$

Adding:

$$e^{i\pi/5} + e^{-i\pi/5} = 2\cos(\pi/5) = \varphi \quad \blacksquare$$

D.4.4 Connection to the Pentagon

The regular pentagon has interior angles of $108° = 3\pi/5$ radians. The diagonal-to-side ratio equals φ.

Proof:

In a regular pentagon with side length 1, let d be the diagonal length. The diagonal and two sides form an isosceles triangle with angles $36°$-$72°$-$72°$ (i.e., $\pi/5$-$2\pi/5$-$2\pi/5$).

By the law of sines:

$$\frac{d}{\sin(3\pi/5)} = \frac{1}{\sin(\pi/5)}$$

Using $\sin(3\pi/5) = \sin(2\pi/5)$ and the identity $\sin(2\theta) = 2\sin\theta\cos\theta$:

$$d = \frac{\sin(2\pi/5)}{\sin(\pi/5)} = \frac{2\sin(\pi/5)\cos(\pi/5)}{\sin(\pi/5)} = 2\cos(\pi/5) = \varphi \quad \blacksquare$$

D.4.5 Self-Similarity of the Pentagon

When you draw all diagonals of a regular pentagon, they form a smaller regular pentagon inside. The ratio of the original to the smaller pentagon is φ^2.

Continuing this process creates a sequence of nested pentagons, each scaled by φ from the previous — a **fractal structure** exhibiting self-similarity at angle $\pi/5$.

D.4.6 The Fibonacci Connection

Theorem: $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi$, where F_n is the Fibonacci sequence.

Proof:

The Fibonacci recurrence $F_{n+1} = F_n + F_{n-1}$ can be written as:

$$\frac{F_{n+1}}{F_n} = 1 + \frac{F_{n-1}}{F_n} = 1 + \frac{1}{F_n/F_{n-1}}$$

If the ratio converges to r, then:

$$r = 1 + \frac{1}{r}$$

$$r^2 = r + 1$$

This is the defining equation of φ. Since ratios are positive, $r = \varphi$. ■

D.5 Summary: The Hierarchy of Constants

Constant	Definition	Emergence Mechanism	Key Property
e	lim$(1+1/n)^n$	Continuous Bayesian update	$\frac{d}{dx}e^x = e^x$
pi	Period of rotation	Normalizing max-entropy dist.	$e^{2\pi i} = 1$
i	$e^{i\pi/2}$	Quarter rotation	$i^2 = -1$
-1	$e^{i\pi}$	Half rotation	Opposition
phi	$2\cos(\pi/5)$	Fifth-rotation + self-reference	$\varphi^2 = \varphi + 1$

All fundamental constants are combinations of e and pi at rational multiples of pi.

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« What we call learning is the universe finding its attractors. »

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