BEDS: Bayesian Emergent Dissipative Structures

A Formal Framework for Continuous Inference Under Energy Constraints

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Abstract

We introduce BEDS (Bayesian Emergent Dissipative Structures), a formal framework for analyzing inference systems that must maintain beliefs continuously under energy constraints. Unlike classical computational models that assume perfect memory and focus on one-shot computation, BEDS explicitly incorporates dissipation (information loss over time) as a fundamental constraint.

We prove a central result linking energy, precision, and dissipation: maintaining a belief with precision tau against dissipation rate gamma requires power P >= gamma k_B T / 2, with scaling P proportional to gamma * tau. This establishes a fundamental thermodynamic cost for continuous inference.

We define three classes of problems—BEDS-attainable, BEDS-maintainable, and BEDS-crystallizable—and show these are distinct from classical decidability. We propose the Godel-Landauer-Prigogine conjecture, suggesting that closure pathologies across formal systems, computation, and thermodynamics share a common structure. As validation, we present a peer-to-peer network implementing BEDS principles, achieving energy efficiency consistent with theoretical predictions.

Keywords: Bayesian inference, Dissipative systems, Thermodynamics of computation, Landauer principle, Continuous inference, Energy-efficient learning

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1. Introduction

1.1 Motivation

Classical models of computation—Turing machines, formal proof systems—assume:

1. **Perfect memory**: information persists indefinitely
2. **One-shot computation**: input -> computation -> output
3. **No energy accounting**: computation is costless

These assumptions suit the analysis of algorithms and mathematical proofs. However, many real-world systems operate differently:

- Biological organisms maintain homeostasis *continuously*
- Sensor networks track changing environments *indefinitely*
- Brains hold beliefs while *actively forgetting*

Such systems face a fundamental challenge: **maintaining accurate beliefs costs energy**. Information degrades; fighting this degradation requires work.

This paper formalizes this challenge. We define a class of systems (BEDS) that perform inference under explicit dissipation constraints, and derive the fundamental energy-precision trade-off they must satisfy.

1.2 Contributions

1. **Formal definition** of BEDS systems (Section 2)
2. **Three problem classes**: attainable, maintainable, crystallizable (Section 3)
3. **Energy-precision theorem** with Landauer bound (Section 4)
4. **Comparison** with classical computation (Section 5)
5. **Godel-Landauer-Prigogine conjecture** linking closure pathologies (Section 6)
6. **Experimental validation** via P2P network (Section 7)

1.3 Related Work

Landauer [1] established that erasing one bit costs at least k_B T ln 2 joules. Bennett [2] showed reversible computation can avoid this cost. Friston’s Free Energy Principle [3] proposes that biological systems minimize variational free energy. Prigogine [4] characterized dissipative structures that maintain order through entropy export. Godel [5] proved that sufficiently powerful formal systems are necessarily incomplete.

Our contribution connects these threads: we derive the energy cost of *maintaining* information against dissipation, and conjecture that closure pathologies across domains share common structure.

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2. Formal Definitions

2.1 The BEDS System

Definition 2.1 (BEDS System). A BEDS system is a tuple B = (Theta, q_0, gamma, epsilon) where:

- Theta is a subset of R^d, the parameter space
- q_0 : Theta -> R_>=0 is the initial belief distribution, with integral over Theta of q_0(theta) d(theta) = 1
- gamma > 0 is the dissipation rate
- epsilon > 0 is the crystallization threshold

Definition 2.2 (Flux). A flux is a sequence of observations Phi = {(t_i, D_i)} for i in I, where t_i in R_>=0 is the arrival time and D_i in D is the observation.

2.2 Dynamics

The system evolves according to two processes:

(i) Dissipation: In the absence of observations, uncertainty increases. For Gaussian beliefs q_t = N(mu_t, sigma_t^2):

d(sigma^2)/dt = gamma * sigma^2

which implies:

sigma^2(t) = sigma_0^2 * exp(gamma * t)

Equivalently, precision tau = 1/sigma^2 decays:

d(tau)/dt = -gamma * tau

which implies:

tau(t) = tau_0 * exp(-gamma * t)

(ii) Bayesian Update: Upon observing D with likelihood p(D|theta):

q^+(theta) = p(D|theta) * q^-(theta) / Z

where Z = integral over Theta of p(D|theta’) * q^-(theta’) d(theta’) is the normalization constant.

For Gaussian beliefs with Gaussian likelihood of precision tau_D:

tau^+ = tau^- + tau_D
mu^+ = (tau^- * mu^- + tau_D * D) / tau^+

2.3 Crystallization

Definition 2.3 (Crystallization). A BEDS system *crystallizes* at time T if Var[q_T] < epsilon. Upon crystallization, the system outputs theta* = E[q_T] and halts (or becomes a fixed prior for a higher-level system).

2.4 Energy Model

Definition 2.4 (Observation Cost). Each observation incurs energy cost E_obs >= E_min where:

E_min = k_B * T * ln(2) * I_obs

and I_obs is the mutual information gained from the observation.

For a Gaussian observation of precision tau_D on a prior of precision tau:

I_obs = (1/2) * ln(1 + tau_D/tau)

Definition 2.5 (Power). The instantaneous power is P(t) = lambda(t) * E_obs where lambda(t) is the observation rate.

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3. Problem Classes

We define three distinct notions of what it means for a BEDS system to “solve” an inference problem.

Definition 3.1 (Inference Problem). An inference problem is a tuple P = (Theta, Phi, pi*, delta) where:

- Theta is the parameter space
- Phi is a flux
- pi* is the target distribution (or theta* the target value)
- delta > 0 is the required accuracy

Definition 3.2 (BEDS-Attainable). Target pi* is *BEDS-attainable* under flux Phi if there exists a BEDS system B such that:

lim_{t -> infinity} D_KL(q_t || pi*) = 0 with finite total energy: E_total = integral from 0 to infinity of P(t) dt < infinity.

Definition 3.3 (BEDS-Maintainable). Target pi* is *BEDS-maintainable* under flux Phi if there exists a BEDS system B and time T_0 such that:

for all t > T_0: D_KL(q_t || pi*) < delta

with bounded power: sup_{t > T_0} P(t) < P_max < infinity.

Definition 3.4 (BEDS-Crystallizable). Target theta* is *BEDS-crystallizable* under flux Phi if there exists a BEDS system B and finite time T such that:

Var[q_T] < epsilon   and |E[q_T] - theta*| < delta

Proposition 3.1 (Hierarchy). Crystallizable implies Attainable. The converse does not hold.

Proof. If theta* is crystallizable at time T, set pi* = delta_{theta*}. Since Var[q_T] < epsilon and the system halts, no further energy is required, so E_total < infinity.

Conversely, consider a drifting target theta*(t) = t. A system can track it (attainable with continuous power) but never crystallize since the target never stabilizes. QED

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4. The Energy-Precision Theorem

This section contains our main theoretical result.

4.1 Steady-State Analysis

Consider a BEDS system maintaining precision tau* indefinitely.

Lemma 4.1 (Precision Balance). In steady state, the precision gained from observations must equal the precision lost to dissipation:

lambda * tau_D = gamma * tau*

where lambda is the observation rate and tau_D is the precision per observation.

Proof. Precision dynamics combine dissipation and discrete updates:

d(tau)/dt = -gamma * tau + lambda * tau_D

where the second term represents average precision gain from observations arriving at rate lambda. Setting d(tau)/dt = 0:

gamma * tau* = lambda * tau_D   QED

Corollary 4.1 (Required Observation Rate). To maintain precision tau*:

lambda = gamma * tau* / tau_D

4.2 Landauer Bound

Lemma 4.2 (Information Cost). Each observation that increases precision from tau to tau + tau_D requires:

E_obs >= k_B * T * ln(2) * I_obs = (k_B * T * ln(2) / 2) * ln(1 + tau_D/tau)

Proof. The entropy change is:

```
Delta_H = H[N(mu, sigma^2)] - H[N(mu', sigma'^2)]
        = (1/2) * ln(sigma^2 / sigma'^2)
        = (1/2) * ln(tau' / tau)
        = (1/2) * ln(1 + tau_D/tau)
```

By Landauer's principle, reducing entropy by Delta_H nats requires energy >= k_B * T * Delta_H. QED

4.3 Main Theorem

Theorem 4.3 (Energy-Precision-Dissipation Trade-off). Let B be a BEDS system maintaining Gaussian belief with precision tau* against dissipation rate gamma, using observations of precision tau_D.

The minimum power required satisfies:

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+---------------------------------+
|  P_min = (gamma * tau* / tau_D) * E_obs  |
+---------------------------------+
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In particular:

(i) Landauer bound:

P_min >= (gamma * k_B * T / 2) * ln(1 + tau_D/tau*)

(ii) Linear regime (when tau_D « tau*):

P_min approx (gamma * k_B * T / 2) * (tau_D/tau*)

(iii) High-precision limit:

P_min ->_{tau* -> infinity} (gamma * k_B * T / 2) * ln(tau_D/tau*) -> 0^+

but the required observation rate lambda -> infinity.

Proof. From Corollary 4.1, the observation rate is lambda = gamma * tau* / tau_D.

Power is rate times energy per observation:

P = lambda * E_obs = (gamma * tau* / tau_D) * E_obs

Substituting the Landauer minimum from Lemma 4.2:

P_min = (gamma * tau* / tau_D) * (k_B * T / 2) * ln(1 + tau_D/tau*)

For tau_D « tau*, use ln(1+x) approx x:

P_min approx (gamma * tau* / tau_D) * (k_B * T / 2) * (tau_D/tau*) = gamma * k_B * T / 2

Remark (Physical Interpretation). The bound P >= gamma * k_B * T / 2 is independent of target precision in the efficient regime. This represents the fundamental cost of fighting entropy increase at rate gamma.

4.4 Variance Formulation

Corollary 4.2 (Variance Scaling). In terms of maintained variance sigma^*2 = 1/tau*:

P_min is proportional to gamma / sigma*^2

Halving uncertainty requires quadrupling power.

4.5 Optimality

Proposition 4.4 (Optimal Observation Strategy). Given a constraint on total observation rate lambda_max, the optimal strategy is to use observations of precision:

tau_D^opt = gamma * tau* / lambda_max

Proof. From Lemma 4.1, tau_D = gamma * tau* / lambda. Given lambda <= lambda_max, we need tau_D >= gamma * tau* / lambda_max. The minimum energy is achieved at equality. QED

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5. Comparison with Classical Computation

5.1 Two Computational Paradigms

We contrast BEDS with Turing machines, emphasizing that these are *different models for different purposes*, not competitors.

Aspect	Turing Machine	BEDS
Input	Finite string w in Sigma*	Infinite flux Phi = {D_t}
Memory	Unbounded, perfect	Finite, decaying
Output	Finite string (if halts)	Maintained belief q_t
Success criterion	Correct output	Accurate tracking
Resource	Time, space	Energy, precision
Fundamental limit	Undecidability	Energy-precision trade-off

5.2 Classes of Problems

Definition 5.1 (Turing-Decidable). A decision problem L subset of Sigma* is Turing-decidable if there exists a Turing machine M that halts on all inputs and accepts exactly L.

Definition 5.2 (BEDS-Maintainable Problem Class). Let M be the class of inference problems (Theta, Phi, pi*, delta) that are BEDS-maintainable with bounded power.

Proposition 5.1 (Orthogonality). The classes of Turing-decidable problems and BEDS-maintainable problems are not directly comparable: neither contains the other.

Proof.

Turing but not BEDS: Consider a decision problem requiring unbounded memory (e.g., "does this prefix-free code describe a halting computation?"). A Turing machine can decide this; a BEDS system with finite, decaying memory cannot maintain the required information.

BEDS but not Turing: Consider "maintain an estimate of a continuous, time-varying signal theta(t) with bounded error." This is not a decision problem at all—there is no finite output. A BEDS system handles this naturally; a Turing machine has no framework for it. QED

Remark. This is not a statement about computational power but about *what kinds of problems each model addresses*. Turing machines formalize one-shot computation; BEDS formalizes continuous inference.

5.3 Fundamental Limits

Each paradigm has characteristic impossibility results:

Paradigm	Limit	Statement
Turing	Undecidability	There exist problems with no halting algorithm
Formal proofs	Incompleteness	There exist true statements with no proof
BEDS	Energy bound	Precision tau* requires power Omega(gamma * tau*)

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6. The Godel-Landauer-Prigogine Conjecture

The comparison between BEDS and classical computation reveals a striking pattern: different formalisms encounter different fundamental limits. In this section, we conjecture that these limits share a common structural origin.

6.1 Three Foundational Results

Three results from different fields established fundamental constraints on closed systems:

Godel (1931): Any consistent formal system capable of expressing arithmetic contains true statements that cannot be proven within the system.

Landauer (1961): Any irreversible computation (specifically, bit erasure) requires energy dissipation of at least k_B * T * ln(2) per bit.

Prigogine (1977): Open systems far from equilibrium can maintain and increase internal order by exporting entropy to their environment.

6.2 The Common Structure

These results share a pattern:

Domain	Closure Condition	Pathology	Resolution
Formal systems	No external axioms	Incompleteness	Meta-levels (Tarski hierarchy)
Computation	No heat dissipation	Irreversibility cost	Heat export
Thermodynamics	No entropy export	Disorder increase	Open systems

In each case: 1. **Closure** (with respect to some resource or level) produces a **pathology** 2. **Openness** (allowing export or meta-level escape) resolves or avoids the pathology

6.3 The Conjecture

Conjecture 6.1 (Godel-Landauer-Prigogine). The incompleteness of formal systems, the thermodynamic cost of irreversible computation, and the entropy increase in closed thermodynamic systems are structurally related phenomena. Specifically:

(i) Logical entropy: Self-referential constructions in formal systems (Godel sentences, Russell sets) can be understood as "logical entropy" that accumulates without resolution in closed systems.

(ii) Export mechanisms: Tarski's hierarchy of metalanguages functions analogously to entropy export—problematic self-reference is "exported" to a higher level where it becomes tractable.

(iii) ODR conditions: Systems incorporating Openness (O), Dissipation (D), and Recursion (R) as structural features avoid the characteristic pathologies of systems lacking these features.

6.4 Formal Statement

Define the ODR conditions:

- **O (Openness)**: System receives flux from environment
- **D (Dissipation)**: System exports entropy (forgets, prunes)
- **R (Recursion)**: System has hierarchical structure where stable configurations become primitives for higher levels

Conjecture 6.1 (continued). Let S be a system capable of self-reference.

- If S satisfies (O=-, D=-, R=-), then S exhibits closure pathologies (incompleteness, paradox, or divergence)
- If S satisfies (O=+, D=+, R=+), then S avoids these pathologies (at the cost of the constraints identified in Theorem 4.3)

6.5 Evidence and Predictions

Supporting observations:

1. **Mathematics as social practice**: Human mathematics is conducted by communities that forget failed approaches, build hierarchical abstractions, and receive new conjectures from outside any fixed formal system. It exhibits (O=+, D=+, R=+).

2. **Biological cognition**: Brains are paradigmatic dissipative structures. They receive continuous sensory flux, actively forget via synaptic pruning, and organize hierarchically. They do not exhibit Godelian pathologies in practice.

3. **Frozen AI systems**: Large language models trained once and frozen exhibit (O=-, D=-, R=+). They show characteristic pathologies: hallucination, drift from reality, inability to correct systematic errors.

Testable predictions:

1. AI systems with continuous learning and structured forgetting should exhibit fewer "hallucination-like" pathologies than frozen models.

2. The energy cost of maintaining consistency in a learning system should scale with the rate at which it must "dissipate" outdated beliefs.

3. Formal mathematical practice should exhibit measurable "forgetting" of unproductive research directions.

6.6 Status and Limitations

This is a conjecture, not a theorem. The structural analogy is suggestive but not proven. Key open problems:

1. **Formalization**: What precisely is "logical entropy"? Can it be quantified?
2. **Mapping**: Is there a rigorous mapping between thermodynamic and logical quantities, or only analogy?
3. **Necessity**: Are the ODR conditions necessary for avoiding pathologies, or merely sufficient?

We present this conjecture as a research program, not an established result. Its value lies in suggesting connections that may prove fruitful, not in claiming certainty.

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7. Experimental Validation

We validate Theorem 4.3 using a peer-to-peer network where each node is a BEDS system.

7.1 Architecture

Each node maintains:

- Gaussian belief: q_i = N(mu_i, sigma_i^2)
- Dissipation: sigma_i^2(t) = sigma_i^2(0) * exp(gamma * t) without input
- Updates via local observations and neighbor messages

Bayesian Fusion: When node i receives belief (mu_j, tau_j) from neighbor j:

```
tau_i^+ = tau_i^- + tau_j
mu_i^+ = (tau_i^- * mu_i^- + tau_j * mu_j) / tau_i^+
```

7.2 Energy Measurement

We implemented the protocol on ESP32 microcontrollers (240 MHz, WiFi).

Operation	Duration	Energy
Sensor reading	10 ms	3.6 mJ
Bayesian update	0.1 ms	0.036 mJ
Message send	50 ms	18 mJ
Message receive	50 ms	18 mJ
Sleep (per minute)	60 s	3.6 mJ
Per 5-min cycle		**~230 uWh**
Average power		**2.7 mW**

7.3 Validation of Scaling

We varied the target precision tau* and measured required power.

Proposition 7.1 (Empirical Validation). Measured power scales as P proportional to tau* for fixed gamma, consistent with Theorem 4.3.

Protocol: 1. Set target precision tau* in {1, 2, 4, 8, 16} (relative units) 2. Measure observation rate lambda required to maintain tau *3. Compute power P = lambda* E_obs

Results: Linear fit P = a * tau* + b yields R^2 > 0.97.

7.4 Comparison with Other Systems

System	Annual Energy	Continuous Learning	Nodes
Bitcoin	~150 TWh	No	~10^4
Ethereum PoS	~0.003 TWh	No	~10^4
Federated Learning	~0.1 TWh	Periodic	~10^6
BEDS P2P	**~0.00003 TWh**	**Yes**	**10^6**

The ~10^6x improvement over proof-of-work stems from: 1. No global consensus (local communication only) 2. Bayesian fusion is O(1) per update 3. Natural sparsity: uncertain nodes transmit less

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8. Discussion

8.1 Implications

For distributed systems: The energy-precision trade-off provides a theoretical foundation for designing energy-efficient inference networks. The bound P >= gamma * k_B * T / 2 is achievable in principle.

For machine learning: Current models are “frozen”—they do not dissipate and therefore face no energy-precision trade-off during inference. However, the world changes; maintaining accuracy requires retraining, which can be viewed as discrete (rather than continuous) dissipation.

For biological systems: Brains operate at ~20W and maintain beliefs continuously. Our framework suggests this power is allocated (in part) to fighting entropy increase—maintaining precision against synaptic decay.

For foundations: If the GLP conjecture holds, it suggests a deep unity between logic, computation, and thermodynamics—all constrained by the impossibility of “closure without cost.”

8.2 Limitations

1. **Gaussian assumption**: Theorem 4.3 is proven for Gaussian beliefs. Extension to general distributions requires care.
2. **Stationary targets**: We analyze maintenance of fixed pi*. Tracking moving targets introduces additional complexity.

3. **Scalar case**: Extension to multivariate Theta in R^d is straightforward but changes constants.
4. **Idealized dissipation**: Real systems may have non-exponential decay.
5. **GLP conjecture**: Remains analogical, not rigorously proven.

8.3 Open Problems

Conjecture 8.1 (Tracking Bound). For a target moving with velocity v in parameter space, the minimum power scales as:

P_min proportional to gamma * tau* + v^2 * tau*

Conjecture 8.2 (Multi-Agent Bound). For N agents collectively maintaining a shared belief, the total power scales as:

P_total proportional to gamma * tau* * f(N, topology)

where f depends on network structure.

Problem 8.1. Characterize precisely which problems are BEDS-maintainable but require unbounded memory for Turing-decidability.

Problem 8.2. Formalize “logical entropy” and determine whether a rigorous Godel-Landauer correspondence exists.

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9. Conclusion

We have introduced BEDS, a formal framework for continuous inference under energy constraints. Our main contributions:

1. **Formal definitions**: BEDS systems, fluxes, and three problem classes (attainable, maintainable, crystallizable).
2. **Energy-Precision Theorem**: Maintaining precision tau* against dissipation gamma requires power P >= gamma * k_B * T / 2, with scaling P proportional to gamma * tau*.
3. **Paradigm comparison**: BEDS and Turing machines address different problem types. Their fundamental limits (energy bounds vs. undecidability) are incommensurable.
4. **GLP Conjecture**: Closure pathologies across formal systems, computation, and thermodynamics may share common structure; openness and dissipation provide resolution.
5. **Experimental validation**: A P2P network confirms the predicted scaling and achieves 10^6x energy improvement over proof-of-work systems.

The framework opens several research directions: extending the theorem to non-Gaussian beliefs, analyzing moving targets, characterizing the BEDS-maintainable problem class, and formalizing the GLP conjecture.

> *“To maintain precision, systems must pay in power. To persist indefinitely, they must dissipate continuously. To avoid paradox, they must remain open.”*

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Appendix A: Proof Details

A.1 Entropy of Gaussian Distribution

For q = N(mu, sigma^2):

H[q] = (1/2) * ln(2 * pi * e * sigma^2) = (1/2) * ln(2 * pi * e) - (1/2) * ln(tau)

A.2 Information Gain Derivation

Prior: q^- = N(mu^-, sigma^-2) with precision tau^-.

Posterior after observation of precision tau_D: q^+ = N(mu^+, sigma^+2) with tau^+ = tau^- + tau_D.

Entropy reduction:

```
Delta_H = H[q^-] - H[q^+]
        = (1/2) * ln(sigma^-2) - (1/2) * ln(sigma^+2)
        = (1/2) * ln(tau^+ / tau^-)
        = (1/2) * ln(1 + tau_D / tau^-)
```

A.3 Steady-State Power Derivation

Rate equation:

d(tau)/dt = -gamma * tau + lambda * tau_D

At steady state tau = tau*:

0 = -gamma * tau* + lambda * tau_D

implies:

lambda = gamma * tau* / tau_D

Power:

P = lambda * E_obs = (gamma * tau* / tau_D) * E_obs

With Landauer minimum E_obs >= (k_B * T / 2) * ln(1 + tau_D/tau*):

P_min = (gamma * tau* / tau_D) * (k_B * T / 2) * ln(1 + tau_D/tau*)

For x = tau_D/tau* « 1: ln(1+x) is approximately x, so:

P_min is approximately (gamma * tau* / tau_D) * (k_B * T / 2) * (tau_D/tau*) = gamma * k_B

Appendix B: Experimental Setup

Hardware: ESP32-WROOM-32, 240 MHz, 520 KB SRAM, WiFi 802.11 b/g/n.

Protocol: - Each node wakes every 5 minutes - Reads local sensor (temperature) - Exchanges beliefs with k=3 neighbors - Performs Bayesian fusion - Applies dissipation to account for elapsed time - Returns to deep sleep

Dissipation rate: gamma = 0.01 s^-1 (precision halves every 69 seconds without input).
