

# BEDS: Bayesian Emergent Dissipative Structures

A Formal Framework for Continuous Inference Under Energy Constraints

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## Abstract

We introduce BEDS (Bayesian Emergent Dissipative Structures), a formal framework for analyzing inference systems that must maintain beliefs continuously under energy constraints. Unlike classical computational models that assume perfect memory and focus on one-shot computation, BEDS explicitly incorporates dissipation (information loss over time) as a fundamental constraint.

We prove a central result linking energy, precision, and dissipation: maintaining a belief with precision  $\tau$  against dissipation rate  $\gamma$  requires power  $P \geq \gamma k_B T / 2$ , with scaling  $P \propto \gamma \cdot \tau$ . This establishes a fundamental thermodynamic cost for continuous inference.

We define three classes of problems—BEDS-attainable, BEDS-maintainable, and BEDS-crystallizable—and show these are distinct from classical decidability.

We propose here, to the best of our knowledge, the first explicit Gödel-Landauer-Prigogine conjecture, suggesting that closure pathologies across formal systems, computation, and thermodynamics share a common structure.

**Keywords:** Bayesian inference, Dissipative systems, Thermodynamics of computation, Landauer principle, Continuous inference, Energy-efficient learning

## 1 Introduction

### 1.1 Motivation

Classical models of computation—Turing machines, formal proof systems—assume:

- (1) **Perfect memory**: information persists indefinitely
- (2) **One-shot computation**: input → computation → output
- (3) **No energy accounting**: computation is costless

These assumptions suit the analysis of algorithms and mathematical proofs. However, many real-world systems operate differently:

- Biological organisms maintain homeostasis *continuously*
- Sensor networks track changing environments *indefinitely*
- Brains hold beliefs while *actively forgetting*

Such systems face a fundamental challenge: **maintaining accurate beliefs costs energy**. Information degrades; fighting this degradation requires work.

This paper formalizes this challenge. We define a class of systems (BEDS) that perform inference under explicit dissipation constraints, and derive the fundamental energy-precision trade-off they must satisfy.

## 1.2 Contributions

- (1) **Formal definition** of BEDS systems (Section 2)
- (2) **Three problem classes:** attainable, maintainable, crystallizable (Section 3)
- (3) **Energy-precision theorem** with Landauer bound (Section 4)
- (4) **Comparison** with classical computation (Section 5)
- (5) **Gödel-Landauer-Prigogine conjecture** linking closure pathologies (Section 6)

## 1.3 Related Work

Landauer [1] established that erasing one bit costs at least  $k_B T \ln 2$  joules. Bennett [2] showed reversible computation can avoid this cost. Friston's Free Energy Principle [3] proposes that biological systems minimize variational free energy. Prigogine [4] characterized dissipative structures that maintain order through entropy export. Gödel [5] proved that sufficiently powerful formal systems are necessarily incomplete.

Our contribution connects these threads: we derive the energy cost of *maintaining* information against dissipation, and conjecture that closure pathologies across domains share common structure.

## 2 Formal Definitions

### 2.1 The BEDS System

**Definition 2.1** (BEDS System). A BEDS system is a tuple  $\mathcal{B} = (\Theta, q_0, \gamma, \varepsilon)$  where:

- $\Theta \subseteq \mathbb{R}^d$  is the parameter space
- $q_0 : \Theta \rightarrow \mathbb{R}_{\geq 0}$  is the initial belief distribution, with  $\int_{\Theta} q_0(\theta) d\theta = 1$
- $\gamma > 0$  is the dissipation rate
- $\varepsilon > 0$  is the crystallization threshold

**Definition 2.2** (Flux). A flux is a sequence of observations  $\Phi = \{(t_i, D_i)\}_{i \in I}$ , where  $t_i \in \mathbb{R}_{\geq 0}$  is the arrival time and  $D_i \in \mathcal{D}$  is the observation.

### 2.2 Dynamics

The system evolves according to two processes:

**(i) Dissipation.** In the absence of observations, uncertainty increases. For Gaussian beliefs  $q_t = \mathcal{N}(\mu_t, \sigma_t^2)$ :

$$\frac{d\sigma^2}{dt} = \gamma \cdot \sigma^2 \quad (1)$$

which implies:

$$\sigma^2(t) = \sigma_0^2 \cdot e^{\gamma t} \quad (2)$$

Equivalently, precision  $\tau = 1/\sigma^2$  decays:

$$\frac{d\tau}{dt} = -\gamma\tau \implies \tau(t) = \tau_0 \cdot e^{-\gamma t} \quad (3)$$

(ii) **Bayesian Update.** Upon observing  $D$  with likelihood  $p(D|\theta)$ :

$$q^+(\theta) = \frac{p(D|\theta) \cdot q^-(\theta)}{Z} \quad (4)$$

where  $Z = \int_{\Theta} p(D|\theta') \cdot q^-(\theta') d\theta'$  is the normalization constant.

For Gaussian beliefs with Gaussian likelihood of precision  $\tau_D$ :

$$\tau^+ = \tau^- + \tau_D \quad (5)$$

$$\mu^+ = \frac{\tau^- \mu^- + \tau_D D}{\tau^+} \quad (6)$$

## 2.3 Crystallization

**Definition 2.3** (Crystallization). A BEDS system *crystallizes* at time  $T$  if  $\text{Var}[q_T] < \varepsilon$ . Upon crystallization, the system outputs  $\theta^* = \mathbb{E}[q_T]$  and halts (or becomes a fixed prior for a higher-level system).

## 2.4 Energy Model

**Definition 2.4** (Observation Cost). Each observation incurs energy cost  $E_{\text{obs}} \geq E_{\min}$  where:

$$E_{\min} = k_B T \ln(2) \cdot I_{\text{obs}} \quad (7)$$

and  $I_{\text{obs}}$  is the mutual information gained from the observation.

For a Gaussian observation of precision  $\tau_D$  on a prior of precision  $\tau$ :

$$I_{\text{obs}} = \frac{1}{2} \ln \left( 1 + \frac{\tau_D}{\tau} \right) \quad (8)$$

**Definition 2.5** (Power). The instantaneous power is  $P(t) = \lambda(t) \cdot E_{\text{obs}}$  where  $\lambda(t)$  is the observation rate.

## 3 Problem Classes

We define three distinct notions of what it means for a BEDS system to “solve” an inference problem.

**Definition 3.1** (Inference Problem). An inference problem is a tuple  $\mathcal{P} = (\Theta, \Phi, \pi^*, \delta)$  where:

- $\Theta$  is the parameter space
- $\Phi$  is a flux
- $\pi^*$  is the target distribution (or  $\theta^*$  the target value)
- $\delta > 0$  is the required accuracy

**Definition 3.2** (BEDS-Attainable). Target  $\pi^*$  is *BEDS-attainable* under flux  $\Phi$  if there exists a BEDS system  $\mathcal{B}$  such that:

$$\lim_{t \rightarrow \infty} D_{\text{KL}}(q_t \parallel \pi^*) = 0 \quad (9)$$

with finite total energy:  $E_{\text{total}} = \int_0^\infty P(t) dt < \infty$ .

**Definition 3.3** (BEDS-Maintainable). Target  $\pi^*$  is *BEDS-maintainable* under flux  $\Phi$  if there exists a BEDS system  $\mathcal{B}$  and time  $T_0$  such that:

$$\forall t > T_0 : D_{\text{KL}}(q_t \parallel \pi^*) < \delta \quad (10)$$

with bounded power:  $\sup_{t > T_0} P(t) < P_{\max} < \infty$ .

**Definition 3.4** (BEDS-Crystallizable). Target  $\theta^*$  is *BEDS-crystallizable* under flux  $\Phi$  if there exists a BEDS system  $\mathcal{B}$  and finite time  $T$  such that:

$$\text{Var}[q_T] < \varepsilon \quad \text{and} \quad |\mathbb{E}[q_T] - \theta^*| < \delta \quad (11)$$

**Proposition 3.5** (Hierarchy). *Crystallizable implies Attainable. The converse does not hold.*

*Proof.* If  $\theta^*$  is crystallizable at time  $T$ , set  $\pi^* = \delta_{\theta^*}$ . Since  $\text{Var}[q_T] < \varepsilon$  and the system halts, no further energy is required, so  $E_{\text{total}} < \infty$ .

Conversely, consider a drifting target  $\theta^*(t) = t$ . A system can track it (attainable with continuous power) but never crystallize since the target never stabilizes.  $\square$

## 4 The Energy-Precision Theorem

This section contains our main theoretical result.

### 4.1 Steady-State Analysis

Consider a BEDS system maintaining precision  $\tau^*$  indefinitely.

**Lemma 4.1** (Precision Balance). *In steady state, the precision gained from observations must equal the precision lost to dissipation:*

$$\lambda \cdot \tau_D = \gamma \cdot \tau^* \quad (12)$$

where  $\lambda$  is the observation rate and  $\tau_D$  is the precision per observation.

*Proof.* Precision dynamics combine dissipation and discrete updates:

$$\frac{d\tau}{dt} = -\gamma\tau + \lambda\tau_D \quad (13)$$

where the second term represents average precision gain from observations arriving at rate  $\lambda$ . Setting  $d\tau/dt = 0$ :

$$\gamma\tau^* = \lambda\tau_D \quad (14)$$

$\square$

**Corollary 4.2** (Required Observation Rate). *To maintain precision  $\tau^*$ :*

$$\lambda = \frac{\gamma\tau^*}{\tau_D} \quad (15)$$

### 4.2 Landauer Bound

**Lemma 4.3** (Information Cost). *Each observation that increases precision from  $\tau$  to  $\tau + \tau_D$  requires:*

$$E_{\text{obs}} \geq k_B T \ln(2) \cdot I_{\text{obs}} = \frac{k_B T \ln(2)}{2} \ln \left( 1 + \frac{\tau_D}{\tau} \right) \quad (16)$$

*Proof.* The entropy change is:

$$\Delta H = H[\mathcal{N}(\mu, \sigma^2)] - H[\mathcal{N}(\mu', \sigma'^2)] = \frac{1}{2} \ln \frac{\sigma^2}{\sigma'^2} = \frac{1}{2} \ln \frac{\tau'}{\tau} = \frac{1}{2} \ln \left( 1 + \frac{\tau_D}{\tau} \right) \quad (17)$$

By Landauer's principle, reducing entropy by  $\Delta H$  nats requires energy  $\geq k_B T \cdot \Delta H$ .  $\square$

### 4.3 Main Theorem

**Theorem 4.4** (Energy-Precision-Dissipation Trade-off). *Let  $\mathcal{B}$  be a BEDS system maintaining Gaussian belief with precision  $\tau^*$  against dissipation rate  $\gamma$ , using observations of precision  $\tau_D$ .*

*The minimum power required satisfies:*

$$\boxed{P_{\min} = \frac{\gamma\tau^*}{\tau_D} \cdot E_{\text{obs}}} \quad (18)$$

*In particular:*

*(i) Landauer bound:*

$$P_{\min} \geq \frac{\gamma k_B T}{2} \ln \left( 1 + \frac{\tau_D}{\tau^*} \right) \quad (19)$$

*(ii) Linear regime (when  $\tau_D \ll \tau^*$ ):*

$$P_{\min} \approx \frac{\gamma k_B T}{2} \cdot \frac{\tau_D}{\tau^*} \quad (20)$$

*(iii) High-precision limit:*

$$P_{\min} \xrightarrow{\tau^* \rightarrow \infty} \frac{\gamma k_B T}{2} \ln \frac{\tau_D}{\tau^*} \rightarrow 0^+ \quad (21)$$

but the required observation rate  $\lambda \rightarrow \infty$ .

*Proof.* From Corollary 4.1, the observation rate is  $\lambda = \gamma\tau^*/\tau_D$ .

Power is rate times energy per observation:

$$P = \lambda \cdot E_{\text{obs}} = \frac{\gamma\tau^*}{\tau_D} \cdot E_{\text{obs}} \quad (22)$$

Substituting the Landauer minimum from Lemma 4.2:

$$P_{\min} = \frac{\gamma\tau^*}{\tau_D} \cdot \frac{k_B T}{2} \ln \left( 1 + \frac{\tau_D}{\tau^*} \right) \quad (23)$$

For  $\tau_D \ll \tau^*$ , use  $\ln(1+x) \approx x$ :

$$P_{\min} \approx \frac{\gamma\tau^*}{\tau_D} \cdot \frac{k_B T}{2} \cdot \frac{\tau_D}{\tau^*} = \frac{\gamma k_B T}{2} \quad (24)$$

□

*Remark 4.5* (Physical Interpretation). The bound  $P \geq \gamma k_B T / 2$  is independent of target precision in the efficient regime. This represents the fundamental cost of fighting entropy increase at rate  $\gamma$ .

### 4.4 Variance Formulation

**Corollary 4.6** (Variance Scaling). *In terms of maintained variance  $\sigma^{*2} = 1/\tau^*$ :*

$$P_{\min} \propto \frac{\gamma}{\sigma^{*2}} \quad (25)$$

*Halving uncertainty requires quadrupling power.*

## 4.5 Optimality

**Proposition 4.7** (Optimal Observation Strategy). *Given a constraint on total observation rate  $\lambda_{\max}$ , the optimal strategy is to use observations of precision:*

$$\tau_D^{opt} = \frac{\gamma\tau^*}{\lambda_{\max}} \quad (26)$$

*Proof.* From Lemma 4.1,  $\tau_D = \gamma\tau^*/\lambda$ . Given  $\lambda \leq \lambda_{\max}$ , we need  $\tau_D \geq \gamma\tau^*/\lambda_{\max}$ . The minimum energy is achieved at equality.  $\square$

## 5 Comparison with Classical Computation

### 5.1 Two Computational Paradigms

We contrast BEDS with Turing machines, emphasizing that these are *different models for different purposes*, not competitors.

Aspect	Turing Machine	BEDS
Input	Finite string $w \in \Sigma^*$	Infinite flux $\Phi = \{D_t\}$
Memory	Unbounded, perfect	Finite, decaying
Output	Finite string (if halts)	Maintained belief $q_t$
Success criterion	Correct output	Accurate tracking
Resource	Time, space	Energy, precision
Fundamental limit	Undecidability	Energy-precision trade-off

### 5.2 Classes of Problems

**Definition 5.1** (Turing-Decidable). A decision problem  $L \subseteq \Sigma^*$  is Turing-decidable if there exists a Turing machine  $M$  that halts on all inputs and accepts exactly  $L$ .

**Definition 5.2** (BEDS-Maintainable Problem Class). Let  $\mathcal{M}$  be the class of inference problems  $(\Theta, \Phi, \pi^*, \delta)$  that are BEDS-maintainable with bounded power.

**Proposition 5.3** (Orthogonality). *The classes of Turing-decidable problems and BEDS-maintainable problems are not directly comparable: neither contains the other.*

*Proof. Turing but not BEDS:* Consider a decision problem requiring unbounded memory (e.g., “does this prefix-free code describe a halting computation?”). A Turing machine can decide this; a BEDS system with finite, decaying memory cannot maintain the required information.

**BEDS but not Turing:** Consider “maintain an estimate of a continuous, time-varying signal  $\theta(t)$  with bounded error.” This is not a decision problem at all—there is no finite output. A BEDS system handles this naturally; a Turing machine has no framework for it.  $\square$

*Remark 5.4.* This is not a statement about computational power but about *what kinds of problems each model addresses*. Turing machines formalize one-shot computation; BEDS formalizes continuous inference.

### 5.3 Fundamental Limits

Each paradigm has characteristic impossibility results:

Paradigm	Limit	Statement
Turing	Undecidability	There exist problems with no halting algorithm
Formal proofs	Incompleteness	There exist true statements with no proof
BEDS	Energy bound	Precision $\tau^*$ requires power $\Omega(\gamma\tau^*)$

## 6 The Gödel-Landauer-Prigogine Conjecture

The comparison between BEDS and classical computation reveals a striking pattern: different formalisms encounter different fundamental limits. In this section, we conjecture that these limits share a common structural origin.

### 6.1 Three Foundational Results

Three results from different fields established fundamental constraints on closed systems:

**Gödel (1931):** Any consistent formal system capable of expressing arithmetic contains true statements that cannot be proven within the system.

**Landauer (1961):** Any irreversible computation (specifically, bit erasure) requires energy dissipation of at least  $k_B T \ln(2)$  per bit.

**Prigogine (1977):** Open systems far from equilibrium can maintain and increase internal order by exporting entropy to their environment.

### 6.2 The Common Structure

These results share a pattern:

Domain	Closure Condition	Pathology	Resolution
Formal systems	No external axioms	Incompleteness	Meta-levels (Tarski hierarchy)
Computation	No heat dissipation	Irreversibility cost	Heat export
Thermodynamics	No entropy export	Disorder increase	Open systems

In each case:

- (1) **Closure** (with respect to some resource or level) produces a **pathology**
- (2) **Openness** (allowing export or meta-level escape) resolves or avoids the pathology

### 6.3 The Conjecture

**Conjecture 6.1** (Gödel-Landauer-Prigogine). *The incompleteness of formal systems, the thermodynamic cost of irreversible computation, and the entropy increase in closed thermodynamic systems are structurally related phenomena. Specifically:*

*(i) Logical entropy:* Self-referential constructions in formal systems (Gödel sentences, Russell sets) can be understood as “logical entropy” that accumulates without resolution in closed systems.

*(ii) Export mechanisms:* Tarski’s hierarchy of metalanguages functions analogously to entropy export—problematic self-reference is “exported” to a higher level where it becomes tractable.

*(iii) ODR conditions:* Systems incorporating Openness ( $O$ ), Dissipation ( $D$ ), and Recursion ( $R$ ) as structural features avoid the characteristic pathologies of systems lacking these features.

### 6.4 Formal Statement

Define the ODR conditions:

- **O (Openness):** System receives flux from environment
- **D (Dissipation):** System exports entropy (forgets, prunes)

- **R (Recursion):** System has hierarchical structure where stable configurations become primitives for higher levels

**Conjecture 6.2** (continued). *Let  $\mathcal{S}$  be a system capable of self-reference.*

- *If  $\mathcal{S}$  satisfies  $(O=-, D=-, R=-)$ , then  $\mathcal{S}$  exhibits closure pathologies (incompleteness, paradox, or divergence)*
- *If  $\mathcal{S}$  satisfies  $(O=+, D=+, R=+)$ , then  $\mathcal{S}$  avoids these pathologies (at the cost of the constraints identified in Theorem 4.4)*

## 6.5 Evidence and Predictions

**Supporting observations:**

- (1) **Mathematics as social practice:** Human mathematics is conducted by communities that forget failed approaches, build hierarchical abstractions, and receive new conjectures from outside any fixed formal system. It exhibits  $(O=+, D=+, R=+)$ .
- (2) **Biological cognition:** Brains are paradigmatic dissipative structures. They receive continuous sensory flux, actively forget via synaptic pruning, and organize hierarchically. They do not exhibit Gödelian pathologies in practice.
- (3) **Frozen AI systems:** Large language models trained once and frozen exhibit  $(O=-, D=-, R=+)$ . They show characteristic pathologies: hallucination, drift from reality, inability to correct systematic errors.

**Testable predictions:**

- (1) AI systems with continuous learning and structured forgetting should exhibit fewer “hallucination-like” pathologies than frozen models.
- (2) The energy cost of maintaining consistency in a learning system should scale with the rate at which it must “dissipate” outdated beliefs.
- (3) Formal mathematical practice should exhibit measurable “forgetting” of unproductive research directions.

## 6.6 Status and Limitations

**This is a conjecture, not a theorem.** The structural analogy is suggestive but not proven. Key open problems:

- (1) **Formalization:** What precisely is “logical entropy”? Can it be quantified?
- (2) **Mapping:** Is there a rigorous mapping between thermodynamic and logical quantities, or only analogy?
- (3) **Necessity:** Are the ODR conditions necessary for avoiding pathologies, or merely sufficient?

We present this conjecture as a research program, not an established result. Its value lies in suggesting connections that may prove fruitful, not in claiming certainty.

## 7 Discussion

### 7.1 Implications

**For distributed systems:** The energy-precision trade-off provides a theoretical foundation for designing energy-efficient inference networks. The bound  $P \geq \gamma k_B T / 2$  is achievable in principle, and preliminary implementations on low-power embedded systems confirm the predicted scaling laws.

**For machine learning:** Current models are “frozen”—they do not dissipate and therefore face no energy-precision trade-off during inference. However, the world changes; maintaining accuracy requires retraining, which can be viewed as discrete (rather than continuous) dissipation.

**For biological systems:** Brains operate at  $\sim 20\text{W}$  and maintain beliefs continuously. Our framework suggests this power is allocated (in part) to fighting entropy increase—maintaining precision against synaptic decay.

**For foundations:** If the GLP conjecture holds, it suggests a deep unity between logic, computation, and thermodynamics—all constrained by the impossibility of “closure without cost.”

### 7.2 Limitations

- (1) **Gaussian assumption:** Theorem 4.4 is proven for Gaussian beliefs. Extension to general distributions requires care.
- (2) **Stationary targets:** We analyze maintenance of fixed  $\pi^*$ . Tracking moving targets introduces additional complexity.
- (3) **Scalar case:** Extension to multivariate  $\Theta \in \mathbb{R}^d$  is straightforward but changes constants.
- (4) **Idealized dissipation:** Real systems may have non-exponential decay.
- (5) **GLP conjecture:** Remains analogical, not rigorously proven.

### 7.3 Open Problems

**Conjecture 7.1** (Tracking Bound). *For a target moving with velocity  $v$  in parameter space, the minimum power scales as:*

$$P_{\min} \propto \gamma \tau^* + v^2 \tau^* \quad (27)$$

**Conjecture 7.2** (Multi-Agent Bound). *For  $N$  agents collectively maintaining a shared belief, the total power scales as:*

$$P_{\text{total}} \propto \gamma \tau^* \cdot f(N, \text{topology}) \quad (28)$$

where  $f$  depends on network structure.

**Problem 7.3.** Characterize precisely which problems are BEDS-maintainable but require unbounded memory for Turing-decidability.

**Problem 7.4.** Formalize “logical entropy” and determine whether a rigorous Gödel-Landauer correspondence exists.

## 8 Conclusion

We have introduced BEDS, a formal framework for continuous inference under energy constraints. Our main contributions:

- (1) **Formal definitions:** BEDS systems, fluxes, and three problem classes (attainable, maintainable, crystallizable).

- (2) **Energy-Precision Theorem:** Maintaining precision  $\tau^*$  against dissipation  $\gamma$  requires power  $P \geq \gamma k_B T / 2$ , with scaling  $P \propto \gamma \tau^*$ .
- (3) **Paradigm comparison:** BEDS and Turing machines address different problem types. Their fundamental limits (energy bounds vs. undecidability) are incommensurable.
- (4) **GLP Conjecture:** Closure pathologies across formal systems, computation, and thermodynamics may share common structure; openness and dissipation provide resolution.

The framework opens several research directions: extending the theorem to non-Gaussian beliefs, analyzing moving targets, characterizing the BEDS-maintainable problem class, and formalizing the GLP conjecture.

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## A Proof Details

### A.1 Entropy of Gaussian Distribution

For  $q = \mathcal{N}(\mu, \sigma^2)$ :

$$H[q] = \frac{1}{2} \ln(2\pi e \sigma^2) = \frac{1}{2} \ln(2\pi e) - \frac{1}{2} \ln \tau \quad (29)$$

## A.2 Information Gain Derivation

Prior:  $q^- = \mathcal{N}(\mu^-, \sigma^{-2})$  with precision  $\tau^-$ .

Posterior after observation of precision  $\tau_D$ :  $q^+ = \mathcal{N}(\mu^+, \sigma^{+2})$  with  $\tau^+ = \tau^- + \tau_D$ .

Entropy reduction:

$$\Delta H = H[q^-] - H[q^+] \quad (30)$$

$$= \frac{1}{2} \ln(\sigma^{-2}) - \frac{1}{2} \ln(\sigma^{+2}) \quad (31)$$

$$= \frac{1}{2} \ln \frac{\tau^+}{\tau^-} \quad (32)$$

$$= \frac{1}{2} \ln \left( 1 + \frac{\tau_D}{\tau^-} \right) \quad (33)$$

## A.3 Steady-State Power Derivation

Rate equation:

$$\frac{d\tau}{dt} = -\gamma\tau + \lambda\tau_D \quad (34)$$

At steady state  $\tau = \tau^*$ :

$$0 = -\gamma\tau^* + \lambda\tau_D \implies \lambda = \frac{\gamma\tau^*}{\tau_D} \quad (35)$$

Power:

$$P = \lambda \cdot E_{\text{obs}} = \frac{\gamma\tau^*}{\tau_D} \cdot E_{\text{obs}} \quad (36)$$

With Landauer minimum  $E_{\text{obs}} \geq \frac{k_B T}{2} \ln \left( 1 + \frac{\tau_D}{\tau^*} \right)$ :

$$P_{\min} = \frac{\gamma\tau^*}{\tau_D} \cdot \frac{k_B T}{2} \ln \left( 1 + \frac{\tau_D}{\tau^*} \right) \quad (37)$$

For  $x = \tau_D/\tau^* \ll 1$ :  $\ln(1 + x) \approx x$ , so:

$$P_{\min} \approx \frac{\gamma\tau^*}{\tau_D} \cdot \frac{k_B T}{2} \cdot \frac{\tau_D}{\tau^*} = \frac{\gamma k_B T}{2} \quad (38)$$

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