

BEDS: Bayesian Emergent Dissipative Structures

A Formal Framework for Continuous Inference Under Energy Constraints

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Abstract

We introduce BEDS (Bayesian Emergent Dissipative Structures), a formal framework for analyzing inference systems that must maintain beliefs continuously under energy constraints. Unlike classical computational models that assume perfect memory and focus on one-shot computation, BEDS explicitly incorporates dissipation (information loss over time) as a fundamental constraint.

We prove a central result linking energy, precision, and dissipation: maintaining a belief with precision τ against dissipation rate γ requires power $P \geq \gamma k_B T / 2$, with scaling $P \propto \gamma \cdot \tau$. This establishes a fundamental thermodynamic cost for continuous inference.

We define three classes of problems—BEDS-attainable, BEDS-maintainable, and BEDS-crystallizable—and show these are distinct from classical decidability.

We propose here, to the best of our knowledge, the first explicit Gödel-Landauer-Prigogine conjecture, suggesting that closure pathologies across formal systems, computation, and thermodynamics share a common structure.

Keywords: Bayesian inference, Dissipative systems, Thermodynamics of computation, Landauer principle, Continuous inference, Energy-efficient learning

1 Introduction

1.1 Motivation

Classical models of computation—Turing machines, formal proof systems—assume:

- (1) **Perfect memory:** information persists indefinitely
- (2) **One-shot computation:** input \rightarrow computation \rightarrow output
- (3) **No energy accounting:** computation is costless

These assumptions suit the analysis of algorithms and mathematical proofs. However, many real-world systems operate differently:

- Biological organisms maintain homeostasis *continuously*
- Sensor networks track changing environments *indefinitely*
- Brains hold beliefs while *actively forgetting*

Such systems face a fundamental challenge: **maintaining accurate beliefs costs energy**. Information degrades; fighting this degradation requires work.

This paper formalizes this challenge. We define a class of systems (BEDS) that perform inference under explicit dissipation constraints, and derive the fundamental energy-precision trade-off they must satisfy.

1.2 Contributions

- (1) **Formal definition** of BEDS systems (Section 2)
- (2) **Three problem classes**: attainable, maintainable, crystallizable (Section 3)
- (3) **Energy-precision theorem** with Landauer bound (Section 4)
- (4) **Comparison** with classical computation (Section 5)
- (5) **Gödel-Landauer-Prigogine conjecture** linking closure pathologies (Section 6)

1.3 Related Work

Landauer [1] established that erasing one bit costs at least $k_B T \ln 2$ joules. Bennett [2] showed reversible computation can avoid this cost. Friston’s Free Energy Principle [3] proposes that biological systems minimize variational free energy. Prigogine [4] characterized dissipative structures that maintain order through entropy export. Gödel [5] proved that sufficiently powerful formal systems are necessarily incomplete.

Our contribution connects these threads: we derive the energy cost of *maintaining* information against dissipation, and conjecture that closure pathologies across domains share common structure.

2 Formal Definitions

2.1 The BEDS System

Definition 2.1 (BEDS System). A BEDS system is a tuple $\mathcal{B} = (\Theta, q_0, \gamma, \varepsilon)$ where:

- $\Theta \subseteq \mathbb{R}^d$ is the parameter space
- $q_0 : \Theta \rightarrow \mathbb{R}_{\geq 0}$ is the initial belief distribution, with $\int_{\Theta} q_0(\theta) d\theta = 1$
- $\gamma > 0$ is the dissipation rate
- $\varepsilon > 0$ is the crystallization threshold

Definition 2.2 (Flux). A flux is a sequence of observations $\Phi = \{(t_i, D_i)\}_{i \in I}$, where $t_i \in \mathbb{R}_{\geq 0}$ is the arrival time and $D_i \in \mathcal{D}$ is the observation.

2.2 Dynamics

The system evolves according to two processes:

(i) Dissipation. In the absence of observations, uncertainty increases. For Gaussian beliefs $q_t = \mathcal{N}(\mu_t, \sigma_t^2)$:

$$\frac{d\sigma^2}{dt} = \gamma \cdot \sigma^2 \tag{1}$$

which implies:

$$\sigma^2(t) = \sigma_0^2 \cdot e^{\gamma t} \tag{2}$$

Equivalently, precision $\tau = 1/\sigma^2$ decays:

$$\frac{d\tau}{dt} = -\gamma\tau \implies \tau(t) = \tau_0 \cdot e^{-\gamma t} \tag{3}$$

(ii) **Bayesian Update.** Upon observing D with likelihood $p(D|\theta)$:

$$q^+(\theta) = \frac{p(D|\theta) \cdot q^-(\theta)}{Z} \quad (4)$$

where $Z = \int_{\Theta} p(D|\theta') \cdot q^-(\theta') d\theta'$ is the normalization constant.

For Gaussian beliefs with Gaussian likelihood of precision τ_D :

$$\tau^+ = \tau^- + \tau_D \quad (5)$$

$$\mu^+ = \frac{\tau^- \mu^- + \tau_D D}{\tau^+} \quad (6)$$

2.3 Crystallization

Definition 2.3 (Crystallization). A BEDS system *crystallizes* at time T if $\text{Var}[q_T] < \varepsilon$. Upon crystallization, the system outputs $\theta^* = \mathbb{E}[q_T]$ and halts (or becomes a fixed prior for a higher-level system).

2.4 Energy Model

Definition 2.4 (Observation Cost). Each observation incurs energy cost $E_{\text{obs}} \geq E_{\min}$ where:

$$E_{\min} = k_B T \ln(2) \cdot I_{\text{obs}} \quad (7)$$

and I_{obs} is the mutual information gained from the observation.

For a Gaussian observation of precision τ_D on a prior of precision τ :

$$I_{\text{obs}} = \frac{1}{2} \ln \left(1 + \frac{\tau_D}{\tau} \right) \quad (8)$$

Definition 2.5 (Power). The instantaneous power is $P(t) = \lambda(t) \cdot E_{\text{obs}}$ where $\lambda(t)$ is the observation rate.

3 Problem Classes

We define three distinct notions of what it means for a BEDS system to “solve” an inference problem.

Definition 3.1 (Inference Problem). An inference problem is a tuple $\mathcal{P} = (\Theta, \Phi, \pi^*, \delta)$ where:

- Θ is the parameter space
- Φ is a flux
- π^* is the target distribution (or θ^* the target value)
- $\delta > 0$ is the required accuracy

Definition 3.2 (BEDS-Attainable). Target π^* is *BEDS-attainable* under flux Φ if there exists a BEDS system \mathcal{B} such that:

$$\lim_{t \rightarrow \infty} D_{\text{KL}}(q_t \| \pi^*) = 0 \quad (9)$$

with finite total energy: $E_{\text{total}} = \int_0^\infty P(t) dt < \infty$.

Definition 3.3 (BEDS-Maintainable). Target π^* is *BEDS-maintainable* under flux Φ if there exists a BEDS system \mathcal{B} and time T_0 such that:

$$\forall t > T_0 : D_{\text{KL}}(q_t \| \pi^*) < \delta \quad (10)$$

with bounded power: $\sup_{t > T_0} P(t) < P_{\max} < \infty$.

Definition 3.4 (BEDS-Crystallizable). Target θ^* is *BEDS-crystallizable* under flux Φ if there exists a BEDS system \mathcal{B} and finite time T such that:

$$\text{Var}[q_T] < \varepsilon \quad \text{and} \quad |\mathbb{E}[q_T] - \theta^*| < \delta \quad (11)$$

Proposition 3.5 (Hierarchy). *Crystallizable implies Attainable. The converse does not hold.*

Proof. If θ^* is crystallizable at time T , set $\pi^* = \delta_{\theta^*}$. Since $\text{Var}[q_T] < \varepsilon$ and the system halts, no further energy is required, so $E_{\text{total}} < \infty$.

Conversely, consider a drifting target $\theta^*(t) = t$. A system can track it (attainable with continuous power) but never crystallize since the target never stabilizes. \square

4 The Energy-Precision Theorem

This section contains our main theoretical result.

4.1 Steady-State Analysis

Consider a BEDS system maintaining precision τ^* indefinitely.

Lemma 4.1 (Precision Balance). *In steady state, the precision gained from observations must equal the precision lost to dissipation:*

$$\lambda \cdot \tau_D = \gamma \cdot \tau^* \quad (12)$$

where λ is the observation rate and τ_D is the precision per observation.

Proof. Precision dynamics combine dissipation and discrete updates:

$$\frac{d\tau}{dt} = -\gamma\tau + \lambda\tau_D \quad (13)$$

where the second term represents average precision gain from observations arriving at rate λ . Setting $d\tau/dt = 0$:

$$\gamma\tau^* = \lambda\tau_D \quad (14)$$

\square

Corollary 4.2 (Required Observation Rate). *To maintain precision τ^* :*

$$\lambda = \frac{\gamma\tau^*}{\tau_D} \quad (15)$$

4.2 Landauer Bound

Lemma 4.3 (Information Cost). *Each observation that increases precision from τ to $\tau + \tau_D$ requires:*

$$E_{\text{obs}} \geq k_B T \ln(2) \cdot I_{\text{obs}} = \frac{k_B T \ln(2)}{2} \ln \left(1 + \frac{\tau_D}{\tau} \right) \quad (16)$$

Proof. The entropy change is:

$$\Delta H = H[\mathcal{N}(\mu, \sigma^2)] - H[\mathcal{N}(\mu', \sigma'^2)] = \frac{1}{2} \ln \frac{\sigma^2}{\sigma'^2} = \frac{1}{2} \ln \frac{\tau'}{\tau} = \frac{1}{2} \ln \left(1 + \frac{\tau_D}{\tau} \right) \quad (17)$$

By Landauer's principle, reducing entropy by ΔH nats requires energy $\geq k_B T \cdot \Delta H$. \square

4.3 Main Theorem

Theorem 4.4 (Energy-Precision-Dissipation Trade-off). *Let \mathcal{B} be a BEDS system maintaining Gaussian belief with precision τ^* against dissipation rate γ , using observations of precision τ_D .*

The minimum power required satisfies:

$$\boxed{P_{\min} = \frac{\gamma\tau^*}{\tau_D} \cdot E_{\text{obs}}} \quad (18)$$

In particular:

(i) Landauer bound:

$$P_{\min} \geq \frac{\gamma k_B T}{2} \ln \left(1 + \frac{\tau_D}{\tau^*} \right) \quad (19)$$

(ii) Linear regime (when $\tau_D \ll \tau^*$):

$$P_{\min} \approx \frac{\gamma k_B T}{2} \cdot \frac{\tau_D}{\tau^*} \quad (20)$$

(iii) High-precision limit:

$$P_{\min} \xrightarrow{\tau^* \rightarrow \infty} \frac{\gamma k_B T}{2} \ln \frac{\tau_D}{\tau^*} \rightarrow 0^+ \quad (21)$$

but the required observation rate $\lambda \rightarrow \infty$.

Proof. From Corollary 4.1, the observation rate is $\lambda = \gamma\tau^*/\tau_D$.

Power is rate times energy per observation:

$$P = \lambda \cdot E_{\text{obs}} = \frac{\gamma\tau^*}{\tau_D} \cdot E_{\text{obs}} \quad (22)$$

Substituting the Landauer minimum from Lemma 4.2:

$$P_{\min} = \frac{\gamma\tau^*}{\tau_D} \cdot \frac{k_B T}{2} \ln \left(1 + \frac{\tau_D}{\tau^*} \right) \quad (23)$$

For $\tau_D \ll \tau^*$, use $\ln(1+x) \approx x$:

$$P_{\min} \approx \frac{\gamma\tau^*}{\tau_D} \cdot \frac{k_B T}{2} \cdot \frac{\tau_D}{\tau^*} = \frac{\gamma k_B T}{2} \quad (24)$$

□

Remark 4.5 (Physical Interpretation). The bound $P \geq \gamma k_B T/2$ is independent of target precision in the efficient regime. This represents the fundamental cost of fighting entropy increase at rate γ .

4.4 Variance Formulation

Corollary 4.6 (Variance Scaling). *In terms of maintained variance $\sigma^{*2} = 1/\tau^*$:*

$$P_{\min} \propto \frac{\gamma}{\sigma^{*2}} \quad (25)$$

Halving uncertainty requires quadrupling power.

4.5 Optimality

Proposition 4.7 (Optimal Observation Strategy). *Given a constraint on total observation rate λ_{\max} , the optimal strategy is to use observations of precision:*

$$\tau_D^{\text{opt}} = \frac{\gamma\tau^*}{\lambda_{\max}} \quad (26)$$

Proof. From Lemma 4.1, $\tau_D = \gamma\tau^*/\lambda$. Given $\lambda \leq \lambda_{\max}$, we need $\tau_D \geq \gamma\tau^*/\lambda_{\max}$. The minimum energy is achieved at equality. \square

5 Comparison with Classical Computation

5.1 Two Computational Paradigms

We contrast BEDS with Turing machines, emphasizing that these are *different models for different purposes*, not competitors.

Aspect	Turing Machine	BEDS
Input	Finite string $w \in \Sigma^*$	Infinite flux $\Phi = \{D_t\}$
Memory	Unbounded, perfect	Finite, decaying
Output	Finite string (if halts)	Maintained belief q_t
Success criterion	Correct output	Accurate tracking
Resource	Time, space	Energy, precision
Fundamental limit	Undecidability	Energy-precision trade-off

5.2 Classes of Problems

Definition 5.1 (Turing-Decidable). A decision problem $L \subseteq \Sigma^*$ is Turing-decidable if there exists a Turing machine M that halts on all inputs and accepts exactly L .

Definition 5.2 (BEDS-Maintainable Problem Class). Let \mathcal{M} be the class of inference problems $(\Theta, \Phi, \pi^*, \delta)$ that are BEDS-maintainable with bounded power.

Proposition 5.3 (Orthogonality). *The classes of Turing-decidable problems and BEDS-maintainable problems are not directly comparable: neither contains the other.*

Proof. **Turing but not BEDS:** Consider a decision problem requiring unbounded memory (e.g., “does this prefix-free code describe a halting computation?”). A Turing machine can decide this; a BEDS system with finite, decaying memory cannot maintain the required information.

BEDS but not Turing: Consider “maintain an estimate of a continuous, time-varying signal $\theta(t)$ with bounded error.” This is not a decision problem at all—there is no finite output. A BEDS system handles this naturally; a Turing machine has no framework for it. \square

Remark 5.4. This is not a statement about computational power but about *what kinds of problems each model addresses*. Turing machines formalize one-shot computation; BEDS formalizes continuous inference.

5.3 Fundamental Limits

Each paradigm has characteristic impossibility results:

Paradigm	Limit	Statement
Turing	Undecidability	There exist problems with no halting algorithm
Formal proofs	Incompleteness	There exist true statements with no proof
BEDS	Energy bound	Precision τ^* requires power $\Omega(\gamma\tau^*)$

6 The Gödel-Landauer-Prigogine Conjecture

The comparison between BEDS and classical computation reveals a striking pattern: different formalisms encounter different fundamental limits. In this section, we conjecture that these limits share a common structural origin.

6.1 Three Foundational Results

Three results from different fields established fundamental constraints on closed systems:

Gödel (1931): Any consistent formal system capable of expressing arithmetic contains true statements that cannot be proven within the system.

Landauer (1961): Any irreversible computation (specifically, bit erasure) requires energy dissipation of at least $k_B T \ln(2)$ per bit.

Prigogine (1977): Open systems far from equilibrium can maintain and increase internal order by exporting entropy to their environment.

6.2 The Common Structure

These results share a pattern:

Domain	Closure Condition	Pathology	Resolution
Formal systems	No external axioms	Incompleteness	Meta-levels (Tarski hierarchy)
Computation	No heat dissipation	Irreversibility cost	Heat export
Thermodynamics	No entropy export	Disorder increase	Open systems

In each case:

- (1) **Closure** (with respect to some resource or level) produces a **pathology**
- (2) **Openness** (allowing export or meta-level escape) resolves or avoids the pathology

6.3 The Conjecture

Conjecture 6.1 (Gödel-Landauer-Prigogine). *The incompleteness of formal systems, the thermodynamic cost of irreversible computation, and the entropy increase in closed thermodynamic systems are structurally related phenomena. Specifically:*

(i) **Logical entropy:** Self-referential constructions in formal systems (Gödel sentences, Russell sets) can be understood as “logical entropy” that accumulates without resolution in closed systems.

(ii) **Export mechanisms:** Tarski’s hierarchy of metalanguages functions analogously to entropy export—problematic self-reference is “exported” to a higher level where it becomes tractable.

(iii) **ODR conditions:** Systems incorporating Openness (O), Dissipation (D), and Recursion (R) as structural features avoid the characteristic pathologies of systems lacking these features.

6.4 Formal Statement

Define the ODR conditions:

- **O (Openness):** System receives flux from environment
- **D (Dissipation):** System exports entropy (forgets, prunes)

- **R (Recursion):** System has hierarchical structure where stable configurations become primitives for higher levels

Conjecture 6.2 (continued). *Let \mathcal{S} be a system capable of self-reference.*

- *If \mathcal{S} satisfies $(O=-, D=-, R=-)$, then \mathcal{S} exhibits closure pathologies (incompleteness, paradox, or divergence)*
- *If \mathcal{S} satisfies $(O=+, D=+, R=+)$, then \mathcal{S} avoids these pathologies (at the cost of the constraints identified in Theorem 4.4)*

6.5 Evidence and Predictions

Supporting observations:

- (1) **Mathematics as social practice:** Human mathematics is conducted by communities that forget failed approaches, build hierarchical abstractions, and receive new conjectures from outside any fixed formal system. It exhibits $(O=+, D=+, R=+)$.
- (2) **Biological cognition:** Brains are paradigmatic dissipative structures. They receive continuous sensory flux, actively forget via synaptic pruning, and organize hierarchically. They do not exhibit Gödelian pathologies in practice.
- (3) **Frozen AI systems:** Large language models trained once and frozen exhibit $(O=-, D=-, R=+)$. They show characteristic pathologies: hallucination, drift from reality, inability to correct systematic errors.

Testable predictions:

- (1) AI systems with continuous learning and structured forgetting should exhibit fewer “hallucination-like” pathologies than frozen models.
- (2) The energy cost of maintaining consistency in a learning system should scale with the rate at which it must “dissipate” outdated beliefs.
- (3) Formal mathematical practice should exhibit measurable “forgetting” of unproductive research directions.

6.6 Status and Limitations

This is a conjecture, not a theorem. The structural analogy is suggestive but not proven. Key open problems:

- (1) **Formalization:** What precisely is “logical entropy”? Can it be quantified?
- (2) **Mapping:** Is there a rigorous mapping between thermodynamic and logical quantities, or only analogy?
- (3) **Necessity:** Are the ODR conditions necessary for avoiding pathologies, or merely sufficient?

We present this conjecture as a research program, not an established result. Its value lies in suggesting connections that may prove fruitful, not in claiming certainty.

7 Discussion

7.1 Implications

For distributed systems: The energy-precision trade-off provides a theoretical foundation for designing energy-efficient inference networks. The bound $P \geq \gamma k_B T/2$ is achievable in principle, and preliminary implementations on low-power embedded systems confirm the predicted scaling laws.

For machine learning: Current models are “frozen”—they do not dissipate and therefore face no energy-precision trade-off during inference. However, the world changes; maintaining accuracy requires retraining, which can be viewed as discrete (rather than continuous) dissipation.

For biological systems: Brains operate at $\sim 20\text{W}$ and maintain beliefs continuously. Our framework suggests this power is allocated (in part) to fighting entropy increase—maintaining precision against synaptic decay.

For foundations: If the GLP conjecture holds, it suggests a deep unity between logic, computation, and thermodynamics—all constrained by the impossibility of “closure without cost.”

7.2 Limitations

- (1) **Gaussian assumption:** Theorem 4.4 is proven for Gaussian beliefs. Extension to general distributions requires care.
- (2) **Stationary targets:** We analyze maintenance of fixed π^* . Tracking moving targets introduces additional complexity.
- (3) **Scalar case:** Extension to multivariate $\Theta \in \mathbb{R}^d$ is straightforward but changes constants.
- (4) **Idealized dissipation:** Real systems may have non-exponential decay.
- (5) **GLP conjecture:** Remains analogical, not rigorously proven.

7.3 Open Problems

Conjecture 7.1 (Tracking Bound). *For a target moving with velocity v in parameter space, the minimum power scales as:*

$$P_{\min} \propto \gamma \tau^* + v^2 \tau^* \quad (27)$$

Conjecture 7.2 (Multi-Agent Bound). *For N agents collectively maintaining a shared belief, the total power scales as:*

$$P_{\text{total}} \propto \gamma \tau^* \cdot f(N, \text{topology}) \quad (28)$$

where f depends on network structure.

Problem 7.3. Characterize precisely which problems are BEDS-maintainable but require unbounded memory for Turing-decidability.

Problem 7.4. Formalize “logical entropy” and determine whether a rigorous Gödel-Landauer correspondence exists.

8 Conclusion

We have introduced BEDS, a formal framework for continuous inference under energy constraints. Our main contributions:

- (1) **Formal definitions:** BEDS systems, fluxes, and three problem classes (attainable, maintainable, crystallizable).

- (2) **Energy-Precision Theorem:** Maintaining precision τ^* against dissipation γ requires power $P \geq \gamma k_B T/2$, with scaling $P \propto \gamma \tau^*$.
- (3) **Paradigm comparison:** BEDS and Turing machines address different problem types. Their fundamental limits (energy bounds vs. undecidability) are incommensurable.
- (4) **GLP Conjecture:** Closure pathologies across formal systems, computation, and thermodynamics may share common structure; openness and dissipation provide resolution.

The framework opens several research directions: extending the theorem to non-Gaussian beliefs, analyzing moving targets, characterizing the BEDS-maintainable problem class, and formalizing the GLP conjecture.

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A Proof Details

A.1 Entropy of Gaussian Distribution

For $q = \mathcal{N}(\mu, \sigma^2)$:

$$H[q] = \frac{1}{2} \ln(2\pi e \sigma^2) = \frac{1}{2} \ln(2\pi e) - \frac{1}{2} \ln \tau \quad (29)$$

A.2 Information Gain Derivation

Prior: $q^- = \mathcal{N}(\mu^-, \sigma^{-2})$ with precision τ^- .

Posterior after observation of precision τ_D : $q^+ = \mathcal{N}(\mu^+, \sigma^{+2})$ with $\tau^+ = \tau^- + \tau_D$.

Entropy reduction:

$$\Delta H = H[q^-] - H[q^+] \quad (30)$$

$$= \frac{1}{2} \ln(\sigma^{-2}) - \frac{1}{2} \ln(\sigma^{+2}) \quad (31)$$

$$= \frac{1}{2} \ln \frac{\tau^+}{\tau^-} \quad (32)$$

$$= \frac{1}{2} \ln \left(1 + \frac{\tau_D}{\tau^-} \right) \quad (33)$$

A.3 Steady-State Power Derivation

Rate equation:

$$\frac{d\tau}{dt} = -\gamma\tau + \lambda\tau_D \quad (34)$$

At steady state $\tau = \tau^*$:

$$0 = -\gamma\tau^* + \lambda\tau_D \implies \lambda = \frac{\gamma\tau^*}{\tau_D} \quad (35)$$

Power:

$$P = \lambda \cdot E_{\text{obs}} = \frac{\gamma\tau^*}{\tau_D} \cdot E_{\text{obs}} \quad (36)$$

With Landauer minimum $E_{\text{obs}} \geq \frac{k_B T}{2} \ln \left(1 + \frac{\tau_D}{\tau^*} \right)$:

$$P_{\text{min}} = \frac{\gamma\tau^*}{\tau_D} \cdot \frac{k_B T}{2} \ln \left(1 + \frac{\tau_D}{\tau^*} \right) \quad (37)$$

For $x = \tau_D/\tau^* \ll 1$: $\ln(1+x) \approx x$, so:

$$P_{\text{min}} \approx \frac{\gamma\tau^*}{\tau_D} \cdot \frac{k_B T}{2} \cdot \frac{\tau_D}{\tau^*} = \frac{\gamma k_B T}{2} \quad (38)$$

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