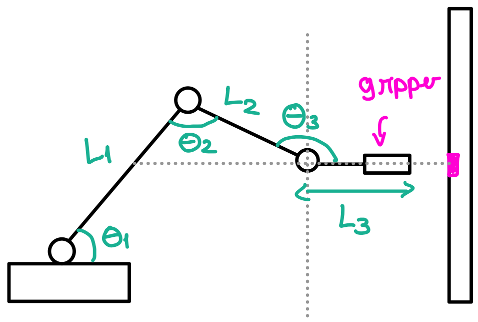
Control of Robotic Arm DOF



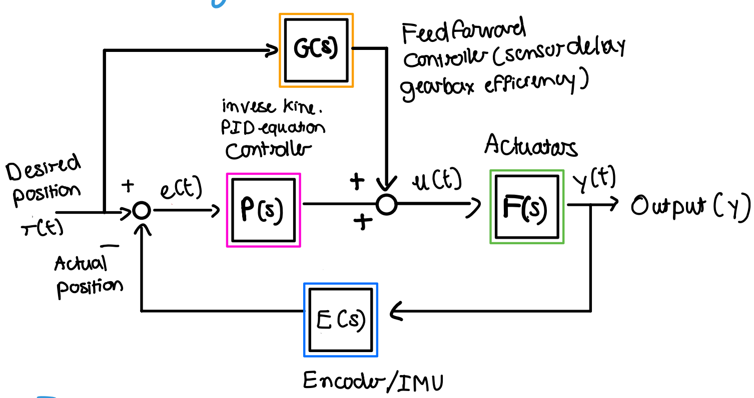
Task:

- Control Robotic Arm w/ one button to move up and down, right and left in straight line

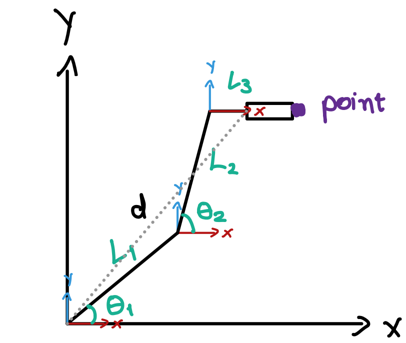
Assumption:

- Planar (x, y only)
- Position control
- Steady State

Control Diagram



Dynamic model:



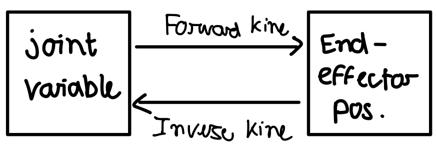
Restriction:

$10° < \theta_1 < 78°$

$10° < \theta_2 < 180°$

$-90° < \theta_3 < 90°$

Feedback:



Logic:

- If button 1 pressed (Left and right), value of y will remain the same as recorded
- If button 2 pressed (up and down), value of x will remain the same as recorded

Dynamic model math:

Lagrangiam Motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

We know:

$$\frac{\partial L}{\partial x} = 0 - mg$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} - 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$m\ddot{x} - (-mg) = 0$$

$$m\ddot{x} + mg = 0$$

$$m\ddot{x} = -mg$$

$$\Rightarrow F = ma$$

Lagragian's motion equation:

$$L = KE - PE$$

$$L = \frac{1}{2} m\dot{x}^2 - mgx + \frac{1}{2} m\dot{y}^2 - mgy$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

Solve dynamic model:

Forward kinematic

$x_3 = L_1\cos(\theta_1) + L_2\cos(\theta_1+\theta_2) + L_3\cos(\theta_1+\theta_2+\theta_3)$

$x_3 = L_1C_1 + L_2C_{12} + L_3C_{123}$

$y_3 = L_1\sin(\theta_1) + L_2\sin(\theta_1+\theta_2) + L_3\sin(\theta_1+\theta_2+\theta_3)$

$y_3 = L_1S_1 + L_2S_{12} + L_3S_{123}$

Let:

$\cos(\theta_1) = C_1$

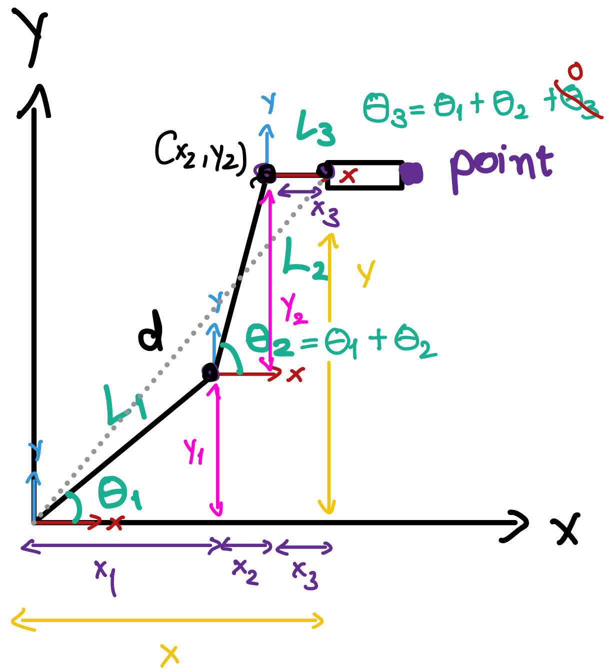
$\cos(\theta_1+\theta_2) = C_{12}$

$\cos(\theta_1+\theta_2+\theta_3) = C_{123}$

$\sin(\theta_1) = S_1$

$\sin(\theta_1+\theta_2) = S_{12}$

$\sin(\theta_1+\theta_2+\theta_3) = S_{123}$



Inverse kinematic:

Following Formula:

$x_1 = L_1C_1$

$y_1 = L_1S_1$

Law of cosine

$x_2 = L_1C_1 + L_2C_{12} = L_1C_1 + L_2C_1C_2 - L_2S_1S_2 = C_1(L_1+L_2C_2) - L_2S_1S_2$ [1]

$y_2 = L_1S_1 + L_2S_{12} = L_1S_1 + L_2S_1C_2 + L_2S_2C_1 = S_1(L_1+L_2C_2) + L_2S_2C_1$ [2]

So, we know from [1] and [2]

$x_2 = C_1(L_1+L_2C_2) - L_2S_1S_2 \Rightarrow x_2^2 = C_1^2(L_1+L_2C_2)^2 + L_2^2S_1^2S_2^2$ [3]

$y_2 = S_1(L_1+L_2C_2) + L_2S_2C_1 \Rightarrow y_2^2 = S_1^2(L_1+L_2C_2)^2 + L_2^2S_2^2C_1^2$ [4]

Using Pythagorean theorem for [3] and [4] $\boxed{\text{SOLVE } \theta_2}$

$d_2^2 = x_2^2 + y_2^2 = C_1^2(L_1+L_2C_2)^2 + L_2^2S_1^2S_2^2 + S_1^2(L_1+L_2C_2)^2 + L_2^2S_2^2C_1^2$

$\Rightarrow x_2^2 + y_2^2 = (L_1+L_2C_2)^2\underbrace{\cancel{[C_1^2+S_1^2]}}_{1 \text{ law of cosine}} + L_2^2S_2^2\underbrace{\cancel{[C_1^2+S_1^2]}}_{1 \text{ law of cosine}}$

$x_2^2 + y_2^2 = (L_1+L_2C_2)^2 + L_2^2S_2^2$

$x_2^2 + y_2^2 = L_1^2 + 2L_1L_2C_2 + L_2^2C_2^2 + L_2^2S_2^2$

$x_2^2 + y_2^2 = L_1^2 + L_2^2\overset{1 \text{ law of cosine}}{\cancel{(C_2^2+S_2^2)}} + 2L_1L_2C_2$

$C_2 = \cos(\theta_2) = \dfrac{(x_2^2+y_2^2) - (L_1^2 - L_2^2)}{2L_1L_2}$

$$\boxed{\theta_2 = \cos^{-1}\left[\dfrac{(x_2^2+y_2^2) - (L_1^2 - L_2^2)}{2L_1L_2}\right]} \quad [5]$$

End-Effector ⚠

x_2 and y_2 is the position of 2nd joint in order to know the position of end-effector

Apply same analogy $\Rightarrow \sin(\theta_2) = \sqrt{1-\cos^2(\theta_2)} \Rightarrow \theta_2 = \boxed{\tan^{-1}\left[\dfrac{\sin(\theta_2)}{\cos(\theta_2)}\right]}$ [6]

$= \boxed{\operatorname{arctan2}(S_2, C_2)}$ [7]

SOLVE θ_1

Recall equation [1] and [2]

$$y_2 = c_1(L_1 + L_2c_2) - s_1L_2s_2$$
$$x_2 = c_1L_2s_2 + s_1(L_1 + L_2c_2)$$

$$\Delta = \begin{bmatrix} L_1 + L_2c_2 & -L_2s_2 \\ L_2s_2 & L_1 + L_2c_2 \end{bmatrix}$$

$$\Delta = x_2^2 + y_2^2 = (L_1 + L_2c_2)^2 + (L_2s_2)^2$$

$$\Rightarrow \Delta\sin\theta_1 = \begin{bmatrix} L_1 + L_2c_2 & x_2 \\ L_2s_2 & y_2 \end{bmatrix} \Rightarrow \frac{\cancel{\Delta}\sin\theta_1}{\cancel{\Delta}} \Rightarrow \sin\theta_1 = \frac{(L_1+L_2c_2)y_2 - L_2s_2x_2}{x_2^2 + y_2^2}$$

$$\Rightarrow \Delta\cos\theta_1 = \begin{bmatrix} x_2 & -L_2s_2 \\ y_2 & L_1 + L_2c_2 \end{bmatrix} \Rightarrow \frac{\cancel{\Delta}\cos\theta_1}{\cancel{\Delta}} \Rightarrow \cos\theta_1 = \frac{(L_1+L_2c_2)x_2 + L_2s_2y_2}{x_2^2 + y_2^2}$$

So now, we have 3x equations:

$$\theta_1 = \sin^{-1}\left[\frac{(L_1+L_2c_2)y_2 - L_2s_2x_2}{x_2^2 + y_2^2}\right] \quad [8] \quad = \cos^{-1}\left[\frac{(L_1+L_2c_2)y_2 + L_2s_2x_2}{x_2^2 + y_2^2}\right] \quad [9]$$

$$\boxed{\theta_1 = \tan^{-1}\left[\frac{\sin(\theta_1)}{\cos(\theta_1)}\right] = \tan^{-1}\left[\frac{(L_1+L_2c_2)y_2 - L_2s_2x_2}{(L_1+L_2c_2)x_2 + L_2s_2y_2}\right] = \operatorname{arctan2}[s_1, c_1]} \quad [10]$$

SOLVE θ_3

We know:

$\phi = 90°$ (limit Range in XY Quadrant)

$$\boxed{\theta_3 = \phi - \theta_2 - \theta_1} \quad [11]$$

Equation:

$$\underbrace{M(q)\ddot{q} + C(q)\dot{q} + g(q)}_{\text{Torque Apply matrix}} = u + \xi$$

Where:

$M(q)$ = mass matrix

$C(q)$ = Coriolis term

$g(q)$ = Gravity term

Term define:

$\dot{q}$: measured vel

$\ddot{q}$: measured acc.

Under global linearity

$$u = K_p e + K_d \dot{e} \quad (1)$$ Derived from Lyapunov Function

We know:

e is error sign of system / compensator

$$\Rightarrow e(\theta) = \theta_f - \theta_i \quad (2)$$

$$\dot{e}(\theta) = \dot{\theta}_f - \dot{\theta}_i \quad (3)$$

θ_f: measured position config. [5×1] matrix joint angles (rad)

θ_i: desired position cofig. [5×1] matrix joint angles

$\dot{\theta}_f$: measured vel [5×1] (rad/s)

$\dot{\theta}_i$: desired vel [5×1] (rad/s)

Combine 1 and 2, we have:

$$u = K_p(\theta_f - \theta_i) + K_d(\dot{\theta}_f - \dot{\theta}_i)$$

Note:

K_p and K_d is PD controller value

End Result:

$$\xi = M(q)\ddot{q} + C(q)\dot{q} + g(q) - K_p(\theta_f - \theta_i) - K_d(\dot{\theta}_f - \dot{\theta}_i)$$