

# Exercise Session 4

## IESM Fall 2024-2025

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## Course Reminders

- Monday 14.10: mock written exam during the lecture time
- Tuesday 15.10: mock exam solutions and Q&A session (we'll gather questions on the Moodle forum by Monday 22:00 14.10)
- **Monday 28.10:** written exam
- Tuesday 05.11: report for exercise 4 due by noon; session for exercise 5

# Report Templates

- To assist in making easily uploadable/readable reports, Simon created Report Templates
- Google Doc or Overleaf (preferable) format
- Whenever you modify code, screenshot the cell/output using the Snipping Tool or use the code environment in latex as described in the hints at the top of the template

## 4. The hartree fock procedure in detail

So far you have run some Hartree Fock calculations using Psi4 and in the lectures we have seen the theoretical derivation of the scheme.

In this exercise, we will recapitulate the HF scheme by implementing it on our own using Psi4. That means we will go through the steps of the HF procedure but we will let Psi4 evaluate the integrals for us.

 [Download introductory slides](#)

Report Template [Google Docs](#)

Report Template [Overleaf](#)



Learning goals



Chapter in script



Resources

## Exercise 4

The Hartree-Fock procedure in detail (Reminder: you can download these slides from the [Exercise page](#))

### Learning goals

Build and diagonalize the Fock matrix

Understand the steps in a HF SCF calculation

Practice implicit summation in numpy

### Chapter in script

Chapter 4 - An introduction to Hartree Fock theory

Appendix C

### Resources

Introduction to Hartree-Fock Molecular Orbital Theory by the Sherrill group: [slides](#) and videos ([part 1](#), [part 2](#))

## Recap of HF Theory

**HF method = approximate many-body wavefunction to a single Slater determinant**

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n) \approx \Psi_{HF} \equiv \frac{1}{\sqrt{N}} |\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2) \dots \phi_n(\mathbf{r}_n)|$$

→  $\Psi_{HF}$  inserted into time-independent Schrödinger equation to find eigenvalue, i.e.  $E = \langle \Psi_{HF} | \hat{H}_{el} | \Psi_{HF} \rangle$ . What is found is a **variational solution**, i.e. HF energy is always above true energy.

# Recap of SCF Method

HF equations (will be derived in detail during Lectures)

$$E_{HF} = \sum_i \langle \phi_i | \hat{h} | \phi_i \rangle + \frac{1}{2} \sum_{i,j} ([\phi_i \phi_i | \phi_j \phi_j] - [\phi_i \phi_j | \phi_j \phi_i]) \quad \forall \phi_i$$

where  $[\dots|\dots]$  integrals contain Coulomb and Exchange operators, whose action on orbital  $\phi_i$  depends on all the other one-electron orbitals  $\phi_j$ . Hence, HF equations have to be solved iteratively until self-consistency (**self consistent field SCF method**)

|                                     | Total Energy      | Delta E      | RMS  [F,P]       |
|-------------------------------------|-------------------|--------------|------------------|
| @DF-UHF iter SAD:                   | -0.85212229561042 | -8.52122e-01 | 0.00000e+00      |
| @DF-UHF iter 1:                     | -1.12493424123815 | -2.72812e-01 | 1.62339e-02 DIIS |
| @DF-UHF iter 2:                     | -1.12669596352817 | -1.76172e-03 | 2.38980e-03 DIIS |
| @DF-UHF iter 3:                     | -1.12673509439903 | -3.91309e-05 | 3.36383e-06 DIIS |
| @DF-UHF iter 4:                     | -1.12673509447661 | -7.75795e-11 | 2.74379e-09 DIIS |
| Energy and wave function converged. |                   |              |                  |

# Hartree-Fock Roothaan Equations

- HF equations are a set of coupled integro-differential equations to determine the HF molecular one-electron orbitals
- If we represent the orbitals in a basis (of AO-like orbitals), the HF equations transform into matrix equations that were first derived by **Roothaan**

$$\Rightarrow \mathbf{FC} = \mathbf{SCE}$$

- Note: as you will see today, this problem can be recasted in an eigenvalue problem via a basis set transformation

$$\Rightarrow \mathbf{F}'\mathbf{C}' = \mathbf{C}'\mathbf{E}'$$

# Building the Fock Matrix

The HF method recasts into a pseudo-eigenvalue problem

$$\mathbb{F}\mathbb{C} = \mathbb{S}\mathbb{C}\mathbb{E}$$

where:

- $\mathbb{F} = \mathbb{H} + 2\mathbb{J} - \mathbb{K}$  is the **Fock matrix**
- $\mathbb{C}$  is the **wavefunction amplitude matrix**
- $\mathbb{S}$  is the **overlap matrix**
- and  $\mathbb{E}$  is the **energy value matrix**

The issue?  $\mathbb{F}$  relies on an orbital solution in order to iteratively solve (with **SCF method**) for the “best” molecular orbitals which make a single Slater determinant description of the system wavefunction. We will begin with a guess and iteratively improve on the guess.



# Performing HF Explicitly



# Overlap Matrix $\mathbb{S}$

$\mathbb{S}$ , the overlap matrix, describes the inter-relationships of the basis set vectors. Other details about  $\mathbb{S}$ :

- The number of basis functions,  $n$ , defines the size and shape of  $\mathbb{S}(n \times n)$
- $\mathbb{S}$  is an identity matrix in the case of orthonormal basis set functions
- By properly transforming the  $\mathbb{S}$  matrix, we can ensure orthonormality

# Implicit Einstein Summation

The einsum function in numpy allows for efficient matrix multiplication.

$$(AB)_{pq} = \sum_i A_{p,i} B_{i,q} \quad \text{explicit summation}$$

$$(AB)_{pq} = A_{p,i} B_{i,q} \quad \text{implicit summation}$$

```
np.einsum('ij,jk->ijk', A, B)
```

