

# Exercise Session 4

## IESM Fall 2022-2023

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## Course Reminders

- Welcome Simon!
- From this week we'll start to have **Lectures/Ex also on Fridays** with deadlines for exercise reports on morning of next exercise session
- Friday lectures in BCH 4310
- Monday 14.11: mock written exam during the lecture time
- Friday 18.11: mock exam solutions and Q&A session (we'll gather questions on the moodle forum by Wednesday 16.11)
- **Monday 21.11:** written exam
- Friday 25.11: report for exercise 4 due by 9am; session for exercise 5

## Exercise 4

The Hartree-Fock procedure in detail (Reminder: you can download these slides from the [Exercise page](#))

### Learning goals

Build and diagonalize the Fock matrix

Understand the steps in a HF SCF calculation

Practice implicit summation in numpy

### Chapter in script

Chapter 4 - An introduction to Hartree Fock theory

Appendix C

### Resources

Introduction to Hartree-Fock Molecular Orbital Theory by the Sherrill group: [slides](#) and videos ([part 1](#), [part 2](#))

## Recap of HF Theory

**HF method = approximate many-body wavefunction to a single Slater determinant**

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n) \approx \Psi_{HF} \equiv \frac{1}{\sqrt{N}} |\phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2) \dots \phi_n(\mathbf{r}_n)|$$

→  $\Psi_{HF}$  inserted into time-independent Schrödinger equation to find eigenvalue, i.e.  $E = \langle \Psi_{HF} | \hat{H}_{el} | \Psi_{HF} \rangle$ . What is found is a **variational solution**, i.e. HF energy is always above true energy.

# Recap of SCF Method

HF equations (will be derived in detail during Lectures)

$$E_{HF} = \sum_i \langle \phi_i | \hat{h} | \phi_i \rangle + \frac{1}{2} \sum_{i,j} ([\phi_i \phi_i | \phi_j \phi_j] - [\phi_i \phi_j | \phi_j \phi_i]) \quad \forall \phi_i$$

where  $[\dots|\dots]$  integrals contain Coulomb and Exchange operators, whose action on orbital  $\phi_i$  depends on all the other one-electron orbitals  $\phi_j$ . Hence, HF equations have to be solved iteratively until self-consistency (**self consistent field SCF method**)

	Total Energy	Delta E	RMS  [F,P]
@DF-UHF iter SAD:	-0.85212229561042	-8.52122e-01	0.00000e+00
@DF-UHF iter 1:	-1.12493424123815	-2.72812e-01	1.62339e-02 DIIS
@DF-UHF iter 2:	-1.12669596352817	-1.76172e-03	2.38980e-03 DIIS
@DF-UHF iter 3:	-1.12673509439903	-3.91309e-05	3.36383e-06 DIIS
@DF-UHF iter 4:	-1.12673509447661	-7.75795e-11	2.74379e-09 DIIS
Energy and wave function converged.			

# Hartree-Fock Roothaan Equations

- HF equations are a set of coupled integro-differential equations to determine the HF molecular one-electron orbitals
- If we represent the orbitals in a basis (of AO-like orbitals), the HF equations transform into matrix equations that were first derived by **Roothaan**

$$\Rightarrow \mathbf{FC} = \mathbf{SCE}$$

- Note: as you will see today, this problem can be recasted in an eigenvalue problem via a basis set transformation

$$\Rightarrow \mathbf{F}'\mathbf{C}' = \mathbf{C}'\mathbf{E}'$$

## Building the Fock Matrix

The HF method recasts into a pseudo-eigenvalue problem

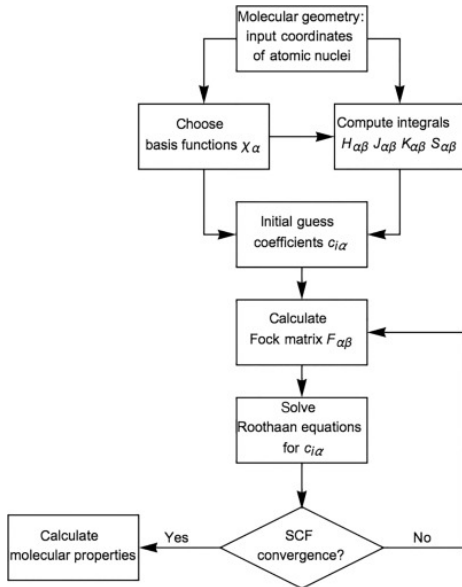
$$\mathbb{F}\mathbb{C} = \mathbb{S}\mathbb{C}\mathbb{E}$$

where:

- $\mathbb{F} = \mathbb{H} + 2\mathbb{J} - \mathbb{K}$  is the **Fock matrix**
- $\mathbb{C}$  is the **wavefunction amplitude matrix**
- $\mathbb{S}$  is the **overlap matrix**
- and  $\mathbb{E}$  is the **energy value matrix**

The issue?  $\mathbb{F}$  relies on an orbital solution in order to iteratively solve (with **SCF method**) for the “best” molecular orbitals which make a single Slater determinant description of the system wavefunction. We will begin with a guess and iteratively improve on the guess.

# Performing HF Explicitly





# Overlap Matrix $\mathbb{S}$

$\mathbb{S}$ , the overlap matrix, describes the inter-relationships of the basis set vectors. Other details about  $\mathbb{S}$ :

- The number of basis functions,  $n$ , defines the size and shape of  $\mathbb{S}(n \times n)$
- $\mathbb{S}$  is an identity matrix in the case of orthonormal basis set functions
- By properly transforming the  $\mathbb{S}$  matrix, we can ensure orthonormality

# Implicit Einstein Summation

The einsum function in numpy allows for efficient matrix multiplication.

$$(AB)_{pq} = \sum_i A_{p,i} B_{i,q} \quad \text{explicit summation}$$

$$(AB)_{pq} = A_{p,i} B_{i,q} \quad \text{implicit summation}$$

```
np.einsum('ij,jk->ijk', A, B)
```



## Exercise 4 - Tips

### Tips!

- If needed, search for commands and variable types in the [Psi4 manual](#)
- Some code blocks just need to be executed. Typically, we use a comment (`#`) in a code block to indicate where edits should be made.
- Today's exercise includes details for the manipulation of matrices used in the HF procedure. As you work, try to consider why these transformations are necessary in order to run a HF method calculation.