Exercise Ex1

Student: Firstname Lastname Sciper: 000000

**Please use this template to submit your answers.**  
If you had to modify code from the notebook, please include the modified code in your submission either as screenshot or in a

\begin{lstlisting}[language=Python]  
\end{lstlisting}

environment.

You only need to include the code cells that you modified.

Note, that references to other parts of the documents aren’t resolved in this template and will show as ??. Check the text of the exercises on website for the reference

**Exercise 1**  
Calculate the vector product with

and, for the same , the scalar product

Your answer here

**Exercise 2**  
Evaluate the matrix product .

Your answer here

**Exercise 3**  
Evaluate the determinant for the matrix .

Your answer here

**Exercise 4**  
Does the exponent of an operator always satisfy the relation ? Start from the definition of the matrix exponential.

Your answer here

**Exercise 5**  
Find the eigenvalues and eigenvectors of the matrix .

Your answer here

**Exercise 6**  
Show that if the product of two Hermitian matrices is also Hermitian, then and commute.

Your answer here

**Exercise 7**  
Explain the connection between the Heisenberg uncertainty principle and the commutation relation.

Your answer here

**Exercise 8**  
What is the meaning of a multiplication of a bra and a ket

and, conversely, an operator formed by a ket and a bra?

Your answer here

**Exercise 9**  
Given a basis for which

where is the Kronecker delta, for any state

$$\begin{aligned}
\Ket{\Psi} = \sum\_j c\_j\Ket{\psi\_j},
\end{aligned}$$

the inner product is defined as

Show that this holds only as long as

where the are the aforementioned expansion coefficients.

Your answer here

**Bonus Exercise 10**  
Prove that, given the above conditions, .

Your answer here

**Exercise 11**  
Diagonalise the matrices and . Specify which one is Hermitian.

Your answer here

**Bonus Exercise 12**  
Prove that the eigenvalues of a Hermitian operator are real.

Your answer here

**Exercise 13**  
Give the position and the momentum operators (consider only one dimension) in the position representation.

Your answer here

**Exercise 14**  
Give the commutator of the position and linear momentum operators in the position representation (consider one dimension only).

Your answer here

**Exercise 15**  
Is the electronic Hamiltonian a linear operator and why (not)?

Your answer here

**Exercise 16**  
Show that, if two operators , commute and if $\Ket{\psi}$ is an eigenvector of , $\hat{\mathrm{B}}\Ket{\psi}$ is an eigenvector of , too, with the same eigenvalue.

**Bonus**: If $\Ket{\psi}$ is part of a set of degenerate eigenvectors, show that the subspace spanned by the eigenvectors of is invariant under the action of .

Your answer here

**Exercise 17**  
Demonstrate that, if two hermitian operators , commute and ${\Ket{\psi\_1}}$, ${\Ket{\psi\_2}}$ are eigenvectors of associated to different eigenvalues, then the matrix element $\Bra{\psi\_1}\hat{\mathrm{B}}\Ket{\psi\_2}$ vanishes.

Your answer here

**Bonus Exercise 18**  
Show that the potential energy operator is multiplicative when applied to the real-space wavefunction.

Your answer here

**Bonus Exercise 19**  
The link between position and momentum representation is given by a Fourier transform. Explain how this relates to the Heisenberg uncertainty principle.

Your answer here

**Exercise 20**  
In a system that consists of only two states (such as an electron spin in a magnetic field, where the electron spin can be in one of two orientations), the Hamiltonian has the following matrix elements: . How would you determine the energy levels and the eigenstates of the system? (You do not need to solve this problem explicitly, merely outlining the procedure is sufficient.)

Your answer here

**Exercise 21**  
Define two vectors, 1 and 2, with two elements each, that are normalized, in the sense , and orthogonal in the sense that .

**Hint:** In numpy a vector v with the two elements 1 and 2 is defined through the command

v=np.array([1,2])

Your answer here

**Exercise 22**  
Show that and are normalized and orthonormal

**Hint:** Here are reported some useful numpy functions to work with vectors:

* v.dot(w) - inner product (scalar product) of two vectors v, w
* v.conj() - complex conjugate of a vector v
* v.conj().dot(w) - inner product of

Your answer here