Lecture – 7

<u>Probability Theory:</u>

- Experiment… Observation…. Measurements
- Hypothesis…. Theory…. Model
- Identify degree of uncertainty in statements

<u>Analysis Workflow:</u>

- Measurements… Estimations… of parameters
- Quantify the estimation
- Test the extent to which the population agrees with the data

<u>Sources of uncertainty:</u>

- Stochasticity
- Incomplete data
- Unknowns (Incomplete modeling)

<u>Probability theory:</u>

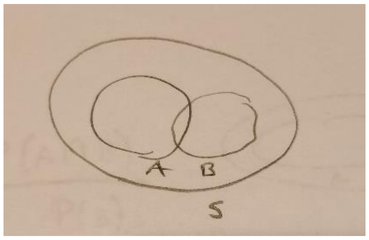
- Analyze the frequency of "events", formally knows as "Frequentist"
    - Probability **P** of an event **X** occurring,
        - **n** repeatable observations of events
        - fraction $p \rightarrow type\ x$

      Ex: Disease in a population

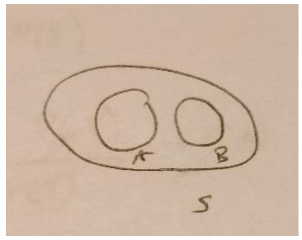
- If you do a test, formally known as "Bayesian"
    - How do you interpret?
    - Accuracy of the test
    - Frequency of disease

<u>Basic Definitions:</u>

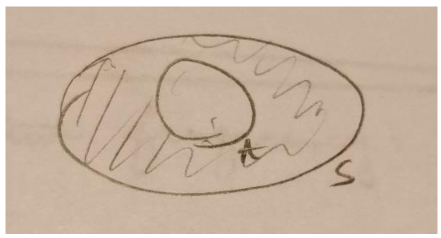
Set **S** with subsets **A,B**



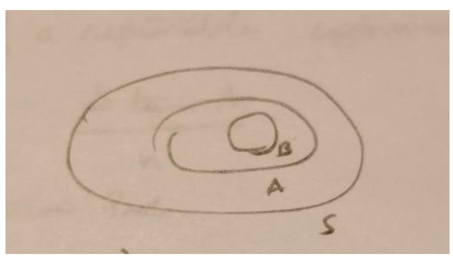
- $P(s) = 1$
- $\forall A\colon\ A \subset S\colon P(A) \geq 0$



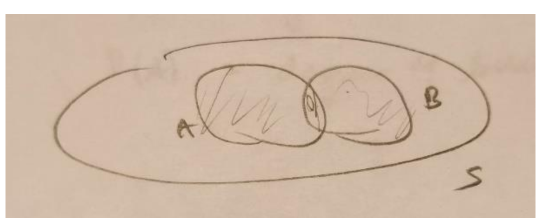
- $If\ A\ \cap B =\ \emptyset, then\ P(A\ \cup B) = P(A) + P(B)$



- $P(\neg A) = 1 - p(A)$
- $P(A\ \cup \neg A) = 1$
- $P(\emptyset) = 0$

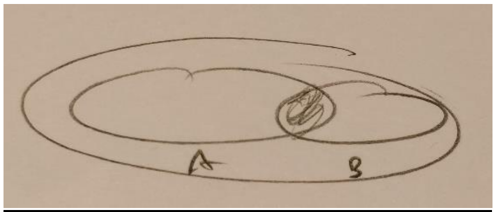


- $If\ A\ \subset B \rightarrow P(A) \leq P(B)$



- $P(A\ \cup B) = P(A) + P(B) - P(A\ \cap B)$

Conditional probability:



$$P(A|B) = \frac{P(A \ \cap B)}{P(B)}$$

$If\ A\ \cup B = \emptyset,$

$$P(A,B) = P(A)\ P(B)$$

$$P(A|B) = \frac{P(A)P(B)}{P(B)} = P(A)$$

Interpretation

1. "Frequentist" => Relative frequency

   A,B,…. Outcome of a repeatable experiment

   $P(A) = \lim_{n \to \infty} \frac{\#\ times\ A}{n}$

2. "Bayesian" => Subjective probability

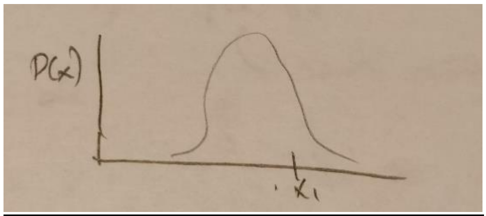
   A,B,… hypothesis (true / false statements)

   P(A) = degree of belief that A is correct (true)

Random Variable:

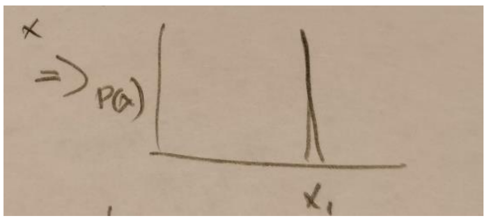
$X \sim \{x_{1,} x_{2,} x_{3,} x_{4,} \ldots \ldots, x_{n,}\}$ where X can have different values (Discrete / Continuous)

Probability Mass Function

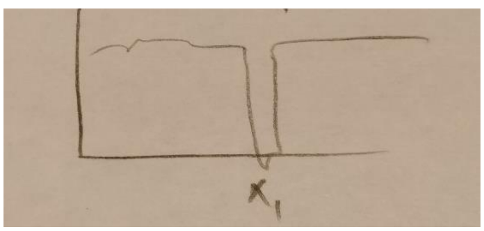


$P(x = x_1) = P(x_1) \neq P(x)$

$If\ P(x_1) = 1 => x\ is\ always\ x_1$



$If\ P(x_1) = 0 => x\ is\ never\ x_1$



$x \sim P(x) \rightarrow x\ is\ drawn\ from\ P(x)$

Properties:

- P domain must have all states of x
  $\forall\, x_1 \in x : 0 \leq P(x_1) \leq 1$
- For discrete, $\sum_{x_1 \in x} P(x) = 1$ (Normalization)
- $P(x \in [x_1, x_1 + \delta x]) = P(x)\delta x$, where P(x) is probability distribution function
- $\int P(x)dx = 1$