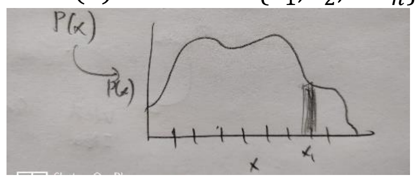
Lecture – 8

- X is a random variable drawn from a *Probability Mass Function* (PDM)

$$X \sim P(x) \; => \; X = \{x_1, x_2, \dots x_n\}$$



- $P(x_1)$ = Probability of getting x_1
  - Frequentist: Fraction of times you get x_1
  - Bayesian: how likely it is to measure x_1
- If x is continuous, then $x = x_1$ is very unlikely

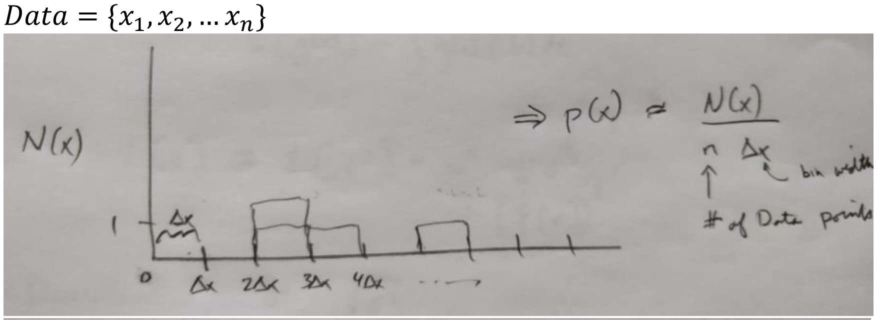
$$P(x \in [x_1, x_1 + \delta x])$$

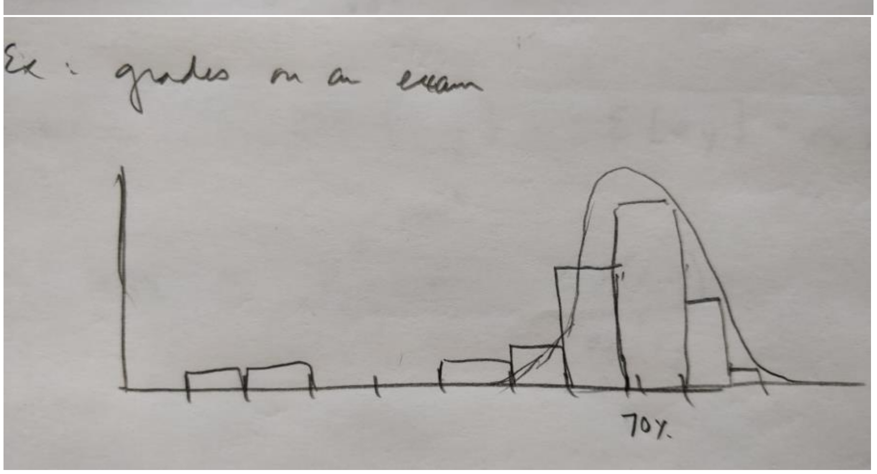
$$\lim_{\delta x \to 0} P(x) = p(x)dx$$

where $P(x)$ is the *Probability Density Function* (PDF)

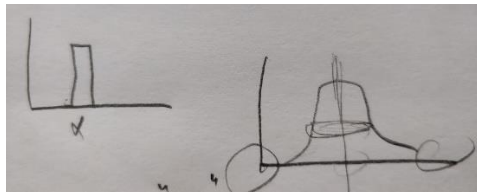
Histogram:

- Plot data in a way that shows the PDF
- PDF = histogram with Infinite data, 0 bin-width & normalized
- $Data = \{x_1, x_2, \dots x_n\}$





- $E(x)$ is a PDF : $\int f(x)dx = 1$



- Expectation value:

$$E[x] = \int xf(x)dx = \mu$$

$$E[g(x)] = \int g(x)\, f(x)\, dx$$

- Variance:

$$V[x] = E[x^2] - \mu^2 = \sigma^2$$

- Standard Deviation:

$$\sigma = \sqrt{\sigma^2}$$

- Covariance:

$$cov[x, y] = E[x, y] - \mu_x \mu_y$$

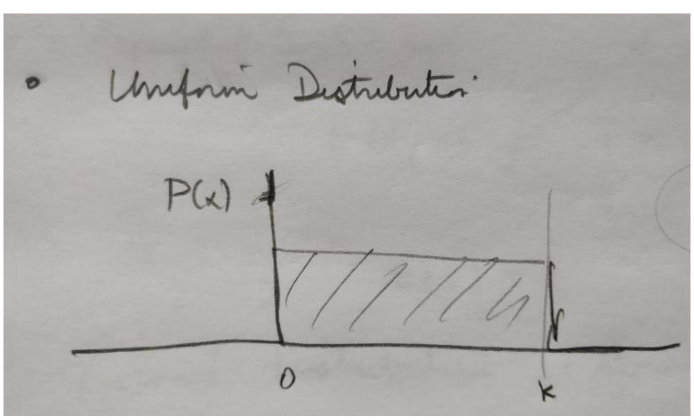
- Correlation coefficient:

$$\mathrm{P}_{x,y} = \frac{cov[x, y]}{\sigma_x, \sigma_y}$$

Common Distributions:

- **Uniform Distribution:**

$$\boldsymbol{P(x)} = \begin{cases} \quad\quad\quad\quad < 0, \;\; 0 \\ 0 \text{ to k}, \;\; \frac{1}{k} \; (equal\ probability\ to\ get\ any\ value) \\ \quad\quad\quad\quad > 0, \;\; 0 \end{cases}$$



- **Binomial Distribution:**

  N independent experiments : true / false (or) succeed / fail (or) 1 / 0

  If N is the no. of trials & n is the no. of successes where $0 \leq n \leq N$, then probability P of the experiment is given by

$$P \rightarrow f(n, N, p) = \frac{N!}{n!\,(N-n)!}\, p^n\, (1-p)^{N-n}$$

$$E[n] = N_p \;\; V[n] = N_p(1-p)$$

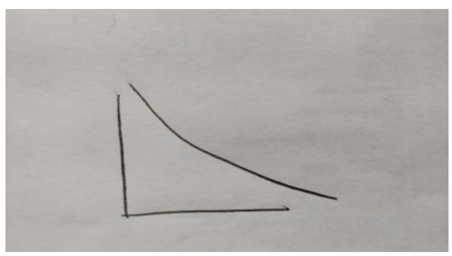
- **Poisson Distribution:**

  Known rate (ν), $N \rightarrow \infty, p \rightarrow 0 => E[n] - N_p = \nu$

$$f(n, \nu) = \frac{\nu^n}{n!}\, e^{-n}$$

$$E[n] = 0 \qquad V[n] = 0 \qquad \sigma = \sqrt{\nu} \;\approx\; \sqrt{n}$$

- **Exponential Distribution:**

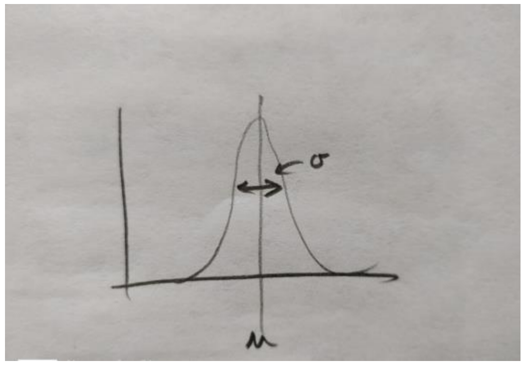


$$\nu \rightarrow \tau = \frac{1}{\nu}$$

$$f(t, \tau) = \begin{cases} \tau\, e^{\frac{-t}{z}}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$E[t] = \tau \qquad V[t] = \tau^2$$

- **Normal Distribution:**



$$f(x, \mu, \sigma) = \frac{1}{2\pi} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$
$$E[x] = \mu$$
$$V[x] = \sigma^2$$

Central Limit theorem:

N independent variable x_i with variance σ_i^2

$y = \sum x_i \quad (Gaussian\ Distribution)$

$E[y] = \sum x_i \quad V[y] = \sum \sigma_i^2$